# Mesons Spectral Functions at Finite Temperature

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**Abstract.** We investigate the thermal spectral densities for (pseudo)scalar and vector currents in the framework of the real time formalism when mass of two quarks are different. Such spectral densities are necessary for the phenomenological investigation of in-medium properties of hadrons. We use the quark propagator at finite temperature and calculate annihilation and scattering parts of spectral densities for above mentioned currents. The investigations show that the thermal contributions are significantly important. The obtained results at $T \to 0$ limit are in good consistency with the vacuum results.

## 1. Introduction

One of the most important subjects of the QCD physics is the phenomenological investigation of in-medium modifications of hadrons. Heavy-ion collisions and reactions have been elementarily used to extract experimental information on in-medium properties of hadrons. Also, there are a lot of different theoretical methods which have been widely used to for this purpose in literature. These methods are effective hadronic models, chiral perturbation theory, lattice theory, low-density theorems, quark models and QCD sum rules. QCD sum rules first introduced by Shifman, Vainshtein and Zakharov [1], and its extension version to finite temperature [2] have been one of the most efficient approaches among these methods. QCD sum rule is obtained by equating the QCD and the phenomenological representations of the correlation function and this method establishes a connection between QCD vacuum structure and hadron properties [3]. Determination of correlation function by using dispersion relations is the first step in QCD sum rules and it is necessary to evaluate the spectral density of mentioned current for this aim. Therefore determination of spectral densities is the main tool of QCD sum rules method. Spectral functions in different cases were studied in the literature [4]-[14].

In this paper, we study the thermal spectral densities for different currents at finite temperature. We calculate the annihilation and scattering parts of the spectral densities for (pseudo)scalar and vector currents for $m_1 \neq m_2$ case using the quark propagator in the real time formulation of the thermal field theory. The investigations show that the thermal contributions are significantly important. Also, we show that our obtained results at $T \to 0$ limit are in good consistency with vacuum results.

## 2. Thermal spectral densities for various currents

The thermal QCD sum rule approach is based on the evaluation of thermal correlator of the interpolating current $J(x) = \bar{q}_1(x)\Gamma q_2(x)$. Two-point thermal correlation function is given by:

$$\Pi\big(q,T\big) = i \int d^4x \; e^{iq\cdot x} Tr\Big(\rho \; \mathcal{T}\big(J(x)J^\dagger(0)\big)\Big), \tag{1}$$

where $\mathcal{T}$ denotes the time ordered product and $\rho = e^{-\beta H}/Tre^{-\beta H}$ is the thermal density matrix of QCD at temperature $T = 1/\beta$. In interpolating current $J(x)$, $\Gamma = I$ or $i\gamma_5$ for scalar and pseudoscalar particles, respectively and $\Gamma = \gamma_\mu$ as for vector particles.

Firstly, we consider the thermal spectral density for pseudo(scalar) particles. $\Pi(q,T)$ for these currents can be written in momentum space as:

$$\Pi(q,T) = 4iN_c \int \frac{d^4k}{(2\pi)^4} (k^2 - k\cdot q - m_1 m_2) D(k,m_1) D(k-q,m_2), \tag{2}$$

where, $D(k)$ is expressed as $D(k,m) = 1/(k^2 - m^2 + i\varepsilon) + 2\pi i n(|k_0|)\delta(k^2 - m^2)$ and in this propagator $n(x) = [\exp(\beta x) + 1]^{-1}$ is the Fermi distribution function. Carrying out the integral over $k_0$, we obtain the imaginary part of the $\Pi(q,T)$ as $Im\Pi(q,T) = L(q_0) + L(-q_0)$, where

$$\begin{aligned}
L(q_0) &= -N_c \int \frac{d\mathbf{k}}{8\pi^2} \frac{\omega_1^2 - \mathbf{k}^2 + \mathbf{k}\cdot\mathbf{q} - \omega_1 q_0 - m_1 m_2}{\omega_1 \omega_2} \Big[ A(n_1, n_2)\delta(q_0 - \omega_1 - \omega_2) \\
&\quad - B(n_1, n_2)\delta(q_0 - \omega_1 + \omega_2)\Big].
\end{aligned} \tag{3}$$

Here $m_1$ and $m_2$ are quark masses, $\omega_1 = \sqrt{\mathbf{k}^2 + m_1^2}$ , $\omega_2 = \sqrt{(\mathbf{k\text{-}q})^2 + m_2^2}$ , $n_1 = n(\omega_1)$, $n_2 = n(\omega_2)$, $A(n_1, n_2) = (1 - n_1)(1 - n_2) + n_1 n_2$, $B(n_1, n_2) = (1 - n_1)n_2 + (1 - n_2)n_1$ and the plus and minus signs in front of $m_1$, $m_2$, correspond to the scalar and pseudoscalar particles, respectively. The term, which does not include the Fermi distribution functions, shows the vacuum contribution. Terms including the Fermi distributions depict medium contributions. The delta-functions in the different terms of Eq. (3) control the regions of non-vanishing imaginary parts of $\Pi(q,T)$ , which define the position of the branch cuts. Taking into account

$$\delta(q_0 - \omega_1 - \omega_2) = \frac{\omega_2}{|\mathbf{k}||\mathbf{q}|} \delta\Big( \cos\theta + \frac{q^2 - 2q_0\omega_1}{2|\mathbf{k}||\mathbf{q}|}\Big) \tag{4}$$

expression (here $\theta$ is angle between of $\mathbf{k}$ and $\mathbf{q}$ momentums) and carrying out some transformations, the annihilation and scattering parts of spectral density is found as:

$$\rho_{a,pert}(s,T) = \rho_0(s)\Big[1 - n\Big(\frac{\sqrt{s}}{2}\Big(1 + \frac{m_1^2 - m_2^2}{s}\Big)\Big) - n\Big(\frac{\sqrt{s}}{2}\Big(1 - \frac{m_1^2 - m_2^2}{s}\Big)\Big)\Big], \tag{5}$$

for $(m_1 + m_2)^2 \leq s \leq \infty$,

$$\rho_{s,pert}(s,T) = \rho_0(s)\Big[n\Big(\frac{\sqrt{s}}{2}\Big(1 + \frac{m_1^2 - m_2^2}{s}\Big)\Big) - n\Big(-\frac{\sqrt{s}}{2}\Big(1 - \frac{m_1^2 - m_2^2}{s}\Big)\Big)\Big], \tag{6}$$

for $0 \leq s \leq (m_1 - m_2)^2$, with $m_1 \geq m_2$. Here, $\rho_0(s)$ is the spectral density in the lowest order of perturbation theory at zero temperature and it is given by

$$\rho_0(s) = \frac{3}{8\pi^2 s} q^2(s)v^n(s), \tag{7}$$

where $q(s) = s - (m_1 - m_2)^2$ and $v(s) = \Big(1 - 4m_1 m_2/q(s)\Big)^{1/2}$. Here $n = 3$ and $n = 1$ for scalar and pseudoscalar particles, respectively.

Similarly, we consider thermal spectral density for vector mesons. The correlation function of vector current in thermal field theory is given by Lorentz invariant functions, $\Pi_l\left(q^2, \omega\right) = \Pi_2/\overline{q}^2$ and $\Pi_t\left(q^2, \omega\right) = -\frac{1}{2}(\Pi_1 + q^2\Pi_2/\overline{q}^2)$. Here $\Pi_1 = g^{\mu\nu}\Pi_{\mu\nu}$, $\Pi_2 = u^\mu\Pi_{\mu\nu}u^\nu$, $\overline{q}^2 = \omega^2 - q^2$, $\omega = u \cdot q$ and $u_\mu$ is four-velocity. Carrying out the integral over $k_0$, we obtain the imaginary part of the $\Pi_2(q, T)$ as $Im\Pi_2 = K(q_0) + K(-q_0)$, where

$$K(q_0) = N_c \int \frac{d\mathbf{k}}{8\pi^2} \frac{\mathbf{k} \cdot \mathbf{q} + q_0\omega_1 - 2\omega_1^2}{\omega_1\omega_2}$$
$$\times \Big[A(n_1, n_2)\delta(q_0 - \omega_1 - \omega_2) - B(n_1, n_2)\delta(q_0 - \omega_1 + \omega_2)\Big]. \tag{8}$$

Using Eq. (4) and carrying out the integral over angle $\theta$, the annihilation and scattering parts of thermal spectral densities for vector current at nonzero momentum can be written as:

$$\rho_{l,a}(s, |\mathbf{q}|) = \frac{3}{16\pi^2} \int_{-\nu}^{\nu} dx(1 - x^2)\Big[1 - 2n_+(s, |\mathbf{q}|)\Big], \tag{9}$$

$$\rho_{l,s}(s, |\mathbf{q}|) = \frac{3}{16\pi^2} \int_{\nu}^{\infty} dx(1 - x^2)\Big[n_+(s, |\mathbf{q}|) - n_-(s, |\mathbf{q}|)\Big], \tag{10}$$

where $n_+(s, |\mathbf{q}|) = n\Big[\frac{1}{2}(|\mathbf{q}|x) + \sqrt{s}\Big]$ and $n_-(s, |\mathbf{q}|) = n\Big[\frac{1}{2}(|\mathbf{q}|x) - \sqrt{s}\Big]$. For $m_1 = m_2$, zero temperature and $|\mathbf{q}| \to 0$ limit cases, Eqs. (5), (6), (9) and (10) are good consistency with the results existing in the literature [14]-[19]. Also, the investigation of obtained results show that thermal contributions are significantly important and therefore the thermal contributions must be taken into account in analysis of mesons properties in medium and interpretation of heavy ion collision experiments.

## 3. Acknowledgement

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