RELIABLE H_∞ CONTROL FOR A CLASS OF SWITCHED NONLINEAR SYSTEMS [1]

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Abstract: This paper focuses on the problem of reliable H_∞ control for a class of switched nonlinear systems with actuator failures among a prespecified subset of actuators. In existing works, the reliable H_∞ design methods are all based on a basic assumption that the never failed actuators must stabilize the given system. But when actuators suffer "serious failure"– the never failed actuators can not stabilize the given system, the standard design methods of reliable H_∞ control do not work. Based on the switching technique, the problem can be solved by means of switching among subsystems or finite candidate controllers.

Keywords: Switched systems; Reliable H_∞ control; Multiple-Lyapunov function; actuator failures; LaSalle's invariance principle

1. INTRODUCTION

In recent years, considerable attention has been paid to switched systems (Branicky, 1998; Liberzon, 2003; Liberzon & Morse, 1999; Sun et al., 2004; Zhao & David, 2004). Switched systems are one of important kinds of hybrid systems. A switched system consists of a number of subsystems, either continuous-time or discrete-time dynamic systems, and a switching law, which orchestrates the switching between the subsystems. The applications in computer disc drives (Gollu & Varaiya, 1989), some robot control systems (Jeon & Tomizuka, 1993), the cart-pendulum systems (Zhao & Spong, 2001), and other engineering systems indicate that switched systems have extensive practice background. Therefore, it has both theoretical significance and practical value to study switched systems.

On the other hand, since failures of control components often occur in real world, classical H_∞ control methods may not provide satisfactory performance, even drive the closed-loop system unstable. To overcome this problem, reliable H_∞ control has made great progress recently (Veillette, 1992; Yang, Wang & Soh, 2001; Yang, Lam & Wang, 1998). In particular, Yang et al. (2001) presented a methodology for the design of reliable H_∞ controller for the case of sensor failures and actuator failures. Yang et al. (1998) solved the reliable H_∞ control problem for affine nonlinear systems by using the Hamilton-Jacobi inequal-

[1] This work was supported by the NSF of China under Grant 60274009, and the SRFDP under Grant 20020145007, and the NSF of Liaoning Province under Grant 20032020.

ity approach. However, these reliable H_∞ design methods are all based on a basic assumption that the never failed actuators must stabilize the given system. This assumption is obviously somehow unpractical. In other words, actuators may suffer "serious failure"– the never failed actuators can not stabilize the given system. In this case, the standard design methods of reliable H_∞ control do not work.

This paper studies the problem of reliable H_∞ control where actuators suffer "serious failure". We assume either a system can be switched among finite subsystems, or a system controller can be switched among finite candidate controllers. Based on the multiple Lyapunov function technique, a sufficient condition for the switched nonlinear systems to be asymptotically stable with H_∞-norm bound is derived for all admissible actuator failures. Furthermore, as a direct application, a hybrid state feedback strategy is proposed to solve the standard H_∞ control problem for nonlinear systems when no single continuous controller is effective. Finally, a numerical example illustrates the effectiveness of the proposed approach.

2. PROBLEM FORMULATION

Consider switched nonlinear systems described by the state-space model of the form:

$$\begin{aligned} \dot{x} &= f_\sigma(x) + g_\sigma(x)u_\sigma + p_\sigma(x)w_\sigma \\ z &= \begin{pmatrix} h_\sigma \\ u_\sigma \end{pmatrix} \end{aligned} \tag{1}$$

where $\sigma : R_+ \to M = \{1, 2, \cdots, m\}$ is the switching signal to be designed, $x \in R^n$ is the state,$u_i = (u_{i1}, \cdots u_{im_i})^T \in R^{m_i}$ and $w_i = (w_{i1}, \cdots w_{iq_i})^T \in R^{q_i}$denote the control input and disturbance input of the i-th subsystem respectively, z is the output to be regulated. Further, let $f_i(x) \in R^n, g_i(x) = (g_{i1}(x), \cdots g_{im_i}(x)) \in R^{n\times m_i}, p_i(x) = (p_{i1}(x), \cdots p_{iq_i}(x)) \in R^{n\times q_i}, h_i(x) = (h_{i1}(x), \cdots h_{ip_i}(x))^T \in R^{p_i}, f_i(0) = 0, h_i(0) = 0, i = 1, 2, \cdots, m$.

We adopt the following notations from (Branicky, 1998) for system (1). In particular, a switching sequence is expressed by

$$\sum = \{x_0; (i_0, t_0), (i_1, t_1), \cdots, (i_j, t_j), \cdots, |i_j \in M, j \in N\}$$

in which t_0 is the initial time, x_0 is the initial state, (i_k, j_k) means that the i_k -th subsystem is activated for $t \in [t_k, t_{k+1})$. Therefore, when $t \in [t_k, t_{k+1})$, the trajectory of the switched system (1) is produced by the i_k-th subsystem. For any $j \in M$,

$$\begin{aligned} \Sigma_t(j) = \{&[t_{j_1}, t_{j_1+1}), [t_{j_2}, t_{j_2+1}), \cdots [t_{j_n}, t_{j_n+1}) \\ &\cdots, \sigma(t) = j, t_{j_k} \le t < t_{j_k+1}, k \in N\} \end{aligned} \tag{2}$$

denotes the sequence of switching times of the j-th subsystem, in which the j-th subsystem is switched on at t_{j_k} and switched off at t_{j_k+1}.

We classify actuators of a given system into two groups. One is a set of actuators susceptible to failures, denoted by $\Theta_i \subseteq \{1, 2, \cdots, m_i\}, i \in M$. The other is a set of actuators robust to failures, denoted by $\bar{\Theta}_i \subseteq \{1, 2, \cdots, m_i\} - \Theta_i, i \in M$. For $\omega_i \subseteq \Theta_i$, introduce the decomposition

$$g_i(x) = g_{\omega_i}(x) + g_{\bar{\omega}_i}(x),$$

where

$$\begin{aligned} g_{\omega_i}(x) =&(\delta_{\omega_i}(1)g_{i1}(x), \delta_{\omega_i}(2)g_{i2}(x), \cdots, \\ &\delta_{\omega_i}(m_i)g_{im_i}(x)) \end{aligned}$$

with δ_{ω_i} defined by:

$$\delta_{\omega_i}(k) = \begin{cases} 1, k \in \omega_i \\ 0, k \notin \omega_i. \end{cases}$$

When actuator failures occur corresponding to $\omega_i \subseteq \Theta_i$ the resulting system can be described by

$$\begin{aligned} \dot{x} &= f_i(x) + g_{\bar{\omega}_i}(x)u_{\bar{\omega}_i} + p_i(x)w_i \\ z_{\bar{\omega}} &= \begin{pmatrix} h_i(x) \\ u_{\bar{\omega}_i} \end{pmatrix} \end{aligned} \tag{3}$$

The following inequalities are obvious and will be used in the sequel:

$$\begin{aligned} g_{\omega_i}(x)g^T_{\omega_i}(x) &\le g_{\Theta_i}(x)g^T_{\Theta_i}(x), \\ g_{\bar{\Theta}_i}(x)g^T_{\bar{\Theta}_i}(x) &\le g_{\bar{\omega}_i}(x)g^T_{\bar{\omega}_i}(x). \end{aligned}$$

Now, the reliable H_∞ control problem for the switched system (1) is stated as follows:

Let a constant $\gamma > 0$ be given. For actuator failures corresponding to any $\omega_i \subseteq \Theta_i$, find a continuous state feedback controller $u_i = u_i(x)$ for each subsystem, and a switching law $i = \sigma(t)$ such that:

(1) The closed-loop system is asymptotically stable when $w_i = 0$.

(2) The output z satisfies $\|z\|_2 \le \gamma\|w_i\|_2$ under the zero initial condition.

Definition(Isidori & Astolfi, 1992). Suppose $f(0) = 0$ and $h(0) = 0$. The pair $\{f, h\}$ is said to be detectable if $x(t)$ is any integral curve of $\dot{x} = f(x)$, then $h(x(t))$ is defined for all $t \ge 0$ and $h(x(t)) \equiv 0$ for all $t \ge 0$ implies $\lim_{t\to\infty} x(t) = 0$.

Remark 1. In the existing standard reliable control problem, the condition that $(f, g_{\bar{\Theta}})$ is a stabilizable pair requisite. This strong condition is no longer needed here for switched systems. In fact, if $(f_j, g_{\bar{\Theta}_j})$ is a stabilizable pair for any $j \in M$, then we can design state feedback controller for the j-th subsystem that makes the system (1) stabilizable with an H_∞ -norm bound γ, and thus the problem becomes trivial.

3. MAIN RESULTS

This section gives a condition for the reliable H_∞ control problem to be solvable, and designs continuous controllers for subsystems and a switching law.

Theorem 1: Let a constant $\gamma > 0$ be given. Suppose that

(1) The pair $\{f_i, h_i\}$ is detectable.
(2) There exist functions $\beta_{ij}(x)(i,j \in M)$ (either all nonnegative or all nonpositive) and radiully unbounded, positive smooth functions $V_i(x), V_i(x(0)) = 0, i \in M$ satisfying the partial differential inequalities

$$\frac{\partial V_i}{\partial x} f_i + \frac{1}{4}\frac{\partial V_i}{\partial x}(\frac{1}{\gamma^2} p_i p_i^T - g_{\bar{\Theta}_i} g_{\bar{\Theta}_i}^T)\frac{\partial^T V_i}{\partial x} + h_i^T h_i + \sum_{j=1}^{m} \beta_{ij}(V_i - V_j) \le 0, i \in M \quad (4)$$

Then, the hybrid state feedback reliable controllers

$$u_i = u_i(x) = -\frac{1}{2} g_i^T(x) \frac{\partial^T V_i}{\partial x}(x), i = 1, 2, \cdots m \quad (5)$$

and the switching law

$$i = \arg\max_{i \in M}\{V_i(x)\} \quad (6)$$

solve the reliable H_∞ control problem.

proof: Consider actuator failures corresponding to any $\omega_i \subseteq \Theta_i$, since the control input $u_i(x)$ is applied to the plant only through normal actuators, it follows that in system (3)

$$u_i = u_{\bar{\omega}_i}(x) = -\frac{1}{2} g_{\bar{\omega}_i}^T(x)\frac{\partial^T V_i}{\partial x}(x)$$

Without loss of generality, suppose $\beta_{ij} \ge 0$. For any fixed $i \in M$, if $x^T(V_i - V_j)x \ge 0, \forall j \in M$ for $x \in R^n$, we have

$$\frac{\partial V_i}{\partial x} f_i + \frac{1}{4}\frac{\partial V_i}{\partial x}(\frac{1}{\gamma^2} p_i p_i^T - g_{\bar{\Theta}_i} g_{\bar{\Theta}_i}^T)\frac{\partial^T V_i}{\partial x} + h_i^T h_i \le 0. \quad (7)$$

Obviously, for $\forall x \in R^n \backslash \{0\}$, there certainly is an $i \in M$ such that $x^T(V_i - V_j)x \ge 0, \forall j \in M$. For any $i \in M$, let

$$\Omega_i = \{x \in R^n | x^T(V_i - V_j)x \ge 0, \forall j \in M\}, \quad (8)$$

then $\bigcup_{i=1}^{m} \Omega_i = R^n \backslash \{0\}$. Construct the sets $\bar{\Omega}_1 = \Omega_1, \cdots, \bar{\Omega}_i = \Omega_i - \bigcup_{j=1}^{i-1} \bar{\Omega}_j, \cdots, \bar{\Omega}_m = \Omega_m - \bigcup_{j=1}^{m-1} \bar{\Omega}_j$.
Obviously, we have
$\bigcup_{i=1}^{m} \bar{\Omega}_i = R^n \backslash \{0\}$, and $\bar{\Omega}_i \bigcap \bar{\Omega}_j = \phi, i \ne j$.

When $x(t) \in \bar{\Omega}_i$, the time-derivative of $V_i(x(t))$ along the trajectory of the system (3) is given by

$$\begin{aligned}
\dot{V}_i(x(t)) &= \frac{\partial V_i}{\partial x}(f_i + g_{\bar{\omega}_i} u_{\bar{\omega}_i} + p_i w_i) \\
&= \frac{\partial V_i}{\partial x}(f_i + p_i w_i + g_i u_i - g_{\omega_i} u_{\omega_i}) \\
&\le \frac{\partial V_i}{\partial x}(f_i + p_i w_i + g_i u_i) \\
&\quad + \frac{1}{4}\frac{\partial V_i}{\partial x} g_{\omega_i} g_{\omega_i}^T \frac{\partial^T V_i}{\partial x} + u_{\omega_i}^T u_{\omega_i} \\
&\le \frac{\partial V_i}{\partial x}(f_i + p_i w_i + g_i u_i) \\
&\quad + \frac{1}{4}\frac{\partial V_i}{\partial x} g_{\Theta_i} g_{\Theta_i}^T \frac{\partial^T V_i}{\partial x} + u_{\omega_i}^T u_{\omega_i} \\
&= \frac{\partial V_i}{\partial x}(f_i + p_i w_i) + \parallel u_i + \frac{1}{2} g_i^T \frac{\partial^T V_i}{\partial x} \parallel^2 \\
&\quad - u_{\bar{\omega}_i}^T u_{\bar{\omega}_i} - \frac{1}{4}\frac{\partial V_i}{\partial x} g_{\bar{\Theta}_i} g_{\bar{\Theta}_i}^T \frac{\partial^T V_i}{\partial x}. \quad (9)
\end{aligned}$$

When $w_i = 0$, substituting (5) into (9) and noticing (7), we have

$$\begin{aligned}
\dot{V}_i(x(t)) &\le \frac{\partial V_i}{\partial x} f_i - u_{\bar{\omega}_i}^T u_{\bar{\omega}_i} - \frac{1}{4}\frac{\partial V_i}{\partial x} g_{\bar{\Theta}_i} g_{\bar{\Theta}_i}^T \frac{\partial^T V_i}{\partial x} \\
&\le -\frac{1}{4\gamma^2}\frac{\partial V_i}{\partial x} p_i p_i^T \frac{\partial^T V_i}{\partial x} - h_i^T h_i - u_{\bar{\omega}_i}^T u_{\bar{\omega}_i} \\
&\le 0.
\end{aligned}$$

Observe that any trajectory satisfying $\dot{V}_i(x(t)) = 0$ for all $t \ge 0$ is necessarily a trajectory of

$$\dot{x} = f_i(x) + g_{\bar{\omega}_i}(x) u_{\bar{\omega}_i}$$

such that $x(t)$ is bounded and $h_i(x(t)) \equiv 0$ for all $t \ge 0$. The detectability of $\{f_i, h_i\}$ gives $\lim_{t\to\infty} x(t) = 0$. Thus, the closed-looped system (1) and (5) is asymptotically stable by LaSalle's invariance principle (Lasalle, 1976).
In the following, we show that the overall L_2-gain from w_i to $z_{\bar{\omega}}$ is less than or equal to γ. We suppose $x(0) = 0$, and without loss of generality, assume that the first subsystem ($\sigma = 1$) is activated at the initial time, i.e. $t_{k_1} = t_0 = 0$. Now we introduce

$$J = \int_0^T (\parallel z_{\bar{\omega}}(t) \parallel^2 - \gamma^2 \parallel w_i(t) \parallel^2) dt.$$

According to the switching sequence (2), when $T \in [t_k, t_{k+1})$

$$\begin{aligned}
J \le& \sum_{j=0}^{k-1}(\int_{t_j}^{t_{j+1}} (\parallel h_{i_j}(t) \parallel^2 + \parallel u_{\bar{\omega}_{i_j}}(t) \parallel^2 \\
&- \gamma^2 \parallel w_{i_j}(t) \parallel^2 + \dot{V}_{i_j}(t)) dt \\
&- (V_{i_j}(x(t_{j+1})) - V_{i_j}(x(t_j)))) + \int_{t_k}^{T} (\parallel h_{i_j}(t) \parallel^2 \\
&+ \parallel u_{\bar{\omega}_{i_j}}(t) \parallel^2 - \gamma^2 \parallel w_{i_j}(t) \parallel^2 + \dot{V}_{i_j}(t)) dt \\
&- (V_{i_k}(x(T)) - V_{i_k}(x(t_k))).
\end{aligned}$$

Note that

$$\dot{V}_{i_j}(t)+ \| h_{i_j}(t) \|^2 + \| u_{\bar{\omega}_{i_j}}(t) \|^2 -\gamma^2 \| w_{i_j}(t) \|^2$$
$$\leq \frac{\partial V_{i_j}}{\partial x}(f_{i_j} + p_{i_j}w_{i_j} + g_{i_j}u_{i_j})$$
$$+ \frac{1}{4}\frac{\partial V_{i_j}}{\partial x} g_{\Theta_{i_j}} g_{\Theta_{i_j}}{}^T \frac{\partial^T V_{i_j}}{\partial x}$$
$$+ \| h_{i_j}(t) \|^2 + \| u_{i_j}(t) \|^2 -\gamma^2 \| w_{i_j}(t) \|^2$$
$$= \frac{\partial V_{i_j}}{\partial x} f_{i_j} + \frac{1}{4\gamma^2}\frac{\partial V_{i_j}}{\partial x} p_{i_j} p_{i_j}^T \frac{\partial^T V_{i_j}}{\partial x}$$
$$- \frac{1}{4}\frac{\partial V_{i_j}}{\partial x} g_{\bar{\Theta}_{i_j}} g_{\bar{\Theta}_{i_j}}^T \frac{\partial^T V_{i_j}}{\partial x} + h_{i_j}^T h_{i_j} +$$
$$\| u_{i_j} + \frac{1}{2} g_{i_j}^T \frac{\partial^T V_{i_j}}{\partial x} \|^2 - \| \gamma w_{i_j} - \frac{1}{2\gamma} p_{i_j}^T \frac{\partial^T V_{i_j}}{\partial x} \|^2 . \quad (10)$$

Substituting(5)into(10), we have

$$\dot{V}_{i_j}(t)+ \| h_{i_j}(t) \|^2 + \| u_{\bar{\omega}_{i_j}}(t) \|^2 -\gamma^2 \| w_{i_j}(t) \|^2$$
$$\leq -\gamma^2 \| w_{i_j} - \frac{1}{\gamma^2} p_{i_j}^T \frac{\partial^T V_{i_j}}{\partial x} \|^2$$
$$\leq 0.$$

Then

$$\begin{aligned} J \leq & -\sum_{j=0}^{k-1}(V_{i_j}(x(t_{j+1})) - V_{i_j}(x(t_j))) \\ & - (V_{i_k}(x(T)) - V_{i_k}(x(t_k))) \\ = & -(V_{i_0}(x(t_1)) - V_{i_0}(x(t_0)) + V_{i_1}(x(t_2)) \\ & - V_{i_1}(x(t_1)) + \cdots + V_{i_{k-1}}(x(t_k)) \\ & - V_{i_{k-1}}(x(t_{k-1}))) - V_{i_k}(x(T)) + V_{i_k}(x(t_k)). \end{aligned}$$

Note that

$$V_{\sigma(t_{k-1})}(t_k) = V_{\sigma(t_k)}(t_k).$$

Therefore

$$\begin{aligned} J & \leq V_{i_0}(x(t_0)) - V_{i_k}(x(T)) \\ & = -V_{i_k}(x(T)) \\ & \leq 0. \end{aligned}$$

Remark 2. The reliable H_∞ control problem for switched nonlinear system is solved by Theorem 1. When $M = \{1\}$ switched system (1) degenerates into a regular nonlinear system and the H_∞ control problem becomes the standard reliable H_∞ control problem for nonlinear systems (Yang et al., 1998).

Remark 3. For the switched linear system

$$\begin{aligned} \dot{x} &= A_i x + B_i u + D_i w, \\ z &= C_i x, \end{aligned}$$

(4) turns to be the matrix inequalities

$$P_i A_i + A_i^T P_i + P_i(\gamma^{-2} D_i D_i^T - \varepsilon^{-1} B_{\bar{\Theta}_i} B_{\bar{\Theta}_i}^T) P_i$$
$$+ C_i^T C_i + \sum_{j=1}^{m} \beta_{ij}(P_i - P_j) < 0, i \in M,$$

where P_i is positive definite matrix, β_{ij} are either all nonnegative or all nonpositive constants. In particular, if $j = 1$, the Riccati inequality follows.

4. HYBRID RELIABLE H_∞ CONTROL FOR NONLINEAR SYSTEMS

In engineering, a continuous reliable H_∞ controller for a nonlinear system may not exist or may be sometimes too complex to implement. Thus, in some control problems, control actions are decided by switching between finite candidate controllers. We try to use this methodology to solve the standard reliable H_∞ control problem for nonlinear systems.

Consider the following nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u + p(x)w \\ z &= \begin{pmatrix} h(x) \\ u \end{pmatrix} \end{aligned} \quad (11)$$

where $x \in R^n$ is the state, u and w denote the control input and disturbance input respectively, z is the output to be regulated,$f(x) \in R^n, g(x) = (g_1(x), \cdots g_m(x)) \in R^{n\times m}, p(x) = (p_1(x), \cdots p_q(x)) \in R^{n\times q}, h(x) = (h_1(x), \cdots h_p(x))^T \in R^p, f(0) = 0, h(0) = 0.$

Suppose that we have exist finite candidate controllers for system (11). When actuator failures occur, none of the individual controller makes the system stabilizable. In particular, we consider the following class of candidate state feedback controllers:

$$\begin{aligned} & u_i = u_i(x) = -\frac{1}{2} g^T(x) \frac{\partial^T V_i}{\partial x}(x), \quad (12) \\ & i = 1, 2, \cdots m, \end{aligned}$$

where V_i will be specified later.

Theorem 2 Let a constant $\gamma > 0$ be given. Suppose that

(1) The pair $\{f, h\}$ is detectable.

(2) There exist functions $\beta_{ij}(x)(i, j \in M)$(either all nonnegative or all nonpositive) and radiully unbounded, positive smooth functions $V_i(x)$, $V_i(x(0)) = 0, i \in M$ satisfying the partial differential inequalities

$$\frac{\partial V_i}{\partial x} f + \frac{1}{4}\frac{\partial V_i}{\partial x}(\frac{1}{\gamma^2} pp^T - g_{\bar{\Theta}} g_{\bar{\Theta}}^T)\frac{\partial^T V_i}{\partial x} + h^T h$$
$$+ \sum_{j=1}^{m} \beta_{ij}(V_i - V_j) \leq 0, i \in M \quad (13)$$

Then, for actuator failures corresponding to any $\omega_i \subseteq \Theta_i$, the hybrid state feedback reliable controller (12) with the switching law (6) solve the reliable H_∞ control problem.

proof. Substituting the designed controllers (12) into the system (11) results in a switched nonlinear system. Then, applying the theorem 1 gives the result.

remark 4. Partial differential inequalities (13) are much easier to satisfy than the Hamilton-Jacobi inequality because the term $\sum_{j=1}^{m} \beta_{ij}(V_i - V_j)$ is added which may change sign when x varies.

In particular, if $j = 1$, (13) degenerate into the Hamilton-Jacobi inequality.

5. EXAMPLE

In this section, we present an example to illustrate the effectiveness of the proposed design method. Consider the following nonlinear switched system:

$$\begin{aligned} \dot{x} &= f_i(x) + g_i(x)u_i + p_i(x)w_i \\ z &= \begin{pmatrix} h_i \\ u_i \end{pmatrix}, \quad i = 1, 2, \end{aligned} \tag{14}$$

where
$f_1(x) = -3x^3, p_1(x) = x, h_1(x) = h_2(x) = x^2,$
$g_1(x) = \left(3\, x^2\right), f_2(x) = -3x^3 + x, p_2(x) = 1,$
$g_2(x) = \left(x^2\ 2\right), \Theta_1 = \{1\}, \Theta_2 = \{2\}.$
It is easy to check that $\{f_i, h_i\}$ is detectable, but $(f_i, g_{\bar{\Theta}_i})$ is not a stabilizable pair, the reliable H_∞ control problem is solvable via switching between subsystems. Now, consider

$$V_1(x) = x^2, V_2(x) = x^4, x \in R^n.$$

Both V_1 and V_2 are globally positive definite and $V_1(0) = V_2(0)$.
Let $\gamma = 1, \beta_1(x) = 3x^2, \beta_2(x) = 5x^2$, then

$$\begin{aligned} &\frac{\partial V_1}{\partial x} f_1 + \frac{1}{4}\frac{\partial V_1}{\partial x}\left(\frac{1}{\gamma^2}p_1p_1^T - g_{\bar{\Theta}_1}g_{\bar{\Theta}_1}^T\right)\frac{\partial^T V_1}{\partial x} + h_1^T h_1 \\ &+ \beta_1(V_1 - V_2) \\ &= 2x(-3x^3) + x^2(x^2 - x^4) + x^4 + 3x^2(x^2 - x^4) \\ &= -x^4 - 4x^6 \\ &\leq 0 \end{aligned}$$

$$\begin{aligned} &\frac{\partial V_2}{\partial x} f_2 + \frac{1}{4}\frac{\partial V_2}{\partial x}\left(\frac{1}{\gamma^2}p_2p_2^T - g_{\bar{\Theta}_2}g_{\bar{\Theta}_2}^T\right)\frac{\partial^T V_2}{\partial x} + h_2^T h_2 \\ &+ \beta_2(V_2 - V_1) \\ &= 4x^3(-x^3 + x) + 4x^6(1 - x^4) + x^4 + 5x^2(x^4 - x^2) \\ &= -3x^6 - 4x^{10} \\ &\leq 0. \end{aligned}$$

So the controllers

$$\begin{aligned} u_1 &= -\frac{1}{2}g_1^T(x)\frac{\partial^T V_1}{\partial x}(x) = \begin{pmatrix} -3x \\ -x^3 \end{pmatrix}, \\ u_2 &= -\frac{1}{2}g_2^T(x)\frac{\partial^T V_2}{\partial x}(x) = \begin{pmatrix} -2x^5 \\ -4x^3 \end{pmatrix}. \end{aligned}$$

and design the switching law by

$$i = \arg\max_{i \in M}\{V_i(x)\}, i = 1, 2.$$

Then, the reliable H_∞ control problem with $\gamma = 1$ is solved.

6. CONCLUSIONS

We have considered the problem of reliable H_∞ control for switched nonlinear systems. In particular, attention is concentrated on actuators suffering "serious failures", which has not been considered in previous reliable works. Based on switching strategy, we design controllers and switching law such that the problem of reliable H_∞ control is solved. Moreover, a hybrid state feedback strategy is proposed to solve the standard H_∞ control problem for nonlinear systems when no single continuous controller is effective. Finally, a numerical example illustrates the effectiveness of the proposed approach.

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