OUTPUT TRACKING CONTROL FOR CLASS OF FUZZY TIME-DELAY SYSTEMS

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Abstract: The output tracking control problem for fuzzy time-delay systems in presence of parameter perturbations has been solved via fuzzy T-S system models and variable-structure control approach. Following the reaching condition, a variable-structure fuzzy control method is proposed accordingly, when the time delay is known and available and when unknown and unavailable. The method guarantees the system operation arrives to the sliding surface in finite time interval and be kept there thereafter while tracking the desired trajectory. The sufficient condition for globally bounded state is derived by using the ISS theory and the LMI method. A simulation example demonstrates the validity and effectiveness of the proposed method.

Keywords: Fuzzy control; output tracking; robustness; sliding-mode control.

1. INTRODUCTION

The tasks of stabilization and tracking control are two typical control problems. In general, tracking problems are more difficult than stabilization problems, especially for nonlinear systems and in the presence of time-delays and time-varying parameters with uncertainties. Recently a lot of research has been devoted to the fusion of computationally intelligent and math-analytical methodologies also including sliding mode control (Kaynak and co-authors, 2001) to solve tracking control problems.

Takagi-Sugeno (T-S) type fuzzy controllers (Takagi and Sugeno, 1985) are known to have been successfully applied to the stabilization control design nonlinear systems (e.g., see Chen and co-authors, 1999; Jadbabai and co-authors, 1998; Tseng and co-authors, 2001; Wang and Lin, 1999). In most of these applications, the T–S fuzzy model has been proved to be a good representation for a certain class of nonlinear dynamical systems (Slotine and Li, 1992). In these studies, a nonlinear plant was represented by a set of linear models interpolated by membership functions (T-S) fuzzy model and then a model-based fuzzy controller was developed to stabilize the plant T-S fuzzy model.

Tracking control designs are important for many practical applications, e.g. in robotic tracking, missile tracking, and aircraft attitude tracking. Yet not too many studies based on the T-S fuzzy model, especially for continuous-time systems exist (Chen and co-authors, 1998; Tseng and co-authors, 2001; Wang and Lin, 1999). Tseng and co-authors (2001) proposed fuzzy tracking control design for nonlinear dynamic systems via T-S fuzzy model. However, their method appeared to be conservative and not applicable to operate practically. Besides, it provides tracking control of nominal T-S fuzzy model only without time-delay and uncertainty has been considered and no robustness is guaranteed.

This paper presents a new solution to the same output tracking control problem for fuzzy time-delay systems in the presence of parameter perturbations.

First, the sliding-mode control synthesis is adopted following variable structure control theory (Chou and Cheng, 2002; Feng and Chiang, 1995; Hung and co-authors, 1993; Utkin, 1978, 1992; Yoo and Chung, 1992). Depending on the reaching condition, the sliding-mode variable-structure control is designed for cases when the time-delay is known and unavailable, respectively. The sufficient condition for globally bounded system state is derived by using input-to-state stability (ISS) theory (Sontag and Wang, 1995) and the LMI method (Boyd and co-authors, 1994). Further, the paper is written as follows. Section 2 presents the formulation of a lass fuzzy time-delay systems. The new design synthesis and the main theoretical contributions are given in Section 3. In Section 4, these are applied to carry out tracking control design for CSTR benchmark process. Thereafter conclusion and references follow.

2. FORMULATION OF THE FUZZY TIME-DELAY SYSTEM MODEL

The Takagi and Sugeno fuzzy dynamic model, employed in this study, is a piecewise interpolation of several linear models through membership functions. The fuzzy model is described by means of fuzzy If-Then rules with appropriate linear dynamical systems in the consequents. It is shown here, this class of models can be employed to deal with nonlinear plants with uncertain parameters in the presence of time delay. Time delay is due to either because of transport feature or approximation of process single-dimensional spatial distribution, thus avoiding infinite-dimensional systems. For such plants, based on i-th rule, the model adopted has the following form:

Plant Rule i $(i = 1, 2, \ldots, r)$

If $q_1(t)$ is m_{i1} and ... and $q_p(t)$ is m_{ip}, Then

$$\dot{x}(t) = (A_{1i} + \Delta A_{1i})x(t) + (A_{2i} + \Delta A_{2i})x(t-t) + Bu(t), \quad (1)$$

$$y(t) = Cx(t). \quad (2)$$

In here, m_{ij} denotes the respective fuzzy set; $x(t) \in R^n$ denotes the state vector; $u(t) \in R^m$ denotes the control input; A_{1i}, A_{2i} denote some constant matrices of compatible dimensions; t is the bounded time-delay in the state variables. ΔA_{1i}, ΔA_{2i} denote the perturbations of system parameters. It is assumed that $0 \le t \le d$, where d denotes a scalar constant. It is further assumed the premise variables are independent of input $u(t)$.

Remark: Given technological facts about sensor and actuators, common matrices B and C in the model (1)-(2) do not impose technical restriction.

The overall fuzzy system model can be created by fuzzy blending of each individual rule as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(q(t))\,[\,(A_{1i} + \Delta A_{1i})x(t) + (A_{2i} + \Delta A_{2i})x(t-t) + Bu(t)\,], \quad (3)$$

$$y(t) = Cx(t), \quad (4)$$

where: $q_i(t)$ $(i = 1, 2, \ldots, p)$ are premise variables constituting $q(t) = [q_1(t), q_2(t), \ldots, q_p(t)]$, and

$$h_i(q(t)) = \frac{w_i(t)}{\sum_{j=1}^{r} w_j(q(t))}, \quad (5)$$

$$w_i(q(t)) = \prod_{j=1}^{r} v_{ij}(q_j(t)), \quad (6)$$

with $v_{ij}(q_j(t))$ the membership grade of $q_j(t)$.

The objectives of this paper are to synthesise a fuzzy sliding mode controller such that the output $y(t)$ of system (3)-(6) tracks a desired reference trajectory $y_d(t)$ in the closed-loop system possessing input-to-state stability (ISS) property. For this purpose, the following assumptions are adopted.

Assumption 1: Matrix CB is nonsingular.

Assumption 2: All the perturbations ΔA_{1i}, ΔA_{2i} satisfy the matching condition that there exist $\|\Delta\bar{A}_{1i}\| \le M_{\Delta A_1}$, $\|\Delta\bar{A}_{2i}\| \le M_{\Delta A_2}$, such that $\Delta A_{1i}(x) = B\Delta\bar{A}_{1i}$, $\Delta A_{2i}(x) = B\Delta\bar{A}_{2i}$, where $M_{\Delta A_1}$, $M_{\Delta A_2}$ are known scalar numbers.

Assumption 3. There exists a known scalar number $q > 1$, such that $\|x(t-t)\| \le q\|x(t)\|$ for $t \in [0, d]$ with d some constant.

This assumption is due to Feng and Jiang (1995) and it is somewhat restrictive, but does not imply necessarily a monotonically increasing state vector.

In the sequel, the concept of ISS and the necessary and sufficient conditions for ISS shall be needed. These are briefly presented through the relevant lemma due to Sontag and Wang (1995). Consider the general nonlinear system

$$\dot{x} = f(x, u) \quad (7)$$

where: $x(t) \in R^n$, $u(t) \in R^m$, $f:\ R^n \times R^m \to R^n$ is a continuous map and satisfies $f(0,0) = 0$.

Lemma 1: System (7) is ISS if and only if there is a smooth function $V:\ r^n \to R_+$ such that there exist K_∞ function and scalars n_1, n_2, and K function abs scalars n_3, n_4, such that:

$$n_1(\|x\|) \le V(x) \le n_2(\|x\|),\ \forall x \in R^n,$$

$\dot{V}(x) \le -n_3(\|x\|)$ so that $V(x) \ge n_4(\|x\|)$.

A function g : $R_+ \to R_+$ is called K function if it is continuous, strictly increasing and satisfies $g(0) = 0$; if g further satisfies $\lim_{t\to\infty} g(t) = \infty$, it is called K_∞ function.

3. OUTPUT TRACKING CONTROL SYNTHESIS

In the case when plant time-delay t is precisely known, the sliding mode can be selected for the system (3)-(6) by means of sliding-mode control theory (Utkin, 1978, 1992) yielding the sliding surface:

$$s(t) = (CB)^{-1}(y(t) - y_r(t)). \qquad (8)$$

Upon defining the tracking error

$$e(t) = y(t) - y_r(t), \qquad (9)$$

Through a lengthy derivation, the following variable structure controller can be obtained:

$$u(t) = -\sum_{i=1}^{r} h_i(q(t))(CB)^{-1}C \cdot$$
$$[(A_{1i} + \Delta A_{1i})x(t) + (A_{2i} + \Delta A_{2i})x(t-t)] -$$
$$- a_1 s(t) - a_2 \operatorname{sgn} s(t) - [M_{\Delta A_1}\|x(t)\| +$$
$$M_{\Delta A_2}\|x(t-t)\|]\operatorname{sgn} s(t) + (CB)^{-1}\dot{y}_r(t), \qquad (10)$$

where a_1, a_2 are two positive scalars. Substitution into plant system model (3)-(6) yields

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(q(t))\{(A_{1i} - B(CB)^{-1}CA_{1i})x(t)$$
$$+ (A_{2i} - B(CB)^{-1}CA_{2i})x(t-t)$$
$$+ B(\Delta\overline{A}_{1i}x(t) + \Delta\overline{A}_{2i}x(t-t))\}$$
$$- B\{a_1 \operatorname{sgn} s(t) + a_2 \operatorname{sgn} s(t) + (M_{\Delta A_1}\|x(t)\|$$
$$M_{\Delta A_2}\|x(t-t)\|)\operatorname{sgn} s(t) - (CB)^{-1}\dot{y}_r(t)\}, \qquad (11)$$

$$y(t) = Cx(t). \qquad (12)$$

Since matrix C has full row rank, a nonsingular matrix T_1 can be found such that $CT_1 = [0 \quad \overline{C}_2]$ $= \overline{C}$, where $\overline{C}_2$ is nonsingular. Let it be used $T^{-1}B = [\overline{B}_1 \quad \overline{B}_2]^{\mathrm{T}} = \overline{B}$, where $\overline{B}_1 \in R^{(n-m)\times m}$, $\overline{B}_2 \in R^{m\times m}$. Since apparently $CB = \overline{C}\overline{B} = \overline{C}_2\overline{B}_2$ it follows matrix $\overline{B}_2$ is a nonsingular one. Therefore the following matrices are well defined:

$$T_2 = \begin{bmatrix} I & -\overline{B}_1\overline{B}_2^{-1} \\ 0 & \overline{C}_2 \end{bmatrix}, \; T_0 = T_2T_1^{-1} = \begin{bmatrix} T_{01} \\ T_{02} \end{bmatrix}, \qquad (13)$$

where $T_{01} = [I \quad -\overline{B}_1\overline{B}_2^{-1}]T_1^{-1}$, $T_{02} = [0 \quad \overline{C}_2]T_1^{-1}$. Further, it can be shown that $T_{02} = C$, $T_{01}B = 0$. Hence for the closed-loop system (11)-(12), the following suitable matrix $T_0^{-1} = [T_{0inv1} \quad T_{0inv2}]$, where $T_{0inv1} \in R^{(n-m)\times m}$ and $T_{0inv2} \in R^{n\times m}$, can be chosen leading to the following ma in result.

Theorem 1: Consider system (3)-(6) along with Assumptions. 1, 2, and 3 satisfied. Suppose that there exist positive-definite matrices P and R such that the inequality

$$\begin{pmatrix} PN_{1i} + N_{1i}^{\mathrm{T}}P + R & PT_{01}A_{2i}T_{0inv1} \\ T_{0inv1}^{\mathrm{T}}A_{2i}^{\mathrm{T}}T_{01}^{\mathrm{T}}P & -R \end{pmatrix} < 0, \; (14)$$

where $N_{1i} = T_{01}A_{1i}T_{0inv1}$, holds true. Then, in the closed-loop, the fuzzy sliding-mode controller (10) guarantees the tracking of desired reference $y_r(t)$ by the plant output and its ISS property.

Proof: The proof is divided into two parts. In part (a) it is shown that output of system (3)-(6) can follow exactly the desired signal $y_r(t)$ after a finite time interval. In part (b) it is shown that the state of the system is bounded globally. For the sake of some simplicity in the derivations, sliding surface function shall be often denoted by symbol s.

(a) *Output tracking*

The derivative of sliding surface $s = s(t)$ along the state trajectory of closed-loop system (11)-(12) is

$$\dot{s}(t) = (CB)^{-1}C\dot{x}(t) - (CB)^{-1}\dot{y}_r(t)$$
$$= \sum_{i=1}^{r} h_i(q(t))(CB)^{-1}C\{A_{1i}x(t) + A_{2i}x(t-t)\}$$
$$+ u(t) + \sum_{i=1}^{r} h_i(q(t))\{\Delta A_{1i}x(t) + \Delta A_{2i}x(t-t)\}$$
$$- (CB)^{-1}\dot{y}_r(t) = - a_1 s - a_2 \operatorname{sgn} s$$
$$- M_{\Delta A_1}\|x(t)\| - M_{\Delta A_2}\|x(t-t)\|$$
$$+ \sum_{i=1}^{r} h_i(q(t))\{\Delta A_{1i}x(t) + \Delta A_{2i}x(t-t)\}.$$

By means of appropriate analysis, when $s_j > 0$, it can be shown:

$$\dot{s}_j = -a_1 s_j - a_2$$
$$- M_{\Delta A_1}\|x(t)\| - M\Delta A_2\|x(t-t)\|$$
$$+ \sum_{i=1}^{r} h_i(q(t))\{\Delta A_{1i}x(t) + \Delta A_{2i}x(t-t)\}_j$$
$$\le -a_1 s_j - a_2 - M_{\Delta A_1}\|x(t)\| - M_{\Delta A_2}\|x(t-t)\|$$
$$+ \sum_{i=1}^{r} h_i(q(t))\{\Delta A_{1i}x(t) + \Delta A_{2i}x(t-t)\}.$$

Hence $\dot{s}_j \le -a_1 s_j - a_2$ whenever $s_j > 0$. Following a similar derivation, it can be shown that $\dot{s}_j \ge -a_1 s_j - a_2$ whenever $s_j < 0$. From these two results, it can be seen that the operation remain confined on the sliding surface and all sliding values s_j shall arrive to zero in a finite time interval and thereafter be kept there.

(b) *Global boundedness of the system state vector*

Let the following transformation of the system state

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = T_0 x(t) = \begin{bmatrix} T_{01}x \\ T_{02}x \end{bmatrix}$$

be introduced. Then the following state equations can be derived:

$$\dot{z}_1(t) = \sum_{i=1}^{r} h_i(q(t))\{T_{01}A_{1i}T_{0inv1}z_1(t) + T_{01}A_{1i}T_{0inv2}z_2(t) + T_{01}A_{2i}T_{0inv1}z_1(t-t) + T_{01}A_{2i}T_{0inv2}z_2(t-t)\}, \quad (15)$$

$$\dot{z}_2(t) = \sum_{i=1}^{r} h_i(q(t))\{CB(\Delta\overline{A}_{1i}x(t) + \Delta\overline{A}_{2i}x(t-t)\} - CB\{a_1 s + a_2 \operatorname{sgn} s + \left(M_{\Delta A_1}\|x(t)\| + M_{\Delta A_2}\|x(t-t)\|\right)\operatorname{sgn} s - (CB)^{-1}\dot{y}_r(t)\}. \quad (16)$$

Notice that it must be $y(t) = y_r(t)$ whenever $t \ge t_r$ (for some finite t_r) due to the previous argument and since $z_2 = T_{02}x = Cx = y$. Hence it remains to investigate (15) and the dynamics of state vector $z_1(t)$.

Fur this purpose, $z_2(t)$ can be viewed as an input to system (15) and the candidate Lyapunov function

$$V(z_1(t)) = z_1^{\mathrm{T}} P z_1 + \int_{t-t}^{t} z_1^{\mathrm{T}}(s) R z_1(s)\, ds$$

in conjunction with the ISS property can be chosen. The time derivative of $V(z_1(t))$ along the trajectory of system (15) can be found:

$$\dot{V}(z_1) = \sum_{i=1}^{r} h_i(q(t)) z_1^T(t)\left(PN_{1i} + N_{1i}^T P\right) z_1(t) + 2\sum_{i=1}^{r} h_i(q(t)) z_1^T(t) T_{01}A_{1i}T_{0inv2}z_2(t) + 2\sum_{i=1}^{r} h_i(q(t)) z_1^T(t) T_{01}A_{2i}T_{0inv1}z_1(t-t) + 2\sum_{i=1}^{r} h_i(q(t)) z_1^T(t) T_{01}A_{2i}T_{0inv2}z_2(t-t) + z_1^T(t) R z_1(t) - z_1^T(t-t) R z_1(t-t)$$

or

$$\dot{V}(z_1) = \sum_{i=1}^{r} h_i(q(t)) \begin{bmatrix} z_1^T(t) & z_1(t-t) \end{bmatrix} \begin{pmatrix} PN_{1i} + N_{1i}^T P + R & PT_{01}A_{2i}T_{0inv1} \\ T_{0inv1}^T A_{2i}^T T_{01}^T P & -R \end{pmatrix} \begin{bmatrix} z_1(t) \\ z_1(t-t) \end{bmatrix}$$

Next, let the following matrix be defined

$$W_i = \begin{pmatrix} PN_{1i} + N_{1i}^T P + R & PT_{01}A_{2i}T_{0inv1} \\ T_{0inv1}^T A_{2i}^T T_{01}^T P & -R \end{pmatrix},$$

and observe the issue of its eigenvalues. Let $l = \min\{l_{\min}(-W_i), 1, 2, \cdots r\}$, $l_{\min}(\cdot)$ denote the minimum eigenvalue of this matrix. Then it follows

$$\dot{V}(z_1) \le -l\left(\|z_1(t)\|^2 + \|z_1(t-t)\|\right) + g_1\|z_1(t)\|\|z_2(t)\| + g_2\|z_1(t)\|\|z_2(t-t)\|$$

where g_1, g_2 are two positive scalars in accordance to Lemma 1. By virtue of Assumption 3, there exist numbers $q > 1$ and $b > 1$ such that

$$\|x(t-t)\| \le q\|x(t)\|,$$

and

$$\dot{V}(z_1) \le -l\|z_1\|^2 + g\|z_1\|\|z_2\| \le -\tfrac{1}{2}l\|z_1\|^2 + (-\tfrac{1}{2}l\|z_1\|^2 + g\|z_1\|\|z_2\|),$$

where $g = g_1 + g_2 b$. Thus, when $\|z_1\| \ge \frac{2g}{l}\|z_2\|$, it follows

$$\dot{V}(z_1) \le -\tfrac{1}{2}l\|z_1\|^2.$$

On the grounds of Lemma 1, the system (15)-(16) is ISS for all $t \ge t_r$. And this completes the proof of Theorem 1. In addition, the above presentation also gives the controller design solution when time-delay t is known.

In almost the same way, the case when time-delay t is unavailable can be resolved. Note first that it is possible to derive the control law

$$u(t) = -\sum_{i=1}^{r} h_i(q(t))(CB)^{-1}CA_{1i} + \sum_{i=1}^{r} h_i(q(t))H_i\|x(t)\|\operatorname{sgn} s(t) - a_1 s(t) - a_2 \operatorname{sgn} s(t) + (CB)^{-1}\dot{y}_r(t), \quad (17)$$

where $H_i = q\left\|(CB)^{-1}CA_{2i}\right\| + M_{\Delta A_1} + qM_{\Delta A_2}$. Clearly, control (17) differs from control (10) of Theorem 1. With regard to the selection of the sliding mode, all the same steps are repeated as in the previously presented derivation and proof of Theorem 1. Hence, next Theorem 2 is stated without treating the proof and any further discussion (see the proof of the previous theorem).

Theorem 2: Consider system (3)-(6) along with Assumptions 1, 2, and 3 satisfied. Suppose that there exist positive-define matrices P and R such that the following inequality

$$\begin{pmatrix} PN_{1i} + N_{1i}^{\mathrm{T}}P + R & PT_{01}A_{2i}T_{0inv1} \\ T_{0inv1}^{\mathrm{T}}A_{2i}^{\mathrm{T}}T_{01}^{\mathrm{T}}P & -R \end{pmatrix} < 0, \quad (18)$$

where $N_{1i} = T_{01}A_{1i}T_{0inv1}$, holds true. Then, in the closed-loop, the fuzzy sliding-mode controller (17) guarantees the tracking of desired reference $y_r(t)$ by the plant output and its ISS property.

4. AN ILLUSTRATIVE EXAMPLE

The benchmark case of a continuously stirred tank reactor (CSTR) has been used to demonstrate the qualities of the proposed tracking control synthesis (Chen and co-authors, 1998). Process dynamics is described by means of the following equations:

$$V\frac{dA}{dt} = lqA_0 + q(1-l)A(t-a) - qA(t) - VK_0\exp(-E/RT(t))A(t),$$

$$VC_r\frac{dA}{dt} = qC_r\,[\,lT_0 + (1-l)T(t-a) - T(t)\,] - V(-\Delta H)K_0\exp(-E/RT(t))A(t) - U(T(t) - T_w)$$

By applying a suitable transformation, one obtains

$$\dot{x}_1(t) = f_1(x) + (\tfrac{1}{l} - 1)x_1(t-t),$$

$$\dot{x}_2(t) = f_2(x) + (\tfrac{1}{l} - 1)x_2(t-t) + bu(t).$$

A steady-state at $x_e(t) = [0.1440;\, 0.8862]$ is considered, and the deviation from it is modelled via defining a new state variables

$$d x_1 = x_1 - x_e(1),\; d x_2 = x_2 - x_e(2).$$

As in the source reference, the following fuzzy system model is presented:.

Rule 1: If $d x_2$ is small (i.e., $d x_2$ is 0.886), Then

$$d\dot{x}(t) = (A_{11} + \Delta A_{11})d x(t) + (A_{21} + \Delta A_{21})d x(t-t) + Bd u$$

Rule 2: If $d x_2$ is medium (i.e. $d x_2$ is 2.7520), Then

$$d\dot{x}(t) = (A_{12} + \Delta A_{12})d x(t) + (A_{22} + \Delta A_{22})d x(t-t) + Bd u$$

Rule 3: If $d x_2$ is large (i.e. $d x_2$ is 4.7052), Then

$$d\dot{x}(t) = (A_{13} + \Delta A_{13})d x(t) + (A_{23} + \Delta A_{23})d x(t-t) + Bd u$$

The simulation experiments were carried out for tracking control of the desired periodical output $y(t) = \sin(t)$ with the initial state $d x(0) = [1 \;\; 1]^T$. The simulation results are presented in Figures I and II, respectively. Results in Fig. I depict the feasible control performance when the time delay is precisely known and available. Results in Figure II depict the feasible control performance when the time delay is not known and unavailable.

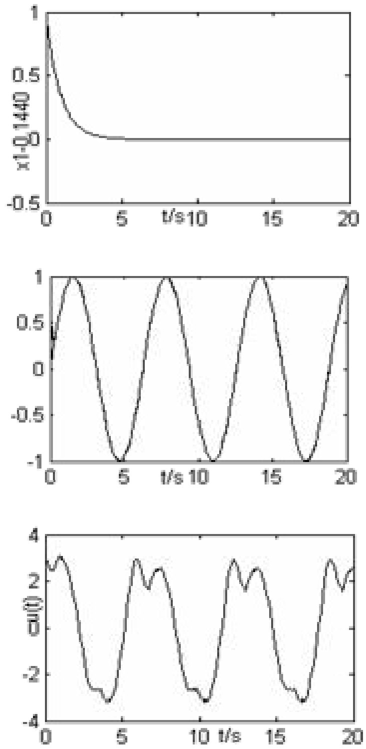

Fig. I. Performance of the overall control system when time-delay is precisely known: (a) The trajectory of $d x_1$; (b) The output tracking trajectory; and (c) System control variable.

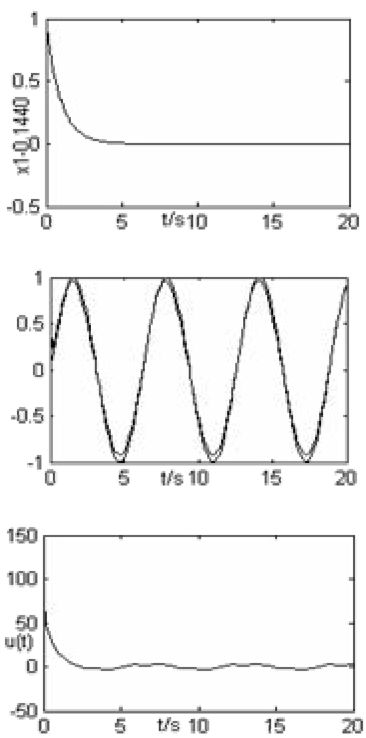

Fig. II. Performance of the overall control system when time-delay is unavailable: (a) The trajectory of $d x_1$; (b) The output tracking trajectory; and (c) System control variable.

Figures I and II demonstrate that the designed controls, based on Theorems 1 and 2, indeed do guarantee global boundedness of the state as well as good output tracking performance in the closed loop. Besides, it seems the chattering phenomenon is attenuated too. It should be noted, however, when the plant time delay is unknown and not available the closed-loop performance is accomplished at the cost of initially almost 10 times more control effort during a short period (less than 2 time units), when the lacking information is compensated for by the fuzzy controller. From the viewpoint of practical use this may cause some troubles, which is a weakness of this synthesis.

5. CONCLUSION

A new output tracking control synthesis for fuzzy time-delay systems in the presence of parameter perturbations is derived via combining T-S fuzzy model, sliding-mode control and Lyapunov function synthesis. First, the sliding-mode control with a sliding surface is designed by using theory of variable structure control. Depending on the reaching conditions the sliding-mode control is designed for cases when the time-delay is available and unavailable, respectively. Sufficient condition for globally bounded system state is derived by using input-to-state stability theory and the LMI method. Designed control guarantees the tracking of the desired reference trajectory by plant outputs.

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