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New delay-dependent stability criteria for recurrent neural networks with time-varying delays

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ABSTRACT

This work is concerned with the delay-dependentstability problem for recurrent neural networks with time-varying delays. A new improved delay-dependent stability criterion expressed in terms of linear matrix inequalities is derived by constructing a dedicated Lyapunov–Krasovskii functional via utilizing Wirtinger inequality and convex combination approach. Moreover, a further improved delay-dependent stability criterion is established by means of a new partitioning method for bounding conditions on the activation function and certain new activation function conditions presented. Finally, the application of these novel results to an illustrative example from the literature has been investigated and their effectiveness is shown via comparison with the existing recent ones.

1. Introduction

During the past several decades, an increasing, revived research activity on recurrent neural networks (RNNs) is taking place because of their successful applications in various areas. These include associative memories, image processing, optimization problems, and pattern recognition as well as other engineering or scientific areas [1–5]. It is well known, the time delay often is a source of the degradation of performance and/or the instability of RNNs. It is therefore that the stability analysis of RNNs with time delays has attracted considerable attention in recent years, e.g. see Refs. [6–11] and references therein.

It should be noted, the existing stability criteria for RNNs with time delays can be classified into the delay-independent ones and the delay-dependent criteria. In general, when the time delay is small, the delay-dependent stability criteria are less conservative than delay-independent ones. For the delay-dependent stability criteria, the maximum delay bound is a very important index for checking the criterion's conservatism. In due course, significant research efforts have been devoted to the reduction of conservatism of the delay-dependent stability criteria for the time-delay RNNs. Following the Lyapunov stability theory, there are two effective ways to reduce the conservatism within in stability analysis of networks and systems. One is the choice of suitable Lyapunov–Krasovskii functional (LKF) and the other one is the estimation of its time derivative.

In recent years, some new techniques of construction of a suitable LKF and estimation of its derivative for delayed neural networks (DNNS) and time delay systems have been presented [12–32,44–48]. Methods for constructing a dedicated LKF include delay-partitioning idea [12–20], triple integral terms [16–25], more information on the activation functions [26], augmented vector [27,28], etc. The proposed methods for estimating the time-derivative of LKF include: Park' inequality [29], Jensen's inequality [30], free-weighing matrices [31], and reciprocally convex optimization [32]. In turn, these methods proved very useful in investigating the stability problems of RNNs with time delays. Among the stability analysis methods, some delay-dependent criteria for the RNNs with time-varying delays have been contributed in works [33–36,42]. For instance, in Ref. [33] the problem of delay-dependent stability has been investigated by considering some semi-positive-definite free matrices. Jensen's inequality combined with convex combination method has been used in Ref. [35]. In Ref. [36] a new improved delay-dependent stability criterion was proposed, which has been derived by constructing a new augmented LKF, containing a triple integral term, also by using Wirtinger-based

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integral inequality and two zero value free matrix equations. However, the introduced free-weighing matrices increase the calculation complexity as well as computational complexity. For the RNNs with interval time-varying delays, work [43] has contributed an improved stability criterion by construction of a suitable augmented LKF and utilization of Wirtinger-based integral inequality with reciprocally convex approach. Following the work [37], both the ability and the performance of neural networks are influenced considerably by the choice of the activation functions. Apparently there is an essential need to look for alternative methods of reducing the conservatism of stability criteria for such neural networks. Thus, the delay-partitioning approach appeared as an effective way to get a tighter bound by calculating the derivative of the LKF, which would lead to better results. However, as the partitioning number of delay increases, the matrix formulation becomes more complex and the dimensionality of the stability criterion grows bigger. Hence the computational burden and computational time consumption growth become a considerable problem. The activation function dividing approach was proposed in work [23], and some new improved delay-dependent criteria for neural networks with time-varying delays have been established. A more general activation function dividing method for delay-dependent stability analysis of DNNs was presented in Ref. [38].

The above motivating discussion has given considerably incentives to utilize a modified approach, albeit making use of the existing knowledge, in order to arrive at less conservative, novel, delay-dependent stability criteria for recurrent neural networks with time-varying delays.

Firstly, a combined convex method is developed for the stability of the recurrent neural network systems with time-varying delays. This method can tackle both the presence of time-varying delays and the variation of delays. As a first novelty, a new LKF is constructed by taking more information on the state and the activation functions as augmented vectors. It has been found by using reciprocal convex approach and Wirtinger inequality to handle the integral term of quadratic quantities. With the new LKF at hand, in Theorem 1, the delay-dependent stability criterion in which both the upper and lower bounds of delay derivative are available is then derived. Secondly, unlike the delay partitioning method, a new dividing approach of the bounding conditions on activation function is utilized in Theorem 2. Considering the time and the improvement of the feasible region, the bounding of activation functions $k_i^- \le (f_i(u)/u) \le k_i^+$ of RNNs with time-varying delays is divided into two subintervals such as to obtain: $k_i^- \le (f_i(u)/u) \le k_i^- + \alpha(k_i^+ - k_i^-)$ and $k_i^- + \alpha(k_i^+ - k_i^-) \le (f_i(u)/u) \le k_i^+$ $(0 \le \alpha \le 1)$, where the two sub-intervals can be either equal or unequal. New activation function conditions for the divided activation functions bounds are proposed and utilized in Theorem 2. Thirdly, by utilizing the results of Theorems 1 and 2, when only the upper bound of the derivative of the time-varying delay is available, the corresponding new results are proposed in Corollaries 1 and 2. Finally, this stability analysis method was applied to a known example from the literature and the respective results computed. These new results were compared with the existing recent ones in order to verify and illustrate the effectiveness of the new method and to demonstrate the improvements obtained. Further, Section 2 presents the problem formulation and Section 3 presents the new main results. Section 4 elaborates on the illustrative example and comparison analysis, while conclusions are drawn and further research outlined in Section 5.

This paper uses the following notations: C^T represents the transposition of matrix C. $\mathbb{R}^n$ denotes n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. $P > 0$ means that P is positive definite. Symbol $*$ represents the elements below the main diagonal of a symmetric block matrix, and $diag\{\cdots\}$ denotes a block diagonal matrix. $Sym(X)$ is defined as $Sym(X) = X + X^T$.

2. Problem formulation

Consider the following recurrent neural networks with discrete time-varying delays:

$$\dot{z}(t) = -Az(t) + f(Wz(t-h(t)) + J) \tag{1}$$

where $z(\cdot) = [z_1(\cdot), ..., z_n(\cdot)]^T$ is the state vector; $f(\cdot) = [f_1(\cdot), ..., f_n(\cdot)]^T$ denote the neuron activation functions; $J = [J_1, ..., J_n]^T \in \mathbb{R}^n$ is a vector representing the bias; $A = diag\{a_1, ..., a_n\} \in \mathbb{R}^{n\times n}$ is a constant matrix of appropriate dimensions; $W = [W_1, ..., W_n]^T \in \mathbb{R}^n$ represents the matrix of connection weights; and $h(t)$ is a time-varying delay having the following bound properties

$$C1: \quad 0 \le h(t) \le h, \; h_D^l \le \dot{h}(t) \le h_D^u < 1,$$

$$C2: \quad 0 \le h(t) \le h, \; \dot{h}(t) \le h_D^u.$$

where $h > 0$ and h_D^l, h_D^u are known constants.

The activation functions $f_i(\cdot)$, $i = 1, ..., n$ are assumed to be bounded and to satisfy the following bound conditions:

$$k_i^- \le \frac{f_i(u) - f_i(v)}{u - v} \le k_i^+, \quad u \ne v, \; i = 1, ..., n \tag{2}$$

where k_i^- and k_i^+ are constants.

In the stability analysis of recurrent neural networks (1), for simplicity, firstly we shift the equilibrium point z^* to the origin by letting $x = z - z^*$. Then the system (1) can be converted into

$$\dot{x}(t) = -Ax(t) + g(Wx(t-h(t))) \tag{3}$$

where $g(\cdot) = [g_1(\cdot), ..., g_n(\cdot)]^T$ and $g(Wx(\cdot)) = f(Wx(\cdot) + z^* + J) - f(Wz^* + J)$ with $g_i(0) = 0$. Notice that functions $g_i(\cdot)\,(i = 1, ..., n)$ satisfy the following bound conditions:

$$k_i^- \le \frac{g_i(u) - g_i(v)}{u - v} \le k_i^+, \quad u \ne v, \; i = 1, ..., n. \tag{4}$$

If $v = 0$ in (4), then these inequalities become

$$k_i^- \le \frac{g_i(u)}{u} \le k_i^+, \quad \forall u \ne 0, \; i = 1, ..., n. \tag{5}$$

The objective of this paper is to explore of asymptotic stability of recurrent neural networks (3) with time-varying delays and to establish a novel analysis method. Before deriving the main results of this contribution, the following lemmas are needed:

Lemma 1. *[32,39]Consider the given positive integers n, m, a positive scalar α in the interval $(0, 1)$, a given $n \times n$ matrix $R > 0$, two matrices W_1 and W_2 in $\mathbb{R}^{n\times m}$. For all vectors ξ in $\mathbb{R}^m$, define the function $\Theta(\alpha, R)$ as*

$$\Theta(\alpha, R) = \frac{1}{\alpha}\xi^T W_1^T R W_1 \xi + \frac{1}{1-\alpha}\xi^T W_2^T R W_2 \xi.$$

Then, if there exists a matrix X in $\mathbb{R}^{n\times n}$ such that $\begin{bmatrix} R & X \\ * & R \end{bmatrix} > 0$, the following inequality holds true:

$$\min_{\alpha \in (0,1)} \Theta(\alpha, R) \ge \begin{bmatrix} W_1\xi \\ W_2\xi \end{bmatrix}^T \begin{bmatrix} R & X \\ * & R \end{bmatrix} \begin{bmatrix} W_1\xi \\ W_2\xi \end{bmatrix}.$$

Lemma 2. *[39] For a given matrix $R > 0$, the following inequality holds for all continuously differentiable functions σ in $[a, b] \to \mathbb{R}^n$:*

$$\int_a^b \dot{\sigma}^T(u) R \dot{\sigma}(u) du \ge \frac{1}{b-a}(\sigma(b) - \sigma(a))^T R (\sigma(b) - \sigma(a)) + \frac{3}{b-a}\delta^T R \delta,$$

where $\delta = \sigma(b) + \sigma(a) - (2/b-a)\int_a^b \sigma(u) du$.

Lemma 3. *[40] Let $\xi \in \mathbb{R}^n$, $\Phi = \Phi^T \in \mathbb{R}^{n\times n}$, and $B \in \mathbb{R}^{m\times n}$ such that rank $(B) < n$. Then, the following statements are equivalent:*

(1) $\xi^T \Phi \xi < 0$, $B\xi = 0$, $\xi \ne 0$,
(2) $(B^\perp)^T \Phi B^\perp < 0$, *where $B^\perp$ is a right orthogonal complement of B.*

Lemma 4. [41] *For symmetric matrices of appropriate dimensions $R>0$ and Ω, and matrix Γ, the following two statements are equivalent: (1) $\Omega-\Gamma R\Gamma^T<0$, and (2) there exists a matrix Π of the appropriate dimension such that*

$$\begin{bmatrix} \Omega+\Gamma\Pi^T+\Pi\Gamma^T & \Pi \\ \Pi^T & -R \end{bmatrix}<0. \tag{6}$$

3. Main results

In this section, the new stability criterion is proposed for the considered class of recurrent neural networks (1), albeit via the equilibrium-shifted representation model (3). For simplicity of matrix representations, the set block entry matrices $e_i (i=1,\dots,13)\in\mathbb{R}^{13n\times n}$ (for example, $e_2^T=[0\ I\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$) are given and defined as follows:

$$\xi^T(t)=\Big[x^T(t)\quad x^T(t-h(t))\quad x^T(t-h)\quad \dot{x}^T(t)\quad \dot{x}^T(t-h(t))\quad \dot{x}^T(t-h)$$
$$g^T(Wx(t))\quad g^T(Wx(t-h(t)))$$
$$g^T(Wx(t-h))\quad \tfrac{1}{h(t)}\textstyle\int_{t-h(t)}^{t}x^T(s)ds\quad \tfrac{1}{h-h(t)}\int_{t-h}^{t-h(t)}x^T(s)ds$$
$$\textstyle\int_{t-h(t)}^{t}g^T(Wx(s))ds\quad \displaystyle\int_{t-h}^{t-h(t)}g^T(Wx(s))ds\Big],$$

$$\omega^T(t)=\Big[x^T(t)\quad x^T(t-h)\quad \int_{t-h(t)}^{t}x^T(s)ds\quad \int_{t-h}^{t-h(t)}x^T(s)ds$$
$$\int_{t-h}^{t}g^T(Wx(s))ds\quad x^T(t-h(t))\Big],$$

$$\alpha^T(t,s)=\Big[x^T(t)\quad x^T(s)\quad \dot{x}^T(s)\quad g^T(Wx(s))\quad x^T(t-h(t))],$$

$$\beta^T(s)=[x^T(s)\quad \dot{x}^T(s)\quad g^T(Wx(s))],$$

$$\Pi_1^0=[e_1\ e_3\ 0\ 0\ e_{12}+e_{13}\ e_2],\Pi_1^1=[0\ 0\ e_{10}\ 0\ 0\ 0],\Pi_1^2=[0\ 0\ 0\ e_{11}\ 0\ 0],$$

$$\Pi_2^0=[e_4\ e_6\ e_1\ -e_3\ e_7-e_9\ 0],$$
$$\Pi_2^1=[0\ 0\ e_2\ e_2\ 0\ e_5],\Pi_3=[e_1\ e_4\ e_7],\Pi_4=[e_2\ e_5\ e_8],$$
$$\Pi_5^0=[0\ 0\ e_1-e_2\ e_{12}\ 0],\Pi_5^1=[e_1\ e_{10}\ 0\ 0\ e_2],$$
$$\Pi_6=[e_4\ 0\ 0\ 0\ e_5],\Pi_7=[e_3\ e_6\ e_9],\Pi_8^0=[0\ 0\ e_2-e_3\ e_{13}\ 0],$$
$$\Pi_8^1=[e_1\ e_{11}\ 0\ 0\ e_2],$$
$$\Pi_9^0=[he_1\ 0\ e_1-e_3\ e_{12}+e_{13}\ he_2],\Pi_9^1=[0\ e_{10}\ 0\ 0\ 0],\Pi_9^2=[0\ e_{11}\ 0\ 0\ 0],$$

$$\Pi_{10}^0=[0\ e_1-e_2\ e_{12}\ 0\ e_2-e_3\ e_{13}],\Pi_{10}^1=[e_{10}\ 0\ 0\ 0\ 0\ 0],$$
$$\Pi_{10}^2=[0\ 0\ 0\ e_{11}\ 0\ 0],$$
$$\Phi_1=Sym([e_7-e_1W^TK_m]D_1We_4^T+[e_1W^TK_p-e_7]D_2We_4^T)$$
$$+Sym([e_9-e_3W^TK_m]D_5We_6^T+[e_3W^TK_p-e_9]D_6We_6^T),$$
$$\Phi_{2|\nabla_d^k|}=(1-\nabla_d^k)Sym\{[e_8-e_2W^TK_m]D_3We_5^T+[e_2W^TK_p-e_8]$$
$$D4We_5^T\},$$
$$\Sigma=Sym([\Pi_1^0P(\Pi_2^{0T}+\Upsilon_{1|\nabla_d^k|}\Pi_2^{1T}))+\Phi_{2|\nabla_d^k|}-(1-\nabla_d^k)[e_1\ \Pi_4\ e_2]$$
$$Q[e_1\ \Pi_4\ e_2]^T+Sym(\Pi_5^0Q\Upsilon_{2|\nabla_d^k|}\Pi_6^T)$$
$$+Sym(\Pi_8^0R\Upsilon_{2|\nabla_d^k|}\Pi_6^T)+(1-\nabla_d^k)[e_1\ \Pi_4\ e_2]R[e_1\ \Pi_4\ e_2]^T$$
$$+Sym(\Pi_9^0N\Upsilon_{2|\nabla_d^k|}\Pi_6^T),$$
$$\Sigma_1=Sym(\Pi_1^1P(\Pi_2^{0T}+\Upsilon_{1|\nabla_d^k|}\Pi_2^{1T}))+\Pi_5^1Q\Upsilon_{2|\nabla_d^k|}\Pi_6^T+\Pi_9^1N\Upsilon_{2|\nabla_d^k|}\Pi_6^T),$$
$$\Sigma_2=Sym(\Pi_1^2P(\Pi_2^{0T}+\Upsilon_{1|\nabla_d^k|}\Pi_2^{1T}))+\Pi_8^1R\Upsilon_{2|\nabla_d^k|}\Pi_6^T+\Pi_9^2N\Upsilon_{2|\nabla_d^k|}\Pi_6^T),$$
$$\Upsilon_{1|\nabla_d^k|}=diag\{I,I,-(1-\nabla_d^k)I,(1-\nabla_d^k)I,I,(1-\nabla_d^k)I\},$$
$$\Upsilon_{2|\nabla_d^k|}=diag\{I,I,I,I,I,(1-\nabla_d^k)I\},k=1,2,\nabla_d^1=h_D^l,\nabla_d^2=h_D^u,$$
$$H=[-A\ 0\ 0-I\ 0\ 0\ 0\ I\ 0\ 0\ 0\ 0\ 0],\Gamma_a=\Pi_{10}^0+h\Pi_{10}^1,\Gamma_b=\Pi_{10}^0+h\Pi_{10}^2,$$

$$\Psi=\Phi_1+[e_1\ \Pi_3\ e_2]Q[e_1\ \Pi_3\ e_2]^T-[e_1\ \Pi_7\ e_2]R[e_1\ \Pi_7\ e_2]^T$$
$$+[e_1\ \Pi_3\ e_2]N[e_1\ \Pi_3\ e_2]^T$$
$$-[e_1\ \Pi_7\ e_2]N[e_1\ \Pi_7\ e_2]+h^2\Pi_3Z\Pi_3^T+h^2e_4Ge_4^T-Y^T\Phi Y,$$

$$Y=[e_1-e_2\ e_1+e_2-2e_{10}\ e_2-e_3\ e_2+e_3-2e_{11}]^T,$$
$$K=-Sym([e_7-e_1W^TK_m]T_1[e_7-e_1W^TK_p]^T$$
$$+[e_8-e_2W^TK_m]T_2[e_8-e_2W^TK_p]^T+[e_9-e_3W^TK_m]T_3[e_9$$
$$-e_3W^TK_p]^T)-Sym([e_7-e_8-(e_1-e_2)W^TK_m]T_4[e_7-e_8$$
$$-(e_1-e_2)W^TK_p]^T$$
$$+[e_8-e_9-(e_2-e_3)W^TK_m]T_5[e_8-e_9-(e_2-e_3)W^TK_p]^T),$$
$$X=\begin{bmatrix} Z & \Lambda \\ * & Z\end{bmatrix},\quad \Phi=\begin{bmatrix} \Xi & S \\ * & \Xi\end{bmatrix},\quad \Xi=\begin{bmatrix} G & 0 \\ 0 & 3G\end{bmatrix},\quad S=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22}\end{bmatrix},$$

$$\Theta_a=-Sym([e_7-e_1W^T(K_m+K_\alpha)]T_1[e_7-e_1W^TK_m]^T$$
$$+[e_8-e_2W^T(K_m+K_\alpha)]T_2[e_8-e_2W^TK_m]^T$$
$$+[e_9-e_3W^T(K_m+K_\alpha)]T_3[e_9-e_3W^TK_m]^T),\quad K_\alpha=\alpha(K_p-K_m),$$

$$\Theta_b=-Sym([e_7-e_1W^TK_p]T_4[e_7-e_1W^T(K_m+K_\alpha)]^T$$
$$+[e_8-e_2W^TK_p]T_5[e_8-e_2W^T(K_m+K_\alpha)]^T$$
$$+[e_9-e_3W^TK_p]T_6[e_9-e_3W^T(K_m+K_\alpha)]^T),$$
$$\Omega_a=-Sym([e_7-e_8-(e_1-e_2)W^T(K_m+K_\alpha)]L_1[e_7-e_8$$
$$-(e_1-e_2)W^TK_m]^T$$
$$+[e_8-e_9-(e_2-e_3)W^T(K_m+K_\alpha)]L_2[e_8-e_9-(e_2-e_3)W^TK_m]^T),$$

$$\Omega_b=-Sym([e_7-e_8-(e_1-e_2)W^T(K_m+K_\alpha)]L_3[e_7-e_8$$
$$-(e_1-e_2)W^TK_p]^T$$
$$+[e_8-e_9-(e_2-e_3)W^T(K_m+K_\alpha)]L_4[e_8-e_9-(e_2-e_3)W^TK_p]^T). \tag{7}$$

Then the first main result is stated as follows:

Theorem 1. *For a given scalar h, any one h_D^l and h_D^u with C1, diagonal matrices $k_p=diag\{k_1^+,\dots,k_n^+\}$ and $k_m=diag\{k_1^-,\dots,k_n^-\}$, system model (3) is asymptotically stable, if there exist the positive definite matrices $P\in\mathbb{R}^{6n\times 6n}$, $Q\in\mathbb{R}^{5n\times 5n}$, $R\in\mathbb{R}^{5n\times 5n}$, $N\in\mathbb{R}^{5n\times 5n}$, $Z\in\mathbb{R}^{3n\times 3n}$, $G\in\mathbb{R}^{n\times n}$, diagonal matrices $D_i=diag(d_{1i},d_{2i},\dots,d_{ni})\geq 0$, $(i=1,\dots,6)$, $T_i=diag(t_{1i},t_{2i},\dots,t_{ni})\geq 0, (i=1,\dots,5)$, and any matrix $\Lambda\in\mathbb{R}^{3n\times 3n}$, with matrices $S_{ij}\in\mathbb{R}^{n\times n}(i,j=1,2)$ and matrix Π of appropriate dimensions, satisfying the following linear matrix inequalities:*

$$\begin{bmatrix} (H^\perp)^T\Omega_1(H^\perp)+(H^\perp)^T\Gamma_a\Pi^T+\Pi\Gamma_a^T(H^\perp) & \Pi \\ * & -X\end{bmatrix}<0, \tag{8}$$

$$\begin{bmatrix} (H^\perp)^T\Omega_2(H^\perp)+(H^\perp)^T\Gamma_b\Pi^T+\Pi\Gamma_b^T(H^\perp) & \Pi \\ * & -X\end{bmatrix}<0, \tag{9}$$

$$X>0,\quad \Phi>0 \tag{10}$$

where $\Omega_1=\Sigma+h\Sigma_1+\Psi+K$, $\Omega_2=\Sigma+h\Sigma_2+\Psi+K$, and Σ, Σ_1, Σ_2, Γ_a, Γ_b, Ψ, K, X, Φ are defined in (7), and $H^\perp$ is the right orthogonal complement of H.

Proof. For positive diagonal matrices $D_i (i=1,\dots,6)$ and positive definite matrices P,Q,R,N,Z,G, we construct the LKF as

$$V=\sum_{i=1}^{6}V_i(x_t) \tag{11}$$

where individual Lyapunov function and Lyapunov–Krasovskii functional are

$$V_1=\omega^T(t)P\omega(t),$$
$$V_2=2\sum_{i=1}^{n}\Big(d_{1i}\int_0^{w_ix_i(t)}(g_i(s)-k_i^-s)ds+d_{2i}\int_0^{w_ix_i(t)}(k_i^+s-g_i(s))ds\Big)$$
$$+2\sum_{i=1}^{n}\Big(d_{3i}\int_0^{w_ix_i(t-h(t))}(g_i(s)-k_i^-s)ds$$
$$+d_{4i}\int_0^{w_ix_i(t-h(t))}(k_i^+s-g_i(s))ds\Big)$$

$$+2\sum_{i=1}^{n}(d_{5i}\int_0^{w_ix_i(t-h)}(g_i(s)-k_i^-s)ds+d_{6i}\int_0^{w_ix_i(t-h)}(k_i^+s-g_i(s))ds),$$

$$V_3=\int_{t-h(t)}^{t}\alpha^T(t,s)Q\alpha(t,s)ds+\int_{t-h}^{t-h(t)}\alpha^T(t,s)R\alpha(t,s)ds,$$

$$V_4=\int_{t-h}^{t}\alpha^T(t,s)N\alpha(t,s)ds,$$

$$V_5=h\int_{t-h}^{t}\int_s^t\beta^T(u)Z\beta(u)duds,$$

$$V_6=h\int_{t-h}^{t}\int_s^t\dot{x}^T(u)G\dot{x}(u)duds.$$

The time derivative of V_1 can be represented as

$$\dot{V}_1=2\omega^T(t)P\dot{\omega}(t)=\xi^T(t)\{Sym((\Pi_1^0+h(t)\Pi_1^1+(h-h(t))\Pi_1^2)P(\Pi_2^0+\Pi_2^1\Upsilon_{1|\dot{h}(t)|})^T)\}\xi(t) \tag{12}$$

where $\Upsilon_{1|\dot{h}(t)|}=diag\{I,I,-(1-\dot{h}(t))I,(1-\dot{h}(t))I,(1-\dot{h}(t))I\}$.

Also, it is fairly easy to calculate

$$\begin{aligned}\dot{V}_2&=2[g(Wx(t))-x(t)WK_m]^TD_1W\dot{x}(t)+2[x(t)WK_p-g(Wx(t))]^TD_2W\dot{x}(t)\\&\quad+(1-\dot{h}(t))\{2[g(Wx(t-h(t)))-x(t-h(t))WK_m]^TD_3W\dot{x}(t-h(t))\\&\quad+2[x(t-h(t))WK_p-g(Wx(t-h(t)))]^TD_4W\dot{x}(t-h(t))\}\\&\quad+2[g(Wx(t-h))-x(t-h)WK_m]^TD_5W\dot{x}(t-h)+2[x(t-h)WK_p\\&\quad-g(Wx(t-h))]^TD_6W\dot{x}(t-h)\\&=\xi^T(t)\{(\Phi_1+\Phi_{2|\dot{h}(t)|})\}\xi(t)\end{aligned} \tag{13}$$

where Φ_1 is defined in (7) and $\Phi_{2|\dot{h}(t)|}=(1-\dot{h}(t))Sym\{[e_8-e_2W^TK_m]D_3We_5^T+[e_2W^TK_p-e_8]D_4We_5^T\}$.

The calculation of $\dot{V}_3$ gives

$$\begin{aligned}\dot{V}_3&=\alpha^T(t,t)Q\alpha(t,t)-(1-\dot{h}(t))\alpha^T(t,t-h(t))Q\alpha(t,t-h(t))\\&\quad+2\int_{t-h(t)}^{t}\alpha^T(t,s)Q\Upsilon_{2|\dot{h}(t)|}\eta ds\\&\quad+(1-\dot{h}(t))\alpha^T(t,t-h(t))R\alpha(t,t-h(t))-\alpha^T(t,t-h)R\alpha(t,t-h)\\&\quad+2\int_{t-h}^{t-h(t)}\alpha^T(t,s)R\Upsilon_{2|\dot{h}(t)|}\eta ds\\&=\xi^T(t)\{[e_1\ \Pi_3\ e_2]Q[e_1\ \Pi_3\ e_2]^T-(1-\dot{h}(t))[e_1\ \Pi_4\ e_2]Q[e_1\ \Pi_4\ e_2]^T\\&\quad+Sym((\Pi_5^0+h(t)\Pi_5^1)Q\Upsilon_{2|\dot{h}(t)|}\Pi_6^T)\\&\quad+(1-\dot{h}(t))[e_1\ \Pi_4\ e_2]R[e_1\ \Pi_4\ e_2]^T-[e_1\ \Pi_7\ e_2]R[e_1\ \Pi_7\ e_2]^T\\&\quad+Sym((\Pi_8^0+(h-h(t))\Pi_8^1)R\Upsilon_{2|\dot{h}(t)|}\Pi_6^T)\}\xi(t)\end{aligned} \tag{14}$$

where $\Upsilon_{2|\dot{h}(t)|}=diag\{I,I,I,I,I,(1-\dot{h}(t))I\}$, $\eta=[\dot{x}(t)\quad 0\quad 0\quad 0\quad \dot{x}(t-h(t))]^T$.

The calculation of $\dot{V}_4$ leads to

$$\begin{aligned}\dot{V}_4&=\alpha^T(t,t)N\alpha(t,t)-\alpha^T(t,t-h)N\alpha(t,t-h)+2\int_{t-h}^{t}\alpha^T(t,s)N\Upsilon_{2|\dot{h}(t)|}\eta ds\\&=\xi^T(t)\{[e_1\ \Pi_3\ e_2]N[e_1\ \Pi_3\ e_2]^T-[e_1\ \Pi_7\ e_2]N[e_1\ \Pi_7\ e_2]^T\\&\quad+Sym((\Pi_9^0+h(t)\Pi_9^1+(h-h(t))\Pi_9^2)N\Upsilon_{2|\dot{h}(t)|}\Pi_6^T)\}\xi(t)\end{aligned} \tag{15}$$

Furthermore, by using Lemma 1 and Jensen's inequality, we can derive

$$\begin{aligned}\dot{V}_5&=h^2\beta^T(t)Z\beta(t)-h\int_{t-h(t)}^{t}\beta^T(s)Z\beta(s)ds-h\int_{t-h}^{t-h(t)}\beta^T(s)Z\beta(s)ds\\&\le h^2\beta^T(t)Z\beta(t)-\left(\frac{h}{h(t)}\right)\left(\int_{t-h(t)}^{t}\beta(s)ds\right)^TZ\left(\int_{t-h(t)}^{t}\beta(s)ds\right)\\&\quad-\left(\frac{h}{h-h(t)}\right)\left(\int_{t-h}^{t-h(t)}\beta(s)ds\right)^TZ\left(\int_{t-h}^{t-h(t)}\beta(s)ds\right)\\&\le h^2\beta^T(t)Z\beta(t)-\begin{bmatrix}\int_{t-h(t)}^{t}\beta(s)ds\\\int_{t-h}^{t-h(t)}\beta(s)ds\end{bmatrix}^T\begin{bmatrix}Z&\Lambda\\ *&Z\end{bmatrix}\begin{bmatrix}\int_{t-h(t)}^{t}\beta(s)ds\\\int_{t-h}^{t-h(t)}\beta(s)ds\end{bmatrix}\\&=\xi^T(t)\left\{h^2\Pi_3Z\Pi_3^T-\Gamma X\Gamma^T\right\}\xi(t)\end{aligned} \tag{16}$$

where $\Gamma=\Pi_{10}^0+h(t)\Pi_{10}^1+(h-h(t))\Pi_{10}^2$.

Finally, the time derivative $\dot{V}_6$ is readily obtained as

$$\dot{V}_6=h^2\dot{x}^T(t)G\dot{x}(t)-h\int_{t-h}^{t}\dot{x}^T(s)G\dot{x}(s)ds \tag{17}$$

According to Lemmas 1 and 2, it can be found that

$$\begin{aligned}-h\int_{t-h}^{t}\dot{x}^T(s)G\dot{x}(s)ds&=-h\int_{t-h(t)}^{t}\dot{x}^T(s)G\dot{x}(s)ds-h\int_{t-h}^{t-h(t)}\dot{x}^T(s)G\dot{x}(s)ds\\&\le-\frac{h}{h(t)}[x(t)-x(t-h(t))]^TG[x(t)-x(t-h(t))]-\frac{h}{h-h(t)}[x(t-h(t))\\&\quad-x(t-h)]^TG[x(t-h(t))-x(t-h)]\\&\quad-\frac{3h}{h(t)}[x(t)+x(t-h(t))-\frac{2}{h(t)}\int_{t-h(t)}^{t}x(s)ds]^TG[x(t)+x(t-h(t))\\&\quad-\frac{2}{h(t)}\int_{t-h(t)}^{t}x(s)ds]\\&\quad-\frac{3h}{h-h(t)}[x(t-h(t))+x(t-h)-\frac{2}{h-h(t)}\\&\quad\int_{t-h}^{t-h(t)}x(s)ds]^TG[x(t-h(t))+x(t-h)-\frac{2}{h-h(t)}\int_{t-h}^{t-h(t)}x(s)ds]\\&=-\frac{h}{h(t)}\begin{bmatrix}x(t)-x(t-h(t))\\x(t)+x(t-h(t))-\frac{2}{h(t)}\int_{t-h(t)}^{t}x(s)ds\end{bmatrix}^T\\&\quad\Xi\begin{bmatrix}x(t)-x(t-h(t))\\x(t)+x(t-h(t))-\frac{2}{h(t)}\int_{t-h(t)}^{t}x(s)ds\end{bmatrix}\\&\quad-\frac{h}{h-h(t)}\begin{bmatrix}x(t-h(t))-x(t-h)\\x(t-h(t))+x(t-h)-\frac{2}{h-h(t)}\int_{t-h}^{t-h(t)}x(s)ds\end{bmatrix}^T\\&\quad\Xi\begin{bmatrix}x(t-h(t))-x(t-h)\\x(t-h(t))+x(t-h)-\frac{2}{h-h(t)}\int_{t-h}^{t-h(t)}x(s)ds\end{bmatrix}\\&\le-\begin{bmatrix}x(t)-x(t-h(t))\\x(t)+x(t-h(t))-\frac{2}{h(t)}\int_{t-h(t)}^{t}x(s)ds\\x(t-h(t))-x(t-h)\\x(t-h(t))+x(t-h)-\frac{2}{h-h(t)}\int_{t-h}^{t-h(t)}x(s)ds\end{bmatrix}^T\\&\quad\Phi\begin{bmatrix}x(t)-x(t-h(t))\\x(t)+x(t-h(t))-\frac{2}{h(t)}\int_{t-h(t)}^{t}x(s)ds\\x(t-h(t))-x(t-h)\\x(t-h(t))+x(t-h)-\frac{2}{h-h(t)}\int_{t-h}^{t-h(t)}x(s)ds\end{bmatrix}\end{aligned}$$

Hence

$$\dot{V}_6(x_t)\le\xi^T(t)\left\{h^2e_4Ge_4^T-Y^T\Phi Y\right\}\xi(t). \tag{18}$$

On the grounds of (4) and (5), for any positive diagonal matrix $T_i=diag(t_{1i},t_{2i},\dots,t_{ni})\ge 0$, $(i=1,\dots,5)$, the following inequality holds true:

$$\begin{aligned}0&\le-2\sum_{i=1}^{n}t_{1i}\left[g_i(W_ix_i(t))-k_i^-W_ix_i(t)\right]\left[g_i(W_ix_i(t))-k_i^+W_ix_i(t)\right]\\&\quad-2\sum_{i=1}^{n}t_{2i}\left[g_i(W_ix_i(t-h(t)))-k_i^-W_ix_i(t-h(t))\right]\left[g_i(W_ix_i(t-h(t)))\right.\\&\quad\left.-k_i^+W_ix_i(t-h(t))\right]\end{aligned}$$

$$-2\sum_{i=1}^{n} t_{3i}\left[g_i(W_ix_i(t-h))-k_i^- W_ix_i(t-h)\right]\left[g_i(W_ix_i(t-h)) - k_i^+ W_ix_i(t-h)\right]$$
$$-2\sum_{i=1}^{n} t_{4i}\left[g_i(W_ix_i(t))-g_i(W_ix_i(t-h(t)))-k_i^-(x_i(t)-x_i(t-h(t)))\right]$$
$$\times\left[g_i(W_ix_i(t))-g_i(W_ix_i(t-h(t)))-k_i^+(x_i(t)-x_i(t-h(t)))\right]-2\sum_{i=1}^{n} t_{5i}$$
$$\times\left[g_i(W_ix_i(t-h(t)))-g_i(W_ix_i(t-h))-k_i^-(x_i(t-h(t))-x_i(t-h))\right]$$
$$\times\left[g_i(W_ix_i(t-h(t)))-g_i(W_ix_i(t-h))-k_i^+(x_i(t-h(t))-x_i(t-h))\right]$$
$$=\xi^T(t)K\xi(t) \tag{19}$$

In order to handle the term $\dot{h}(t)$, which occurred in the above derivative, let define quantity ∇_d in the following set:

$$\Psi_d := \{\nabla_d | \nabla_d \in conv\{\nabla_d^1, \nabla_d^2\}\} \tag{20}$$

where $conv$ denotes the convex hull, $\nabla_d^1 = h_D^l$, and $\nabla_d^2 = h_D^u$. Then, there exists a parameter $\theta > 0$ such that $\dot{h}(t)$ can be expressed as convex combination of the vertices as follows:

$$\dot{h}(t) = \theta\nabla_d^1 + (1-\theta)\nabla_d^2. \tag{21}$$

If a matrix $M_{|\dot{h}(t)|}$ is affine dependent on $\dot{h}(t)$, then $M_{|\dot{h}(t)|}$ can be expressed as convex combinations of the vertices

$$M_{|\dot{h}(t)|} = \theta M_{|\nabla_d^1|} + (1-\theta)M_{|\nabla_d^2|} \tag{22}$$

It follows from (22), if a stability condition is affine dependent on $\dot{h}(t)$, then the only need is to check the vertex values of $\dot{h}(t)$ instead of checking all the values of $\dot{h}(t)$ [43].

From expressions (12)–(22), we can get

$$\dot{V} \le \xi^T(t)(\Omega - \Gamma X \Gamma^T)\xi(t) \tag{23}$$

where $\Omega = \Sigma + h(t)\Sigma_1 + (h-h(t))\Sigma_2 + \Psi + K$, and Γ is defined in (16).

By virtue of Lemma 3, it follows that $\xi^T(t)\{\Omega - \Gamma X\Gamma^T\}\xi(t) < 0$ with $0 = H\xi(t)$ is equivalent to $(H^\perp)^T(\Omega - \Gamma X\Gamma^T)(H^\perp) < 0$. Then by Lemma 4, inequality $(H^\perp)^T(\Omega - \Gamma X\Gamma^T)(H^\perp) < 0$ is equivalent to the inequality

$$\begin{bmatrix} (H^\perp)^T\Omega(H^\perp) + (H^\perp)^T\Gamma\Pi^T + \Pi\Gamma^T(H^\perp) & \Pi \\ * & -X \end{bmatrix} < 0, \tag{24}$$

where Π is a matrix of appropriate dimensions. Based on inequality (24) and the convex optimization approach, we can find precisely that inequality (24) holds if and only if inequalities (8)–(10) do hold. Thus, then system (3) is asymptotically stable and hence the system (1) too. This completes the proof.

Remark 1. Recently, the reciprocally convex optimization technique and the Wirtinger inequality was proposed in Refs. [32,39] respectively, and these two methods were utilized in deriving (18). In Lemma 2, it can be noticed the term $(1/(b-a))(\sigma(b)-\sigma(a))^T R(\sigma(b)-\sigma(a))$ is equal to Jensen's inequality and the newly appeared term $(3/(b-a))\delta^T R\delta$ can reduce the LKF enlargement of the estimation. The usage of reciprocally convex optimization method avoids the enlargement of $h(t)$ and $h-h(t)$ while only introduces matrices S, Λ. Then, the convex optimization method is used to handle $\dot{V}(x_t)$. During the above proof procedure, the dedicated construction of LKF (11) does have full information on the recurrent neural network system dynamics. It is therefore that the conservatism is reduced.

Remark 2. In Theorem 1, firstly, the terms $(1/h-h(t))\int_{t-h}^{t-h(t)} x^T(s)ds$ and $(1/h(t))\int_{t-h(t)}^{t} x^T(s)ds$ are used for the vector $\xi(t)$. This treatment can separate the time derivative of the LKF into yields $h(t)$-dependent and $(h-h(t))$-dependent. Secondly, the states $x(t-h(t))$ and $x(t-h)$ are taken as intervals of integral terms, as shown in the second and third terms of V_2. Therefore considerably more information on the cross terms in $(g(Wx(t-h(t)))$, $x(t-h(t))$, $\dot{x}(t-h(t))$ and $(g(Wx(t-h))$, $x(t-h)$, $\dot{x}(t-h)$ are being utilized. Thirdly, notice the introduction of $x(t)$, $x(t-h(t))$ as integral terms in V_3, V_4, and of the term $\int_{t-h}^{t-h(t)} \alpha^T(t,s)R\alpha(t,s)ds$ in V_3, which before have not proposed in the literature. These considerations highlight the main differences in the construction of the LKF candidate in this paper.

Remark 3. In the stability criteria for delayed neural networks, many works choose the delay-partitioning number as two as a kind of a tradeoff between the computational burden and the improvement of feasible region in stability conditions. However, when the condition $0 \le h(t) \le h$ is divided into $0 \le h(t) \le h/2$ and $h/2 \le h(t) \le h$, the matrix formulation becomes more complex and the dimension of stability conditions grows larger because of more augmented vector. Inspired by work [23] on the activation functions dividing method for neural networks with time-varying delays, we have divided the bounding of the activation function $k_i^- \le f_i(u)/u \le k_i^+$ for the considered time-varying delay RNNs into $k_i^- \le f_i(u)/u \le k_i^- + \alpha(k_i^+ - k_i^-)$ and $k_i^- + \alpha(k_i^+ - k_i^-) \le f_i(u)/u \le k_i^+$, $0 \le \alpha \le 1$. This new activation partitioning method for time-varying delay RNNs is more general and less conservative. The new bounding partitioning approach is utilized instead of using delay-partitioning method; this latter technique is used in the subsequent Theorem 2. Thus through Theorems 1 and 2 less conservative stability criteria are derived.

Now, based on the results of Theorem 1, a new stability criterion for system (3) is introduced by utilizing the new bounding partitioning approach.

Theorem 2. *For the given scalars $0 \le \alpha \le 1$ and h, any one h_D^l and h_D^u satisfying C1, and diagonal matrices $k_p = diag\{k_1^+, ..., k_n^+\}$ and $k_m = diag\{k_1^-, ..., k_n^-\}$, system (3) is asymptotically stable, if there exist positive definite matrices $P \in \mathbb{R}^{6n\times 6n}$, $Q \in \mathbb{R}^{5n\times 5n}$, $R \in \mathbb{R}^{5n\times 5n}$, $N \in \mathbb{R}^{5n\times 5n}$, $Z \in \mathbb{R}^{3n\times 3n}$, $G \in \mathbb{R}^{n\times n}$, diagonal matrices $D_i = diag(d_{1i}, d_{2i}, \dots, d_{ni}) \ge 0$, $(i = 1, ..., 6)$, $T_i = diag(t_{1i}, t_{2i}, \dots, t_{ni}) \ge 0$, $(i = 1, ..., 6)$, $L_i = diag(l_{1i}, l_{2i}, \dots, l_{ni}) \ge 0$, $(i = 1, ..., 4)$, and any matrix $\Lambda \in \mathbb{R}^{3n\times 3n}$, along with matrices $S_{ij} \in \mathbb{R}^{n\times n}(i,j = 1,2)$ and Π of appropriate dimensions, satisfying the following linear matrix inequalities*

$$\begin{bmatrix} (H^\perp)^T\Theta_1(H^\perp) + (H^\perp)^T\Gamma_a\Pi^T + \Pi\Gamma_a^T(H^\perp) & \Pi \\ * & -X \end{bmatrix} < 0 \quad \forall\Delta = a,b \tag{25}$$

$$\begin{bmatrix} (H^\perp)^T\Theta_2(H^\perp) + (H^\perp)^T\Gamma_b\Pi^T + \Pi\Gamma_b^T(H^\perp) & \Pi \\ * & -X \end{bmatrix} < 0 \quad \forall\Delta = a,b \tag{26}$$

$$X > 0, \quad \Phi > 0 \tag{27}$$

where $\Theta_1 = \Sigma + h\Sigma_1 + \Psi + \Theta_\Delta + \Omega_\Delta$, $\Theta_2 = \Sigma + h\Sigma_2 + \Psi + \Theta_\Delta + \Omega_\Delta$, $\forall\Delta = a,b$ and Σ, Σ_1, Σ_2, Ψ, Γ_a, Γ_b, X, Φ, Θ_a, Θ_b, Ω_a, Ω_b are defined in (7), and where $H^\perp$ is the right orthogonal complement of H.

Proof. While considering the same Lyapunov–Krasovskii functional as proposed in Theorem 1, we divide the bounding on activation function (5) into two sub-intervals, thus denoting the Case 1 and the Case 2 within this proof.

Case 1: Notice

$$k_i^- \le \frac{g_i(u)-g_i(v)}{u-v} \le k_i^- + \alpha(k_i^+ - k_i^-), \quad 0 \le \alpha \le 1 \tag{28}$$

which by choosing $v=0$, it is equivalent to

$$[g_i(u)-k_i^- u][g_i(u)-(k_i^- +\alpha(k_i^+ -k_i^-))u]<0. \tag{29}$$

From (29), for any positive definite diagonal matrices $T_1=diag(t_{11},t_{12},\dots,t_{1n})\geq 0$, $T_2=diag(t_{21},t_{22},\dots,t_{2n})\geq 0$, and $T_3=diag(t_{31},t_{32},\dots,t_{3n})\geq 0$ the following inequality is satisfied:

$$\begin{aligned}0\leq & -2\sum_{i=1}^{n} t_{1i}\left[g_i(W_ix_i(t))-k_i^- W_ix_i(t)\right]\left[g_i(W_ix_i(t))\right.\\ & \left.-(k_i^- +\alpha(k_i^+ -k_i^-))W_ix_i(t)\right]-2\sum_{i=1}^{n} t_{2i}\left[g_i(W_ix_i(t-h(t)))\right.\\ & \left.-k_i^- W_ix_i(t-h(t))\right]\left[g_i(W_ix_i(t-h(t)))\right.\\ & \left.-(k_i^- +\alpha(k_i^+ -k_i^-))W_ix_i(t-h(t))\right]-2\sum_{i=1}^{n} t_{3i}\left[g_i(W_ix_i(t-h))\right.\\ & \left.-k_i^- W_ix_i(t-h)\right]\left[g_i(W_ix_i(t-h))-(k_i^- +\alpha(k_i^+ -k_i^-))W_ix_i(t-h)\right]\\ = & \ \xi^T(t)\Theta_a\xi(t)\end{aligned} \tag{30}$$

For (28), the following conditions are fulfilled:

$$k_i^- \leq \frac{g_i(W_ix_i(t))-g_i(W_ix_i(t-h(t)))}{W_ix_i(t)-W_ix_i(t-h(t))}\leq k_i^- +\alpha(k_i^+ -k_i^-),$$
$$k_i^- \leq \frac{g_i(W_ix_i(t-h(t)))-g_i(W_ix_i(t-h))}{W_ix_i(t-h(t))-W_ix_i(t-h)}\leq k_i^- +\alpha(k_i^+ -k_i^-) \tag{31}$$

For $i=1,\cdots,n$, the above two conditions are equivalent to

$$\begin{aligned}&\left[g_i(W_ix_i(t))-g_i(W_ix_i(t-h(t)))-k_i^- W_i(x_i(t)-x_i(t-h(t)))\right]\\ &\quad\times\left[g_i(W_ix_i(t))-g_i(W_ix_i(t-h(t)))-(k_i^- +\alpha(k_i^+ -k_i^-))W_i(x_i(t)\right.\\ &\quad\left.-x_i(t-h(t)))\right]\leq 0,\\ &\left[g_i(W_ix_i(t-h(t)))-g_i(W_ix_i(t-h))-k_i^- W_i(x_i(t-h(t))-x_i(t-h))\right]\\ &\quad\times\left[g_i(W_ix_i(t-h(t)))-g_i(W_ix_i(t-h))\right.\\ &\quad\left.-(k_i^- +\alpha(k_i^+ -k_i^-))W_i(x_i(t-h(t))-x_i(t-h))\right]\leq 0\end{aligned} \tag{32}$$

Therefore, for any positive definite matrices $L_1=diag\{l_{11},\cdots,l_{1n}\}$, $L_2=diag\{l_{21},\cdots,l_{2n}\}$, the following inequality holds true:

$$\begin{aligned}0\leq & -2\sum_{i=1}^{n}\left\{l_{1i}\left[g_i(W_ix_i(t))-g_i(W_ix_i(t-h(t)))-k_i^- W_i(x_i(t)-x_i(t-h(t)))\right]\right.\\ & \times\left[g_i(W_ix_i(t))-g_i(W_ix_i(t-h(t)))-(k_i^- +\alpha(k_i^+ -k_i^-))W_i(x_i(t)\right.\\ & \left.\left.-x_i(t-h(t)))\right]\right\}-2\sum_{i=1}^{n}\left\{l_{2i}\left[g_i(W_ix_i(t-h(t)))-g_i(W_ix_i(t-h))\right.\right.\\ & \left.-k_i^- W_i(x_i(t-h(t))-x_i(t-h))\right].\\ & \times\left[g_i(W_ix_i(t-h(t)))-g_i(W_ix_i(t-h))-(k_i^- +\alpha(k_i^+ -k_i^-))W_i(x_i(t-h(t))\right.\\ & \left.\left.-x_i(t-h))\right]\right\}=\xi^T(t)\Omega_a\xi(t)\end{aligned} \tag{33}$$

From the proof of Theorem 1, when $k_i^- \leq (g_i(u)-g_i(v)/u-v)\leq k_i^- +\alpha(k_i^+ -k_i^-)$, then an upper bound of $\dot{V}$ can be found as

$$\dot{V}\leq\xi^T(t)\{\Theta+\Theta_a+\Omega_a-\Gamma X\Gamma^T\}\xi(t). \tag{34}$$

with $0=H\xi(t)$, where $\Theta=\Sigma+h(t)\Sigma_1+(h-h(t))\Sigma_2+\Psi$, and Γ as defined in (16).
Case 2: Notice

$$k_i^- +\alpha(k_i^+ -k_i^-)\leq\frac{g_i(u)-g_i(v)}{u-v}\leq k_i^+ \tag{35}$$

For this case, let define positive definite diagonal matrices $T_4=diag(t_{41},t_{42},\dots,t_{4n})\geq 0$, $T_5=diag(t_{51},t_{52},\dots,t_{5n})\geq 0$, $T_6=diag(t_{61},t_{62},\dots,t_{6n})\geq 0$ and $L_3=diag\{l_{31},\cdots,l_{3n}\}$, $L_4=diag\{l_{41},\cdots,l_{4n}\}$. Then by applying a similar procedure as the one used in Case 1, ultimately we obtain

$$\dot{V}\leq\xi^T(t)\{\Theta+\Theta_b+\Omega_b-\Gamma X\Gamma^T\}\xi(t). \tag{36}$$

with $0=H\xi(t)$.

Thus, for $k_i^- \leq f_i(u)/u\leq k_i^+$ an upper bound of $\dot{V}$ is obtained as follows:

$$\dot{V}\leq\xi^T(t)\{\Theta+\Theta_\Delta+\Omega_\Delta-\Gamma X\Gamma^T\}\xi(t)\quad\forall\Delta=a,b \tag{37}$$

where Θ_Δ, $\Omega_\Delta(\forall\Delta=a,b)$. Similarly as in the proof of Theorem 1, inequality (37) holds precisely if and only if inequalities (25) and (26) are satisfied. Thus the feasibility of satisfying inequalities (25)–(27) means the recurrent neural network (3) is asymptotically stable, and so is network (1). This completes the proof.

Remark 4. In Theorem 1, we consider that $h(t)$ satisfies C1, but it should be noted there are many systems satisfying the condition C2. Therefore we can introduce Corollary 1 in order to analyze the stability of recurrent neural networks with the condition C2 by setting $D_3,D_4=0$, $R=0$ and changing LKF terms V_1, V_2, V_3, V_4.

In Corollary 1 below, block entry matrices $\tilde{e}_i\in\mathbb{R}^{12n\times n}$, will be used and the following notations are defined for the sake of simplicity of matrix notation:

$$\begin{aligned}\tilde{\xi}^T(t)=&[x^T(t)\quad x^T(t-h(t))\quad x^T(t-h)\quad \dot{x}^T(t)\quad \dot{x}^T(t-h)\quad g^T(Wx(t))\\ & g^T(Wx(t-h(t)))\quad g^T(Wx(t-h))\\ & \tfrac{1}{h(t)}\textstyle\int_{t-h(t)}^{t}x^T(s)ds\quad \tfrac{1}{h-h(t)}\int_{t-h}^{t-h(t)}x^T(s)ds\quad \int_{t-h(t)}^{t}g^T(Wx(s))ds\\ & \left.\int_{t-h}^{t-h(t)}g^T(Wx(s))ds\right],\quad \tilde{\omega}^T(t)=\left[x^T(t)\quad x^T(t-h)\quad \int_{t-h}^{t}x^T(s)ds\right.\\ & \left.\int_{t-h}^{t}g^T(Wx(s))ds\right],\quad \tilde{H}=[-A\ 0\ 0\ -I\ 0\ 0\ I\ 0\ 0\ 0\ 0\ 0],\\ \tilde{\Pi}_1^0=&[\tilde{e}_1\ \tilde{e}_3\ 0\ \tilde{e}_{11}+\tilde{e}_{12}],\tilde{\Pi}_1^1=[0\ 0\ \tilde{e}_9\ 0],\tilde{\Pi}_1^2=[0\ 0\ \tilde{e}_{10}\ 0],\\ \tilde{\Pi}_2=&[\tilde{e}_4\ \tilde{e}_5\ \tilde{e}_1-\tilde{e}_3\ \tilde{e}_6-\tilde{e}_8],\quad \tilde{\Pi}_3=[\tilde{e}_1\tilde{e}_6],\\ \tilde{\Pi}_4=&[\tilde{e}_2\ \tilde{e}_7],\tilde{\Pi}_5=[\tilde{e}_1\ \tilde{e}_4\ \tilde{e}_6],\tilde{\Pi}_6=[\tilde{e}_3\ \tilde{e}_5\ \tilde{e}_8],\\ \tilde{\Pi}_7^0=&[0\ \tilde{e}_1-\tilde{e}_2\ \tilde{e}_{11}\ 0\ \tilde{e}_2-\tilde{e}_3\ \tilde{e}_{12}],\quad \tilde{\Pi}_7^1=[\tilde{e}_9\ 0\ 0\ 0\ 0\ 0],\\ \tilde{\Pi}_7^2=&[0\ 0\ 0\ \tilde{e}_{10}\ 0\ 0],\tilde{\Gamma}_a=\tilde{\Pi}_7^0+h\tilde{\Pi}_7^1,\tilde{\Gamma}_b=\tilde{\Pi}_7^0+h\tilde{\Pi}_7^2,\\ \tilde{\Phi}_1=&Sym([\tilde{e}_6-\tilde{e}_1W^TK_m]D_1W\tilde{e}_4^T+[\tilde{e}_1W^TK_p-\tilde{e}_6]D_2W\tilde{e}_4^T)\\ & +Sym([\tilde{e}_8-\tilde{e}_3W^TK_m]D_5W\tilde{e}_5^T+[\tilde{e}_3W^TK_p-\tilde{e}_8]D_6W\tilde{e}_5^T),\\ \tilde{\Psi}=&\tilde{\Sigma}+\tilde{\Phi}_1+\tilde{\Pi}_3\tilde{Q}\tilde{\Pi}_3^T-(1-h_D^u)\tilde{\Pi}_4\tilde{Q}\tilde{\Pi}_4^T+\tilde{\Pi}_5\tilde{N}\tilde{\Pi}_5^T\\ & -\tilde{\Pi}_6\tilde{N}\tilde{\Pi}_6^T+h^2\tilde{\Pi}_5Z\tilde{\Pi}_5^T+h^2\tilde{e}_4G\tilde{e}_4^T-\tilde{Y}^T\Phi\tilde{Y},\\ \tilde{\Sigma}=&Sym(\tilde{\Pi}_1^0\tilde{P}\tilde{\Pi}_2^T),\tilde{\Sigma}_1=Sym(\tilde{\Pi}_1^1\tilde{P}\tilde{\Pi}_2^T),\quad \tilde{\Sigma}_2=Sym(\tilde{\Pi}_1^2\tilde{P}\tilde{\Pi}_2^T),\\ \tilde{Y}=&[\tilde{e}_1-\tilde{e}_2\ \tilde{e}_1+\tilde{e}_2-2\tilde{e}_9\ \tilde{e}_2-\tilde{e}_3\ \tilde{e}_2+\tilde{e}_3-2\tilde{e}_{10}]^T\\ \tilde{K}=&-Sym([\tilde{e}_6-\tilde{e}_1W^TK_m]T_1[\tilde{e}_6-\tilde{e}_1W^TK_p]^T\\ & +[\tilde{e}_7-\tilde{e}_2W^TK_m]T_2[\tilde{e}_7-\tilde{e}_2W^TK_p]^T\\ & +[\tilde{e}_8-\tilde{e}_3W^TK_m]T_3[\tilde{e}_8-\tilde{e}_3W^TK_p]^T)\\ & -Sym([\tilde{e}_6-\tilde{e}_7-(\tilde{e}_1-\tilde{e}_2)W^TK_m]T_4[\tilde{e}_6-\tilde{e}_7-(\tilde{e}_1-\tilde{e}_2)W^TK_p]^T\\ & +[\tilde{e}_7-\tilde{e}_8-(\tilde{e}_2-\tilde{e}_3)W^TK_m]T_5[\tilde{e}_7-\tilde{e}_8-(\tilde{e}_2-\tilde{e}_3)W^TK_p]^T),\\ \tilde{\Theta}_a=&-Sym([\tilde{e}_6-\tilde{e}_1W^T(K_m+K_\alpha)]T_1[\tilde{e}_6-\tilde{e}_1W^TK_m]^T\\ & +[\tilde{e}_7-\tilde{e}_2W^T(K_m+K_\alpha)]T_2[\tilde{e}_7-\tilde{e}_2W^TK_m]^T\\ & +[\tilde{e}_8-\tilde{e}_3W^T(K_m+K_\alpha)]T_3[\tilde{e}_8-\tilde{e}_3W^TK_m]^T),\\ \tilde{\Theta}_b=&-Sym([\tilde{e}_6-\tilde{e}_1W^TK_p]T_4[\tilde{e}_6-\tilde{e}_1W^T(K_m+K_\alpha)]^T\\ & +[\tilde{e}_7-\tilde{e}_2W^TK_p]T_5[\tilde{e}_7-\tilde{e}_2W^T(K_m+K_\alpha)]^T\\ & +[\tilde{e}_8-\tilde{e}_3W^TK_p]T_6[\tilde{e}_8-\tilde{e}_3W^T(K_m+K_\alpha)]^T),\\ \tilde{\Omega}_a=&-Sym([\tilde{e}_6-\tilde{e}_7-(\tilde{e}_1-\tilde{e}_2)W^T(K_m+K_\alpha)]L_1[\tilde{e}_6-\tilde{e}_7\\ & -(\tilde{e}_1-\tilde{e}_2)W^TK_m]^T+[\tilde{e}_7-\tilde{e}_8-(\tilde{e}_2-\tilde{e}_3)W^T(K_m+K_\alpha)]L_2[\tilde{e}_7\\ & -\tilde{e}_8-(\tilde{e}_2-\tilde{e}_3)W^TK_m]^T),\end{aligned}$$

Table 1
Delay bounds h with different h_D.

Methods	Condition of $\dot{h}(t)$	$h_D=0.0$	$h_D=0.1$	$h_D=0.5$	$h_D=0.9$	Unknown
[11]	$\dot{h}(t) \le h_D$	1.3323	0.8245	0.3733	0.2343	0.2313
[33]	$\dot{h}(t) \le h_D$	1.3323	0.8402	0.4264	0.3214	0.3209
[34]	$\dot{h}(t) \le h_D$	1.5330	0.9331	0.4268	–	0.3215
[35]	$-h_D \le \dot{h}(t) \le h_D$	–	0.8411	0.4267	0.3227	0.3215
[36] (Theorem 1)	$-h_D \le \dot{h}(t) \le h_D$	1.5575	1.0389	0.5478	0.4602	–
[36] (Corollary 1)	$\dot{h}(t) \le h_D$	1.5575	0.9430	0.4417	0.3632	0.3632
Theorem 1	$-h_D \le \dot{h}(t) \le h_D$	1.8899	1.1240	0.5698	0.4737	–
Theorem 2($\alpha=0.7$)	$-h_D \le \dot{h}(t) \le h_D$	2.1082	1.1778	0.5824	0.4824	–
Corollary 1	$\dot{h}(t) \le h_D$	1.6386	0.9956	0.4464	0.3800	0.3695
Corollary 2 ($\alpha=0.7$)	$\dot{h}(t) \le h_D$	1.8211	1.0401	0.4535	0.3781	0.3781

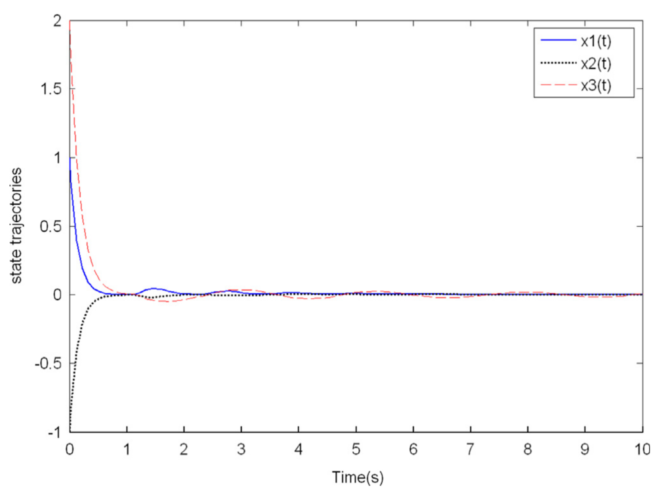

Fig. 1. State trajectories of the system of Example 1.

$$\tilde{\Omega}_b = -Sym([\tilde{e}_6-\tilde{e}_7-(\tilde{e}_1-\tilde{e}_2)W^T(K_m+K_\alpha)]L_3[\tilde{e}_6-\tilde{e}_7 -(\tilde{e}_1-\tilde{e}_2)W^TK_p]^T+[\tilde{e}_7-\tilde{e}_8-(\tilde{e}_2-\tilde{e}_3)W^T(K_m+K_\alpha)]L_4[\tilde{e}_7 -\tilde{e}_8-(\tilde{e}_2-\tilde{e}_3)W^TK_p]^T) \tag{38}$$

Corollary 1. *For the given scalars h and h_D^u satisfying C2, diagonal matrices $k_p=diag\{k_1^+,....,k_n^+\}$ and $k_m=diag\{k_1^-,....,k_n^-\}$, system (3) is asymptotically stable, if there exist positive matrices $\tilde{P}\in\mathbb{R}^{4n\times 4n}$, $\tilde{N}\in\mathbb{R}^{3n\times 3n}$, $\tilde{Q}\in\mathbb{R}^{2n\times 2n}$, $Z\in\mathbb{R}^{3n\times 3n}$, $G\in\mathbb{R}^{n\times n}$, diagonal matrices $D_i=diag(d_{1i},d_{2i},\dots,d_{ni})\ge 0,(i=1,\dots,6)$, $T_i=diag(t_{1i},t_{2i},\dots,t_{ni})\ge 0$, $(i=1,...,5)$, and any matrix $\Lambda\in\mathbb{R}^{3n\times 3n}$ along with matrices $S_{ij}\in\mathbb{R}^{n\times n}(i,j=1,2)$, and $\tilde{\Pi}$ of appropriate dimensions, satisfying the following linear matrix inequalities:*

$$\begin{bmatrix} (\tilde{H}^\perp)^T\tilde{\Omega}_1(\tilde{H}^\perp)+(\tilde{H}^\perp)^T\tilde{\Gamma}_a\tilde{\Pi}^T+\tilde{\Pi}\tilde{\Gamma}_a^T(\tilde{H}^\perp) & \tilde{\Pi} \\ * & -X \end{bmatrix}<0 \tag{39}$$

$$\begin{bmatrix} (\tilde{H}^\perp)^T\tilde{\Omega}_2(\tilde{H}^\perp)+(\tilde{H}^\perp)^T\tilde{\Gamma}_b\tilde{\Pi}^T+\tilde{\Pi}\tilde{\Gamma}_b^T(\tilde{H}^\perp) & \tilde{\Pi} \\ * & -X \end{bmatrix}<0 \tag{40}$$

$$X>0,\quad \Phi>0 \tag{41}$$

where $\tilde{\Omega}_1=h\tilde{\Sigma}_1+\tilde{\Psi}+\tilde{K}$, $\tilde{\Omega}_2=h\tilde{\Sigma}_2+\tilde{\Psi}+\tilde{K}$, and X, Φ are defined in (7), $\tilde{\Sigma}_1$, $\tilde{\Sigma}_2$, $\tilde{\Gamma}_a$, $\tilde{\Gamma}_b$, $\tilde{\Psi}$, $\tilde{K}$ and $\tilde{H}^\perp$ is the right orthogonal complement of $\tilde{H}$ are defined in (38).

Proof. Notice

$$\tilde{V}(x_t)=\sum_{i=1}^{6}\tilde{V}_i(x_t)$$

where

$$\tilde{V}_1=\tilde{\omega}^T(t)\tilde{P}\tilde{\omega}(t) \tag{42}$$

$$\tilde{V}_2=2\sum_{i=1}^{n}(d_{1i}\int_0^{w_ix_i(t)}(g_i(s)-k_i^-s)ds+d_{2i}\int_0^{w_ix_i(t)}(k_i^+s-g_i(s))ds) +2\sum_{i=1}^{n}(d_{5i}\int_0^{w_ix_i(t-h)}(g_i(s)-k_i^-s)ds+d_{6i}\int_0^{w_ix_i(t-h)}(k_i^+s-g_i(s))ds) \tag{43}$$

$$\tilde{V}_3=\int_{t-h(t)}^{t}\begin{bmatrix} x(s)\\ g(Wx(s))\end{bmatrix}^T\tilde{Q}\begin{bmatrix} x(s)\\ g(Wx(s))\end{bmatrix}ds \tag{44}$$

$$\tilde{V}_4=\int_{t-h}^{t}\beta^T(s)\tilde{N}\beta(s)ds, \tilde{V}_5=V_5, \tilde{V}_6=V_6. \tag{45}$$

Therefore, we can get

$$\dot{\tilde{V}}(x_t)\le\tilde{\xi}^T(t)(\tilde{\Omega}-\tilde{\Gamma}X\tilde{\Gamma}^T)\tilde{\xi}(t)$$

where $\tilde{\Omega}=h(t)\tilde{\Sigma}_1+(h-h(t))\tilde{\Sigma}_2+\tilde{\Psi}+\tilde{K}$, $\tilde{\Gamma}=\tilde{\Pi}_7^0+h(t)\tilde{\Pi}_7^1+(h-h(t))\tilde{\Pi}_7^2$. Further the proof follows similar steps as before for deriving (24). Thus, we can see that inequalities (39)–(41) do guarantee the asymptotic stability of recurrent neural networks (3) hence the networks (1) too.

Remark 5. Also for Theorem 2, we can introduce Corollary 2 in order to analyze the stability of recurrent neural networks with the condition C2 applicable by setting $D_3,D_4=0$, $Q,R=0$ and changing LKF terms V_1,V_2,V_3,V_4. The proof is very similar to the proof of Corollary 1, and thus omitted here.

Corollary 2. *For the given scalars $0\le\alpha\le 1$ and h, h_D^u satisfying C2, diagonal matrices $k_p=diag\{k_1^+,....,k_n^+\}$ and $k_m=diag\{k_1^-,....,k_n^-\}$, system (3) is asymptotically stable, if there exist positive matrices $\tilde{P}\in\mathbb{R}^{4n\times 4n}$, $\tilde{N}\in\mathbb{R}^{3n\times 3n}$, $\tilde{Q}\in\mathbb{R}^{2n\times 2n}$, $Z\in\mathbb{R}^{3n\times 3n}$, $G\in\mathbb{R}^{n\times n}$, diagonal matrices $D_i=diag(d_{1i},d_{2i},\dots,d_{ni})\ge 0,(i=1,\dots,6)$, $T_i=diag(t_{1i},t_{2i},\dots,t_{ni})\ge 0$, $(i=1,...,6)$, $L_i=diag(l_{1i},l_{2i},\dots,l_{ni})\ge 0$, $(i=1,...,4)$, and any matrix $\Lambda\in\mathbb{R}^{3n\times 3n}$ along with matrices $S_{ij}\in\mathbb{R}^{n\times n}(i,j=1,2)$, and $\tilde{\Pi}$ of appropriate dimensions, satisfying the following linear matrix inequalities:*

$$\begin{bmatrix} (\tilde{H}^\perp)^T\tilde{\Theta}_1(\tilde{H}^\perp)+(\tilde{H}^\perp)^T\tilde{\Gamma}_a\tilde{\Pi}^T+\tilde{\Pi}\tilde{\Gamma}_a^T(\tilde{H}^\perp) & \tilde{\Pi} \\ * & -X \end{bmatrix}<0 \quad \forall\Delta=a,b \tag{46}$$

$$\begin{bmatrix} (\tilde{H}^\perp)^T\tilde{\Theta}_2(\tilde{H}^\perp)+(\tilde{H}^\perp)^T\tilde{\Gamma}_b\tilde{\Pi}^T+\tilde{\Pi}\tilde{\Gamma}_b^T(\tilde{H}^\perp) & \tilde{\Pi} \\ * & -X \end{bmatrix}<0 \quad \forall\Delta=a,b \tag{47}$$

$$X>0,\quad \Phi>0 \tag{48}$$

where $\tilde{\Theta}_1=h\tilde{\Sigma}_1+\tilde{\Psi}+\tilde{\Theta}_\Delta+\tilde{\Omega}_\Delta$, $\tilde{\Theta}_2=h\tilde{\Sigma}_2+\tilde{\Psi}+\tilde{\Theta}_\Delta+\tilde{\Omega}_\Delta$, $\forall\Delta=a,b$ and X, Φ are defined in (7), $\tilde{\Sigma}_1$, $\tilde{\Sigma}_2$, $\tilde{\Gamma}_a$, $\tilde{\Gamma}_b$, $\tilde{\Psi}$, $\tilde{\Theta}_a$, $\tilde{\Theta}_b$, $\tilde{\Omega}_a$, $\tilde{\Omega}_b$, and $\tilde{H}^\perp$ is the right orthogonal complement of $\tilde{H}$ are defined in (38).

Though it should be noted, in some cases, the information on the derivative of the delay may not be available. Then the criterion for such a situation can be derived from Corollaries 1 and 2 by setting $\tilde{Q}=0$.

4. Illustrative example

In this section, the results of applying the proposed stability method to an example from the literature (e.g., see Ref. [11] for instance) are presented via a comparison analysis with those of the previous relevant methods to show its effectiveness and demonstrate the improvements. These results are given below in terms of the calculations in Table 1 and the computer simulations in Fig. 1.

Example 1. Consider a recurrent neural network of class (3) that is defined by the following parameter matrices:

$$A=\begin{bmatrix}7.3458 & 0 & 0\\ 0 & 6.9987 & 0\\ 0 & 0 & 5.5949\end{bmatrix},\quad W=\begin{bmatrix}13.6014 & -2.9616 & -0.6936\\ 7.4736 & 21.6810 & 3.2100\\ 0.7290 & -2.6334 & -20.1300\end{bmatrix},$$

$$K_m=diag\{0,0,0\},\quad K_p=diag\{0.3680,0.1795,0.2876\}. \tag{49}$$

For this recurrent neural network in which the condition $-h_D\le \dot{h}(t)\le h_D$ applied, the results obtained by means of Theorems 1 and 2 are summarized in Table 1, and given in comparison with the existing recent ones. It can be seen that, as compared to those in works [11,33–36], our results have improved the feasible region where asymptotic stability holds. It is worth pointing out the results based on Theorem 2 clearly provide larger delay bounds than those of Theorem 1 when $\alpha=0.7$. This fact also clearly demonstrates the effectiveness of the method with partitioning the bounding conditions on the activation functions. For the case of C2, the results obtained by Corollaries 1 and 2 are shown in Table 1 too. Again it is seen our results are less conservative than the existing ones.

The responses shown in Fig. 1 are obtained setting $x(0)=[1,\ -1,\ 2]^T$ for the recurrent neural network with a time-varying delay in Example 1, where the following quantities were defined: $h=1.1778$ for $h_D=-h_D=0.1$, $h(t)=0.1\ \sin(t)+1.0778\le 1.1778$, $g(x(t))=[0.3680\tanh(x_1(t)),\quad 0.1795\tanh(x_2\ (t)),\quad 0.2876\ \tanh(x_3(t))]^T$. These results verify the asymptotic stability of the considered class of time-varying delay RNNs obtained by means of the theorems proved in the previous section.

5. Conclusions

The problem of delay-dependent stability conditions for recurrent neural network (RNN) systems with time-varying delays has been investigated and new method derived. Less conservative delay-dependent stability criteria, which are expressed in terms of LMIs, are derived by using a novel method of partitioning the bounding conditions on network's activation function and a novel Lyapunov–Krasovskii functional (KLF), especially derived for this purpose. This new proposed method of stability analysis for the time-varying delay RNNs has been applied to the illustrative example taken from the literature. The obtained results are summarized in a comparison table with those in the recent literature and also verified by the asymptotic stability of state responses obtained via computer simulation. The presented results clearly demonstrate reduced conservativeness and response improvements.

This new methodological approach can be extended to other stability analysis problems for all kinds of neural networks, e.g. for stability problems involving H-infinite performance, passivity, and dissipativity too. In addition, by applying the main idea to the control synthesis problem for dynamic networks, such as stochastic delayed complex networks and Markovian jumping delayed complex networks, the feasible stability region can be enhanced. These aspects will be studied in future works. Also, it is worth noting, constructing a more suitable LKF and reducing the calculation enlargement in estimating the derivative also needs further investigation.

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