Thermal behaviors of light unflavored tensor mesons in the framework of QCD sum rule

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Abstract. In this paper, we investigated the sensitivity of the masses and decay constants of $f_2(1270)$ and $a_2(1320)$ tensor mesons to the temperature using OCD sum rule approach. In our calculations, we take into account new additional operators appearing in operator product expansion (OPE). At the end of numerical analyses we show that at deconfinement temperature the decay constants and masses decreased by 6% and 96% of their vacuum values, respectively. Our results on the masses and decay constants at zero temperature of the tensor mesons are consistent with the experimental data as well as the vacuum sum rules predictions.

1. Introduction

Recent years, in order to understand properties of matter at high energies heavy-ion collision experiments are performed [1-6]. These experiments have a critical role on investigating hadron structures. Up to now, there were not enough experimental results on quark-gluon plasma (QGP), but with increasing energies on heavy-ion colliders, theoretical studies on the strong interaction at finite temperature have become more important. According to thermal QCD, around critical temperature, $T_c = 175 MeV$, a transition occurs from hadronic matter to QGP phase which probably exists in early universe and neutron stars. Hence, theoretical efforts on thermal QCD calculations are important to understand the phase diagram and other properties of strong interactions [7-24].

In order to investigate Chiral symetry at high temperatures one needs to modify hadronic parameters for medium. There are many works on the medium corrections of hadron parameters by using different approaches such as chiral model [17,19], coupled channel [19] and sum rules [20-23] approaches. But still the thermal behaviors of tensor mesons are not known well. In this study, we use the thermal QCD sum rule method which is one of the most efficient tools [12,13] to calculate hadronic parameters of light unflavored tensor mesons. According to this method, hadron parameters are evaluated by using interpolating currents and operator product expansion approach (OPE). To extend the QCD sum rule method to finite temperatures, we assume that the Wilson expansion and the quark-hadron duality approximation are valid, but the quark and gluon condensates are changed with thermal expectation expressions [13]. In thermal QCD sum rule, the Lorentz invariance is broken and in the Wilson expansion appear extra operators which are expressed in terms of 4-vector velocity of the media and the energy-momentum tensor

[14-16]. Taking into account these new additional operators at finite temperature sum rule is obtained.

2. QCD sum rule for unflavored light mesons
In this section our aim is to obtain spectral density and non-perturbative part of correlation function to show how the physical quantities changes with the temperature. To calculate the spectral density we need to compute two point correlation function which is given as

$$\Pi_{\mu\nu,\alpha\beta}(q,T) = i \int d^4x e^{iq\cdot(x-y)} \langle \mathcal{T}[J_{\mu\nu}(x)\bar{J}_{\alpha\beta}(y)]\rangle \mid_{y=0}, \tag{1}$$

where $J_{\mu\nu}$ is the interpolating current of the tensor mesons as the following form:

$$\begin{aligned}
J_{\mu\nu}^{f_2(a_2)}(x) &= \frac{i}{2\sqrt{2}}\Big[\bar{u}(x)\gamma_\mu D_\nu(x)u(x) + \bar{u}(x)\gamma_\nu D_\mu(x)u(x) \\
&\pm \bar{d}(x)\gamma_\mu D_\nu(x)d(x) \pm \bar{d}(x)\gamma_\nu D_\mu(x)d(x)\Big].
\end{aligned} \tag{2}$$

Here terms have positive sign for the f_2 meson and terms have negative sign for the a_2 meson in our calculations. Also $D_\mu(x)$ contain four-derivatives, acting on the left and right, simultaneously, with respect to the space-time. After applying derivatives we will put $y = 0$. Using the free quark propagator for light quarks we obtained the spectral density in the following form:

$$\rho_{f_2(a_2)}(s) = \frac{(m_u^2 + m_d^2)s}{32\pi^2} + \frac{3s^2}{160\pi^2}. \tag{3}$$

Using the non-perturbative parts of quark propagator as follows

$$\begin{aligned}
S_q^{ij}(x-y) &= -\frac{\langle\bar{q}q\rangle}{12}\delta_{ij} - \frac{(x-y)^2}{192}m_0^2\langle\bar{q}q\rangle\Big[1 - i\frac{m_q}{6}(\slashed{x}-\slashed{y})\Big]\delta_{ij} \\
&+ \frac{i}{3}\Big[(\slashed{x}-\slashed{y})\Big(\frac{m_q}{16}\langle\bar{q}q\rangle - \frac{1}{12}\langle u\Theta^f u\rangle\Big) + \frac{1}{3}\Big(u\cdot(x-y)\,\slashed{u}\langle u\Theta^f u\rangle\Big)\Big]\delta_{ij},
\end{aligned} \tag{4}$$

we obtain the non-perturbative contributions which contain additional operators arising at finite temperature. $\Theta_{\mu\nu}^f$ is the fermionic part of the energy momentum tensor and u_μ is the four-velocity of the heat bath in Eq. (4). After matching the hadronic and OPE representations of correlator thermal sum rules for the decay constants obtained as:

$$\frac{f_{f_2(a_2)}^2(T)m_{f_2(a_2)}^6(T)}{m_{f_2(a_2)}^2 - q^2} = \int_{(m_u+m_d)^2}^{s_0(T)} ds \frac{\rho_{f_2(a_2)}(s)}{s-q^2} + \Pi_{f_2(a_2)}^{non-pert}, \tag{5}$$

where non-perturbative contributions can be written as

$$\Pi_{f_2(a_2)}^{non-pert} = \frac{m_d m_0^2}{144q^2}\langle\bar{d}d\rangle + \frac{m_u m_0^2}{144q^2}\langle\bar{u}u\rangle - \frac{2\langle u\Theta^f u\rangle(q\cdot u)^2}{9q^2}. \tag{6}$$

3. Conclusions and Discussions
Last step is to determine the working region of two parameters: Borel mass parameter M^2 and the hadronic threshold at zero temperature $s_0(T=0)$. Determination of these parameters is important due to sum rules results should be stable under their small variations. In this work, we have chosen the intervals $s_0 = (2.2-2.5)GeV^2$ and $s_0 = (2.4-2.7)GeV^2$ for the continuum

thresholds in f_2 and a_2 cases, respectively. Also we have chosen the working region of Borel mass as $1.4 GeV^2 \leq M^2 \leq 3 GeV^2$.

Taking into account the temperature dependencies of hadronic threshold, energy density and quark condensates, we obtained that masses and decay constants are well described by the following fit functions:

$$m_{f_2} = -1.0549 \times 10^{-5} e^{72.6744T} + 1.26519, \tag{7}$$

$$f_{f_2} = -4.28022 \times 10^{-6} e^{42.8082T} + 0.04258, \tag{8}$$

and

$$m_{a_2} = -1.25459 \times 10^{-5} e^{71.8390T} + 1.3221, \tag{9}$$

$$f_{a_2} = -4.06115 \times 10^{-6} e^{42.6621T} + 0.04232. \tag{10}$$

Here decay constants, masses and temperature are expressed in units of GeV. These parameterizations are valid only in the interval $0 \leq T \leq 0.16 GeV$.

Our investigations show that the values of masses and decay constants are stable until temperature $0.1\ GeV$ but after this point they start to decrease with altering the temperature. Also at deconfinement temperature, the decay constants and masses decreased by 6% and 96% of their vacuum values, respectively

Our results show that at zero temperature the masses and decay constants are $m_{f_2} = 1.28 \pm 0.08 GeV$, $f_{f_2} = 0.041 \pm 0.002$, $m_{a_2} = 1.33 \pm 0.10 GeV$ and $f_{a_2} = 0.042 \pm 0.002$. Our results for masses are compatible with the vacuum sum rules predictions. Our predictions on the temperature behaviors of decay constants and masses can be verified in the future experiments.

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