

# Friedmann Equation for Brans Dicke Cosmology

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## Abstract.

In the context of Brans-Dicke scalar tensor theory of gravitation, the cosmological Friedmann equation which relates the expansion rate  $H$  of the universe to the various fractions of energy density is analyzed rigorously. It is shown that Brans-Dicke scalar tensor theory of gravitation brings a negligible correction to the matter density component of Friedmann equation. Besides, in addition to  $\Omega_\Lambda$  and  $\Omega_M$  in standard Einstein cosmology, another density parameter,  $\Omega_\Delta$ , is expected by the theory inevitably. Some cosmological consequences of such non-familiar case are examined as far as recent observational results are concerned. Theory implies that if  $\Omega_\Delta$  is found to be nonzero, data can favor this model and hence this theory turns out to be the most powerful candidate in place of the standard Einstein cosmological model with cosmological constant. Such replacement will enable more accurate predictions for the rate of change of Newtonian gravitational constant in the future.

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Recent observational data have strongly confirmed that we live in an accelerating universe [1] and have made it possible to determine the composition of the universe [2]-[4]. According to these observations, nearly seventy percent of the energy density in the universe is unclustered (dark energy) and has negative pressure by which it is driving an accelerated expansion [5]-[8]. Furthermore, the energy density of the vacuum is much smaller than the estimated values so far. By itself, acceleration seems to be much more understandable in the context of general relativity (cosmological constant) [9] and quantum field theory (quantum zero point energy); however, the very small and non-zero energy scale implied by the observations is not quite comprehensible. Because of these conceptual problems associated with the cosmological constant [10]-[13], alternative treatments to the problem have been produced and they are being used widely in the literature nowadays [14]-[17]. For more detailed explanation about a number of approaches proposed so far and recent progress made towards understanding the nature of this dark energy see [18]. In some of these treatments, mostly, a scalar field  $\phi$  is considered together with a suitably chosen  $V(\phi)$  to make the vacuum energy vary with time.

To get a model in which the current value of the cosmological constant can be expressed in a more natural way; namely, without need of any fine tuning, in

the literature, there exist a number of studies on accelerated models in Brans Dicke theory [19]-[25]. For example, Sen *et al* [26] have found the potential relevant to power law expansion in Brans-Dicke (BD) cosmology whereas Arık and Çalık [27] have shown that BD theory of gravity with the standard mass term potential  $(1/2)m^2\phi^2$  is a beneficial theory in both explaining the rapid primordial and slow late-time inflation. In this regard, we have chosen the underlying theory as a scalar tensor theory, especially, BD scalar tensor theory of gravitation since scalar-tensor theories are the most serious alternative to standard general relativity. The theory is parameterized by a dimensionless constant  $\omega$ , as  $\omega \rightarrow \infty$  BD theory approaches the Einstein theory [28]. In BD model, Lagrangian is defined by as in the following form

$$L_{BD} = \sqrt{-g} \left[ \left( \varphi R - \omega \frac{1}{\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right) + L_M(\Psi) \right], \quad (1)$$

where the dimensionless  $\omega$  is the only parameter of the theory, and  $L_M$  is the matter Lagrangian. According to our metric convention, (+ - - -), Equation (1) turns out to be

$$L_{BD} = \sqrt{-g} \left[ \left( -\varphi R + \omega \frac{1}{\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right) + L_M(\Psi) \right]. \quad (2)$$

In particular, it is expected that  $\varphi(t, \vec{x})$  is spatially uniform and evolves slowly only with cosmic time  $t$  such that  $\varphi(t, \vec{x}) \rightarrow \varphi(t)$ . As another point, the second term on the right hand side of (1) appears to be kinetic term of the scalar field but it is in an unlikely form, since the presence of the  $\varphi^{-1}$  which seems to indicate a singularity, and the presence of the coupling constant in multiplicative form is undesired. However, whole term can be transformed into the standard canonical form by re-defining the scalar field  $\varphi$  by introducing a new field  $\phi$ , and a new constant  $\epsilon$  in such a way that

$$\varphi = \frac{1}{8\omega} \phi^2 \quad (3)$$

where  $\epsilon = \frac{1}{4\omega}$ .

In this new form, BD Lagrangian is redefined as

$$L_{BD} = \sqrt{-g} \left[ -\frac{1}{8\omega} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_M(\Psi) \right], \quad (4)$$

where the signs of the non-minimal coupling term and the kinetic energy term are properly adopted to (+ - - -) metric signature in such a way that as  $g^{\mu\nu} \sim \eta^{00}$ , the kinetic term,  $\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$  becomes  $\frac{1}{2} \dot{\phi}^2$ . Since the definition of  $\omega$  is not changed, the limit  $\omega$  approaching infinity is the same in both cases. In this limit  $\varphi$  remains constant whereas, as one can see from the above relation the limit of  $\phi^2$  is singular. Since the matter part of lagrangian density  $L_M$  does not contain neither  $\phi$  nor  $\omega$  it does not take any part in this transformation and therefore is unchanged. Present limits of the constant  $\omega$  based on time-delay experiments [29]-[31] require  $\omega > 10^4 \gg 1$ . Besides, since these theories have been found as the low energy limit of string theory and they provide appealing models for inflation, scalar tensor theories enable an interesting arena

where the standard model can be tested. Hence, in this work, we aim to calculate the corrections, in the context of BD cosmology, to the famous Friedmann equation

$$\left(\frac{H}{H_0}\right)^2 = \Omega_\Lambda + \Omega_R \left(\frac{a_0}{a}\right)^2 + \Omega_M \left(\frac{a_0}{a}\right)^3 \quad (5)$$

which relates the expansion rate  $H = \dot{a}/a$  of the universe to the energy density. The fractional density parameter,  $\Omega_i$ , is defined as the ratio of the energy density to the critical energy density which is a special required density in order to make the geometry of the universe flat. The Friedmann equation is used for fitting the Hubble parameter,  $H$ , to the measured density parameters ( $\Omega_\Lambda$ ,  $\Omega_R$ ,  $\Omega_M$ ) of the universe in such a way that  $\Omega_\Lambda + \Omega_R + \Omega_M = 1$ . According to recent observational results for the present universe, we have  $\Omega_\Lambda \simeq 0.75$ ,  $\Omega_M \simeq 0.25$ ,  $\Omega_R \simeq 0$  [32]. In the light of these values, one can conclude that the universe is mostly filled with non-baryonic matter and it seems that this non baryonic matter is responsible for the expansion of the universe solely. In the context of (BD) theory [33] whose scalar potential term consists of only a mass term and matter fields, the action in the canonical form is given by

$$S = \int d^4x \sqrt{g} \left[ -\frac{1}{8\omega} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 + L_M \right]. \quad (6)$$

The signs of non-minimal coupling term and the kinetic energy term are properly adopted to  $(+ - - -)$  metric signature. As in the cosmological approximation,  $\phi$  is spatially uniform, but varies slowly with time. As long as the dynamical scalar field  $\phi$  varies slowly,  $G_{eff}$ , the effective gravitational constant, is defined as  $G_{eff}^{-1} = \frac{2\pi}{\omega} \phi^2$  by replacing the non-minimal coupling term  $\frac{1}{8\omega} \phi^2 R$  with the Einstein-Hilbert term  $\frac{1}{16\pi G_N} R$  where  $R$  is the Ricci scalar in Einstein relativity. In natural units where  $c = \hbar = 1$ , we define the Planck-length,  $L_P^2 = \omega/2\pi\phi_0^2$  where  $\phi_0$  is the present value of the scalar field  $\phi$ . Thus, the dimension of the scalar field is  $L^{-1}$  so that the dimension of  $G_{eff}$  is  $L^2$ .  $L_M$  is decoupled from  $\phi$  as was assumed in the original BD theory. Hence, considering  $\phi$  does not couple to  $L_M$  as a matter field, we may consider a classical perfect fluid with the energy-momentum tensor  $T_\nu^\mu = diag(\rho, -p, -p, -p)$  where  $p$  is the pressure. The gravitational field equations derived from the variation of the action (6) with respect to the Robertson-Walker metric are

$$\frac{3}{4\omega} \phi^2 \left( \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) - \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} m^2 \phi^2 + \frac{3}{2\omega} \frac{\dot{a}}{a} \dot{\phi} \phi = \rho_M \quad (7)$$

$$\frac{-1}{4\omega} \phi^2 \left( 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) - \frac{1}{\omega} \frac{\dot{a}}{a} \dot{\phi} \phi - \frac{1}{2\omega} \ddot{\phi} \phi - \left( \frac{1}{2} + \frac{1}{2\omega} \right) \dot{\phi}^2 + \frac{1}{2} m^2 \phi^2 = p_M \quad (8)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a} \dot{\phi} + \left[ m^2 - \frac{3}{2\omega} \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) \right] \phi = 0 \quad (9)$$

where  $k$  is the curvature parameter with  $k = -1, 0, 1$  corresponding to open, flat, closed universes respectively and  $a(t)$  is the scale factor of the universe (dot denotes  $d/dt$ ),  $M$  denotes everything except the  $\phi$  field. The right hand side of the  $\phi$  equation (9) is set to be zero due to the assumption that the matter Lagrangian  $L_M$  is independent of

the scalar field  $\phi$ . Since the matter lagrangian affects the first two equations (7-8) and when  $R$  is solved from the first two equations and substituted into the  $\phi$  equation (9) this equation depends on the trace of the energy momentum tensor  $T_{\mu\nu}$ . Indeed this is how the BD field equations are sometimes written:

$$\ddot{\phi}\phi + \dot{\phi}^2 - \left(\frac{2\omega}{2\omega+3}\right) m^2 \phi^2 + 3\frac{\dot{a}}{a}\dot{\phi}\phi + \left(\frac{2\omega}{2\omega+3}\right) (3p - \rho) = 0. \quad (10)$$

But in any case the system of equations and the solutions to the system of equations are independent of this phenomenon. Instead of working with the field equations (7-9) stated in terms of  $\phi(t)$ ,  $a(t)$  and their derivatives with respect to the cosmological time  $t$ . We define the fractional rate of change of  $\phi$  as  $F(a) = \dot{\phi}/\phi$  and the Hubble parameter as  $H(a) = \dot{a}/a$ , and rewrite the left hand-side of the field equations (7-9) in terms of  $H(a)$ ,  $F(a)$  and their derivatives with respect to the scale size of the universe  $a$  (prime denotes  $\frac{d}{da}$ )

$$H^2 - \frac{2\omega}{3} F^2 + 2 H F + \frac{k}{a^2} - \frac{2\omega}{3} m^2 = \left(\frac{4\omega}{3}\right) \frac{\rho_M}{\phi^2} \quad (11)$$

$$H^2 + \left(\frac{2\omega}{3} + \frac{4}{3}\right) F^2 + \frac{4}{3} H F + \frac{2a}{3} (H H' + H F') + \frac{k}{3a^2} - \frac{2\omega}{3} m^2 = \left(\frac{-4\omega}{3}\right) \frac{p_M}{\phi^2} \quad (12)$$

$$H^2 - \frac{\omega}{3} F^2 - \omega H F + a \left(\frac{H H'}{2} - \frac{\omega}{3} H F'\right) + \frac{k}{2a^2} - \frac{\omega}{3} m^2 = 0. \quad (13)$$

From these three equations it can be shown that the continuity equation for the matter-energy excluding the BD scalar field is also satisfied with the help of the  $\phi$  equation (13)

$$\dot{\rho}_M + 3 \left(\frac{\dot{a}}{a}\right) (p_M + \rho_M) = 0 \quad (14)$$

and hence, instead of considering the  $p$  equation (12) solely as one of the dynamical equations to be satisfied, we choose continuity equation in addition to the density equation and the  $\phi$  equation to be satisfied in any cosmological case we want to explain. That is because once the continuity equation is satisfied than  $p$  equation must already be satisfied automatically provided that  $\dot{a}$  is nonzero. To eliminate the  $\phi$  dependence in (11), we take the time derivative of both sides of the  $\rho$  equation and after some rearrangements, we get (11) purely in terms of  $H(a)$ ,  $F(a)$ ,  $\rho(a)$  and their derivatives with respect to  $a$ .

$$\begin{aligned} & H'(H^2 + HF) + F'(H^2 - \frac{2\omega}{3}HF) \\ &= \frac{H^3}{2} \left(\frac{\rho'}{\rho}\right) + \frac{2\omega}{3a} F^3 + H^2 F \left[\left(\frac{\rho'}{\rho}\right) - \frac{1}{a}\right] + F^2 H \left[-\frac{2}{a} - \frac{\omega}{3} \left(\frac{\rho'}{\rho}\right)\right] \\ &+ \frac{k}{a^2} \left[H \left(\left(\frac{\rho'}{2\rho}\right) + \frac{1}{a}\right) - \frac{F}{a}\right] - \omega m^2 \left[H \left(\frac{\rho'}{3\rho}\right) - \frac{2F}{3a}\right]. \end{aligned} \quad (15)$$

After rewriting (13) in the following form

$$3aHH' - 2\omega HaF' = -6H^2 + 2\omega F^2 + 6\omega HF - \frac{3k}{a^2} + 2\omega m^2 \quad (16)$$

we solve (15, 16) for  $H'$ ,  $F'$  and get the general form of the solution in the sense that once the curvature constant  $k$  and energy density in terms of  $a$  is given than  $H$  and  $F$  can be solved from the following equations:

$$H' = \frac{[\omega a(\rho'/\rho) - 6]}{(2\omega + 3) aH} H^2 - \frac{[4\omega^2 + 2\omega + 2a\omega^2(\rho'/3\rho)]}{(2\omega + 3) aH} F^2 + \frac{[8\omega + 2a\omega(\rho'/\rho)]}{(2\omega + 3) aH} HF - \frac{[2\omega^2 a(\rho'/3\rho) - 2\omega]}{(2\omega + 3) aH} m^2 + k \frac{[2\omega + \omega a(\rho'/2) - 3]}{(2\omega + 3) a^3 H} \quad (17)$$

$$F' = \frac{[3a(\rho'/2\rho) + 6]}{(2\omega + 3) aH} H^2 - \frac{[8\omega + a\omega(\rho'/\rho) + 6]}{(2\omega + 3) aH} F^2 - \frac{[6\omega - 3a(\rho'/\rho) - 3]}{(2\omega + 3) aH} HF - \frac{[\omega a(\rho'/\rho) + 2\omega]}{(2\omega + 3) aH} m^2 + k \frac{[6 + 3a(\rho'/2\rho)]}{(2\omega + 3) a^3 H}. \quad (18)$$

Hence, in the present epoch, to discover how the Hubble parameter  $H$  changes with the scale size of the universe  $a$ , we assume that the present universe is mostly flat and it necessarily obeys the  $p_M = 0$  equation of state . Using (14), we find that the energy density  $\rho$  evolves with  $a$  in the same manner as in standard Einstein cosmology when the universe is solely governed by matter,

$$\rho = \frac{C}{a^3} \quad (19)$$

where  $C$  is an integration constant. Setting  $k = 0$  and inserting this energy density into (17, 18), we get the following form of the equations to be solved:

$$H' = \frac{-1}{H(2\omega + 3)a} [3(2 + \omega)H^2 + 2\omega(\omega + 1)F^2 - 2\omega HF - 2\omega(\omega + 1)m^2] \quad (20)$$

$$F' = \frac{1}{H(2\omega + 3)a} \left[ \frac{3}{2}H^2 - (5\omega + 6)F^2 - 6(1 + \omega)HF + \omega m^2 \right]. \quad (21)$$

With the transformation  $u = \left(\frac{a\alpha}{a}\right)^\alpha$  , we rewrite (20, 21) in terms of  $H(u)$ ,  $F(u)$  and their derivatives with respect to  $u$

$$\frac{dH}{du} = \frac{1}{\alpha H(2\omega + 3)u} [3(2 + \omega)H^2 + 2\omega(\omega + 1)F^2 - 2\omega HF - 2\omega(\omega + 1)m^2] \quad (22)$$

$$\frac{dF}{du} = \frac{-1}{\alpha H(2\omega + 3)u} \left[ \frac{3}{2}H^2 - (5\omega + 6)F^2 - 6(1 + \omega)HF + \omega m^2 \right]. \quad (23)$$

Since these coupled equations are hard enough to be solved analytically for  $H$  and  $F$ , our approach is to determine a perturbative solution in which both  $H$  and  $F$  vary about some constants  $H_\infty$  and  $F_\infty$  respectively:

$$H = H_\infty + H_1 u + H_2 u^2 + \dots \quad (24)$$

$$F = F_\infty + F_1 u + F_2 u^2 + \dots \quad (25)$$

where  $H_\infty$ ,  $F_\infty$ ,  $H_1$ ,  $F_1$ ,  $\alpha$ , are all constants to be determined from the theory and from fitting the Hubble parameter,  $H$ , to the measured density parameters ( $\Omega_\Lambda$ ,  $\Omega_R$ ,  $\Omega_M$ ) of the universe via Friedmann equation. Plugging this perturbative solution into (22, 23)

and keeping only the zeroth, first, second order terms of  $u$  and neglecting higher terms, we end up with two sets of solutions in the zeroth order

$$H_\infty = \frac{\sqrt{\omega}(2\omega + 2)m}{\sqrt{(6\omega^2 + 17\omega + 12)}}; F_\infty = \frac{H_\infty}{2(\omega + 1)} \quad (26)$$

and

$$H_\infty = \frac{2\sqrt{3\omega}m}{3\sqrt{3\omega + 4}}; F_\infty = \frac{3}{2}H_\infty. \quad (27)$$

Comparing the first order terms of  $u$ , on the other hand, provides two linearly dependent equations for which the only possible solution is the trivial solution of  $H_1 = 0$  and  $F_1 = 0$ ,

$$\{[6(\omega + 2) - \alpha(2\omega + 3)]H_\infty - 2\omega F_\infty\}H_1 + [-2\omega H_\infty + 4\omega(\omega + 1)F_\infty]F_1 = 0 \quad (28)$$

$$[-3H_\infty + 6(\omega + 1)F_\infty]H_1 + \{[6(\omega + 1) - \alpha(2\omega + 3)]H_\infty + 2(5\omega + 6)F_\infty\}F_1 = 0. \quad (29)$$

Since the solution in which  $H_1$  and  $F_1$  are nonzero is much more plausible for our aim, the coefficient matrix is properly constructed from (28, 29) and its determinant is set to be zero to get the value of  $\alpha$  for which  $H_1$  and  $F_1$  need not be zero simultaneously. We get two different  $\alpha$  values

$$\alpha = 3 + \frac{1}{1 + \omega} \quad (30)$$

and

$$\alpha \sim \sqrt{\omega} \quad (31)$$

corresponding to the solution sets (26) and (27) respectively. In this regard, we note two things here:

- Concerning the solution of  $H$  we seek for, the solution (30) is much more precious than the solution (31) which approaches to infinity as  $\omega$  becomes infinitely large. On the other hand, in the same limit, (30) gives  $\alpha = 3$  which is the well known term in a matter dominated universe solution of standard Einstein cosmology.
- The correction factor  $1/(1 + \omega)$  in the solution (30) is solely coming from the exact solutions of the field equations of BD theory.

Two linearly dependent equations are available when one compares the second order terms of  $u$ ;

$$\begin{aligned} & \{[6(\omega + 2) - 2\alpha(2\omega + 3)]H_\infty - 2\omega F_\infty\}H_2 + [4\omega(\omega + 1)F_\infty - 2\omega H_\infty]F_2 \\ & = [\alpha(2\omega + 3) - 3(\omega + 2)]H_1^2 - 2\omega(\omega + 1)F_1^2 + 2\omega H_1 F_1 \end{aligned} \quad (32)$$

$$\begin{aligned} & [3H_\infty - 6(\omega + 1)F_\infty]H_2 + \{[2\alpha(2\omega + 3) - 6(\omega + 1)]H_\infty - 2(5\omega + 6)F_\infty\}F_2 \\ & = -\frac{3}{2}H_1^2 + F_1^2 + [6(\omega + 1) - \alpha(2\omega + 3)]H_1 F_1. \end{aligned} \quad (33)$$

Letting  $\alpha = 3 + 1/(\omega + 1)$  and  $F_\infty = H_\infty/2(\omega + 1)$  in (32, 33) gives  $H_2$  and  $F_2$  only in terms of  $H_\infty, H_1, F_1$ ;

$$H_2 = \frac{[-(3\omega^2 + 8\omega + 6)H_1^2 + 2\omega(\omega + 1)^2 F_1^2 - 2\omega(\omega + 1)H_1 F_1]}{(3\omega + 4)(2\omega + 3)H_\infty} \quad (34)$$

$$F_2 = \frac{\left[ \frac{-3(\omega+1)}{2} H_1^2 + (\omega + 1) F_1^2 - (5\omega + 6) H_1 F_1 \right]}{(3\omega + 4)(2\omega + 3) H_\infty}. \quad (35)$$

Hence, with these perturbation constants found from theory, we can express  $H$  and  $F$  as

$$H = H_\infty + H_1 \left( \frac{a_0}{a} \right)^{3+\frac{1}{\omega+1}} + H_2 \left( \frac{a_0}{a} \right)^{6+\frac{2}{\omega+1}} + \dots \quad (36)$$

$$F = F_\infty + F_1 \left( \frac{a_0}{a} \right)^{3+\frac{1}{\omega+1}} + F_2 \left( \frac{a_0}{a} \right)^{6+\frac{2}{\omega+1}} + \dots \quad (37)$$

where (26) gives

$$H_\infty = [2(\omega + 1) \sqrt{\omega m}] / \sqrt{(6\omega^2 + 17\omega + 12)} \quad (38)$$

and

$$F_\infty = (\sqrt{\omega m}) / \sqrt{(6\omega^2 + 17\omega + 12)}. \quad (39)$$

To proceed one step further, we rewrite the standard Friedmann equation (5) with the extra term having new density parameter,  $\Omega_\Delta$ ,

$$\left( \frac{H}{H_0} \right)^2 = \Omega_\Lambda + \Omega_M \left( \frac{a_0}{a} \right)^{3+\frac{1}{\omega+1}} + \Omega_\Delta \left( \frac{a_0}{a} \right)^{6+\frac{2}{\omega+1}} \quad (40)$$

and we fit all theory parameters to the observational density parameters;

$$\Omega_\Lambda = \frac{H_\infty^2}{H_\Sigma^2}, \quad (41)$$

$$\Omega_M = \frac{2H_\infty H_1}{H_\Sigma^2}, \quad (42)$$

$$\Omega_\Delta = \frac{H_1^2 + 2H_\infty H_2}{H_\Sigma^2}, \quad (43)$$

where

$$H_\Sigma^2 = H_\infty^2 + 2H_\infty(H_1 + H_2) + H_1^2. \quad (44)$$

With these relations above and the constraint  $\Omega_\Lambda + \Omega_M + \Omega_\Delta = 1$ , we can express theoretical parameters  $H_1$ ,  $H_2$  in terms of the observational density parameters  $\Omega_\Lambda$ ,  $\Omega_M$  and  $H_\infty$ :

$$H_1 = \frac{\Omega_M}{2\sqrt{\Omega_\Lambda}} H_\infty \quad (45)$$

$$H_2 = \left[ 2 - \left( \sqrt{2\Omega_\Lambda} + \frac{\Omega_M}{\sqrt{2\Omega_\Lambda}} \right)^2 \right] H_\infty. \quad (46)$$

Now at this stage, investigation of two cases for  $\Omega_\Delta$  can be meaningful:

We first set  $\Omega_\Delta \simeq 0$ , ie;  $\Omega_M + \Omega_\Lambda = 1$  consistent with today's universe density compositions and  $H_2 \neq 0$ . By using recent observational results on density parameters  $\Omega_M \simeq 0.27$ ,  $\Omega_\Lambda \simeq 0.73$  [35]-[36] together with (45) and (46), we determine

$$H_1 = \frac{0.23}{2\sqrt{0.73}}H_\infty \simeq 0.15H_\infty, \quad (47)$$

$$H_2 = -\frac{H_1^2}{2H_\infty} \simeq -\frac{(0.15H_\infty)^2}{2H_\infty} \simeq -0.01H_\infty < 0. \quad (48)$$

Now, if (36) is satisfied for  $H = H_0$ , we get

$$H_\infty \simeq 0.88H_0 \quad (49)$$

where  $H_0$  is the present value of the Hubble parameter [34] and we may estimate mass of BD scalar field,  $m$ , for a fixed  $H_0$  as  $m \lesssim 10^{-2}H_0$  by using the relation  $H_\infty \simeq 0.88H_0 \simeq 0.82m\omega^{1/2}$  for  $\omega \rightarrow \infty$ . Furthermore, by using (34), (35), (47), (48) simultaneously, we get  $F_1 \simeq 0.08H_\infty/\omega$ ,  $F_2 \simeq -0.04H_\infty/\omega$  for  $\omega \rightarrow \infty$ . Remembering that  $F_\infty \simeq H_\infty/2\omega$ , we may say that  $F_\infty$  in (37) is the dominating term in today's universe. This shows us that similar to the expansion rate of the universe  $H$ , the rate of change of the Newtonian gravitational constant has approached the asymptotic regime.

On the contrary, when we set  $\Omega_\Delta \neq 0$  but  $\Omega_\Delta > 0$  ie;  $\Omega_M + \Omega_\Lambda + \Omega_\Delta = 1$  together with  $H_2 \simeq 0$ , from (46), we immediately get the relation

$$\Omega_M = 2\sqrt{\Omega_\Lambda} \left(1 - \sqrt{\Omega_\Lambda}\right), \quad (50)$$

and via this relation, we are able to estimate  $\Omega_\Lambda$  and  $\Omega_\Delta$  as in the following portions if the recent measured observational result on matter density parameter  $\Omega_M \simeq 0.27$  [35] is kept fixed,

$$\begin{aligned} \Omega_\Lambda &\simeq 0.71 \\ \Omega_\Delta &\simeq 0.02. \end{aligned} \quad (51)$$

To figure out the reliance of such theory based density parameters and to show how this model can pass the observational constraint of the SNIa data [37], we have compared our model of  $\Omega_M = 2\sqrt{\Omega_\Lambda} (1 - \sqrt{\Omega_\Lambda})$  proposing  $\Omega_\Delta \simeq 0.02$ ,  $\Omega_M \simeq 0.27$  and  $\Omega_\Lambda \simeq 0.71$  with LCDM model of  $\Omega_\Lambda = 1 - \Omega_M$  proposing  $\Omega_M = 0.276 \pm 0.026$  and  $\Omega_\Lambda = 0.724 \pm 0.026$  according to wmap+SNIa data (**ref**). From such comparison, it can be seen that in an asymptotic regime, although  $\Omega_\Delta \neq 0$ , SNIa data do not exclude our model as far as the range of uncertainties proposed by WMAP data team are valid. Besides, under this non-familiar case, we get other theory parameters  $F_1 \simeq 0.2H_\infty/\sqrt{\omega}$ ,  $F_2 \simeq -7 \times 10^{-3}H_\infty/\omega$  as  $\omega \rightarrow \infty$ . Remembering that  $F_\infty \simeq H_\infty/2\omega$ , we see that  $F_1$  is the dominating term in (37), namely, the rate of change of the Newtonian gravitational constant  $\left(\dot{G}_N/G_N\right)$  has not approached to the asymptotic regime yet. However, theory predicts that when the size of the universe exceeds  $a \gg 0.6\omega^{1/6}a_0$ , then the term  $F_\infty$  will become dominant so that asymptotic regime will be satisfied for  $\left(\dot{G}_N/G_N\right)$ .

As a last case, we investigate the possibility that  $\Omega_\Delta < 0$  theoretically, ie;  $\Omega_\Lambda + \Omega_M > 1$ . According to SNIa data, this case is important in the sense that if theory



assures that  $\Omega_\Delta < 0$  then this will imply that data favors this model instead of standard Einstein cosmological model with cosmological constant. However, when we use (34) and (43) simultaneously to make  $\Omega_\Delta < 0$  we see that for  $F_1 = H_1/2(\omega + 1)$ ,  $H_2$  attains its most negative value of  $H_2 = -H_1^2/2H_\infty$  and for this value of  $H_2$ ,  $\Omega_\Delta$  exactly is being equal to zero instead of being equal to negative value. With this result we note that BD theory can not be forced to have  $\Omega_\Delta < 0$  so that data can favor this model instead of standard Einstein cosmological model with cosmological constant.

Hence, in the light of these examinations on  $\Omega_\Delta$ , we may conclude that measurement of  $\Omega_\Delta$  will be important in two respects. Firstly, if  $\Omega_\Delta$  is found to have positive value, this will indicate that data can favor this model instead of standard Einstein cosmological model with cosmological constant. Secondly, if that is so, making much more accurate predictions for the rate of change of  $G_N$  could be plausible.

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