Analysis of the $B_q \to D_q(D_q^*)P$ and $B_q \to D_q(D_q^*)V$ decays within the factorization approach in QCD

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Abstract

Using the factorization approach and considering the contributions of the current-current, QCD penguin and electroweak penguin operators at the leading approximation, the decay amplitudes and decay widths of $B_q \to D_q(D_q^*)P$ and $B_q \to D_q(D_q^*)V$ transitions, where $q = u, d, s$ and P and V are pseudoscalar and vector mesons, are calculated in terms of the transition form factors of the $B_q \to D_q$ and $B_q \to D_q^*$. Having computed those form factors in three-point QCD sum rules, the branching fraction for these decays are also evaluated. A comparison of our results with the predictions of the perturbative QCD as well as the existing experimental data is presented.

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1 Introduction

With the chances that a lot of B_q mesons will be produced in B factories [1, 2], it would be possible to check the two-body non-leptonic charmed decay modes $B_q \to D_q(D_q^*)P$ and $B_q \to D_q(D_q^*)V$. Analyzing of such type decays could give valuable information about the origin of the CP violation, hadronic flavor changing neutral currents, test of the standard model (SM), constraints on new physic parameters as well as strong interactions among the participating particles which provides valuable tests of the QCD factorization framework.

Theoretically, analyzing of the two-body B-decay amplitudes have been started using the framework of so called "naive factorization" [3–7]. This method for some decay channels is replaced by QCD factorization [8, 9] since it could not predict direct CP asymmetries in those decay modes. First, the QCD factorization approach had been applied for the simplest charmless $B \to \pi\pi$ and $B \to \pi K$ decays [8, 10–13] then extended to the vector and exotic mesons in final states [14–17] and η or η' with a pseudoscalar or vector kaon [18]. In [19–21], decay modes of B_s meson are discussed. A comprehensive study of the exclusive hadronic B-meson decays into the final states containing two pseudoscalar mesons (PP) or a pseudoscalar and a vector meson (PV) is discussed in [22]. The Charmless anti-$B_s \to VV$ decays has also been analyzed in QCD factorization in [23]. The hard-scattering kernels relevant to the negative-helicity decay amplitude in B decays to two vector mesons are calculated in [24] in the same framework. The two-body hadronic decays of B mesons into pseudoscalar and axial vector mesons have been studied within the framework of QCD factorization in [25]. A detailed study of charmless two-body B decays into final states involving two vector mesons (V V) or two axial-vector mesons (AA) or one vector and one axial-vector meson (V A) has also been done within the framework of QCD factorization in [26]. Considering the contributions of both current-current and penguin operators, the amplitudes and branching ratios are recently estimated at the leading approximation for $B_c \to B^*P, BV$ in [27].

In the present work, taking into account the contributions of the current-current, QCD penguin and electroweak penguin operators at the leading approximation, we describe the charmed decays $B_q \to D_q(D_q^*)P$ and $B_q \to D_q(D_q^*)V$ in the framework of the QCD factorization method. First, using the factorization method, we calculate the decay amplitudes and decay widths of these decays in terms of the transition form factors of the $B_q \to D_q$ and $B_q \to D_q^*$. Having calculated these transition form factors in the framework of the QCD sum rules in our previous works in [28, 29], we calculate the branching ratio of these decays. In order to estimate the approximate branching ratios and to have a sense of the order of amplitudes, we make a rough approximation, i.e. at the leading order of α_s. Within this approximation, the hard-scattering kernel functions become very simple and equal to unity [27]. In this approximation, the long-distance interactions between the $P(V)$ and $B_q - D_q(D_q^*)$ system could be neglected. The higher order α_s corrections might not be small, but calculation of these contributions is not as easy as the light systems in final state and needs much more efforts. Hence, for obtaining the exact results on the considered transitions, those effects should be encountered in the future works. There are several methods in which such type contributions can be studied, QCD Factorization [8–10, 18, 22, 24], Perturbative QCD [30] and Soft-Collinear Effective Theory [31, 32]. For more detail analysis of NNLO corrections to $B \to light - light$ systems and higher order QCD corrections to

the charmless B decays see also [33–38]. Note that, some of the considered decays in this paper have been analyzed in the framework of the perturbative QCD in (PQCD) [30] and for some of them, we have some experimental data [39].

The outline of the paper is as follows: In section 2, we calculate the decay amplitudes and decay widths for $B_q \to D_q(D_q^*)P$ and $B_q \to D_q(D_q^*)V$ transitions. Finally, section 3 is devoted to the numerical analysis, a comparison of our results with the predictions of the PQCD as well as the existing experimental data and discussion.

2 Decay amplitudes and decay widths

In the present section, we study the decay amplitudes and decay widths for $B_q \to D_q(D_q^*)P$ and $B_q \to D_q(D_q^*)V$ decays, where $q = u, d, s$, $P = \pi, K, D_{q'}$ and $V = K^*, D_{q'}^*$ ($q^{'} = d, s$). At the quark level, the effective Hamiltonian for $B_q \to D_q(D_q^*)\pi(K, K^*)$ is given by

$$H_{eff} = \frac{G_F}{\sqrt{2}}\left\{V_{cb}V_{uq'}^*(C_1O_1^u + C_2O_2^u)\right\}. \quad (1)$$

Here O_1^u and O_2^u are quark operators and are given by

$$O_1^u = (\bar{q_i'}u_i)_{V-A}(\bar{c}_jb_j)_{V-A}, \quad O_2^u = (\bar{q_i'}u_j)_{V-A}(\bar{c}_jb_i)_{V-A}, \quad (2)$$

where $q^{'} = d, s$ and $(\bar{q}_1q_2)_{V\pm A} = \bar{q}_1\gamma^\mu(1 \pm \gamma_5)q_2$. However, the effective Hamiltonian for $B_q \to D_q(D_q^*)\ D_{q'}$ and $B_q \to D_q(D_q^*)\ D_{q'}^*$ at the quark level can be written as

$$H_{eff} = \frac{G_F}{\sqrt{2}}\left\{V_{cb}V_{cq'}^*(C_1O_1^c + C_2O_2^c) + V_{tb}V_{tq'}^*\sum_{n=3}^{10}C_nO_n\right\}. \quad (3)$$

Here O_n are quark operators and are given by

$$\begin{aligned}
O_1^c &= (\bar{q_i'}c_i)_{V-A}(\bar{c}_jb_j)_{V-A}, & O_2^c &= (\bar{q_i'}c_j)_{V-A}(\bar{c}_jb_i)_{V-A},\\
O_{3(5)} &= (\bar{q_i'}b_i)_{V-A}\textstyle\sum_q(\bar{q}_jq_j)_{V-(+)A}, & O_{4(6)} &= (\bar{q_i'}b_j)_{V-A}\textstyle\sum_q(\bar{q}_jq_i)_{V-(+)A}, \quad (4)\\
O_{7(9)} &= (\bar{q_i'}b_i)_{V-A}\textstyle\sum_q\frac{3}{2}e_q(\bar{q}_jq_j)_{V+(-)A}, & O_{8(10)} &= (\bar{q_i'}b_j)_{V-A}\textstyle\sum_q\frac{3}{2}e_q(\bar{q}_jq_i)_{V+(-)A},
\end{aligned}$$

where $\sum_q$ sums over $q = u, d, c, s, b$ and i and j are color indices. The operators O_1 and O_2 are called the current-current operators, $O_3...O_6$ are QCD penguin operators and $O_7...O_{10}$ are called the electroweak penguin operators.

The Wilson coefficients C_n have been calculated in different schemes [40–43]. In this paper we will use consistently the naive dimensional regularization (NDR) scheme. The values of C_n at $\mu \approx m_b$ with the next-to-leading order (NLO) QCD corrections are given by [40–43]

$$\begin{aligned}
C_1 &= 1.117\ , & C_2 &= -0.257\ ,\\
C_3 &= 0.017\ , & C_4 &= -0.044\ ,\\
C_5 &= 0.011\ , & C_6 &= -0.056\ , \quad (5)\\
C_7 &= -1\times10^{-5}\ , & C_8 &= 5\times10^{-4}\ ,\\
C_9 &= -0.010\ , & C_{10} &= 0.002\ .
\end{aligned}$$

The decay width and the branching ratio of the nonleptonic process $B_q \to D_q(D^*{}_q)M$, where M stands for the P or V mesons, is given by:

$$\begin{aligned}\Gamma(B_q \to D_q(D^*{}_q)M) &= \frac{1}{16\pi m^3_{B_q}}|\mathcal{A}|^2\sqrt{\lambda(m^2_{B_q}, m^2_{D_q(D^*_q)}, m^2_M)}\\ Br(B_q \to D_q(D^*{}_q)M) &= \tau_{B_q}\Gamma(B_q \to D_q(D^*{}_q)M)\end{aligned} \tag{6}$$

where, $\lambda(x,y,z) = x^2+y^2+z^2-2xy-2xz-2yz$ is usual triangle function.

To obtain the decay width, we should calculate the amplitude $\mathcal{A}$. This amplitude is obtained using the factorization method and the definition of the related matrix elements in terms of form factors for the $B_q \to D_q$ and $B_q \to D^*_q$ weak transitions as:

$$< D_q(p')|\bar{c}\gamma_\mu(1-\gamma_5)b|B_q(p) >= (p+p')_\mu f_+^{B_q\to D_q}(q^2) + \ (p-p')_\mu f_-^{B_q\to D_q}(q^2), \tag{7}$$

$$< D^*_q(p',\varepsilon) \mid \overline{c}\gamma_\mu b \mid B_q(p) >= \frac{-2f_1^{B_q\to D^*_q}(q^2)}{m_{D^*_q}+m_{B_q}}\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p'^\beta \tag{8}$$

$$\begin{aligned}< D^*_q(p',\varepsilon) \mid \overline{c}\gamma_\mu\gamma_5 b \mid B_q(p) > &= -i\Big[f_0^{B_q\to D^*_q}(q^2)(m_{D^*_q}+m_{B_q})\varepsilon^*_\mu\\ +\frac{f_2^{B_q\to D^*_q}(q^2)}{m_{D^*_q}+m_{B_q}}(\varepsilon^* p)(p+p')_\mu &+ \frac{f_3^{B_q\to D^*_q}(q^2)}{m_{D^*_q}+m_{B_q}}(\varepsilon^* p)(p-p')_\mu\Big],\end{aligned} \tag{9}$$

where q^2 is transferred momentum square, $q^2=(p-p')^2$, and p and p' are momentum of the initial and final meson states, respectively.

We obtain the $\mathcal{A}$ as following:

for $B_q \to D_q P$ $(P=\pi, K)$ and $B_q \to D_q D_{q'}$:

$$\begin{aligned}\mathcal{A}^{B_{d(s)}\to D_{d(s)}P} &= i\frac{G_F}{\sqrt{2}}\ V_{cb}V^*_{uq'}\ a_1\ f_P\ F_1^{B_{d(s)}\to D_{d(s)}}(m^2_P),\\ \mathcal{A}^{B_u\to D_u P} &= i\frac{G_F}{\sqrt{2}}\ V_{cb}V^*_{uq'}\Big[a_1\ f_P\ F_1^{B_u\to D_u}(m^2_P) + a_2\ f_{D_u}\ F_1^{B_u\to P}(m^2_{D_u})\Big]\\ \mathcal{A}^{B_q\to D_q D_{q'}} &= i\frac{G_F}{\sqrt{2}}\ f_{D_{q'}}\ F_1^{B_q\to D_q}(m^2_{D_{q'}})\ [V_{cb}V^*_{cq'}\ a_1\ - V_{tb}V^*_{tq'}(a_4+a_{10}\\ &\quad +R_{q'}(a_6+a_8))],\end{aligned} \tag{10}$$

where,

$$\begin{aligned}F_1^{B_q\to P_1}(m^2_{P_2}) &= (m^2_{B_q}-m^2_{P_1})f_+^{B_q\to P_1}(m^2_{P_2}) + m^2_{P_2}f_-^{B_q\to P_1}(m^2_{P_2}),\\ R_{q'} &= \frac{2m^2_{D_{q'}}}{(m_b-m_c)(m_{q'}+m_c)},\end{aligned} \tag{11}$$

for $B_q \to D_q K^*$ and $B_q \to D_q D^*_{q'}$:

$$\begin{aligned}
\mathcal{A}^{B_{d(s)}\to D_{d(s)}K^*} &= \frac{2\ m_{K^*}\ f_{K^*} G_F}{\sqrt{2}}\ V_{cb}V^*_{us}\ a_1\ (\ p\ .\varepsilon^*_{K^*}) f_+^{B_{d(s)}\to D_{d(s)}}(m^2_{K^*}),\\
\mathcal{A}^{B_u\to D_u K^*} &= \frac{G_F}{\sqrt{2}}\ V_{cb}V^*_{us}\ (p\ .\varepsilon^*_{K^*})\Big[2a_1\ m_{K^*}\ f_{K^*}\ f_+^{B_u\to D_u}(m^2_{K^*}) - a_2\ f_{D_u} F_2^{B_u\to K^*}(m^2_{D_u})\Big],\\
\mathcal{A}^{B_q\to D_q D^*_{q'}} &= \frac{2\ m_{D^*_{q'}}\ f_{D^*_{q'}} G_F}{\sqrt{2}}\ (p\ .\varepsilon^*_{D^*_{q'}}) f_+^{B_q\to D_q}(m^2_{D^*_{q'}})[V_{cb}V^*_{cq'}\ a_1 - V_{tb}V^*_{tq'}(a_4+a_{10})],
\end{aligned} \tag{12}$$

where,

$$F_2^{B_q\to V}(m^2_P) = f_0^{B_q\to V}(m^2_P)(m_{B_q}+m_V) + f_2^{B_q\to V}(m^2_P)(m_{B_q}-m_V) + \frac{f_3^{B_q\to V}(m^2_P)}{(m_{B_q}+m_V)} m^2_P, \tag{13}$$

for $B_q \to D^*_q P(P=\pi,K)$,and $B_q \to D^*_q D_{q'}$:

$$\begin{aligned}
\mathcal{A}^{B_q\to D^*_{d(s)}P} &= -\frac{G_F}{\sqrt{2}}\ V_{cb}V^*_{uq'}\ a_1\ f_P\ (p\ .\varepsilon^*_{D^*})\ F_2^{B_q\to D^*_{d(s)}}(m^2_P),\\
\mathcal{A}^{B_u\to D^*_u P} &= -\frac{G_F}{\sqrt{2}}\ V_{cb}V^*_{uq'}(p\ .\varepsilon^*_{D^*})\ [a_1\ f_P\ F_2^{B_u\to D^*_u}(m^2_P) - 2a_2\ f_{D^*_u}\ f_+^{B_u\to P}(m^2_{D^*_u})],\\
\mathcal{A}^{B_q\to D^*_q D_{q'}} &= -\frac{G_F}{\sqrt{2}}\ f_{D_{q'}}\ (p\ .\varepsilon^*_{D^*_q})\ F_2^{B_q\to D^*_q}(m^2_{D_{q'}})[V_{cb}V^*_{cq'}\ a_1\ - V_{tb}V^*_{tq'}(a_4+a_{10}\\
&\quad + R^{'}_{q'}(a_6+a_8))],
\end{aligned} \tag{14}$$

with

$$R^{'}_{q'} = -\frac{2m^2_{D_{q'}}}{(m_b+m_c)(m_{q'}+m_c)}, \tag{15}$$

and for $B_q \to D^*_q K^*$ and $B_q \to D^*_q D^*_{q'}$:

$$\begin{aligned}
\mathcal{A}^{B_{d(s)}\to D^*_{d(s)}K^*} &= i\frac{m_{K^*}\ f_{K^*} G_F}{\sqrt{2}}\ V_{cb}V^*_{us}\ a_1\ \Big[F_3^{B_{d(s)}\to D^*_{d(s)}}(m^2_{K^*})\ (\varepsilon^*_{D^*}.\varepsilon^*_{K^*})\\
&\quad + F_4^{B_{d(s)}\to D^*_{d(s)}}(m^2_{K^*})\ (p\ .\varepsilon^*_{D^*})(p\ .\varepsilon^*_{K^*}) - iF_5^{B_{d(s)}\to D^*_{d(s)}}(m^2_{K^*})\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\mu}_{K^*}\varepsilon^{*\nu}_{D^*}p^\rho p^\sigma_{D^*}\Big],\\
\mathcal{A}^{B_u\to D^*_u K^*} &= i\frac{G_F}{\sqrt{2}}\ V_{cb}V^*_{us}\Big(\ a_1\ m_{K^*}\ f_{K^*}\ \Big[F_3^{B_u\to D^*_u}(m^2_{K^*})\ (\varepsilon^*_{D^*}.\varepsilon^*_{K^*})\\
&\quad + F_4^{B_u\to D^*_u}(m^2_{K^*})\ (p\ .\varepsilon^*_{D^*})(p\ .\varepsilon^*_{K^*}) - iF_5^{B_u\to D^*_u}(m^2_{K^*})\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\mu}_{K^*}\varepsilon^{*\nu}_{D^*}p^\rho p^\sigma_{D^*}\Big]\\
&\quad + a_2\ m_{D^*}\ f_{D^*}\ \Big[F_3^{B_u\to K^*}(m^2_{D^*})\ (\varepsilon^*_{K^*}.\varepsilon^*_{D^*}) + F_4^{B_u\to K^*}(m^2_{D^*})\ (p\ .\varepsilon^*_{K^*})(p\ .\varepsilon^*_{D^*})\\
&\quad - iF_5^{B_u\to K^*}(m^2_{D^*})\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\mu}_{D^*}\varepsilon^{*\nu}_{K^*}p^\rho {p_{K^*}}^\sigma\Big]\Big),
\end{aligned}$$

$$\mathcal{A}^{B_q\to D_q^* D_{q'}^*} = i\frac{G_F}{\sqrt{2}}\, m_{D_{q'}^*}\, f_{D_{q'}^*}\, \Big[F_3^{B_q\to D_q^*}(m_{K^*}^2)\ (\varepsilon_{D_q^*}^*.\varepsilon_{D_{q'}^*}^*) + F_4^{B_q\to D_q^*}(m_{D_{q'}^*}^2)\ (p\ .\varepsilon_{D_q^*}^*)(p\ .\varepsilon_{D_{q'}^*}^*) - iF_5^{B_q\to D_q^*}(m_{D_{q'}^*}^2)\varepsilon_{\mu\nu\rho\sigma}\varepsilon_{D_{q'}^*}^{*\mu}\varepsilon_{D_q^*}^{*\nu}p^\rho p_{D_q^*}^\sigma\Big] \times[V_{cb}V_{cq'}^*\ a_1\ - V_{tb}V_{tq'}^*(a_4+a_{10})], \tag{16}$$

where

$$\begin{aligned} F_3^{B_q\to V_1}(m_{V_2}^2) &= (m_{B_q}+m_{V_1})f_0^{B_q\to V_1}(m_{V_2}^2), \\ F_4^{B_q\to V_1}(m_{V_2}^2) &= \frac{2f_2^{B_q\to V_1}(m_{V_2}^2)}{(m_{B_q}+m_{V_1})}, \\ F_5^{B_q\to V_1}(m_{V_2}^2) &= \frac{-2f_1^{B_q\to V_1}(m_{V_2}^2)}{(m_{B_q}+m_{V_1})}. \end{aligned} \tag{17}$$

In the above expressions, the ε^*, $\varepsilon^{'*}$, ε_K stand for the polarization of the D_q^*, $D_{q'}^*$ and K^* mesons, respectively. The quantities a_i, are given in terms of the coefficient C_i,

$$a_i = C_i + \frac{1}{N_c}C_{i+1}\ (i=odd); \qquad a_i = C_i + \frac{1}{N_c}C_{i-1}\ (i=even), \tag{18}$$

where i runs from $i=1,...,10$ and N_c is number of color in QCD. In the above equation, the a_1 and a_2 are both related to the coefficients $C_{1,2}$, which are very large comparing with the other Wilson coefficients, but we will keep all coefficients to get ride of further approximation.

Now we can calculate the decay widths for $B_q \to D_q(D_q^*)P$ and $B_q \to D_q(D_q^*)V$ decays. The explicit expressions for decay widths are given as follow:

$$\Gamma(B_{d(s)} \to D_{d(s)}P(P=\pi,K)) = \frac{G_F^2}{32\ \pi m_{B_{d(s)}}^3}|V_{cb}V_{uq'}^*|^2\ a_1^2\ f_P^2\ \sqrt{\lambda(m_{B_{d(s)}}^2, m_{D_{d(s)}}^2, m_P^2)} \times[F_1^{B_{d(s)}\to D_{d(s)}}(m_P^2)]^2, \tag{19}$$

$$\Gamma(B_u \to D_u P(P=\pi,K)) = \frac{G_F^2}{32\ \pi m_{B_u}^3}|V_{cb}V_{uq'}^*|^2\sqrt{\lambda(m_{B_u}^2, m_{D_u}^2, m_P^2)}\ \Big[a_1^2\ f_P^2\ [F_1^{B_u\to D_u}(m_P^2)]^2 + a_2^2\ f_{D_u}^2\ [F_1^{B_u\to P}(m_{D_u}^2)]^2 + 2\ a_1\ a_2\ f_{D_u}\ f_P\ [F_1^{B_u\to D_u}(m_P^2)\ F_1^{B_u\to P}(m_{D_u}^2)]\ \Big], \tag{20}$$

$$\Gamma(B_q \to D_q D_{q'}) = \frac{G_F^2}{32\ \pi m_{B_q}^3}f_{D_{q'}}^2\ \sqrt{\lambda(m_{B_q}^2, m_{D_q}^2, m_{D_{q'}}^2)}[F_1^{B_q\to D_q}(m_{D_{q'}}^2)]^2 \times\Big|V_{cb}V_{cq'}^*\ a_1\ - V_{tb}V_{tq'}^*[a_4+a_{10}+R_{q'}(a_6+a_8)]\Big|^2, \tag{21}$$

$$\Gamma(B_{d(s)} \to D_{d(s)} K^*) = \frac{G_F^2}{32\ \pi m_{B_{d(s)}}^3} |V_{cb}V_{us}^*|^2\ a_1^2\ f_{K^*}^2 \lambda(m_{B_{d(s)}}^2, m_{D_{d(s)}}^2, m_{K^*}^2)^{\frac{3}{2}} \times [f_+^{B_{d(s)}\to D_{d(s)}}(m_{K^*}^2)]^2, \quad (22)$$

$$\Gamma(B_u \to D_u K^*) = \frac{G_F^2}{32\ \pi m_{B_u}^3} |V_{cb}V_{us}^*|^2 \frac{\lambda(m_{B_u}^2, m_{D_u}^2, m_{K^*}^2)^{\frac{3}{2}}}{4m_{K^*}^2} \Big[4\ a_1^2\ m_{K^*}^2\ f_{K^*}^2\ [f_+^{B_u\to D_u}(m_{K^*}^2)]^2 + a_2^2\ f_{D_u}^2\ \left[F_2^{B_u\to K^*}(m_{D_u}^2)\right]^2 - 4\ a_1\ a_2\ m_{K^*}\ f_{K^*} f_{D_u}\ f_+^{B_u\to D_u}(m_{K^*}^2)\ F_2^{B_u\to K^*}(m_{D_u}^2)\Big], \quad (23)$$

$$\Gamma(B_q \to D_q D_{q'}^*) = \frac{G_F^2}{32\ \pi m_{B_q}^3} f_{D_{q'}^*}^2 \lambda(m_{B_q}^2, m_{D_q}^2, m_{D_{q'}^*}^2)^{\frac{3}{2}}\ [f_+^{B_q\to D_q}(m_{D_{q'}^*}^2)]^2 \times \left|V_{cb}V_{cq'}^*\ a_1\ - V_{tb}V_{tq'}^*[a_4 + a_{10}]\right|^2, \quad (24)$$

$$\Gamma(B_{d(s)} \to D_{d(s)}^* K^*) = \frac{G_F^2}{32\ \pi m_{B_{d(s)}}^3} |V_{cb}V_{us}^*|^2\ a_1^2\ m_{K^*}^2 f_{K^*}^2\ \lambda(m_{B_{d(s)}}^2, m_{D_{d(s)}^*}^2, m_{K^*}^2)^{\frac{1}{2}} \Bigg(\Big[F_3^{B_{d(s)}\to D_{d(s)}^*}(m_{K^*}^2)\Big]^2 \Big[\frac{\lambda(m_{B_{d(s)}}^2, m_{D_{d(s)}^*}^2, m_{K^*}^2)}{4m_{D_{d(s)}^*}^2 m_{K^*}^2} + 3\Big] + \Big[F_4^{B_{d(s)}\to D_{d(s)}^*}(m_{K^*}^2)\Big]^2 \Big[\frac{\lambda(m_{B_{d(s)}}^2, m_{D_{d(s)}^*}^2, m_{K^*}^2)^2}{16m_{D_{d(s)}^*}^2 m_{K^*}^2}\Big] + \Big[F_5^{B_{d(s)}\to D_{d(s)}^*}(m_{K^*}^2)\Big]^2 \Big[\frac{\lambda(m_{B_{d(s)}}^2, m_{D_{d(s)}^*}^2, m_{K^*}^2)}{2}\Big] + \Big[F_3^{B_{d(s)}\to D_{d(s)}^*}(m_{K^*}^2)\Big]\Big[F_4^{B_{d(s)}\to D_{d(s)}^*}(m_{K^*}^2)\Big] \times (m_{B_{d(s)}}^2 - m_{D_{d(s)}^*}^2 - m_{K^*}^2)\Big[\frac{\lambda(m_{B_{d(s)}}^2, m_{D_{d(s)}^*}^2, m_{K^*}^2)}{4m_{D_{d(s)}^*}^2 m_{K^*}^2}\Big]\Bigg), \quad (25)$$

$$\Gamma(B_u \to D_u^* K^*) = \frac{G_F^2}{32\ \pi m_{B_u}^3} |V_{cb}V_{us}^*|^2\ \lambda(m_{B_u}^2, m_{D_u^*}^2, m_{K^*}^2)^{\frac{1}{2}} \Bigg(\Big[a_1\ m_{K^*} f_{K^*} F_3^{B_u\to D_u^*}(m_{K^*}^2) + a_2\ m_{D_u^*} f_{D_u^*} F_3^{B_u\to K^*}(m_{D_u^*}^2)\Big]^2 \Big[\frac{\lambda(m_{B_u}^2, m_{D_u^*}^2, m_{K^*}^2)}{4m_{D_u^*}^2 m_{K^*}^2} + 3\Big] + \Big[a_1\ m_{K^*} f_{K^*} F_4^{B_u\to D_u^*}(m_{K^*}^2) + a_2\ m_{D_u^*} f_{D_u^*} F_4^{B_u\to K^*}(m_{D_u^*}^2)\Big]^2 \times \Big[\frac{\lambda(m_{B_u}^2, m_{D_u^*}^2, m_{K^*}^2)^2}{16m_{D_u^*}^2 m_{K^*}^2}\Big] + \Big[a_1\ m_{K^*} f_{K^*} F_5^{B_u\to D_u^*}(m_{K^*}^2)$$

$$
\begin{aligned}
&+a_2\ m_{D_u^*} f_{D_u^*} F_5^{B_u\to K^*}(m^2_{D_u^*})\Big]^2\Big[\frac{\lambda(m^2_{B_u},m^2_{D_u^*},m^2_{K^*})}{2}\Big]\\
&+\Big[a_1\ m_{K^*} f_{K^*} F_3^{B_u\to D_u^*}(m^2_{K^*})+a_2\ m_{D_u^*} f_{D_u^*} F_3^{B_u\to K^*}(m^2_{D_u^*})\Big]\\
&\times\Big[a_1\ m_{K^*} f_{K^*} F_4^{B_u\to D_u^*}(m^2_{K^*})+a_2\ m_{D_u^*} f_{D_u^*} F_4^{B_u\to K^*}(m^2_{D_u^*})\Big]\\
&\times(m^2_{B_u}-m^2_{D_u^*}-m^2_{K^*})\Big[\frac{\lambda(m^2_{B_u},m^2_{D_u^*},m^2_{K^*})}{4m^2_{D_u^*}m^2_{K^*}}\Big]\Big),
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
\Gamma(B_q\to D_q^* D_{q'}^*) \;=\;& \frac{G_F^2}{32\ \pi m^3_{B_q}}\ m^2_{D_{q'}^*} f^2_{D_{q'}^*}\ \lambda(m^2_{B_q},m^2_{D_q^*},m^2_{D_{q'}^*})^{\frac{1}{2}}\\
&\Bigg(\Big[F_3^{B_q\to D_q^*}(m^2_{D_{q'}^*})\Big]^2\Big[\frac{\lambda(m^2_{B_q},m^2_{D_q^*},m^2_{D_{q'}^*})}{4m^2_{D_q^*}m^2_{D_{q'}^*}}+3\Big]\\
&+\Big[F_4^{B_q\to D_q^*}(m^2_{D_{q'}^*})\Big]^2\Big[\frac{\lambda(m^2_{B_q},m^2_{D_q^*},m^2_{D_{q'}^*})^2}{16m^2_{D_q^*}m^2_{D_{q'}^*}}\Big]\\
&+\Big[F_5^{B_q\to D_q^*}(m^2_{D_{q'}^*})\Big]^2\Big[\frac{\lambda(m^2_{B_q},m^2_{D_q^*},m^2_{D_{q'}^*})}{2}\Big]\\
&+\Big[F_3^{B_q\to D_q^*}(m^2_{D_{q'}^*})\Big]\Big[F_4^{B_q\to D_q^*}(m^2_{D_{q'}^*})\Big]\\
&\times(m^2_{B_q}-m^2_{D_q^*}-m^2_{D_{q'}^*})\Big[\frac{\lambda(m^2_{B_q},m^2_{D_q^*},m^2_{D_{q'}^*})}{4m^2_{D_q^*}m^2_{D_{q'}^*}}\Big]\Bigg)\\
&\times\Big|V_{cb}V^*_{cq'}\ a_1\ -V_{tb}V^*_{tq'}[a_4+a_{10}]\Big|^2,
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
\Gamma(B_{d(s)}\to D^*_{d(s)}P(P=\pi,K)) \;=\;& \frac{G_F^2}{128\ \pi m^3_{B_{d(s)}} m^2_{D^*_{d(s)}}}|V_{cb}V^*_{uq'}|^2\ a_1^2\ f_P^2\lambda(m^2_{B_{d(s)}},m^2_{D^*_{d(s)}},m^2_P)^{\frac{3}{2}}\\
&[F_2^{B_{d(s)}\to D^*_{d(s)}}(m^2_P)]^2,
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
\Gamma(B_u\ \to\ D_u^* P(P=\pi,K)) &= \frac{G_F^2}{32\ \pi m^3_{B_u}}|V_{cb}V^*_{uq'}|^2\frac{\lambda(m^2_{B_u},m^2_{D_u^*},m^2_P)^{\frac{3}{2}}}{4m^2_{D_u^*}}\\
&\times\Big(4\ a_2^2\ m^2_{D_u^*}\ f^2_{D_u^*}\ [f_+^{B_u\to P}(m^2_{D_u^*})]^2+a_1^2\ f_P^2\ [F_2^{B_u\to D_u^*}(m^2_P)]^2\\
&-4\ a_1\ a_2\ m_{D_u^*}\ f_P f_{D_u^*}\ f_+^{B_u\to P}(m^2_{D_u^*})\ F_2^{B_u\to D_u^*}(m^2_P)\Big),
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
\Gamma(B_q\to D_q^* D_{q'}) \;=\;& \frac{G_F^2}{128\ \pi m^3_{B_q} m^2_{D_q^*}} f^2_{D_{q'}}\lambda(m^2_{B_q},m^2_{D_q^*},m^2_{D_{q'}})^{\frac{3}{2}}\ [F_2^{B_q\to D_q^*}(m^2_{D_{q'}})]^2\\
&\times\Big|V_{cb}V^*_{cq'}\ a_1\ -V_{tb}V^*_{tq'}[a_4+a_{10}+R'_{q'}(a_6+a_8)]\Big|^2.
\end{aligned}
\tag{30}
$$

m_π	m_K	$m_{D^\pm}$	$m_{\bar{D}^0}$	m_{D_s}	$m_{K^*(892)}$
139.570	493.677 ± 0.016	1869.62 ± 0.20	1864.84 ± 0.17	1968.49 ± 0.34	891.66 ± 0.26
$m_{D^{*\pm}}$	$m_{\bar{D}^{*0}}$	$m_{D^*{}_s}$	$m_{B^\pm}$	m_{B^0}	m_{B_s}
2010.27 ± 0.17	2006.97 ± 0.19	2112.3 ± 0.5	5279.2 ± 0.3	5279.5 ± 0.3	5366.3 ± 0.6

Table 1: Values of the masses in MeV [39].

f_π[39]	f_K[39]	$f_{D^\pm}$[39]	$f_{\bar{D}^0}$[39]	f_{D_s}[39]	f_{K^*}[44]
130.7 ± 0.46	159.8 ± 1.84	222.6 ± 19.5	222.6 ± 19.5	294 ± 27	217 ± 5
$f_{D^{*\pm}}$[39]	$f_{\bar{D}^{*0}}$[39]	$f_{D^*{}_s}$[45]	$f_{B^\pm}$[39]	f_{B^0}[39]	f_{B_s}[46]
230 ± 20	230 ± 20	266 ± 32	176 ± 42	176 ± 42	206 ± 10

Table 2: Values of the decay constants in MeV.

3 Numerical analysis

This section encompasses our numerical analysis, comparison of our results with the predictions of the PQCD as well as the existing experimental data and discussion. The expressions of the amplitudes and decay widths depict that the main input parameters entering the expressions are Wilson coefficients presented in the section 2, elements of the CKM matrix, leptonic decay constants, Borel parameters M_1^2 and M_2^2 as well as the continuum thresholds s_0 and s_0' [28, 29]. In further numerical analysis, we choose the numerical values as presented in the Tables 1, 2 and 3. The Borel mass squares M_1^2 and M_2^2 and continuum thresholds s_0 and s_0' are auxiliary parameters, hence the physical quantities should be independent of them. The parameters s_0 and s_0', are determined from the conditions that guarantees the sum rules for form factors to have the best stability in the allowed M_1^2 and M_2^2 region. The working regions for M_1^2 and M_2^2 as well as the values for continuum thresholds are determined in [28, 29]. Here, we choose the values $s_0 = 35 \pm 5\ GeV^2$, $s_0^{'} = 7 \pm 1\ GeV^2$, $M_1^2 = 17.0 \pm 2.5\ GeV^2$, $M_2^2 = 7 \pm 1\ GeV^2$ from those working values for auxiliary parameters. The values of the form factors $f_\pm$ and $f_{0,1,2,3}$ at different values of q^2 which we need in the expressions for decay widths are presented in Tables 4 and 5, respectively. Using the expressions for total decay widths, the values of branching fractions for $B_q \to D_q P$, $B_q \to D_q^* P$, $B_q \to D_q V$ and $B_q \to D_q^* V$ are found. We depict the values of the branching ratios in Tables 6, 7, 8 and 9. Here, we should stress that, as we mentioned before, our results depicted in the Tables are approximate results since we considered the observable

$\lvert V_{ud}\rvert$	$\lvert V_{us}\rvert$	$\lvert V_{cd}\rvert$	$\lvert V_{cs}\rvert$	$\lvert V_{cb}\rvert$
0.97377 ± 0.00027	0.2257 ± 0.0021	0.230 ± 0.011	0.957 ± 0.110	0.0416 ± 0.0006
$\lvert V_{ub}\rvert$	$\lvert V_{td}\rvert$	$\lvert V_{tb}\rvert$	$\lvert V_{ts}\rvert$	
0.00431 ± 0.00030	0.0074 ± 0.0008	0.77 ± 0.18	0.0406 ± 0.0027	

Table 3: Values of the elements of the CKM matrix [39].

only at the leading order of α_s. To obtain more exact results the higher order α_s corrections should be taken into account. However, the presented uncertainties in the results are belong to the uncertainties in the values of the input parameters as well as variations in form factors which are related to the errors in determination of the auxiliary parameters namely, Borell mass parameters M_1^2 and M_2^2 and continuum thresholds s_0 and s_0'. These Tables also include a comparison of our results with the existing predictions of the PQCD as well as the experimental data. From these Tables, we see a good consistency among two non-perturbative approaches and the experiment in order of magnitude. In many cases, the presented results out of order of magnitude from three approaches coincide especially, when we consider the uncertainties in the results. The best consistency between our results and predictions of the PQCD is related to the $B_s^0 \to D_s^{*\pm} K^{\mp}$ transition and $B_s^0 \to D_s{}^{\pm} K^*(892)^{\mp}$ transition shows the biggest discrepancy between two methods. Our central value prediction on $B^0 \to \bar{D}^{*\pm} K^{\mp}$ is approximately the same as the experimental result, however, the central experimental result on the branching ratio of $B_s^0 \to D_s^{*\pm} D_s^{*\mp}$ depicts a big discrepancy comparing that of our prediction. The presented predictions from PQCD are related to the charm-charmless cases in the final states and we have no predictions on the charm-charm cases from this approach. In this approach, the wave functions of the participating mesons, which are available with higher order corrections, have been used to calculate the amplitudes [30]. Therefor, over all agreement between our results and predictions of PQCD for charm-light cases in the final state and the experimental data for both charm-light and charm-charm cases, could be considered as a good test of the QCD factorization at leading order of α_s for related transitions. However, for exact comparison, much more efforts are needed in the future works, which may include the higher order corrections. Our results of some decay modes which have not been measured in the experiment can be tested in the future experiments at LHCb and other B factories.

In conclusion, using the QCD factorization approach and taking into account the contributions of the current-current, QCD penguin and the electroweak penguin operators at the leading approximation, the decay amplitudes and decay widths of $B_q \to D_q(D_q^*)P$ and $B_q \to D_q(D_q^*)V$ transitions were calculated in terms of the transition form factors of the $B_q \to D_q(D_q^*)$. Having computed those form factors in the framework of the three-point QCD sum rules in our previous works, the branching fraction for these decays were also evaluated. A comparison of our results with the predictions of the perturbative QCD as well as the existing experimental data was presented. Our results are over all in a good agreement with the predictions of the PQCD and the existing experimental data. Our predictions on some transitions, which have no experimental data can be checked by future experiments at LHCb or other B factories. To get more exact results from the QCD factorization method, higher order α_s corrections should be considered in the future works.

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q^2	$m^2_{\pi^\pm}$	$m^2_{K^\pm}$	$m^2_{K^*(892)^\pm}$	$m^2_{D^\pm}$	$m^2_{D_s}$	$m^2_{D^{*\pm}}$	$m^2_{D_s^*}$
$f_+^{B^+\to\bar{D}^0}(q^2)$	0.59 ± 0.14	0.60 ± 0.15	0.63 ± 0.16	0.86 ± 0.22	0.92 ± 0.23	0.96 ± 0.24	1.08 ± 0.27
$f_-^{B^+\to\bar{D}^0}(q^2)$	-0.20 ± 0.05	-0.21 ± 0.05	-0.22 ± 0.06	-0.38 ± 0.10	-0.44 ± 0.11	-0.49 ± 0.12	-0.69 ± 0.17
$f_+^{B^0\to D^+}(q^2)$	0.58 ± 0.15	0.59 ± 0.15	0.63 ± 0.17	0.86 ± 0.22	0.92 ± 0.22	0.95 ± 0.23	1.08 ± 0.27
$f_-^{B^0\to D^+}(q^2)$	-0.20 ± 0.05	-0.21 ± 0.05	-0.22 ± 0.06	-0.37 ± 0.10	-0.44 ± 0.11	-0.49 ± 0.12	-0.69 ± 0.17
$f_+^{B_s^0\to D_s^+}(q^2)$	0.26 ± 0.06	0.27 ± 0.06	0.28 ± 0.07	0.35 ± 0.09	0.38 ± 0.10	0.39 ± 0.10	0.42 ± 0.11
$f_-^{B_s^0\to D_s^+}(q^2)$	-0.11 ± 0.03	-0.12 ± 0.03	-0.13 ± 0.03	-0.15 ± 0.04	-0.16 ± 0.04	-0.17 ± 0.05	-0.18 ± 0.05

Table 4: The values of form factors $f_\pm$ at different values of q^2.

q^2	$m^2_{\pi^\pm}$	$m^2_{K^\pm}$	$m^2_{K^*(892)^\pm}$	$m^2_{D^\pm}$	$m^2_{D_s}$	$m^2_{D^{*\pm}}$	$m^2_{D_s^*}$
$f_0^{B^+\to\bar{D}^{*0}}(q^2)$	0.76 ± 0.19	0.78 ± 0.19	0.80 ± 0.20	0.97 ± 0.24	1.01 ± 0.25	1.09 ± 0.26	1.13 ± 0.27
$f_1^{B^+\to\bar{D}^{*0}}(q^2)$	0.62 ± 0.15	0.63 ± 0.15	0.67 ± 0.16	0.98 ± 0.24	1.08 ± 0.26	1.14 ± 0.27	1.27 ± 0.29
$f_2^{B^+\to\bar{D}^{*0}}(q^2)$	0.90 ± 0.22	0.96 ± 0.23	0.99 ± 0.24	1.50 ± 0.38	1.60 ± 0.40	1.82 ± 0.46	2.00 ± 0.50
$f_3^{B^+\to\bar{D}^{*0}}(q^2)$	-1.51 ± 0.38	-1.62 ± 0.40	-1.65 ± 0.40	-2.01 ± 0.50	-2.30 ± 0.55	-2.50 ± 0.60	-2.65 ± 0.61
$f_0^{B^0\to D^{*+}}(q^2)$	0.76 ± 0.19	0.78 ± 0.19	0.81 ± 0.20	0.97 ± 0.24	1.02 ± 0.25	1.10 ± 0.26	1.13 ± 0.27
$f_1^{B^0\to D^{*+}}(q^2)$	0.61 ± 0.15	0.63 ± 0.15	0.66 ± 0.16	0.98 ± 0.24	1.07 ± 0.26	1.14 ± 0.27	1.27 ± 0.29
$f_2^{B^0\to D^{*+}}(q^2)$	0.90 ± 0.22	0.95 ± 0.23	0.99 ± 0.24	1.51 ± 0.38	1.61 ± 0.40	1.82 ± 0.46	2.01 ± 0.50
$f_3^{B^0\to D^{*+}}(q^2)$	-1.52 ± 0.38	-1.62 ± 0.40	-1.66 ± 0.40	-2.01 ± 0.50	-2.31 ± 0.55	-2.51 ± 0.60	-2.65 ± 0.61
$f_0^{B_s^0\to D_s^{*+}}(q^2)$	0.38 ± 0.10	0.40 ± 0.11	0.41 ± 0.11	0.58 ± 0.15	0.62 ± 0.16	0.67 ± 0.17	0.70 ± 0.18
$f_1^{B_s^0\to D_s^{*+}}(q^2)$	0.33 ± 0.08	0.36 ± 0.08	0.40 ± 0.11	0.67 ± 0.17	0.71 ± 0.17	0.74 ± 0.18	0.76 ± 0.18
$f_2^{B_s^0\to D_s^{*+}}(q^2)$	0.43 ± 0.11	0.47 ± 0.12	0.50 ± 0.13	0.81 ± 0.21	0.85 ± 0.22	0.88 ± 0.23	0.91 ± 0.23
$f_3^{B_s^0\to D_s^{*+}}(q^2)$	-0.67 ± 0.17	-0.69 ± 0.17	-0.72 ± 0.18	-1.29 ± 0.32	-1.42 ± 0.34	-1.49 ± 0.35	-1.55 ± 0.36

Table 5: The values of form factors $f_{0,1,2,3}$ at different values of q^2.

$B_q \to D_q P$	present work	PQCD [30]	Exp [39]
$B^\pm \to \bar{D}^0\pi^\pm$	$(5.95 \pm 1.95) \times 10^{-3}$	$5.11^{+2.95+0.43+0.15}_{-2.07-0.75-0.15} \times 10^{-3}$	$(4.92 \pm 0.20) \times 10^{-3}$
$B^\pm \to \bar{D}^0 K^\pm$	$(4.31 \pm 1.52) \times 10^{-4}$	$4.00^{+2.35+0.63+0.12}_{-1.64-0.93-0.12} \times 10^{-4}$	$(4.08 \pm 0.24) \times 10^{-4}$
$B^\pm \to \bar{D}^0 D^\pm$	$(3.44 \pm 1.22) \times 10^{-4}$	–	$(4.80 \pm 1.00) \times 10^{-4}$
$B^\pm \to \bar{D}^0 D_s{}^\pm$	$(2.03 \pm 0.85) \times 10^{-2}$	–	(1.09 ± 0.27) %
$B^0 \to \bar{D}^\pm \pi^\mp$	$(5.69 \pm 1.70) \times 10^{-3}$	$2.69^{+1.78+0.55+0.08}_{-1.17-0.73-0.08} \times 10^{-3}$	$(3.40 \pm 0.90) \times 10^{-3}$
$B^0 \to \bar{D}^\pm K^\mp$	$(3.53 \pm 1.23) \times 10^{-4}$	$2.43^{1.56+0.63+0.07}_{-1.01-0.71-0.07} \times 10^{-4}$	$(2.00 \pm 0.60) \times 10^{-4}$
$B^0 \to \bar{D}^\pm D^\mp$	$(2.87 \pm 0.89) \times 10^{-4}$	–	$(1.90 \pm 0.60) \times 10^{-4}$
$B^0 \to \bar{D}^\pm D_s{}^\mp$	$(8.88 \pm 2.82) \times 10^{-3}$	–	$(6.50 \pm 2.10) \times 10^{-3}$
$B_s^0 \to D_s{}^\pm \pi^\mp$	$(1.42 \pm 0.57) \times 10^{-3}$	$2.13^{+1.14+0.69+0.06}_{-0.81-0.68-0.06} \times 10^{-3}$	$(3.80 \pm 0.30) \times 10^{-3}$
$B_s^0 \to D_s{}^\pm K^\mp$	$(1.03 \pm 0.51) \times 10^{-4}$	$1.71^{+0.92+0.58+0.05}_{-0.65-0.55-0.05} \times 10^{-4}$	–
$B_s^0 \to D_s{}^\pm D^\mp$	$(1.20 \pm 0.73) \times 10^{-4}$	–	–
$B_s^0 \to D_s{}^\pm D_s{}^\mp$	$(2.17 \pm 0.82) \times 10^{-3}$	–	–

Table 6: Values for the branching ratio of $B_q \to D_q P$.

$B \to D_q^* P$	present work	PQCD [30]	Exp [39]
$B^\pm \to \bar{D}^{*0}\pi^\pm$	$(4.89 \pm 1.52) \times 10^{-3}$	$5.04^{+2.92+0.44+0.15}_{-2.04-0.73-0.15} \times 10^{-3}$	$(4.60 \pm 0.40) \times 10^{-3}$
$B^\pm \to \bar{D}^{*0}K^\pm$	$(3.38 \pm 1.04) \times 10^{-4}$	$3.60^{+2.33+0.62+0.12}_{-1.62-0.92-0.12} \times 10^{-4}$	$(3.70 \pm 0.40) \times 10^{-4}$
$B^\pm \to \bar{D}^{*0}D^\pm$	$(2.57 \pm 0.88) \times 10^{-4}$	–	–
$B^\pm \to \bar{D}^{*0}D_s^\pm$	$(11.03 \pm 2.91) \times 10^{-3}$	–	$(10.00 \pm 4.00) \times 10^{-3}$
$B^0 \to \bar{D}^{*\pm}\pi^\mp$	$(3.45 \pm 1.75) \times 10^{-3}$	$2.60^{+1.73+0.53+0.07}_{-1.14-0.70-0.07} \times 10^{-3}$	$(2.76 \pm 0.21) \times 10^{-3}$
$B^0 \to \bar{D}^{*\pm}K^\mp$	$(2.08 \pm 0.68) \times 10^{-4}$	$2.37^{+1.52+0.62+0.07}_{-0.99-0.69-0.07} \times 10^{-4}$	$(2.14 \pm 0.20) \times 10^{-4}$
$B^0 \to \bar{D}^{*\pm}D^\mp$	$(3.14 \pm 1.46) \times 10^{-4}$	–	$(9.30 \pm 1.50) \times 10^{-4}$
$B^0 \to \bar{D}^{*\pm}D_s^\mp$	$(8.69 \pm 2.88) \times 10^{-3}$	–	$(8.80 \pm 1.60) \times 10^{-3}$
$B_s^0 \to D_s^{*\pm}\pi^\mp$	$(2.11 \pm 0.73) \times 10^{-3}$	$2.42^{+1.12+0.78+0.07}_{-0.72-0.77-0.07} \times 10^{-3}$	–
$B_s^0 \to D_s^{*\pm}K^\mp$	$(1.59 \pm 0.67) \times 10^{-4}$	$1.65^{+0.90+0.56+0.05}_{-0.63-0.53-0.05} \times 10^{-4}$	–
$B_s^0 \to D_s^{*\pm}D^\mp$	$(0.30 \pm 0.11) \times 10^{-4}$	–	–
$B_s^0 \to D_s^{*\pm}D_s^\mp$	$(2.54 \pm 0.57) \times 10^{-3}$	–	–

Table 7: Values for the branching ratio of $B_q \to D_q^* P$.

$B_q \to D_q V$	present work	PQCD [30]	Exp [39]
$B^{\pm} \to \bar{D}^0 K^*(892)^{\pm}$	$(2.90 \pm 0.88) \times 10^{-4}$	$6.49^{+3.86+0.12+0.20}_{-2.68-1.58-0.20} \times 10^{-4}$	$(6.30 \pm 0.80) \times 10^{-4}$
$B^{\pm} \to \bar{D}^0 D^{*\pm}$	$(5.61 \pm 1.88) \times 10^{-4}$	–	$(4.60 \pm 0.90) \times 10^{-4}$
$B^{\pm} \to \bar{D}^0 D_s^{*\pm}$	$(7.01 \pm 2.09) \times 10^{-3}$	–	$(7.20 \pm 2.60) \times 10^{-3}$
$B^0 \to \bar{D}^{\pm} K^*(892)^{\mp}$	$(3.20 \pm 1.15) \times 10^{-4}$	$4.07^{+2.61+0.94+0.12}_{-1.69-1.11-0.12} \times 10^{-4}$	$(4.50 \pm 0.70) \times 10^{-4}$
$B^0 \to \bar{D}^{\pm} D^{*\mp}$	$(8.18 \pm 2.84) \times 10^{-4}$	–	–
$B^0 \to \bar{D}^{\pm} D_s^{*\mp}$	$(9.23 \pm 2.67) \times 10^{-3}$	–	$(8.60 \pm 3.40) \times 10^{-3}$
$B_s^0 \to D_s{}^{\pm} K^*(892)^{\mp}$	$(0.50 \pm 0.22) \times 10^{-4}$	$3.02^{+1.62+0.88+0.10}_{-1.16-0.91-0.10} \times 10^{-4}$	–
$B_s^0 \to D_s{}^{\pm} D^{*\mp}$	$(1.07 \pm 0.59) \times 10^{-4}$	–	–
$B_s^0 \to D_s{}^{\pm} D_s^{*\mp}$	$(2.62 \pm 0.93) \times 10^{-3}$	–	–

Table 8: Values for the branching ratio of $B_q \to D_q V$.

$B_q \to D_q^* V$	present work	PQCD[30]	Exp[39]
$B^\pm \to \bar{D}^{*0} K^*(892)^\pm$	$(5.07 \pm 2.61) \times 10^{-4}$	$6.82^{+4.14+1.22+0.21}_{-2.80-1.65-0.21} \times 10^{-4}$	$(8.30 \pm 1.50) \times 10^{-4}$
$B^\pm \to \bar{D}^{*0} D^{*\pm}$	$(0.11 \pm 0.07) \times 10^{-2}$	–	< 1.1 %
$B^\pm \to \bar{D}^{*0} D_s^{*\pm}$	$(6.85 \pm 2.98) \times 10^{-2}$	–	2.20 ± 0.70 %
$B^0 \to \bar{D}^{*\pm} K^*(890)^\mp$	$(3.55 \pm 1.25) \times 10^{-4}$	$4.88^{+3.18+1.16+0.15}_{-2.08-1.41-0.15} \times 10^{-4}$	$(3.30 \pm 0.60) \times 10^{-4}$
$B^0 \to \bar{D}^{*\pm} D^{*\mp}$	$(8.78 \pm 2.50) \times 10^{-4}$	–	$(8.30 \pm 1.01) \times 10^{-4}$
$B^0 \to \bar{D}^{*\pm} D_s^{*\mp}$	$(8.17 \pm 2.93) \times 10^{-2}$	–	(1.79 ± 0.16) %
$B_s^0 \to D_s^{*\pm} K^*(890)^\mp$	$(1.63 \pm 0.86) \times 10^{-4}$	$3.47^{+1.96+1.07+0.11}_{-1.35-1.66-0.11} \times 10^{-4}$	–
$B_s^0 \to D_s^{*\pm} D^{*\mp}$	$(6.76 \pm 2.69) \times 10^{-4}$	–	–
$B_s^0 \to D_s^{*\pm} D_s^{*\mp}$	$(2.77 \pm 0.76) \times 10^{-2}$	–	(23^{+21}_{-13}) %

Table 9: Values for the branching ratio of $B_q \to D_q^* V$.