

Magnetic dipole moment of the light tensor mesons in light cone QCD sum rules

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Abstract

The magnetic dipole moments of the light tensor mesons f_2 , a_2 and strange $K_2^{*0}(1430)$ tensor meson are calculated in the framework of the light cone QCD sum rules. It is observed that the values of the magnetic dipole moment for the charged tensor particles are considerably different from zero. These values are very close to zero for the light neutral f_2 and a_2 tensor mesons, while it has a small nonzero value for the neutral strange $K_2^{*0}(1430)$ tensor meson.

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1 Introduction

Investigation of the properties of the hadrons such as electromagnetic form factors and multipole moments can shed light in understanding their internal structure, as well as their geometric shape. The electromagnetic properties of non-tensor mesons, as well as light and heavy baryons have been widely discussed in the literature. The electromagnetic form factors of the pseudoscalar π mesons are investigated extensively (see [1]–[6] and references therein). Form factors of the vector mesons are studied in [7]–[11], as well as in lattice QCD [6, 7, 12]. The magnetic and quadrupole moments of light vector and axial–vector mesons in light cone QCD sum rules (LCSR) are investigated in [13] (for more about LCSR, see [14] and [15]). It should be noted that static properties of baryons within the LCSR method are studied in [16] and [17]. However these properties of tensor mesons have received less interest and further detailed analysis is needed in this respect. So far, only the mass and decay constants of the light, unflavored tensor mesons within QCD sum rules is studied [18]. Recently, similar calculation is extended to cover the strange tensor mesons [19]. In the present work, we calculate the magnetic dipole moment of the light tensor mesons in the LCSR method.

The outline of the paper is as follows: In section 2, the sum rules for the the magnetic dipole moments of the light tensor mesons are obtained in the framework of the LCSR method. Section 3 is devoted to the numerical analysis of the magnetic dipole moment and discussion.

2 Light cone QCD sum rules for the magnetic moment of the light tensor mesons

In order to obtain the sum rules for the magnetic dipole moment, we consider the following correlation function:

$$\Pi_{\mu\nu\alpha\beta} = i \int d^4x e^{ipx} \langle 0 | \mathcal{T} \{ j_{\mu\nu}(x) \bar{j}_{\alpha\beta}(0) \} | 0 \rangle_\gamma, \quad (1)$$

where, \mathcal{T} is the time ordering, $j_{\mu\nu}$ and $\bar{j}_{\alpha\beta}$ are the interpolating currents corresponding to the initial and final states, with p being the momentum of the final state and q is the momentum transfer, and γ is the external electromagnetic field, respectively. The interpolating current $j_{\mu\nu}$ of the ground state tensor mesons is given as:

$$j_{\mu\nu} = \frac{i}{2} \left[\bar{q}_1(x) \gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu(x) q_2(x) + \bar{q}_1(x) \gamma_\nu \overleftrightarrow{\mathcal{D}}_\mu(x) q_2(x) \right], \quad (2)$$

where $\overleftrightarrow{\mathcal{D}}_\mu(x)$ represents the derivative with respect to x_μ acting on the right and left sides simultaneously, which is defined as:

$$\overleftrightarrow{\mathcal{D}}_\mu(x) = \frac{1}{2} \left[\overrightarrow{\mathcal{D}}_\mu(x) - \overleftarrow{\mathcal{D}}_\mu(x) \right]. \quad (3)$$

The covariant derivative defined in Eq. (3) can be written in terms of the normal derivative and the external (vacuum) gluon fields as follows:

$$\overrightarrow{\mathcal{D}}_\mu(x) = \overrightarrow{\partial}_\mu(x) - i \frac{g}{2} \lambda^a A_\mu^a(x),$$

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$$\overleftarrow{\mathcal{D}}_\mu(x) = \overleftarrow{\partial}_\mu(x) + i\frac{g}{2}\lambda^a A_\mu^a(x), \quad (4)$$

where λ^a are the Gell–Mann matrices.

In further analysis we will use the Fock–Schwinger gauge. The main advantage of this gauge is that the external field is expressed in terms of the field strength tensor, i.e., in the Fock–Schwinger gauge, where the condition $x^\mu A_\mu^a(x) = 0$ is imposed, we have:

$$A_\mu^a(x) = \int_0^1 d\alpha \alpha x_\beta G_{\beta\mu}^a(\alpha x). \quad (5)$$

It follows from Eq. (2) that, the current of the tensor mesons contains derivatives, and therefore we take into account the initial state at point y , and then after carrying out calculations, we set it to zero.

We can now proceed to obtain the sum rules for the magnetic dipole moment of the light tensor mesons. This sum rules are obtained from the following three steps.

- The correlation function in Eq. (1) is calculated in hadronic language, so-called the physical or phenomenological side. In order to obtain the expression from the physical side, the correlation function is saturated with a tower of tensor mesons having the same quantum numbers as the interpolating currents.
- The aforementioned correlation function is calculated in quark–gluon language called the theoretical or QCD side. In this representation, the correlator is calculated in deep Euclidean region with the help of the operator product expansion (OPE), where the short and long distance quark–gluon interactions are factored out. The short distance effects are calculated using the perturbation theory, whereas the long distance effects are parametrized in terms of the photon distribution amplitudes (DA's).
- The two representations of the correlation function in the above steps are equated through the dispersion relation. To suppress the contribution of the higher states and continuum, the Borel transformation and continuum subtraction are applied to both sides of the equality.

Let us first calculate the physical side of the correlation function. Inserting the complete sets of mesonic states into correlation function in Eq. (1), and isolating the ground state, we obtain:

$$\Pi_{\mu\nu\alpha\beta} = i \frac{\langle 0 | j_{\mu\nu} | T(p, \varepsilon) \rangle \langle T(p, \varepsilon) | T(p+q, \varepsilon) \rangle_\gamma \langle T(p+q, \varepsilon) | j_{\alpha\beta} | 0 \rangle}{p^2 - m_T^2} + \dots, \quad (6)$$

where $T(p, \varepsilon)$ denotes the light tensor mesons with momentum p and polarization tensor ε . Here, \dots represents the contribution of the higher states and the continuum. The vacuum to the mesonic state of the interpolating current is parametrized in terms of the decay constant and the polarization tensor as:

$$\langle 0 | j_{\mu\nu} | T(p, \varepsilon) \rangle = m_T^3 g_T \varepsilon_{\mu\nu}. \quad (7)$$

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The transition matrix element $\langle T(p, \varepsilon) | T(p + q, \varepsilon) \rangle_\gamma$ in terms of the form factors is determined as follows:

$$\begin{aligned} \langle T(p, \varepsilon) | T(p + q, \varepsilon) \rangle_\gamma &= \varepsilon_{\alpha'\beta'}^*(p) \left\{ 2(\varepsilon' \cdot p) \left[g^{\alpha'\rho} g^{\beta'\sigma} F_1 - g^{\beta'\sigma} \frac{q^{\alpha'} q^\rho}{2m_T^2} F_3 + \frac{q^{\alpha'} q^\rho}{2m_T^2} \frac{q^{\beta'} q^\sigma}{2m_T^2} F_5 \right] \right. \\ &\quad \left. + (\varepsilon'^\sigma q^{\beta'} - \varepsilon'^{\beta'} q^\sigma) \left[g^{\alpha'\rho} F_2 - \frac{q^{\alpha'} q^\rho}{2m_T^2} F_4 \right] \right\} \varepsilon_{\rho\sigma}(p + q), \end{aligned} \quad (8)$$

where $F_i(q^2)$ are the form factors and ε' is the photon polarization vector.

However, in the experiments it is more convenient to use the set of form factors which correspond to a definite multipole in a given reference frame. Relations between the two sets of form factors for the arbitrary integer spin (as well as arbitrary half-integer) case are obtained in [20]. In our case, i.e., for the real photon case $q^2 = 0$, these relations are rather simple and are as follows:

$$\begin{aligned} F_1(0) &= G_{E_0}(0), \\ F_2(0) &= G_{M_1}(0), \\ F_3(0) &= -2G_{E_0}(0) + G_{E_2}(0) + G_{M_1}(0), \\ F_4(0) &= -G_{M_1}(0) + G_{M_3}(0), \\ F_5(0) &= G_{E_0}(0) - G_{E_2}(0) - G_{M_1}(0) + G_{E_4}(0) + G_{M_3}(0), \end{aligned} \quad (9)$$

where $G_{E_\ell}(0)$ and $G_{M_\ell}(0)$ are the electric and magnetic multipoles.

Using Eqs. (8) and (9), the transition matrix element $\langle T(p, \varepsilon) | T(p + q, \varepsilon) \rangle_\gamma$ can be written in terms of the electric and magnetic multipoles as:

$$\begin{aligned} \langle T(p, \varepsilon) | T(p + q, \varepsilon) \rangle_\gamma &= \varepsilon_{\alpha'\beta'}^*(p) \left\{ 2(\varepsilon' \cdot p) \left[g^{\alpha'\rho} g^{\beta'\sigma} G_{E_0} - \frac{q^{\alpha'} q^\rho}{2m_T^2} g^{\beta'\sigma} (-2G_{E_0} + G_{E_2} + G_{M_1}) \right. \right. \\ &\quad \left. \left. + \frac{q^{\alpha'} q^\rho}{2m_T^2} \frac{q^{\beta'} q^\sigma}{2m_T^2} (G_{E_0} - G_{E_2} - G_{M_1} + G_{E_4} + G_{M_3}) \right] \right. \\ &\quad \left. + (\varepsilon'^\sigma q^{\beta'} - \varepsilon'^{\beta'} q^\sigma) \left[g^{\alpha'\rho} G_{M_1} - \frac{q^{\alpha'} q^\rho}{2m_T^2} (-G_{M_1} + G_{M_3}) \right] \right\} \varepsilon_{\rho\sigma}(p + q). \end{aligned} \quad (10)$$

Substituting Eqs. (7) and (10) into Eq. (6) and performing summation over the polarizations of spin-2 particles using the relation,

$$\begin{aligned} \varepsilon_{\mu\nu}(p) \varepsilon_{\alpha\beta}^*(p) &= \frac{1}{2} \left(-g_{\mu\alpha} + \frac{p_\mu p_\alpha}{m_T^2} \right) \left(-g_{\nu\beta} + \frac{p_\nu p_\beta}{m_T^2} \right) \\ &\quad + \frac{1}{2} \left(-g_{\mu\beta} + \frac{p_\mu p_\beta}{m_T^2} \right) \left(-g_{\nu\alpha} + \frac{p_\nu p_\alpha}{m_T^2} \right) \\ &\quad - \frac{1}{3} \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_T^2} \right) \left(-g_{\alpha\beta} + \frac{p_\alpha p_\beta}{m_T^2} \right), \end{aligned} \quad (11)$$

one gets the final expression of the correlation function on the physical side. Obviously, the correlation function contains many independent structures encountered, and among these structures we can choose any independent one for determination of the multipole form factors. In the present work, we restrict ourselves to calculate the magnetic dipole form factor only and for this aim the structure $(\varepsilon'^{\beta} q^{\nu} - \varepsilon'^{\nu} q^{\beta})g^{\mu\alpha}$ is chosen. The choice of this structure is dictated by the fact that it does not get contribution from the contact terms (for a discussion about contact terms see [21]). Separating out the coefficient of this structure we get the final expression for the correlation function in physical side in terms of the magnetic dipole moment as:

$$\Pi = i \frac{m_T^6 g_T^2}{(p^2 - m_T^2)[(p+q)^2 - m_T^2]} \left\{ \frac{1}{4} G_{M_1} + \text{other structures} \right\} + \dots \quad (12)$$

On the QCD side, the correlation function is calculated in deep Euclidean region where $p^2 \rightarrow -\infty$ and $(p+q)^2 \rightarrow -\infty$, via the OPE. After contracting out all quark pairs, we obtain the following representation of the correlation function on the theoretical side:

$$\begin{aligned} \Pi_{\mu\nu\alpha\beta} = & \frac{-i}{16} \int e^{ip \cdot x} d^4x \left\langle \gamma \left| \left\{ S_{q_1}(y-x) \gamma_{\mu} \left[\vec{\partial}_{\nu}(x) \vec{\partial}_{\beta}(y) \right. \right. \right. \right. \\ & - \left. \left. \left. \vec{\partial}_{\nu}(x) \vec{\partial}_{\beta}(y) - \vec{\partial}_{\nu}(x) \vec{\partial}_{\beta}(y) + \vec{\partial}_{\nu}(x) \vec{\partial}_{\beta}(y) \right] S_{q_2}(x-y) \gamma_{\alpha} \right\} \right| 0 \right\rangle \\ & + \{ \beta \leftrightarrow \alpha \} + \{ \nu \leftrightarrow \mu \} + \{ \beta \leftrightarrow \alpha, \nu \leftrightarrow \mu \} . \end{aligned} \quad (13)$$

In order to proceed with the analysis, we need to know the explicit expressions for the light quark propagator. The light quark propagator in the external field is calculated in [3, 4], which has the form:

$$\begin{aligned} S_q(x-y) = & S^{free}(x-y) - \frac{\langle qq \rangle}{12} \left[1 - i \frac{m_q}{4} (\not{x} - \not{y}) \right] - \frac{(x-y)^2}{192} m_0^2 \langle qq \rangle \left[1 - i \frac{m_q}{6} (\not{x} - \not{y}) \right] \\ & - i g_s \int_0^1 du \left\{ \frac{\not{x} - \not{y}}{16\pi^2(x-y)^2} G_{\mu\nu}(u(x-y)) \sigma^{\mu\nu} - u(x^\mu - y^\mu) G_{\mu\nu}(u(x-y)) \gamma^\nu \right. \\ & \left. \times \frac{i}{4\pi^2(x-y)^2} - i \frac{m_q}{32\pi^2} G_{\mu\nu}(u(x-y)) \sigma^{\mu\nu} \left[\ln \left(-\frac{(x-y)^2 \Lambda^2}{4} + 2\gamma_E \right) \right] \right\} , \end{aligned} \quad (14)$$

where Λ is the scale parameter and we choose it as a factorization scale, i.e., $\Lambda = (0.5 - 1) \text{ GeV}$ [22]. Here, we would like to made the following remark about the expression of the quark propagator. The complete light cone expansion of the light quark propagator is given in [23] and it gets contributions from nonlocal $\bar{q}Gq$, $\bar{q}G^2q$ and $\bar{q}q\bar{q}q$ operators, where $G_{\mu\nu}$ is the gluon field strength tensor. In the present work, we take into account the operators with only one gluon field and neglect the contributions coming from the $\bar{q}G^2q$ and $\bar{q}q\bar{q}q$ operators. Formally, ignoring from these terms can be justified on the basis of an expansion in conformal spin [24].

The free quark operator is given as:

$$S^{free}(x-y) = \frac{i \not{x} - \not{y}}{2\pi^2(x-y)^4} - \frac{m_q}{4\pi^2(x-y)^2} . \quad (15)$$

In order to evaluate Eq. (13) we substitute the light quark propagator, we first take the derivatives with respect to x and y , and then set $y = 0$. The correlation function receives contributions from the following three sources:

- Perturbative contributions.
- “Mixed contributions”. This contribution takes place when photon interacts with the quark fields perturbatively and the other quark fields interact with the QCD vacuum, i.e., they form condensates.
- Long distance contributions, i.e., photon is radiated at long distance.

The perturbative contribution can be obtained from Eq. (13) by replacing on of the propagators by:

$$S(x - y) = \int d^4z S^{free}(x - z) \not{A}(z) S^{free}(z - y) , \quad (16)$$

and the other propagator is chosen as the free quark propagator.

In order to calculate the mixed contribution, it is enough to replace one of the propagators in Eq. (13) by Eq. (16). The remaining propagator is replaced by the quark condensates.

Calculation of long distance contribution proceeds as follows. One of the quark propagator is replaced by

$$S_{\alpha\beta}^{ab}(x - y) = -\frac{1}{4}\bar{q}^a(x)\Gamma_j q^b(y) (\Gamma_j)_{\alpha\beta} ,$$

where $\Gamma_j = \left\{ I, \gamma_\mu, \gamma_5, i\gamma_5\gamma_\mu, \sigma_{\mu\nu}/\sqrt{2} \right\}$ are the full set of Dirac matrices. Since a photon interacts with quark fields at long distance there appears the matrix elements of nonlocal operators $\bar{q}(x)\Gamma q'(y)$ and $\bar{q}(x)G_{\mu\nu}\Gamma q'(y)$ between the vacuum and photon state. These are the matrix elements of the photon DA's. The other remaining propagator is either replaced by the free quark operator, or by the quark condensate.

The matrix elements of the nonlocal operators $\bar{q}\Gamma q'$ and $\bar{q}G_{\mu\nu}\Gamma q'$ between one photon and the vacuum states are determined in terms of photon DA's in the following way [25]:

$$\begin{aligned} \langle \gamma(q) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle &= -ie_q \bar{q}q (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int_0^1 du e^{i\bar{u}qx} \left(\chi \varphi_\gamma(u) + \frac{x^2}{16} \mathbb{A}(u) \right) \\ &\quad - \frac{i}{2(qx)} e_q \langle \bar{q}q \rangle \left[x_\nu \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) - x_\mu \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \right] \int_0^1 du e^{i\bar{u}qx} h_\gamma(u) \\ \langle \gamma(q) | \bar{q}(x) \gamma_\mu q(0) | 0 \rangle &= e_q f_{3\gamma} \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \int_0^1 du e^{i\bar{u}qx} \psi^v(u) \\ \langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle &= -\frac{1}{4} e_q f_{3\gamma} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^\nu q^\alpha x^\beta \int_0^1 du e^{i\bar{u}qx} \psi^a(u) \\ \langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q}q \rangle \left(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu \right) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{S}(\alpha_i) \end{aligned}$$

$$\begin{aligned}
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu} i\gamma_5(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q}q \rangle \left(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu \right) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \tilde{\mathcal{S}}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) \gamma_\alpha \gamma_5 q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) i\gamma_\alpha q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= e_q \langle \bar{q}q \rangle \left\{ \left[\left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\nu} - \frac{1}{qx} (q_\alpha x_\nu + q_\nu x_\alpha) \right) q_\beta \right. \right. \\
&- \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\nu} - \frac{1}{qx} (q_\beta x_\nu + q_\nu x_\beta) \right) q_\alpha \\
&- \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\mu} - \frac{1}{qx} (q_\alpha x_\mu + q_\mu x_\alpha) \right) q_\beta \\
&+ \left. \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\mu} - \frac{1}{qx} (q_\beta x_\mu + q_\mu x_\beta) \right) q_\alpha \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_1(\alpha_i) \\
&+ \left[\left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) q_\nu \right. \\
&- \left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\nu\beta} - \frac{1}{qx} (q_\nu x_\beta + q_\beta x_\nu) \right) q_\mu \\
&- \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) q_\nu \\
&+ \left. \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\nu\alpha} - \frac{1}{qx} (q_\nu x_\alpha + q_\alpha x_\nu) \right) q_\mu \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_2(\alpha_i) \\
&+ \frac{1}{qx} (q_\mu x_\nu - q_\nu x_\mu) (\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_3(\alpha_i) \\
&+ \left. \frac{1}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_4(\alpha_i) \right\}, \tag{17}
\end{aligned}$$

where $\varphi_\gamma(u)$ is the leading twist 2, $\psi^v(u)$, $\psi^a(u)$, \mathcal{A} and \mathcal{V} are the twist 3 and $h_\gamma(u)$, \mathbb{A} , \mathcal{T}_i ($i = 1, 2, 3, 4$) are the twist 4 photon DA's, respectively and χ is the magnetic susceptibility of the quark fields. The photon DA's are calculated in [25], and we will give their explicit forms in the following section.

The measure $\mathcal{D}\alpha_i$ is defined as

$$\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g). \tag{18}$$

Carrying out the above-mentioned calculations and separating out the coefficient of the structure $\varepsilon''^\beta q^\nu g^{\mu\alpha}$ from the QCD side of the correlation function and equating this result to the coefficient of the same structure from physical side, the sum rules for the magnetic moments of the light tensor mesons are obtained. In order to suppress the contribution of the higher states and continuum, we apply double Borel transformation with respect to the variables p^2 and $(p+q)^2$. As a result of the above procedure we obtain:

$$\begin{aligned}
G_{M_1}(q^2 = 0) = & \frac{4}{m_T^6 g_T^2} e^{m_T^2/M^2} \left\{ \frac{-1}{24} M^2 E_0(x) \left[e_{q_1} m_{q_2} \langle \bar{q}_1 q_1 \rangle \left(\mathbb{A}(u_0) + 2(u + u_0) i_1(h_\gamma) - 2\tilde{i}_1(h_\gamma) \right) \right. \right. \\
& - e_{q_2} m_{q_1} \langle \bar{q}_2 q_2 \rangle \left(\mathbb{A}(\bar{u}_0) + 2(\bar{u} + u_0) i_2(h_\gamma) + \tilde{i}_2(h_\gamma) \right) \left. \right] \\
& + \frac{1}{32\pi^2} M^4 E_1(x) (e_{q_1} - e_{q_2}) m_{q_1} m_{q_2} \left(\gamma_E + \ln \frac{\Lambda^2}{M^2} \right) \\
& + \frac{1}{48\pi^2} M^4 E_1(x) (3 - 4u_0) (e_{q_1} - e_{q_2}) m_{q_1} m_{q_2} \\
& - \frac{1}{48} f_{3\gamma} M^4 E_1(x) \left[e_{q_2} \left(8i_2(\psi_v) - \psi_a(\bar{u}_0) + (\bar{u} + u_0) (4\psi_v(\bar{u}_0) + \psi'_a(\bar{u}_0)) \right) \right. \\
& - e_{q_1} \left(8i_1(\psi_v) - \psi_a(u_0) + (u + u_0) (4\psi_v(u_0) - \psi'_a(u_0)) \right) \left. \right] \\
& - \frac{1}{240\pi^2} M^6 E_2(x) (5 - 18u_0) (e_{q_1} - e_{q_2}) \\
& + \frac{1}{32\pi^2} (e_{q_1} - e_{q_2}) m_{q_1} m_{q_2} \tilde{j}(s_0) \\
& + \frac{1}{16M^2} m_0^2 \left(e_{q_2} m_{q_2} \langle \bar{q}_1 q_1 \rangle - e_{q_1} m_{q_1} \langle \bar{q}_2 q_2 \rangle \right) \left[j(s_0) + M^2 \left(2\gamma_E + \ln \frac{\Lambda^2}{M^2} \right) \right] \\
& - \frac{1}{32\pi^2} M^4 E_1(x) (e_{q_1} - e_{q_2}) m_{q_1} m_{q_2} \\
& - \left. \frac{1}{72} m_0^2 [e_{q_2} \langle \bar{q}_1 q_1 \rangle (m_{q_1} - 3m_{q_2}) + e_{q_1} \langle \bar{q}_2 q_2 \rangle (3m_{q_1} - m_{q_2})] \right\}, \tag{19}
\end{aligned}$$

where

$$\begin{aligned}
i_1(f(u')) &= \int_{u_0}^1 du' f(u'), \\
\tilde{i}_1(f(u')) &= \int_{u_0}^1 du' (u' - u_0) f(u'), \\
i_2(f(u')) &= \int_0^{\bar{u}_0} du' f(u'), \\
\tilde{i}_2(f(u')) &= \int_0^{\bar{u}_0} du' (u' - \bar{u}_0) f(u'), \\
j(s_0) &= \int_0^{s_0} ds \left(\ln \frac{s}{\Lambda^2} e^{-s/M^2} \right), \\
\tilde{j}(s_0) &= \int_0^{s_0} ds \left(s \ln \frac{s}{\Lambda^2} e^{-s/M^2} \right), \\
E_n(x) &= 1 - e^{-x} \sum_{k=0}^n \frac{x^k}{k!} = \frac{1}{n!} \int_0^x dx' x'^n e^{-x'},
\end{aligned}$$

with $x = s_0/M^2$, s_0 being the continuum threshold, and the Borel parameter M^2 is defined as:

$$M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}, \tag{20}$$

and

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2} \approx \frac{m_i^2}{m_i^2 + m_f^2}, \quad (21)$$

where, m_i and m_f are the mass of the initial and final states. Remembering that the mass of the initial and final states are the same, therefore it is quite natural to expect that the Borel mass parameters should be very close, hence we set $M_1^2 = M_2^2 = 2M^2$ and $u_0 = 1/2$, which means that the quark and antiquark each carries half of the photon's momentum. Here, we should say that terms proportional to the gluon field strength tensor and quark condensates are small compared to the main nonperturbative contribution comes from leading twist distribution amplitudes existing in light cone QCD sum rules approach. Therefore, we have omitted numerically ignorable terms proportional to the gluon field strength tensor from the expression of the magnetic dipole moment in Eq. (19).

3 Numerical analysis

This section encompasses our numerical analysis on the magnetic dipole moment of the light tensor mesons, G_{M_1} . The parameters used in the analysis of the sum rules are as follows: $\langle \bar{u}u \rangle(1 \text{ GeV}) = \langle \bar{d}d \rangle(1 \text{ GeV}) = -(0.243)^3 \text{ GeV}^3$, $\langle \bar{s}s \rangle(1 \text{ GeV}) = 0.8 \langle \bar{u}u \rangle(1 \text{ GeV})$, $m_0^2(1 \text{ GeV}) = (0.8 \pm 0.2) \text{ GeV}^2$ [26], $m_{f_2} = (1275 \pm 1.2) \text{ MeV}$, $m_{a_2} = (1318.3 \pm 0.6) \text{ MeV}$, $m_{K_2^*}(1430) = (1425.6 \pm 1.5) \text{ MeV}$ [27] and $f_{3\gamma} = -0.0039 \text{ GeV}^2$ [25]. The magnetic susceptibility is chosen as $\chi(1 \text{ GeV}) = -(3.15 \pm 0.3) \text{ GeV}^{-2}$ [25]. From sum rules for the magnetic dipole G_{M_1} , it is clear that we also need to know the decay constants of the light unflavored and strange tensor mesons. Their values are taken as $g_T = 0.04$ [18] and $g_T = 0.050 \pm 0.002$ [19], respectively. The explicit forms of the photon DA's which are needed in the numerical calculations are as follows [25]:

$$\begin{aligned} \varphi_\gamma(u) &= 6u\bar{u} \left[1 + \varphi_2(\mu) C_2^{\frac{3}{2}}(u - \bar{u}) \right], \\ \psi^v(u) &= 3[3(2u - 1)^2 - 1] + \frac{3}{64}(15w_\gamma^V - 5w_\gamma^A)[3 - 30(2u - 1)^2 + 35(2u - 1)^4], \\ \psi^a(u) &= [1 - (2u - 1)^2][5(2u - 1)^2 - 1] \frac{5}{2} \left(1 + \frac{9}{16}w_\gamma^V - \frac{3}{16}w_\gamma^A \right), \\ \mathcal{A}(\alpha_i) &= 360\alpha_q\alpha_{\bar{q}}\alpha_g^2 \left[1 + w_\gamma^A \frac{1}{2}(7\alpha_g - 3) \right], \\ \mathcal{V}(\alpha_i) &= 540w_\gamma^V(\alpha_q - \alpha_{\bar{q}})\alpha_q\alpha_{\bar{q}}\alpha_g^2, \\ h_\gamma(u) &= -10(1 + 2\kappa^+)C_2^{\frac{1}{2}}(u - \bar{u}), \\ \mathbb{A}(u) &= 40u^2\bar{u}^2(3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2)[u\bar{u}(2 + 13u\bar{u}) + 2u^3(10 - 15u + 6u^2)\ln(u) \\ &\quad + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln(\bar{u})], \\ \mathcal{T}_1(\alpha_i) &= -120(3\zeta_2 + \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \\ \mathcal{T}_2(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)[(\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)], \\ \mathcal{T}_3(\alpha_i) &= -120(3\zeta_2 - \zeta_2^+)(\alpha_{\bar{q}} - \alpha_q)\alpha_{\bar{q}}\alpha_q\alpha_g, \end{aligned}$$

$$\begin{aligned}
\mathcal{T}_4(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)[(\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)] , \\
\mathcal{S}(\alpha_i) &= 30\alpha_g^2\{(\kappa + \kappa^+)(1 - \alpha_g) + (\zeta_1 + \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \\
&\quad + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\} , \\
\tilde{\mathcal{S}}(\alpha_i) &= -30\alpha_g^2\{(\kappa - \kappa^+)(1 - \alpha_g) + (\zeta_1 - \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \\
&\quad + \zeta_2[3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\}. \tag{22}
\end{aligned}$$

The constants entering the above DA's are borrowed from [5] whose values are $\varphi_2(1 \text{ GeV}) = 0$, $w_\gamma^V = 3.8 \pm 1.8$, $w_\gamma^A = -2.1 \pm 1.0$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\zeta_1^+ = 0$ and $\zeta_2^+ = 0$.

The sum rules for the magnetic dipole moment of the tensor mesons contain two more auxiliary parameters, namely, continuum threshold s_0 and Borel parameter M^2 . The continuum threshold is not completely arbitrary and it is related to the energy of the excited states. From our analysis we observe that G_{M_1} is practically independent of s_0 in the interval $(m_T + 0.3)^2 \leq s_0 \leq (m_T + 0.6)^2$. Note that, this region of s_0 practically coincides with the region of s_0 used in analysis of the mass sum rules for the light tensor mesons (for details see [28]). The working region for the Borel parameter M^2 is obtained by the following procedure: The upper limit of M^2 is determined by requiring that the series of the light cone expansion with increasing twist should be convergent. The lower limit of M^2 is obtained by the fact that the contribution of the higher states and continuum to the correlation function should be small enough. The above requirements restrict the working region of the Borel parameter to $1 \text{ GeV}^2 \leq M^2 \leq 3 \text{ GeV}^2$.

Using the photon DA's and the working regions of s_0 and M^2 , we obtain the values of the magnetic dipole moments of the light tensor mesons for both the charged and neutral cases, which are presented in Table 1.

Tensor Meson	$G_{M_1} (e/2m_T)$
f_2^\pm	2.1 ± 0.5
f_2^0	0.0 ± 0.0
a_2^\pm	1.88 ± 0.4
a_2^0	0.0 ± 0.0
$K_2^{*\pm}(1430)$	0.75 ± 0.08
$K_2^{*0}(1430)$	0.076 ± 0.008

Table 1: Magnetic dipole moments of the tensor mesons in units of $e/2m_T$.

The values of the magnetic dipole moments presented in Table 1 are in units of $e/2m_T$, and the quoted errors are due to the uncertainties in the input parameters, that is, the parameters entering the photon DA's, as well as, the working region for the continuum threshold s_0 , and the Borel parameter M^2 . We see from the table that the value of the magnetic dipole moments for the charged tensor mesons are considerably different from zero, which can be attributed to the response of the tensor mesons to an external magnetic field. The magnetic dipole moments for the neutral and unflavored light tensor mesons are very close to zero, while it has a nonzero small value for the $K_2^{*0}(1430)$ tensor meson.

Finally, let us compare our results on magnetic moments of light tensor mesons with the existing lattice QCD results [12]. For example, the magnetic moment of $K_2^{*\pm}(1430)$ mesons in lattice QCD change between ± 0.5 and ± 0.8 , depending on effective mass of the π meson and our analysis predicts $\pm(0.75 \pm 0.08)$. We see that our prediction is in good agreement with the lattice results, especially for the large effective π meson mass case. Our result on the magnetic moment of the neutral $K_2^{*0}(1430)$ meson is $\pm(0.076 \pm 0.008)$, which is slightly higher compared to the lattice QCD prediction ± 0.05 . Note that in calculations, the SU(2) flavor symmetry is implied. The nonzero value of the magnetic moment of $K_2^{*0}(1430)$ is due to the SU(3) flavor symmetry breaking.

In summary, the magnetic dipole moment of the light tensor mesons have been calculated in the framework of the LCSR using the photon DA's. We observe that the magnetic moments of charged light tensor mesons are practically 2.5–3 times larger compared to that of the magnetic moments of the charged strange tensor mesons. The magnetic moment of the neutral strange tensor meson is also nonzero, but its value is small. However, the values of the magnetic dipole moments of the light, neutral, unflavored tensor mesons are very close to zero.

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