

# Flavor Changing Neutral Currents Transition of the $\Sigma_Q$ to Nucleon in Full QCD and Heavy Quark Effective Theory

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The loop level flavor changing neutral currents transitions of the  $\Sigma_b \rightarrow n l^+ l^-$  and  $\Sigma_c \rightarrow p l^+ l^-$  are investigated in full QCD and heavy quark effective theory in the light cone QCD sum rules approach. Using the most general form of the interpolating current for  $\Sigma_Q$ ,  $Q = b$  or  $c$ , as members of the recently discovered sextet heavy baryons with spin 1/2 and containing one heavy quark, the transition form factors are calculated using two sets of input parameters entering the nucleon distribution amplitudes. The obtained results are used to estimate the decay rates of the corresponding transitions. Since such type transitions occurred at loop level in the standard model, they can be considered as good candidates to search for the new physics effects beyond the SM.

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## I. INTRODUCTION

The  $\Sigma_b \rightarrow n l^+ l^-$  and  $\Sigma_c \rightarrow p l^+ l^-$  are governed by flavor changing neutral currents (FCNC) transitions of  $b \rightarrow d$  and  $c \rightarrow u$ , respectively. These transitions are described via electroweak penguin and weak box diagrams in the standard model (SM) and they are sensitive to new physics contributing to penguin operators. Looking for SUSY particles [1], light dark matter [2] and also probable fourth generation of the quarks is possible by investigating such loop level transitions. This transitions are also good framework to reliable determination of the  $V_{tb}$ ,  $V_{td}$ ,  $V_{cb}$ , and  $V_{bu}$  as members of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, CP and T violations and polarization asymmetries. The  $\Sigma_{b,c}$  as members of the spin 1/2 sextet heavy baryons containing a single heavy bottom or charm quark are considered by their most general interpolating currents which generalize the Ioffe current for these baryons. In the recent years, important experimental progresses has been made in the spectroscopy of the heavy baryons containing heavy b or c quark [3–10]. Having the heavy quark makes these states be experimentally narrow, so their isolation and detection are easy comparing with the light baryons. Experimentally, investigation of the semileptonic decays of the heavy baryons, may be considered at large hadron collider (LHC) in the future, hence theoretical calculations of the decay properties can play crucial role in this respect.

In our two recent works, we analyzed the tree level semileptonic decays of  $\Sigma_b$  to proton [11] and  $\Lambda_b(\Lambda_c) \rightarrow p(n)l\nu$  [12] in light cone QCD sum rules. In full theory, these tree level transitions in the SM are described via six form factors (for details and more about the works devoted to the semileptonic decays of the heavy baryons using different phenomenological methods see [11, 12] and references therein). In the present work, considering the long and short distance effects, we calculate the 12 form factors entering the semileptonic loop level  $\Sigma_b \rightarrow n l^+ l^-$  and  $\Sigma_c \rightarrow p l^+ l^-$  transitions using the light cone QCD sum rules in full theory as well as heavy quark effective theory (HQET). The short distance effects are calculated using the perturbation theory and long distance contributions are expanded in terms of the nucleon distribution amplitudes (DA's) with increasing twists near the light cone,  $x^2 \simeq 0$ . We use the value of the eight independent parameters entering to the nucleon DA's from two different sources: predicted using a simple model in which the deviation from the asymptotic DAs is taken to be 1/3 of that suggested by the QCD sum rule estimates [13] and obtained via lattice QCD [14–16]. Using the obtained form factors, we predict the corresponding transition rates. Investigation of these decays can also give essential information about the internal structure of  $\Sigma_{b,c}$  baryons as well as the nucleon DA's.

The layout of the paper is as follows: in section II, we introduce the theoretical framework to calculate the form factors in light cone QCD sum rules method in full theory. The HQET relations among the form factors are also introduced in this section. Section III is devoted to the numerical analysis of the form factors and their extrapolation in terms of the transferred momentum squared,  $q^2$ , their HQET limit and our predictions for the decay rates obtained in two different sets of parameters entering the nucleon distribution amplitudes.

## II. LIGHT CONE QCD SUM RULES FOR TRANSITION FORM FACTORS

Form factors play essential role in analyzing the  $\Sigma_b \rightarrow n \ l^+l^-$  and  $\Sigma_c \rightarrow p \ l^+l^-$  transitions. At quark level, these decays proceed by loop  $b \ (c) \rightarrow d \ (u)$  transition, which can be described by the following effective Hamiltonian:

$$\mathcal{H}_{eff} = \frac{G_F \alpha V_{Q'Q} V_{Q'q}^*}{2\sqrt{2} \pi} \left\{ C_9^{eff} \bar{q}\gamma_\mu(1 - \gamma_5)Q\bar{l}\gamma^\mu l + C_{10} \bar{q}\gamma_\mu(1 - \gamma_5)Q\bar{l}\gamma^\mu\gamma_5 l - 2m_Q C_7 \frac{1}{q^2} \bar{q}i\sigma_{\mu\nu}q^\nu(1 + \gamma_5)Q\bar{l}\gamma^\mu l \right\}, \quad (1)$$

where,  $Q'$  refers to the  $u, c, t$  for bottom case and  $d, s, b$  for charm case, respectively. The main contributions come from the heavy quarks, so we will consider  $Q' = t$  and  $Q' = b$  respectively for the  $\Sigma_b \rightarrow n \ l^+l^-$  and  $\Sigma_c \rightarrow p \ l^+l^-$  transitions. The amplitude of the considered transitions can be obtained by sandwiching the above Hamiltonian between the initial and final states. To proceed, we need to know the the matrix elements  $\langle N | J_\mu^{tr,I} | \Sigma_Q \rangle$  and  $\langle N | J_\mu^{tr,II} | \Sigma_Q \rangle$ , where  $J_\mu^{tr,I}(x) = \bar{q}(x)\gamma_\mu(1 - \gamma_5)Q(x)$  and  $J_\mu^{tr,II}(x) = \bar{q}(x)i\sigma_{\mu\nu}q^\nu(1 + \gamma_5)Q(x)$  are transition currents entering to the Hamiltonian. From the general philosophy of the QCD sum rules, to obtain sum rules for the physical quantities we start considering the following correlation functions:

$$\begin{aligned} \Pi_\mu^I(p, q) &= i \int d^4x e^{iqx} \langle N(p) | T \{ J_\mu^{tr,I}(x) \bar{J}^{\Sigma_Q}(0) \} | 0 \rangle, \\ \Pi_\mu^{II}(p, q) &= i \int d^4x e^{iqx} \langle N(p) | T \{ J_\mu^{tr,II}(x) \bar{J}^{\Sigma_Q}(0) \} | 0 \rangle, \end{aligned} \quad (2)$$

where,  $J^{\Sigma_Q}$  is interpolating currents of  $\Sigma_{b(c)}$  baryon and  $p$  denotes the proton (neutron) momentum and  $q = (p+q) - p$  is the transferred momentum. The main idea in QCD sum rules is to calculate the aforementioned correlation functions in two different ways:

- In theoretical side, the time ordering product of the initial state and transition current is expanded in terms of nucleon distribution amplitudes having different twists via the operator product expansion (OPE) at deep Euclidean region. By OPE the short and large distance effects are separated. The short distance contribution is calculated using the perturbation theory, while the long distance phenomena are parameterized in terms of nucleon DA's.
- From phenomenological or physical side, they are calculated in terms of the hadronic parameters via saturating them with a tower of hadrons with the same quantum numbers as the interpolating currents.

To get the sum rules for the physical quantities, the two above representations of the correlation functions are equated through the dispersion relation. To suppress the contribution of the higher states and continuum and isolate the ground state, the Borel transformation as well as continuum subtraction through quark-hadron duality assumption are applied to both sides of the sum rules expressions.

The first task is to calculate the aforementioned correlation function from QCD side in deep Euclidean region where  $(p+q)^2 \ll 0$ . To proceed, the explicit expression of the interpolating field of the  $\Sigma_Q$  baryon is needed. Considering the quantum numbers, the most general form of interpolating current creating the  $\Sigma_Q$  from the vacuum can be written as

$$\begin{aligned} J^{\Sigma_b}(x) &= \frac{-1}{\sqrt{2}} \epsilon^{abc} \left[ \left\{ q_1^{Ta}(x) C Q^b(x) \right\} \gamma_5 q_2^c(x) - \left\{ Q^{Ta}(x) C q_2^b(x) \right\} \gamma_5 q_1^c(x) \right. \\ &\quad \left. + \beta \left\{ \left\{ q_1^{Ta}(x) C \gamma_5 Q^b(x) \right\} q_2^c(x) - \left\{ Q^{Ta}(x) C \gamma_5 q_2^b(x) \right\} q_1^c(x) \right\} \right], \end{aligned} \quad (3)$$

where,  $C$  is the charge conjugation operator and  $\beta$  is an arbitrary parameter with  $\beta = -1$  corresponding to the Ioffe current,  $q_1$  and  $q_2$  are the  $u$  and  $d$  quarks, respectively and  $a, b, c$  are the color indices. Using the transition currents, and  $J^{\Sigma_Q}$  and contracting out all quark pairs via the Wick's theorem, we obtain the following representations of the correlation functions in QCD side:

$$\begin{aligned} \Pi_\mu^I &= \frac{-i}{\sqrt{2}} \epsilon^{abc} \int d^4x e^{iqx} \left\{ \left( \left[ (C)_{\beta\eta}(\gamma_5)_{\rho\phi} - (C)_{\phi\beta}(\gamma_5)_{\rho\eta} \right] + \beta \left[ (C\gamma_5)_{\beta\eta}(I)_{\rho\phi} \right. \right. \right. \\ &\quad \left. \left. - (C\gamma_5)_{\phi\beta}(I)_{\rho\eta} \right] \right) \left[ \gamma_\mu(1 - \gamma_5) \right]_{\sigma\theta} \left. \right\} S_Q(-x)_{\beta\sigma} \langle N(p) | \bar{d}_\eta^a(0) \bar{d}_\theta^b(x) \bar{u}_\phi^c(0) | 0 \rangle, \end{aligned} \quad (4)$$

$$\begin{aligned} \Pi_{\mu}^{II} = & \frac{-i}{\sqrt{2}} \epsilon^{abc} \int d^4x e^{iqx} \left\{ \left( \left[ (C)_{\beta\eta}(\gamma_5)_{\rho\phi} - (C)_{\phi\beta}(\gamma_5)_{\rho\eta} \right] + \beta \left[ (C\gamma_5)_{\beta\eta}(I)_{\rho\phi} \right. \right. \right. \\ & \left. \left. \left. - (C\gamma_5)_{\phi\beta}(I)_{\rho\eta} \right] \right) \left[ i\sigma_{\mu\nu} q^{\nu} (1 + \gamma_5) \right]_{\sigma\theta} \right\} S_Q(-x)_{\beta\sigma} \langle N(p) | \bar{d}_{\eta}^a(0) \bar{d}_{\theta}^b(x) \bar{u}_{\phi}^c(0) | 0 \rangle, \end{aligned} \quad (5)$$

where,  $S_Q(x)$  is the heavy quark propagator and its expression is given as [17]:

$$S_Q(x) = S_Q^{free}(x) - ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 dv \left[ \frac{k + m_Q}{(m_Q^2 - k^2)^2} G^{\mu\nu}(vx) \sigma_{\mu\nu} + \frac{1}{m_Q^2 - k^2} vx_{\mu} G^{\mu\nu} \gamma_{\nu} \right]. \quad (6)$$

where

$$S_Q^{free} = \frac{m_Q^2}{4\pi^2} \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} - i \frac{m_Q^2}{4\pi^2 x^2} K_2(m_Q \sqrt{-x^2}), \quad (7)$$

and  $K_i$  are the Bessel functions. When doing calculations, we neglect the terms proportional to the gluon field strength tensor because they are contributed mainly to the four and five particle distribution functions and expected to be very small in our case [18–20]. The matrix element  $\langle N(p) | \epsilon^{abc} \bar{d}_{\eta}^a(0) \bar{d}_{\theta}^b(x) \bar{u}_{\phi}^c(0) | 0 \rangle$  appearing in Eqs. (4,5) denotes the nucleon wave function, which is given in terms of some calligraphic functions [13, 18–21]:

$$\begin{aligned} & 4 \langle 0 | \epsilon^{abc} d_{\alpha}^a(a_1x) d_{\beta}^b(a_2x) u_{\gamma}^c(a_3x) | N(p) \rangle \\ = & S_1 m_N C_{\alpha\beta}(\gamma_5 N)_{\gamma} + S_2 m_N^2 C_{\alpha\beta}(\not{x} \gamma_5 N)_{\gamma} \\ + & P_1 m_N (\gamma_5 C)_{\alpha\beta} N_{\gamma} + P_2 m_N^2 (\gamma_5 C)_{\alpha\beta} (\not{x} N)_{\gamma} + (\mathcal{V}_1 + \frac{x^2 m_N^2}{4} \mathcal{V}_1^M) (\not{p} C)_{\alpha\beta} (\gamma_5 N)_{\gamma} \\ + & \mathcal{V}_2 m_N (\not{p} C)_{\alpha\beta} (\not{x} \gamma_5 N)_{\gamma} + \mathcal{V}_3 m_N (\gamma_{\mu} C)_{\alpha\beta} (\gamma^{\mu} \gamma_5 N)_{\gamma} + \mathcal{V}_4 m_N^2 (\not{x} C)_{\alpha\beta} (\gamma_5 N)_{\gamma} \\ + & \mathcal{V}_5 m_N^2 (\gamma_{\mu} C)_{\alpha\beta} (i\sigma^{\mu\nu} x_{\nu} \gamma_5 N)_{\gamma} + \mathcal{V}_6 m_N^3 (\not{x} C)_{\alpha\beta} (\not{x} \gamma_5 N)_{\gamma} + (\mathcal{A}_1 \\ + & \frac{x^2 m_N^2}{4} \mathcal{A}_1^M) (\not{p} \gamma_5 C)_{\alpha\beta} N_{\gamma} + \mathcal{A}_2 m_N (\not{p} \gamma_5 C)_{\alpha\beta} (\not{x} N)_{\gamma} + \mathcal{A}_3 m_N (\gamma_{\mu} \gamma_5 C)_{\alpha\beta} (\gamma^{\mu} N)_{\gamma} \\ + & \mathcal{A}_4 m_N^2 (\not{x} \gamma_5 C)_{\alpha\beta} N_{\gamma} + \mathcal{A}_5 m_N^2 (\gamma_{\mu} \gamma_5 C)_{\alpha\beta} (i\sigma^{\mu\nu} x_{\nu} N)_{\gamma} + \mathcal{A}_6 m_N^3 (\not{x} \gamma_5 C)_{\alpha\beta} (\not{x} N)_{\gamma} \\ + & (\mathcal{T}_1 + \frac{x^2 m_N^2}{4} \mathcal{T}_1^M) (p^{\nu} i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma^{\mu} \gamma_5 N)_{\gamma} + \mathcal{T}_2 m_N (x^{\mu} p^{\nu} i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma_5 N)_{\gamma} \\ + & \mathcal{T}_3 m_N (\sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\nu} \gamma_5 N)_{\gamma} + \mathcal{T}_4 m_N (p^{\nu} \sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\rho} x_{\rho} \gamma_5 N)_{\gamma} \\ + & \mathcal{T}_5 m_N^2 (x^{\nu} i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma^{\mu} \gamma_5 N)_{\gamma} + \mathcal{T}_6 m_N^2 (x^{\mu} p^{\nu} i\sigma_{\mu\nu} C)_{\alpha\beta} (\not{x} \gamma_5 N)_{\gamma} \\ + & \mathcal{T}_7 m_N^2 (\sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\nu} \not{x} \gamma_5 N)_{\gamma} + \mathcal{T}_8 m_N^3 (x^{\nu} \sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\rho} x_{\rho} \gamma_5 N)_{\gamma}. \end{aligned} \quad (8)$$

The calligraphic functions have not definite twists but they can be expressed in terms of the nucleon distribution amplitudes (DA's) with definite and increasing twists by the help of the scalar product  $px$  and the parameters  $a_i$ ,  $i = 1, 2, 3$ . The explicit expressions for scalar, pseudo-scalar, vector, axial vector and tensor DA's for nucleons are given in Tables I, II, III, IV and V, respectively.

$S_1 = S_1$
$2px S_2 = S_1 - S_2$

TABLE I: Relations between the calligraphic functions and nucleon scalar DA's.

Each distribution amplitude  $F(a_i px) = S_i, P_i, V_i, A_i, T_i$  can be expressed as:

$$F(a_i px) = \int dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1) e^{ipx \Sigma_i x_i a_i} F(x_i). \quad (9)$$

$\mathcal{P}_1 = P_1$
$2px\mathcal{P}_2 = P_1 - P_2$

TABLE II: Relations between the calligraphic functions and nucleon pseudo-scalar DA's.

$\mathcal{V}_1 = V_1$
$2px\mathcal{V}_2 = V_1 - V_2 - V_3$
$2\mathcal{V}_3 = V_3$
$4px\mathcal{V}_4 = -2V_1 + V_3 + V_4 + 2V_5$
$4px\mathcal{V}_5 = V_4 - V_3$
$4(px)^2\mathcal{V}_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6$

TABLE III: Relations between the calligraphic functions and nucleon vector DA's.

where,  $x_i$  with  $i = 1, 2$  and  $3$  are longitudinal momentum fractions carried by the participating quarks. Using the nucleon wave functions, which their explicit expressions are calculated in [13] and the expression for the heavy quark propagator, and after performing the Fourier transformation, the final expressions of the correlation functions for both vertexes are found in terms of the nucleon DA's in QCD or theoretical side. For simplicity, we present the explicit expressions of the nucleon DA's in the Appendix.

The next step is to calculate the phenomenological or physical sides of the correlation functions. Saturating the correlation functions with a complete set of the initial state, isolating the ground state and performing the integral over  $x$ , we get:

$$\Pi_\mu^I(p, q) = \sum_s \frac{\langle N(p) | J_\mu^{tr,I}(0) | \Sigma_Q(p+q, s) \rangle \langle \Sigma_Q(p+q, s) | \bar{J}^{\Sigma_Q}(0) | 0 \rangle}{m_{\Sigma_Q}^2 - (p+q)^2} + \dots, \quad (10)$$

$$\Pi_\mu^{II}(p, q) = \sum_s \frac{\langle N(p) | J_\mu^{tr,II}(0) | \Sigma_Q(p+q, s) \rangle \langle \Sigma_Q(p+q, s) | \bar{J}^{\Sigma_Q}(0) | 0 \rangle}{m_{\Sigma_Q}^2 - (p+q)^2} + \dots, \quad (11)$$

where, the ... denotes the contribution of the higher states and continuum. The baryonic to the vacuum matrix element of the interpolating current, i.e.,  $\langle \Sigma_Q(p+q, s) | \bar{J}^{\Sigma_Q}(0) | 0 \rangle$  can be parameterized in terms of the residue,  $\lambda_{\Sigma_Q}$  as:

$$\langle \Sigma_Q(p+q, s) | \bar{J}^{\Sigma_Q}(0) | 0 \rangle = \lambda_{\Sigma_Q} \bar{u}_{\Sigma_Q}(p+q, s). \quad (12)$$

To proceed, we also need to know the transition matrix elements,  $\langle N(p) | J_\mu^{tr,I} | \Sigma_Q(p+q, s) \rangle$  and  $\langle N(p) | J_\mu^{tr,II} | \Sigma_Q(p+q, s) \rangle$ . In full theory, they are parameterized in terms of 12 transition form factors,  $f_i, g_i, f_i^T$  and  $g_i^T$  with  $i = 1 \rightarrow 3$  by the following way:

$$\begin{aligned} \langle N(p) | J_\mu^{tr,I}(x) | \Sigma_Q(p+q) \rangle &= \bar{N}(p) \left[ \gamma_\mu f_1(Q^2) + i\sigma_{\mu\nu} q^\nu f_2(Q^2) + q^\mu f_3(Q^2) - \gamma_\mu \gamma_5 g_1(Q^2) - i\sigma_{\mu\nu} \gamma_5 q^\nu g_2(Q^2) \right. \\ &\quad \left. - q^\mu \gamma_5 g_3(Q^2) \right] u_{\Sigma_Q}(p+q), \end{aligned} \quad (13)$$

and

$$\begin{aligned} \langle N(p) | J_\mu^{tr,II}(x) | \Sigma_Q(p+q) \rangle &= \bar{N}(p) \left[ \gamma_\mu f_1^T(Q^2) + i\sigma_{\mu\nu} q^\nu f_2^T(Q^2) + q^\mu f_3^T(Q^2) + \gamma_\mu \gamma_5 g_1^T(Q^2) + i\sigma_{\mu\nu} \gamma_5 q^\nu g_2^T(Q^2) \right. \\ &\quad \left. + q^\mu \gamma_5 g_3^T(Q^2) \right] u_{\Sigma_Q}(p+q), \end{aligned} \quad (14)$$

where  $Q^2 = -q^2$ . Here,  $N(p)$  and  $u_{\Sigma_Q}(p+q)$  are the spinors of nucleon and  $\Sigma_Q$ , respectively. Using Eqs. (10), (11), (12), (13) and (14) and performing summation over spins of the  $\Sigma_Q$  baryon using

$$\sum_s u_{\Sigma_Q}(p+q, s) \bar{u}_{\Sigma_Q}(p+q, s) = \not{p} + \not{q} + m_{\Sigma_Q}, \quad (15)$$

$\mathcal{A}_1 = A_1$
$2px\mathcal{A}_2 = -A_1 + A_2 - A_3$
$2\mathcal{A}_3 = A_3$
$4px\mathcal{A}_4 = -2A_1 - A_3 - A_4 + 2A_5$
$4px\mathcal{A}_5 = A_3 - A_4$
$4(px)^2\mathcal{A}_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6$

TABLE IV: Relations between the calligraphic functions and nucleon axial vector DA's.

$\mathcal{T}_1 = T_1$
$2px\mathcal{T}_2 = T_1 + T_2 - 2T_3$
$2\mathcal{T}_3 = T_7$
$2px\mathcal{T}_4 = T_1 - T_2 - 2T_7$
$2px\mathcal{T}_5 = -T_1 + T_5 + 2T_8$
$4(px)^2\mathcal{T}_6 = 2T_2 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8$
$4px\mathcal{T}_7 = T_7 - T_8$
$4(px)^2\mathcal{T}_8 = -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8$

TABLE V: Relations between the calligraphic functions and nucleon tensor DA's.

we obtain the following expressions

$$\begin{aligned} \Pi_\mu^I(p, q) = & \frac{\lambda_{\Sigma_Q}}{m_{\Sigma_Q}^2 - (p+q)^2} \bar{N}(p) \left[ \gamma_\mu f_1(Q^2) + i\sigma_{\mu\nu} q^\nu f_2(Q^2) + q^\mu f_3(Q^2) - \gamma_\mu \gamma_5 g_1(Q^2) - i\sigma_{\mu\nu} \gamma_5 q^\nu g_2(Q^2) \right. \\ & \left. - q^\mu \gamma_5 g_3(Q^2) \right] (\not{p} + \not{q} + m_{\Sigma_Q}) + \dots \end{aligned} \quad (16)$$

and

$$\begin{aligned} \Pi_\mu^{II}(p, q) = & \frac{\lambda_{\Sigma_Q}}{m_{\Sigma_Q}^2 - (p+q)^2} \bar{N}(p) \left[ \gamma_\mu f_1^T(Q^2) + i\sigma_{\mu\nu} q^\nu f_2^T(Q^2) + q^\mu f_3^T(Q^2) + \gamma_\mu \gamma_5 g_1^T(Q^2) + i\sigma_{\mu\nu} \gamma_5 q^\nu g_2^T(Q^2) \right. \\ & \left. + q^\mu \gamma_5 g_3^T(Q^2) \right] (\not{p} + \not{q} + m_{\Sigma_Q}) + \dots \end{aligned} \quad (17)$$

Using the relation

$$\bar{N} \sigma_{\mu\nu} q^\nu u_{\Sigma_Q} = i \bar{N} [(m_N + m_{\Sigma_Q}) \gamma_\mu - (2p + q)_\mu] u_{\Sigma_Q}, \quad (18)$$

in Eqs. (16) and Eqs. (17), we attain the final expressions for the physical side of the correlation functions:

$$\begin{aligned} \Pi_\lambda^I(p, q) = & \frac{\lambda_{\Sigma_Q}}{m_{\Sigma_Q}^2 - (p+q)^2} \bar{N}(p) \left[ 2f_1(Q^2) p_\mu + \left\{ -f_1(Q^2)(m_N - m_{\Sigma_Q}) + f_2(Q^2)(m_N^2 - m_{\Sigma_Q}^2) \right\} \gamma_\mu \right. \\ & + \left\{ f_1(Q^2) - f_2(Q^2)(m_N + m_{\Sigma_Q}) \right\} \gamma_\mu \not{q} + 2f_2(Q^2) p_\mu \not{q} + \left\{ f_2(Q^2) + f_3(Q^2) \right\} (m_N + m_{\Sigma_Q}) q_\mu \\ & + \left\{ f_2(Q^2) + f_3(Q^2) \right\} q_\mu \not{q} + 2g_1(Q^2) p_\mu \gamma_5 - \left\{ g_1(Q^2)(m_N + m_{\Sigma_Q}) - g_2(Q^2)(m_N^2 - m_{\Sigma_Q}^2) \right\} \gamma_\mu \gamma_5 + \\ & \left\{ g_1(Q^2) - g_2(Q^2)(m_N - m_{\Sigma_Q}) \right\} \gamma_\mu \not{q} \gamma_5 + 2g_2(Q^2) p_\mu \not{q} \gamma_5 + \left\{ g_2(Q^2) + g_3(Q^2) \right\} (m_N - m_{\Sigma_Q}) q_\mu \gamma_5 \\ & \left. + \left\{ g_2(Q^2) + g_3(Q^2) \right\} q_\mu \not{q} \gamma_5 \right] + \dots \end{aligned} \quad (19)$$

and

$$\begin{aligned}
 \Pi_{\lambda}^{II}(p, q) = & \frac{\lambda_{\Sigma_Q}}{m_{\Sigma_Q}^2 - (p+q)^2} \bar{N}(p) \left[ 2f_1^T(Q^2)p_{\mu} + \left\{ -f_1^T(Q^2)(m_N - m_{\Sigma_Q}) + f_2^T(Q^2)(m_N^2 - m_{\Sigma_Q}^2) \right\} \gamma_{\mu} \right. \\
 & + \left\{ f_1^T(Q^2) - f_2^T(Q^2)(m_N + m_{\Sigma_Q}) \right\} \gamma_{\mu} \not{q} + 2f_2^T(Q^2)p_{\mu} \not{q} + \left\{ f_2^T(Q^2) + f_3^T(Q^2) \right\} (m_N + m_{\Sigma_Q})q_{\mu} \\
 & + \left\{ f_2^T(Q^2) + f_3^T(Q^2) \right\} q_{\mu} \not{q} - 2g_1^T(Q^2)p_{\mu}\gamma_5 + \left\{ g_1^T(Q^2)(m_N + m_{\Sigma_Q}) - g_2^T(Q^2)(m_N^2 - m_{\Sigma_Q}^2) \right\} \gamma_{\mu}\gamma_5 - \\
 & \left\{ g_1^T(Q^2) - g_2^T(Q^2)(m_N - m_{\Sigma_Q}) \right\} \gamma_{\mu} \not{q}\gamma_5 - 2g_2^T(Q^2)p_{\mu} \not{q}\gamma_5 - \left\{ g_2^T(Q^2) + g_3^T(Q^2) \right\} (m_N - m_{\Sigma_Q})q_{\mu}\gamma_5 \\
 & \left. - \left\{ g_2^T(Q^2) + g_3^T(Q^2) \right\} q_{\mu} \not{q}\gamma_5 \right] + \dots
 \end{aligned} \tag{20}$$

In order to calculate the form factors or their combinations,  $f_1, f_2, f_2 + f_3, g_1, g_2$  and  $g_2 + g_3$ , we will choose the independent structures  $p_{\mu}, p_{\mu} \not{q}, q_{\mu} \not{q}, p_{\mu}\gamma_5, p_{\mu} \not{q}\gamma_5$ , and  $q_{\mu} \not{q}\gamma_5$  from Eq. (19), respectively. The same structures are chosen to calculate the form factors or their combinations labeled by T in the second correlation function in Eq. (20).

Having computed both sides of the correlation functions, it is time to obtain the sum rules for the related form factors. Equating the coefficients of the corresponding structures from both sides of the correlation functions through the dispersion relations and applying Borel transformation with respect to  $(p+q)^2$  to suppress the contribution of the higher states and continuum, one can obtain sum rules for the form factors  $f_1, f_2, f_3, g_1, g_2, g_3, f_1^T, f_2^T, f_3^T, g_1^T, g_2^T$  and  $g_3^T$ . In heavy quark effective theory (HQET), where  $m_Q \rightarrow \infty$ , the number of independent form factors is reduced to two, namely,  $F_1$  and  $F_2$ . In this limit, the transition matrix element can be parameterized in terms of these two form factors in the following way [22, 23]:

$$\langle N(p) | \bar{d}\Gamma b | \Sigma_Q(p+q) \rangle = \bar{N}(p)[F_1(Q^2) + \not{q}F_2(Q^2)]\Gamma u_{\Sigma_Q}(p+q), \tag{21}$$

where,  $\Gamma$  is any Dirac matrices and  $\not{q} = \frac{p \cdot q}{m_{\Sigma_Q}}$ . Here we should mention that the above relation is exact for  $\Lambda$ -like baryons, where the light degrees of freedom are spinless. For the  $\Sigma$  like baryons this relation cannot hold exactly and has to be replaced by a more complicated relation. In the present work, we will use the above approximate relation for the considered transitions. Comparing this matrix element and our definitions of the form factors in Eqs. (13) and (14), we get the following relations among the form factors in HQET limit (see also [24, 25])

$$\begin{aligned}
 f_1 &= g_1 = f_2^T = g_2^T = F_1 + \frac{m_N}{m_{\Lambda_b}} F_2 \\
 f_2 &= g_2 = f_3 = g_3 = \frac{F_2}{m_{\Sigma_Q}} \\
 f_1^T &= g_1^T = \frac{F_2}{m_{\Sigma_Q}} q^2 \\
 f_3^T &= -\frac{F_2}{m_{\Sigma_Q}} (m_{\Sigma_Q} - m_N) \\
 g_3^T &= \frac{F_2}{m_{\Sigma_Q}} (m_{\Sigma_Q} + m_N)
 \end{aligned} \tag{22}$$

Looking at the above relations, we see that it is possible to write all form factors in terms of  $f_1$  and  $f_2$ , so we will present the explicit expressions for these two form factors in the Appendix and give extrapolation of the other form factors in finite mass as well as HQET in terms of  $q^2$  in the numerical analysis section.

The expressions of the sum rules for form factors show that we need to know also the residue  $\lambda_{\Sigma_Q}$ . This residue is determined in [26]:

$$-\lambda_{\Sigma_Q}^2 e^{-m_{\Sigma_Q}^2/M_B^2} = \int_{m_q^2}^{s_0} e^{\frac{-s}{M_B^2}} \rho(s) ds + e^{\frac{-m_Q^2}{M_B^2}} \Gamma, \tag{23}$$

where,

$$\begin{aligned} \rho(s) = & (\langle \bar{d}d \rangle + \langle \bar{u}u \rangle) \frac{(\beta^2 - 1)}{64\pi^2} \left\{ \frac{m_0^2}{4m_Q} (6\psi_{00} - 13\psi_{02} - 6\psi_{11}) + 3m_Q (2\psi_{10} - \psi_{11} - \psi_{12} + 2\psi_{21}) \right\} \\ & + \frac{m_Q^4}{2048\pi^4} [5 + \beta(2 + 5\beta)] [12\psi_{10} - 6\psi_{20} + 2\psi_{30} - 4\psi_{41} + \psi_{42} - 12\ln(\frac{s}{m_Q^2})], \end{aligned} \quad (24)$$

and

$$\Gamma = \frac{(\beta - 1)^2}{24} \langle \bar{d}d \rangle \langle \bar{u}u \rangle \left[ \frac{m_Q^2 m_0^2}{2M_B^4} + \frac{m_0^2}{4M_B^2} - 1 \right]. \quad (25)$$

Here,  $\psi_{nm} = \frac{(s-m_Q^2)^n}{s^m (m_Q^2)^{n-m}}$  are some dimensionless functions.

### III. NUMERICAL RESULTS

This section deals with the numerical analysis of the form factors as well as the total decay rate of the loop level  $\Sigma_b \rightarrow n\ell^+\ell^-$  and  $\Sigma_c \rightarrow p\ell^+\ell^-$  transitions in both full theory and HQET limit. In obtaining numerical values, we use the following inputs for masses and quark condensates:  $\langle \bar{u}u \rangle(1 \text{ GeV}) = \langle \bar{d}d \rangle(1 \text{ GeV}) = -(0.243)^3 \text{ GeV}^3$ ,  $m_n = 0.939 \text{ GeV}$ ,  $m_p = 0.938 \text{ GeV}$ ,  $m_b = 4.7 \text{ GeV}$ ,  $m_c = 1.23 \text{ GeV}$ ,  $m_{\Sigma_b} = 5.805 \text{ GeV}$ ,  $m_{\Sigma_c} = 2.4529 \text{ GeV}$  and  $m_0^2(1 \text{ GeV}) = (0.8 \pm 0.2) \text{ GeV}^2$  [27]. From the sum rules expressions for the form factors, it is clear that the nucleon DA's (see Appendix) are the main input parameters. These DA's contain eight independent parameters, namely,  $f_N$ ,  $\lambda_1$ ,  $\lambda_2$ ,  $V_1^d$ ,  $A_1^u$ ,  $f_1^d$ ,  $f_1^u$  and  $f_2^d$ . All of these parameters have been calculated in the framework of the light cone QCD sum rules [13] and most of them are now available in lattice QCD [14–16] (see Table VI). Here, we should stress that in [13] those parameters are obtained both as QCD sum rules and asymptotic sets, but to improve the agreement with experimental data on nucleon form factors, a set of parameters is obtained using a simple model in which the deviation from the asymptotic DAs is taken to be 1/3 of that suggested by the QCD sum rule estimates (see [13]). We will use this set of parameters in this paper and refer it as set1 (see Table VI). In the following, we also will denote the lattice QCD input parameters by set2.

	set1 [13]	set2 or Lattice QCD [14–16]
$f_N$	$(5.0 \pm 0.5) \times 10^{-3} \text{ GeV}^2$	$(3.234 \pm 0.063 \pm 0.086) \times 10^{-3} \text{ GeV}^2$
$\lambda_1$	$-(2.7 \pm 0.9) \times 10^{-2} \text{ GeV}^2$	$(-3.557 \pm 0.065 \pm 0.136) \times 10^{-2} \text{ GeV}^2$
$\lambda_2$	$(5.4 \pm 1.9) \times 10^{-2} \text{ GeV}^2$	$(7.002 \pm 0.128 \pm 0.268) \times 10^{-2} \text{ GeV}^2$
$V_1^d$	0.30	$0.3015 \pm 0.0032 \pm 0.0106$
$A_1^u$	0.13	$0.1013 \pm 0.0081 \pm 0.0298$
$f_1^d$	0.33	–
$f_1^u$	0.09	–
$f_2^d$	0.25	–

TABLE VI: The values of the 8 independent parameters entering the nucleon DA's. The first errors in lattice values are statistical and the second errors correspond to the uncertainty due to the Chiral extrapolation and renormalization. For last tree parameters, the values are not available in lattice and we will use the set1 values for both sets of data.

The explicit expressions for the form factors also show their dependency to three auxiliary mathematical objects, namely, continuum threshold  $s_0$ , Borel mass parameter  $M_B^2$  and general parameter  $\beta$  entering to the most general form of the interpolating current of the initial state. The form factors as physical quantities should be independent of these parameters, hence we need to look for working regions for them. The working region for Borel mass squared is determined as follows: the upper limit of  $M_B^2$  is chosen demanding that the series of the light cone expansion with increasing twist should be convergent. The lower limit is determined from condition that the higher states and continuum contributions constitute a small fraction of total dispersion integral. Both conditions are satisfied in the

regions  $15 \text{ GeV}^2 \leq M_B^2 \leq 30 \text{ GeV}^2$  and  $4 \text{ GeV}^2 \leq M_B^2 \leq 12 \text{ GeV}^2$  for bottom and charm cases, respectively. The value of the continuum threshold  $s_0$  is not completely arbitrary and it is correlated to the first excited state with quantum numbers of the initial particle interpolating current. Our numerical calculations show that the form factors weakly depend on the continuum threshold in the interval,  $(m_{\Sigma_Q} + 0.5)^2 \leq s_0 \leq (m_{\Sigma_Q} + 0.7)^2$ . To obtain the working region for  $\beta$  at which the form factors are practically independent of it, we look for the variation of the form factors with respect to  $\cos\theta$  in the interval  $-1 \leq \cos\theta \leq 1$  which is equivalent to the  $-\infty \leq \beta \leq \infty$ , where  $\beta = \tan\theta$ . As a result, the interval  $-0.5 \leq \cos\theta \leq 0.6$  is obtained for  $\beta$  for both charm and bottom cases. In this interval, the dependency on this parameter is weak.

The next step is to discuss the behaviour of the form factors in terms of the  $q^2$ . The sum rules predictions for the form factors are not reliable in the whole physical region. To be able to extend the results for the form factors to the whole physical region, we look for a parametrization of the form factors such that in the reliable region which is approximately  $1 \text{ GeV}$  below the perturbative cut, the original form factors and their fit parametrization coincide each other. Our numerical results lead to the following extrapolation for the form factors in terms of  $q^2$ :

$$f_i(q^2)[g_i(q^2)] = \frac{a}{\left(1 - \frac{q^2}{m_{fit}^2}\right)} + \frac{b}{\left(1 - \frac{q^2}{m_{fit}^2}\right)^2}, \quad (26)$$

where the fit parameters  $a$ ,  $b$  and  $m_{fit}$  in full theory and HQET limit are given in Tables VII, VIII, IX and X using two sets for the independent parameters. These Tables, show poles of the form factors outside the allowed physical region. Therefore, the form factors are analytic in the full physical interval. In principle, we can use fit parametrization either with single pole or double poles. However, when we combine them the accuracy of the fitting becomes very high, specially when the pole is the same for two parts. We could start from  $f_i(q^2) = \frac{a}{1 - q^2/m_1^2} + \frac{b}{1 - q^2/m_{fit}^2}$ , however for all form factors  $m_{fit}$  gets too close to  $m_1$ , so the fit becomes numerically unstable. In such a case, it is appropriate to expand the above relation to first order in  $m_{fit} - m_1$ , which gives the Eq. (26) used to extrapolate the factors over the whole range of  $q^2$ . For the same situation in  $B \rightarrow D$  mesonic transition see for instance [28, 29]. The values of form factors at  $q^2 = 0$  are presented in Tables XI and XII in both full theory and HQET for bottom and charm cases, respectively. In extraction of the values of form factors at  $q^2 = 0$ , the mean values of the form factors obtained from the quoted ranges for the auxiliary parameters have been considered. When we look at these Tables, we see that although the values for the eight independent input parameters for two sets are close to each other but the results for the central values of some form factors differ in two sets, considerably. The numerical results show that the result of sum rules are very sensitive to these parameters specially  $f_N$ ,  $\lambda_2$  and  $A_1^u$ . Within the errors, the quoted values become close to each other for both sets. The numerical analysis depicts also that all form factors approximately satisfy the HQET limit relations in Eq. (22) within the errors for both sets of input parameters and bottom case,  $Q = b$  at  $q^2 = 0$ . However for the charm case,  $Q = c$  although some of the relations are satisfied but most of them are violated at  $q^2 = 0$ . This is an expected result since the  $m_c \rightarrow \infty$  limit is not as reasonable as the  $m_b \rightarrow \infty$ .

Our next task is to calculate the total decay rate of the FCNC  $\Sigma_b \rightarrow pl^+l^-$  and  $\Sigma_c \rightarrow nl^+l^-$  transitions in the full allowed physical region, namely,  $4m_l^2 \leq q^2 \leq (m_{\Sigma_{b,c}} - m_{p,n})^2$ . To derive the expression for the decay rate, we will make the following assumptions (see also [30]): the CLEO predicts the value  $R = \frac{F_2}{F_1} = -0.25 \pm 0.14 \pm 0.08$  for the ratio of the form factors of  $\Lambda_c \rightarrow \Lambda e e_\nu$  at HQET limit [31]. This result shows that  $|F_2| < |F_1|$  and considering Eq. (22), the form factors  $f_1$ ,  $g_1$ ,  $f_1$ ,  $f_2^T$  and  $g_2^T$  are expected to be large comparing to the other form factors since they are proportional to the  $F_1$ . Moreover, it is clear from the considered Hamiltonian as well as the definition of the transition matrix elements in terms of the form factors that the form factors labeled by T are related to the Wilson coefficient  $C_7$  which is about one order of magnitude smaller than the other coefficients entered to the Hamiltonian, i.e.,  $C_9$  and  $C_{10}$ , hence their effects expected to be small. As a result of the above procedure, the following results for decay width describing such transitions is obtained [30]:

$$\frac{d\Gamma}{ds}(\Sigma_Q \rightarrow Nl^+l^-) = \frac{G_F^2 \alpha_{em}^2 |V_{Q'Q} V_{Q'q}^*|^2}{384\pi^5} m_{\Sigma_Q}^5 \sqrt{\phi(s)} \sqrt{1 - \frac{4m_l^2}{q^2}} f^2 R_{\Sigma_Q}(s), \quad (27)$$

where

$$R_{\Sigma_Q}(s) = \Gamma_1(s) + \Gamma_2(s) + \Gamma_3(s) \quad (28)$$



	set1			set2		
	a	b	$m_{fit}$	a	b	$m_{fit}$
$f_1$	-0.16	0.29	5.70	0.027	0.044	5.02
$f_2$	0.008	-0.02	5.96	0.018	-0.024	7.96
$f_3$	0.011	-0.024	6.34	-0.003	-0.003	6.45
$g_1$	-0.21	0.33	5.73	-0.13	0.20	5.29
$g_2$	0.008	-0.02	5.92	-0.01	0.003	5.72
$g_3$	0.005	-0.02	5.87	0.014	-0.023	7.83
$f_1^T$	-0.06	0.023	5.22	-0.029	-0.018	5.12
$f_2^T$	-0.16	0.29	6.47	0.069	-0.017	5.69
$f_3^T$	-0.18	0.25	8.81	0.084	-0.023	5.13
$g_1^T$	-0.15	0.14	5.02	-0.01	-0.028	5.11
$g_2^T$	-0.20	0.31	5.24	0.026	0.04	4.72
$g_3^T$	0.17	-0.25	5.76	0.11	-0.18	5.33

TABLE VII: Parameters appearing in the fit function of the form factors in full theory for  $\Sigma_b \rightarrow n\ell^+\ell^-$ .

	set1			set2		
	a	b	$m_{fit}$	a	b	$m_{fit}$
$f_1$	0.08	0.097	1.53	-0.12	0.18	1.52
$f_2$	-0.009	-0.056	1.57	-0.01	-0.029	1.53
$f_3$	-0.025	0.012	1.61	0.008	-0.047	1.56
$g_1$	-0.015	0.31	1.59	-0.038	0.21	1.60
$g_2$	-0.008	-0.12	1.55	0.002	-0.14	1.61
$g_3$	-0.026	-0.13	1.53	-0.024	-0.14	1.52
$f_1^T$	-0.23	0.19	1.52	0.09	-0.097	1.58
$f_2^T$	0.066	0.067	1.63	0.12	0.13	1.55
$f_3^T$	0.15	0.006	1.56	0.21	0.032	1.65
$g_1^T$	-0.45	0.29	1.59	-0.17	0.09	1.62
$g_2^T$	0.009	0.08	1.57	-0.026	0.14	1.59
$g_3^T$	-0.09	-0.11	1.54	-0.07	-0.16	1.56

TABLE VIII: Parameters appearing in the fit function of the form factors in full theory for  $\Sigma_c \rightarrow p\ell^+\ell^-$ .

	set1			set2		
	a	b	$m_{fit}$	a	b	$m_{fit}$
$f_1$	-0.22	0.4	4.96	0.037	0.06	5.13
$f_2$	0.009	-0.024	5.13	0.021	-0.028	5.28
$f_3$	0.009	-0.02	5.72	-0.003	-0.002	5.67
$g_1$	-0.029	0.19	5.32	-0.18	0.28	5.57
$g_2$	0.008	-0.02	5.41	-0.01	0.003	5.35
$g_3$	0.005	-0.018	4.87	0.013	-0.021	5.06
$f_1^T$	-0.065	0.025	5.16	-0.03	0.019	5.25
$f_2^T$	-0.24	0.43	5.04	0.104	-0.026	5.13
$f_3^T$	-0.19	0.27	5.13	0.091	-0.025	5.54
$g_1^T$	-0.14	0.13	5.11	-0.009	-0.011	5.14
$g_2^T$	-0.03	0.17	5.58	0.04	0.06	5.47
$g_3^T$	0.21	-0.27	5.16	0.1	-0.17	4.96

TABLE IX: Parameters appearing in the fit function of the form factors at HQET limit for  $\Sigma_b \rightarrow n\ell^+\ell^-$ .

	set1			set2		
	a	b	$m_{fit}$	a	b	$m_{fit}$
$f_1$	0.02	0.13	1.64	-0.17	0.25	1.55
$f_2$	-0.011	-0.077	1.76	-0.014	-0.04	1.51
$f_3$	-0.034	0.017	1.73	0.011	-0.065	1.62
$g_1$	-0.021	0.43	1.68	-0.053	0.29	1.65
$g_2$	-0.008	-0.12	1.57	-0.002	-0.14	1.63
$g_3$	-0.26	0.10	1.62	-0.022	-0.13	1.57
$f_1^T$	-0.097	0.05	1.58	0.098	-0.1	1.65
$f_2^T$	0.099	0.1	1.69	0.18	0.196	1.59
$f_3^T$	0.14	0.009	1.51	0.32	-0.048	1.71
$g_1^T$	-0.4	0.26	1.66	-0.15	0.08	1.57
$g_2^T$	0.014	0.12	1.63	-0.039	0.21	1.63
$g_3^T$	-0.08	-0.1	1.58	-0.064	-0.15	1.60

TABLE X: Parameters appearing in the fit function of the form factors at HQET limit for  $\Sigma_c \rightarrow p\ell^+\ell^-$ .

	Full Theory		HQET	
	set1	set2	set1	set2
$f_1(0)$	$0.14 \pm 0.04$	$0.07 \pm 0.02$	$0.19 \pm 0.05$	$0.10 \pm 0.03$
$f_2(0)$	$-0.012 \pm 0.003$	$-0.006 \pm 0.002$	$-0.014 \pm 0.004$	$-0.005 \pm 0.002$
$f_3(0)$	$-0.013 \pm 0.003$	$-0.006 \pm 0.002$	$-0.011 \pm 0.002$	$-0.005 \pm 0.002$
$g_1(0)$	$0.12 \pm 0.03$	$0.07 \pm 0.02$	$0.17 \pm 0.04$	$0.10 \pm 0.03$
$g_2(0)$	$-0.012 \pm 0.003$	$-0.007 \pm 0.002$	$-0.012 \pm 0.004$	$-0.007 \pm 0.002$
$g_3(0)$	$-0.014 \pm 0.004$	$-0.009 \pm 0.003$	$-0.013 \pm 0.004$	$-0.008 \pm 0.003$
$f_1^T(0)$	$-0.03 \pm 0.01$	$-0.04 \pm 0.01$	$-0.03 \pm 0.01$	$-0.010 \pm 0.003$
$f_2^T(0)$	$0.13 \pm 0.04$	$0.052 \pm 0.020$	$0.19 \pm 0.05$	$0.079 \pm 0.003$
$f_3^T(0)$	$0.07 \pm 0.02$	$0.061 \pm 0.020$	$0.08 \pm 0.03$	$0.066 \pm 0.021$
$g_1^T(0)$	$-0.012 \pm 0.003$	$-0.03 \pm 0.01$	$-0.012 \pm 0.004$	$-0.020 \pm 0.006$
$g_2^T(0)$	$0.11 \pm 0.03$	$0.066 \pm 0.021$	$0.16 \pm 0.04$	$0.10 \pm 0.03$
$g_3^T(0)$	$-0.07 \pm 0.02$	$-0.073 \pm 0.025$	$-0.07 \pm 0.02$	$-0.066 \pm 0.021$

TABLE XI: The values of the form factors at  $q^2 = 0$  for  $\Sigma_b \rightarrow n\ell^+\ell^-$ .

	Full Theory		HQET	
	set1	set2	set1	set2
$f_1(0)$	$0.19 \pm 0.05$	$0.05 \pm 0.02$	$0.16 \pm 0.05$	$0.069 \pm 0.022$
$f_2(0)$	$-0.066 \pm 0.021$	$-0.04 \pm 0.01$	$-0.078 \pm 0.023$	$-0.047 \pm 0.014$
$f_3(0)$	$-0.013 \pm 0.003$	$-0.039 \pm 0.012$	$-0.010 \pm 0.003$	$-0.034 \pm 0.011$
$g_1(0)$	$0.30 \pm 0.09$	$0.17 \pm 0.06$	$0.40 \pm 0.12$	$0.24 \pm 0.06$
$g_2(0)$	$-0.12 \pm 0.03$	$-0.14 \pm 0.04$	$-0.12 \pm 0.04$	$-0.14 \pm 0.04$
$g_3(0)$	$-0.16 \pm 0.05$	$-0.16 \pm 0.05$	$-0.15 \pm 0.05$	$-0.15 \pm 0.04$
$f_1^T(0)$	$-0.039 \pm 0.012$	$-0.007 \pm 0.002$	$-0.042 \pm 0.013$	$-0.0020 \pm 0.0007$
$f_2^T(0)$	$0.14 \pm 0.04$	$0.25 \pm 0.07$	$0.21 \pm 0.06$	$0.38 \pm 0.12$
$f_3^T(0)$	$0.15 \pm 0.05$	$0.24 \pm 0.07$	$0.16 \pm 0.04$	$0.26 \pm 0.08$
$g_1^T(0)$	$-0.16 \pm 0.05$	$-0.08 \pm 0.03$	$-0.14 \pm 0.05$	$-0.07 \pm 0.02$
$g_2^T(0)$	$0.09 \pm 0.03$	$0.10 \pm 0.03$	$0.14 \pm 0.05$	$0.15 \pm 0.05$
$g_3^T(0)$	$-0.20 \pm 0.07$	$-0.23 \pm 0.06$	$-0.18 \pm 0.05$	$-0.21 \pm 0.08$

TABLE XII: The values of the form factors at  $q^2 = 0$  for  $\Sigma_c \rightarrow p\ell^+\ell^-$ .

and

$$\begin{aligned} \Gamma_1(s) = & -6\sqrt{r}s \left[ -2\hat{m}_Q\rho \left( 1 + 2\frac{m_l^2}{q^2} \right) \text{Re}C_9^{\text{eff}}C_7^* + \delta \left( \left( 1 + 2\frac{m_l^2}{q^2} \right) |C_9^{\text{eff}}|^2 + \left( 1 - 6\frac{m_l^2}{q^2} \right) |C_{10}|^2 \right) \right] \\ & + \left[ -2r \left( 1 + 2\frac{m_l^2}{q^2} \right) - 4t^2 \left( 1 - \frac{m_l^2}{q^2} \right) + 3(1+r)t \right] \times \left[ (2\hat{m}_Q\rho)^2 |C_7|^2 + |C_9^{\text{eff}}|^2 + |C_{10}|^2 \right] \\ & + 6\hat{m}_l^2 t \left[ (2\hat{m}_Q\rho)^2 |C_7|^2 + |C_9^{\text{eff}}|^2 - |C_{10}|^2 \right], \end{aligned} \quad (29)$$

$$\begin{aligned} \Gamma_2(s) = & 6\sqrt{r}(1-t) \left\{ 4 \left( 1 + 2\frac{m_l^2}{q^2} \right) \hat{m}_Q^2 \rho |C_7|^2 + \rho s \left[ \left( 1 + 2\frac{m_l^2}{q^2} \right) |C_9^{\text{eff}}|^2 + \left( 1 - 2\frac{m_l^2}{q^2} \right) |C_{10}|^2 \right] \right\} \\ & + 12 \left( 1 + 2\frac{m_l^2}{q^2} \right) \hat{m}_Q (t-r) (1+s\rho^2) \text{Re}C_9^{\text{eff}}C_7^*, \end{aligned} \quad (30)$$

$$\begin{aligned} \Gamma_3(s) = & 12 \left( 1 + 2\frac{m_l^2}{q^2} \right) \hat{m}_Q \sqrt{r} s \rho \text{Re}C_9^{\text{eff}}C_7^* - \left[ 2t^2 \left( 1 + 2\frac{m_l^2}{q^2} \right) + 4r \left( 1 - \frac{m_l^2}{q^2} \right) - 3(1+r)t \right] \\ & \times \left[ \frac{4\hat{m}_Q^2}{s} |C_7|^2 + s\rho^2 \left( |C_9^{\text{eff}}|^2 + |C_{10}|^2 \right) \right] - 6\hat{m}_l^2 (2r - (1+r)t) \left[ \left( \frac{2\hat{m}_Q}{s} \right)^2 |C_7|^2 + \rho^2 \left( |C_9^{\text{eff}}|^2 - |C_{10}|^2 \right) \right]. \end{aligned} \quad (31)$$

Here,  $G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$  is the Fermi coupling constant,  $\bar{f} = \frac{f_1+g_1}{2}$ ,  $\rho = m_{\Sigma_Q} \frac{f_2+g_2}{f_1+g_1}$ ,  $\delta = \frac{f_1-g_1}{f_1+g_1}$ ,  $s = \frac{q^2}{m_{\Sigma_Q}^2}$ ,  $\hat{m}_Q = \frac{m_Q}{m_{\Sigma_Q}}$ ,  $\hat{m}_l = \frac{m_l}{m_{\Sigma_Q}}$ ,  $r = \frac{m_N^2}{m_{\Sigma_Q}^2}$ ,  $t = \frac{1}{2m_{\Sigma_Q}^2} [m_{\Sigma_Q}^2 + m_N^2 - q^2]$  and  $m_l$  is the lepton mass. For the Wilson coefficients, we use  $C_7 = -0.313, C_9 = 4.344, C_{10} = -4.669$  [32]. Here we should mention that the Wilson coefficient  $C_9^{\text{eff}}$  receives long distance contributions from  $J/\psi$  family, in addition to short distance contributions. In the present work, we do not take into account the long distance effects. The elements of the CKM matrix  $V_{tb} = (0.77_{-0.24}^{+0.18})$ ,  $V_{td} = (8.1 \pm 0.6) \times 10^{-3}$ ,  $V_{bc} = (41.2 \pm 1.1) \times 10^{-3}$  and  $V_{bu} = (3.93 \pm 0.36) 10^{-3}$  have also been used [33].

Using the formula for the decay rate the final results as shown in Table XIII are obtained. From this table, we see

	$\Sigma_b \rightarrow n e^+ e^-$	$\Sigma_b \rightarrow n \mu^+ \mu^-$	$\Sigma_b \rightarrow n \tau^+ \tau^-$	$\Sigma_c \rightarrow p e^+ e^-$	$\Sigma_c \rightarrow p \mu^+ \mu^-$
Full (set1)	$(4.26 \pm 1.27) \times 10^{-20}$	$(2.08 \pm 0.70) \times 10^{-20}$	$(1.0 \pm 0.3) \times 10^{-22}$	$(5.59 \pm 1.78) \times 10^{-25}$	$(9.7 \pm 2.7) \times 10^{-26}$
Full (set2)	$(5.4 \pm 1.6) \times 10^{-21}$	$(2.64 \pm 0.79) \times 10^{-21}$	$(4.01 \pm 1.25) \times 10^{-23}$	$(1.35 \pm 0.35) \times 10^{-25}$	$(2.36 \pm 0.80) \times 10^{-26}$
HQET(set1)	$(8.20 \pm 3.04) \times 10^{-20}$	$(4.25 \pm 2.07) \times 10^{-20}$	$(6.26 \pm 2.46) \times 10^{-22}$	$(7.99 \pm 3.07) \times 10^{-25}$	$(1.50 \pm 0.58) \times 10^{-25}$
HQET(set2)	$(1.10 \pm 0.33) \times 10^{-20}$	$(5.67 \pm 1.73) \times 10^{-21}$	$(1.16 \pm 0.46) \times 10^{-22}$	$(2.50 \pm 0.81) \times 10^{-25}$	$(4.30 \pm 1.36) \times 10^{-26}$

TABLE XIII: Values of the  $\Gamma(\Sigma_{b,c} \rightarrow n, p \ell^+ \ell^-)$  in GeV for different leptons and two sets of input parameters.

that: a) The value of the decay rate decreases by increasing in the lepton mass. This is reasonable since the phase space in for example  $\tau$  case is smaller than that of the electron and  $\mu$  cases. b) The order of magnitude on decay rate of bottom case shows the possibility of the experimental studies on the  $\Sigma_b \rightarrow n \ell^+ \ell^-$  transition, specially the  $\mu$  case, at large hadron collider (LHC) in the near future. The lifetime of the  $\Sigma_b$  is not exactly known yet but if we consider its lifetime approximately the same order of the b-baryon admixture ( $\Lambda_b, \Xi_b, \Sigma_b, \Omega_b$ ) lifetime, which is  $\tau = (1.319_{0.038}^{+0.039}) \times 10^{-12} \text{ s}$  [33], the branching fraction is obtained in  $10^{-7}$  order. Any measurements in this respect and comparison of the results with the predictions of the present work can give essential information about the nature of the  $\Sigma_Q$  baryon, nucleon distribution amplitudes and search for the new physics beyond the standard model.

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## Appendix

In this Appendix, the explicit expressions for the form factors  $f_1$  and  $f_2$  for  $b$  case as well as the nucleon DA's are given:

$$\begin{aligned}
 f_1(Q^2) = & \frac{1}{\sqrt{2}\lambda_{\Sigma_b}} e^{m_{\Sigma_b}^2/M_B^2} \left( \int_{t_0}^1 dx_2 \int_0^{1-x_2} dx_1 e^{-s(x_2, Q^2)/M_B^2} \frac{1}{2\sqrt{2}} \left[ m_b \left\{ (1+3\beta)\mathcal{H}_{19}(x_i) - 2(-1+\beta)\mathcal{H}_{17}(x_i) \right. \right. \right. \\
 & \left. \left. - (3+\beta)\mathcal{H}_5(x_i) \right\} - m_N x_2 \left\{ \mathcal{H}_{12, -11, -13, 19_2, -5, 7}(x_i) + \beta\mathcal{H}_{11, 13, -17_2, -19_8, 3_2, 5, -7}(x_i) \right\} \right] \\
 & + \int_{t_0}^1 dx_2 \int_0^{1-x_2} dx_1 \int_{t_0}^{x_2} dt_1 e^{-s(t_1, Q^2)/M_B^2} \left[ - \frac{m_N^4 m_b}{M_B^4 t_1^3 \sqrt{2}} (-1+\beta)x_2 \mathcal{H}_{22}(x_i) \right. \\
 & - \frac{m_N^2}{M_B^4 t_1^2 2\sqrt{2}} \left\{ m_N^3 x_2 \left[ (-1+\beta)\mathcal{H}_{-10, 16}(x_i) + 2\beta\mathcal{H}_{24}(x_i) \right] + \left[ 4m_N^3 x_2 + m_b \{ Q^2 + s(t_1, Q^2) \} (-1+\beta)x_2 \right. \right. \\
 & \left. \left. - m_N^2 m_b (-1+\beta)(2+3x_2) \right] \mathcal{H}_{22}(x_i) \right\} + \frac{m_N^2}{M_B^4 t_1 2\sqrt{2}} \left\{ m_N x_2 \left[ Q^2 + s(t_1, Q^2) \mathcal{H}_{16, -22_4}(x_i) + (-1+\beta)\mathcal{H}_{10}(x_i) \right. \right. \\
 & \left. \left. - \beta\mathcal{H}_{16, 24_2}(x_i) \right] + m_b (-1+\beta) \left[ Q^2 (1+3x_2) + s(t_1, Q^2) (1+x_2) \right] \mathcal{H}_{22}(x_i) + m_N^2 m_b \left[ x_2 (1+3\beta)\mathcal{H}_{16}(x_i) \right. \right. \\
 & \left. \left. + 2(-1+\beta)\mathcal{H}_{24}(x_i) + (3+\beta)\mathcal{H}_{10}(x_i) - (-1+\beta)(3+x_2)\mathcal{H}_{22}(x_i) \right] - m_N^3 \left[ (-1+\beta)(1+x_2)\mathcal{H}_{10, -16}(x_i) \right. \right. \\
 & \left. \left. - 2\{\beta(1+x_2)\mathcal{H}_{24}(x_i) + (2+4x_2)\mathcal{H}_{22}(x_i)\} \right\} + \frac{m_N^2}{M_B^4 2\sqrt{2}} \left\{ -3m_b Q^2 (-1+\beta)\mathcal{H}_{22}(x_i) \right. \right. \\
 & \left. \left. + m_N^2 m_b \left[ (-1+\beta)\mathcal{H}_{22, -24_2}(x_i) - (1+3\beta)\mathcal{H}_{16}(x_i) - (3+\beta)\mathcal{H}_{10}(x_i) \right] + m_N^3 \left[ (-8+2t_1 - 2x_2)\mathcal{H}_{22}(x_i) \right. \right. \right. \\
 & \left. \left. + (-1+\beta)\mathcal{H}_{10, -16}(x_i) - 2\beta\mathcal{H}_{24}(x_i) \right] + m_N \left[ Q^2 (-1+\beta) (-1+t_1 - x_2)\mathcal{H}_{10, -16}(x_i) \right. \right. \\
 & \left. \left. + Q^2 (-6t_1 + 6x_2 + 4 + 2\beta)\mathcal{H}_{22}(x_i) + 2Q^2 \beta (1-t_1 + x_2)\mathcal{H}_{24}(x_i) \right] \right\} + \frac{m_N^3}{M_B^2 t_1^2 2\sqrt{2}} \left\{ \mathcal{H}_{6, -18_3, 20}(x_i) \right. \right. \\
 & \left. \left. + (-1+\beta)\mathcal{H}_{12}(x_i) - \mathcal{H}_{6, -13, 18}(x_i) \right\} + \frac{m_N}{M_B^2 t_1 4\sqrt{2}} \left\{ [Q^2 + s(t_1, Q^2)] \left[ (3+25\beta)\mathcal{H}_{20}(x_i) + 2(-1+\beta)\mathcal{H}_{-6, 12}(x_i) \right] \right. \right. \\
 & \left. \left. - (5+\beta)\mathcal{H}_{18}(x_i) - m_N^2 \left[ 2(-1+\beta)\mathcal{H}_{-6, 12}(x_i) - (11+3\beta)\mathcal{H}_{18}(x_i) + (5+67\beta)\mathcal{H}_{20}(x_i) \right] \right. \right. \\
 & \left. \left. + 2x_2 \left[ (-1+\beta)\mathcal{H}_{-10, 16}(x_i) + \beta\mathcal{H}_{24}(x_i) \right] - m_N m_b \left[ \mathcal{H}_{6_6, -8_3, -9_3, 12_2, 14, 15, -20_4, 21_4}(x_i) + 4x_2 (-1+\beta)\mathcal{H}_{22}(x_i) \right. \right. \right. \\
 & \left. \left. + \beta\mathcal{H}_{6_2, -8, -9, 12_6, 14_3, 15_3, 20_4, -21_4}(x_i) \right] \right\} + \frac{m_N}{M_B^2 4\sqrt{2}} \left\{ Q^2 \left[ \mathcal{H}_{-6_2, +12_2, 18_9, -20_3}(x_i) + \beta\mathcal{H}_{6_2, -12_2, 18_3, -20_{47}}(x_i) \right] \right. \right. \\
 & \left. \left. + 4m_N (-1+\beta)\mathcal{H}_{22}(x_i) + s(t_1, Q^2) \left[ \mathcal{H}_{18_3, -20}(x_i) + \beta\mathcal{H}_{18, -20_{21}}(x_i) \right] \right. \right. \\
 & \left. \left. + m_N^2 \left[ \beta\mathcal{H}_{4_4, 8, -9, -10_2, 14, -15, 16_2, -18, 20_{41}, -21_4, 23_{16}, 24_4}(x_i) + \mathcal{H}_{-2_4, -8, 9, 10_2, -14, 15, -16_2, -18_3, 20, -23_4}(x_i) \right. \right. \right. \\
 & \left. \left. + 8(t_1 - x_2)\mathcal{H}_{22}(x_i) \right] \right\} + \frac{m_N}{t_1 4\sqrt{2}} \left\{ 2(-1+\beta)\mathcal{H}_{-6, 12}(x_i) + (1+5\beta)\mathcal{H}_{20}(x_i) - (3+\beta)\mathcal{H}_{18}(x_i) \right\} \\
 & + \frac{m_N}{4\sqrt{2}} \left\{ (1+21\beta)\mathcal{H}_{20}(x_i) - (3+\beta)\mathcal{H}_{18}(x_i) \right\} + \int_{t_0}^1 dx_2 \int_0^{1-x_2} dx_1 e^{-s_0/M_B^2} \left[ \frac{m_N^4 t_0^2}{(Q^2 + m_N^2 t_0^2)^3 \sqrt{2}} (t_0 - x_2) \left\{ \right. \right. \\
 & \left. \left[ m_N^2 m_b (-1+\beta)(-2+t_0)(-1+t_0) + 2m_N^3 t_0 \{ 2 + (-4+t_0)t_0 \} - 2m_N t_0^2 \{ Q^2(-2+3t_0) + (-2+t_0)s(s_0, Q^2) \} \right. \right. \\
 & \left. \left. - m_b (-1+\beta)t_0 \{ Q^2(-1+3t_0) + (-1+t_0)s(s_0, Q^2) \} \right] \mathcal{H}_{22}(x_i) + m_N t_0 \left( \{ m_N^2 (-1+\beta)(-1+t_0) \right. \right. \\
 & \left. \left. - m_N m_b t_0 (3+\beta) + (-1+\beta)t_0 [Q^2(-1+t_0) - s(s_0, Q^2)] \} \mathcal{H}_{10}(x_i) - \left[ m_N^2 (-1+\beta)(-1+t_0) + m_N m_b t_0 (1+3\beta) \right. \right. \right. \\
 & \left. \left. + (-1+\beta)t_0 \{ Q^2(-1+t_0) - s(s_0, Q^2) \} \right] \mathcal{H}_{16}(x_i) + 2 \left[ -m_N m_b t_0 (-1+\beta) + m_N^2 \beta (1-t_0) + \beta t_0 (Q^2 (1-t_0) \right. \right. \\
 & \left. \left. + s(s_0, Q^2)) \right] \mathcal{H}_{24}(x_i) \right\} + \frac{m_N^2}{(Q^2 + m_N^2 t_0^2)^2 2\sqrt{2}} (t_0 - x_2) \left\{ \left[ m_N^2 m_b (-1+\beta)(-2+t_0)(-1+t_0) \right. \right. \right. \\
 & \left. \left. + 2m_N^3 t_0 \{ 2 + (-4+t_0)t_0 \} - 2m_N t_0^2 \{ Q^2(-2+3t_0) + (-2+t_0)s(s_0, Q^2) \} - m_b t_0 (-1+\beta) \{ Q^2(-1+3t_0) \right. \right. \\
 & \left. \left. + (-1+t_0)s(s_0, Q^2) \} \right] \mathcal{H}_{22}(x_i) + m_N t_0 \left[ \{ m_N^2 (-1+\beta)(-1+t_0) - m_N m_b t_0 (3+\beta) + (-1+\beta)t_0 [Q^2(-1+t_0) \right. \right. \\
 & \left. \left. - s(s_0, Q^2)] \} \mathcal{H}_{10}(x_i) - \{ m_N^2 (-1+\beta)(-1+t_0) + m_N m_b t_0 (1+3\beta) + (-1+\beta)t_0 (Q^2(-1+t_0) - s(s_0, Q^2)) \} \right] \mathcal{H}_{16}(x_i) \right\} \\
 & \left. \left. \left. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{m_N}{(Q^2 + m_N^2 t_0^2) 4\sqrt{2} M_B^2 t_0} \left\{ 2m_N(t_0 - x_2) \left[ m_N^2 m_b (-1 + \beta)(-2 + t_0)(-1 + t_0) + 2m_N^3 t_0(2 + (-4 + t_0)t_0) \right. \right. \\
 & + m_b t_0(-1 + \beta) \{ Q^2(1 - 3t_0) + 2M_B^2 t_0 + (1 - t_0)s(s_0, Q^2) \} + 2m_N t_0^2 \{ Q^2(2 - 3t_0) \} + 2M_B^2 t_0 \\
 & + (2 - t_0)s(s_0, Q^2) \left. \right] \mathcal{H}_{22}(x_i) + t_0 \left[ m_N^2 M_B^2 \mathcal{H}_{62, -122, -186, 202}(x_i) + \beta \mathcal{H}_{-62, 122, -182, 2026}(x_i) \right] + 2m_N^4 (1 - t_0) \mathcal{H}_{10, 16}(x_i) \\
 & + m_N^2 M_B^2 t_0 \mathcal{H}_{-62, 122, 1811, -205}(x_i) + m_N m_b M_B^2 t_0 \mathcal{H}_{-66, 83, 93, -122, -14, -15, 204, -214}(x_i) + M_B^2 Q^2 t_0 \mathcal{H}_{62, -122, -185, 203}(x_i) \\
 & + m_N^4 \beta t_0 \mathcal{H}_{-102, 162, 244}(x_i) + m_N^2 M_B^2 \beta t_0 \mathcal{H}_{62, -122, 183, -2067}(x_i) + m_N m_b M_B^2 \beta t_0 \mathcal{H}_{-62, 8, 9, -126, -143, -153, -204, 214}(x_i) \\
 & + M_B^2 Q^2 \beta t_0 \mathcal{H}_{-62, 122, -18, 2025}(x_i) + 2m_N^4 t_0 \mathcal{H}_{-10, 16}(x_i) + m_N^2 m_b t_0^2 \mathcal{H}_{-106, -162, 244}(x_i) \\
 & + m_N^2 M_B^2 t_0^2 \mathcal{H}_{-24, -8, 9, 102, -14, 15, -162, -183, 20, -234}(x_i) + 2m_N^2 Q^2 t_0^2 \mathcal{H}_{10, -16}(x_i) + M_B^2 Q^2 t_0^2 \mathcal{H}_{-62, 122, 189, -203}(x_i) \\
 & + m_N^4 \beta t_0^2 \mathcal{H}_{102, -162, -244}(x_i) - m_N^3 m_b \beta t_0^2 \mathcal{H}_{102, 166, 244}(x_i) + m_N^2 M_B^2 \beta t_0^2 \mathcal{H}_{44, 8, -9, -102, 14, -15, 162, -18, 2041, -214, 2316, 244}(x_i) \\
 & + m_N^2 Q^2 \beta t_0^2 \mathcal{H}_{-102, 162, 244}(x_i) + M_B^2 Q^2 \beta t_0^2 \mathcal{H}_{62, -122, 183, -2047}(x_i) - 2Q^2 t_0^3 \mathcal{H}_{10, -16}(x_i) + Q^2 \beta t_0^3 \mathcal{H}_{102, -162, -244}(x_i) \\
 & + M_B^2 t_0 s(s_0, Q^2) \mathcal{H}_{62, -122, -185, 203}(x_i) + M_B^2 \beta t_0 s(s_0, Q^2) \mathcal{H}_{-62, 122, -18, 2025}(x_i) + 2m_N^2 t_0 s(s_0, Q^2) \mathcal{H}_{10, -16}(x_i) \\
 & + M_B^2 t_0^2 s(s_0, Q^2) \mathcal{H}_{183, -20}(x_i) + m_N^2 \beta t_0^2 s(s_0, Q^2) \mathcal{H}_{-102, 162, 244}(x_i) + M_B^2 \beta t_0^2 s(s_0, Q^2) \mathcal{H}_{18, -2021}(x_i) \\
 & - 2m_N^2 x_2 \left[ \{ (-1 + \beta)(-1 + t_0) - m_N m_b t_0(3 + \beta) + (-1 + \beta)t_0[-M_B^2 - Q^2(1 - t_0) + s(s_0, Q^2)] \} \mathcal{H}_{10}(x_i) \right. \\
 & - \left. \left[ m_N^2(-1 + \beta)(-1 + t_0) + m_N m_b t_0(1 + 3\beta) + (-1 + \beta)t_0\{-M_B^2 - Q^2(1 - t_0) - s(s_0, Q^2)\} \right] \mathcal{H}_{16}(x_i) \right. \\
 & \left. + 2 \left[ -m_N m_b t_0(-1 + \beta) + m_N^2 \beta(1 - t_0) + \beta t_0 \{ M_B^2 + Q^2(1 - t_0) + s(s_0, Q^2) \} \right] \mathcal{H}_{24}(x_i) \right] \left. \right\}, \tag{32}
 \end{aligned}$$

$$\begin{aligned}
 f_2(Q^2) & = \frac{1}{\sqrt{2}\lambda_{\Sigma_b}} e^{m_{\Sigma_b}^2/M_B^2} \left( \int_{t_0}^1 dx_2 \int_0^{1-x_2} dx_1 e^{-s(x_2, Q^2)/M_B^2} \frac{1}{2\sqrt{2}x_2} \left[ \mathcal{H}_{11, -172, 5}(x_i) - \beta \mathcal{H}_{11, -1712, 5}(x_i) \right] \right. \\
 & + \int_{t_0}^1 dx_2 \int_0^{1-x_2} dx_1 \int_{t_0}^{x_2} dt_1 e^{-s(t_1, Q^2)/M_B^2} \left[ -\frac{m_N^4}{M_B^4 t_1^3 \sqrt{2}} (3 + \beta) x_2 \mathcal{H}_{22}(x_i) + \frac{m_N^2}{M_B^4 t_1^2 2\sqrt{2}} \left\{ m_N m_b x_2 \left[ (1 + 3\beta) \mathcal{H}_{16}(x_i) \right. \right. \right. \\
 & + 2(-1 + \beta) \mathcal{H}_{24}(x_i) + (3 + \beta) \mathcal{H}_{10}(x_i) \left. \right] + 2 \left[ -m_N m_b x_2(-1 + \beta) - \{ Q^2 + s(t_1, Q^2) \} (3 + \beta) x_2 + m_N^2 (3 + \beta) \right. \\
 & + (5 + \beta) x_2 \left. \right] \mathcal{H}_{22}(x_i) \left. \right\} + \frac{m_N^2}{M_B^4 t_1 2\sqrt{2}} \left\{ m_N m_b \left[ (1 + 3\beta) \mathcal{H}_{16}(x_i) + 2(-1 + \beta) \mathcal{H}_{24}(x_i) + (3 + \beta) \mathcal{H}_{10}(x_i) \right] \right. \\
 & \left. + 2 \left[ m_N m_b (1 - \beta) - s(t_1, Q^2) (3 + \beta + x_2) + m_N^2 (5 + \beta + x_2) - Q^2 (3 + \beta + (4 + \beta) x_2) \right] \mathcal{H}_{22}(x_i) \right\} \\
 & + \frac{m_N^2}{M_B^4 \sqrt{2}} \left\{ \left[ m_N^2 - (4 + \beta) Q^2 - s(t_1, Q^2) \right] \mathcal{H}_{22}(x_i) \right\} + \frac{m_N}{M_B^2 t_1^2 2\sqrt{2}} \left\{ -m_b \left[ n \mathcal{H}_{63, 12, -18, -20}(x_i) + \beta \mathcal{H}_{6, 123, 18, 20}(x_i) \right] \right. \\
 & - 2(2 + \beta) x_2 \mathcal{H}_{22}(x_i) \left. \right\} + \frac{m_N^2}{M_B^2 t_1 4\sqrt{2}} \left\{ \mathcal{H}_{-24, -8, 9, 15, -182, -202, 228, -234}(x_i) + (-1 + \beta) \mathcal{H}_{14}(x_i) \right. \\
 & - \left. \beta \mathcal{H}_{-44, -8, 9, 15, 182, -2022, 214, -224, -2316}(x_i) - 8 \mathcal{H}_{22}(x_i) \right\} + \frac{m_N^2 \sqrt{2}}{M_B^2} \mathcal{H}_{22}(x_i) \left. \right] \\
 & + \int_{t_0}^1 dx_2 \int_0^{1-x_2} dx_1 e^{-s_0/M_B^2} \left[ \frac{m_N^4 t_0^2}{(Q^2 + m_N^2 t_0^2)^3 \sqrt{2}} (t_0 - x_2) \left\{ -m_N m_b t_0 \left[ (1 + 3\beta) \mathcal{H}_{16}(x_i) + 2(-1 + \beta) \mathcal{H}_{24}(x_i) \right. \right. \right. \\
 & + (3 + \beta) \mathcal{H}_{10}(x_i) + 2(m_N m_b (-1 + \beta) t_0 + m_N^2 (3 + \beta - (5 + \beta) t_0 + t_0^2) + t_0(Q^2(3 + \beta) - (4 + \beta) t_0) \\
 & + (3 + \beta - t_0)s(s_0, Q^2)) \left. \right] \mathcal{H}_{22}(x_i) \left. \right\} - \frac{m_N^2}{(Q^2 + m_N^2 t_0^2)^2 2\sqrt{2}} (t_0 - x_2) \left\{ m_N m_b t_0 \left[ (1 + 3\beta) \mathcal{H}_{16}(x_i) + 2(-1 + \beta) \mathcal{H}_{24}(x_i) \right. \right. \\
 & + (3 + \beta) \mathcal{H}_{10}(x_i) \left. \right] + 2 \left[ -m_N m_b (-1 + \beta) t_0 + Q^2 t_0 \{-3 - \beta + (4 + \beta) t_0\} + m_N^2 \{-3 + \beta(-1 + t_0) - (-5 + t_0) t_0\} \right. \\
 & \left. + t_0(-3 - \beta + t_0)s(s_0, Q^2) \right] \mathcal{H}_{22}(x_i) \left. \right\} + \frac{m_N}{(Q^2 + m_N^2 t_0^2) 4\sqrt{2} M_B^2 t_0} \left\{ 4m_N(t_0 - x_2) \left[ m_N m_b (-1 + \beta) t_0 + m_N^2 \{ 3 + \beta \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & -(5 + \beta)t_0 + t_0^2\} + t_0\{M_B^2(2 + \beta + 2t_0) + Q^2\{3 + \beta - (4 + \beta)t_0\} + (3 + \beta - t_0)s(s_0, Q^2)\}\mathcal{H}_{22}(x_i) \\
 & -t_0\left[m_b M_B^2\{\mathcal{H}_{6_6,12_2,-20_2}(x_i) + \beta\mathcal{H}_{6_2,12_6,20_2}(x_i)\} + m_N^2 m_b t_0\{\mathcal{H}_{10_6,16_2,-24_4}(x_i) + \beta\mathcal{H}_{10_2,16_6,24_4}(x_i)\}\right. \\
 & + m_N M_B^2 t_0\{\mathcal{H}_{24_4,8,-9,14,-15,20_2,23_4}(x_i) + \beta\mathcal{H}_{-4_4,-8,9,-14,15,-20_{22},21_4,-23_{16}}(x_i)\} + 2M_B^2\{m_b(-1 + \beta) \\
 & \left. + m_N(1 + \beta)t_0\}\mathcal{H}_{18}(x_i) - 2m_N^2 m_b x_2\{(3 + \beta)\mathcal{H}_{10}(x_i) + (1 + 3\beta)\mathcal{H}_{16}(x_i) + 2(-1 + \beta)\mathcal{H}_{24}(x_i)\}\right]\Bigg\}, \quad (33)
 \end{aligned}$$

where

$$\begin{aligned}
 \mathcal{H}(x_i) &= \mathcal{H}(x_1, x_2, 1 - x_1 - x_2), \\
 s(y, Q^2) &= (1 - y)m_N^2 + \frac{(1 - y)}{y}Q^2 + \frac{m_b^2}{y}. \quad (34)
 \end{aligned}$$

The  $t_0 = t_0(s_0, Q^2)$  is the solution of the equation  $s(t_0, Q^2) = s_0$ , i.e.,

$$t_0(s_0, Q^2) = \frac{m_N^2 - Q^2 + \sqrt{4m_N^2(m_b^2 + Q^2) + (m_N^2 - Q^2 - s_0)^2} - s_0}{2m_N^2}. \quad (35)$$

Here,  $s_0$  is continuum threshold,  $M_B^2$  is the Borel mass parameter. In calculations, the following short hand notations for the functions  $\mathcal{H}_{\pm i_a, \pm j_b, \dots} = \pm a\mathcal{H}_i \pm b\mathcal{H}_j \dots$  are used, and  $\mathcal{H}_i$  functions are written in terms of the DA's in the following way:

$$\begin{aligned}
 \mathcal{H}_1 &= S_1 & \mathcal{H}_2 &= S_{1,-2} \\
 \mathcal{H}_3 &= P_1 & \mathcal{H}_4 &= P_{1,-2} \\
 \mathcal{H}_5 &= V_1 & \mathcal{H}_6 &= V_{1,-2,-3} \\
 \mathcal{H}_7 &= V_3 & \mathcal{H}_8 &= -2V_{1,-5} + V_{3,4} \\
 \mathcal{H}_9 &= V_{4,-3} & \mathcal{H}_{10} &= -V_{1,-2,-3,-4,-5,6} \\
 \mathcal{H}_{11} &= A_1 & \mathcal{H}_{12} &= -A_{1,-2,3} \\
 \mathcal{H}_{13} &= A_3 & \mathcal{H}_{14} &= -2A_{1,-5} - A_{3,4} \\
 \mathcal{H}_{15} &= A_{3,-4} & \mathcal{H}_{16} &= A_{1,-2,3,4,-5,6} \\
 \mathcal{H}_{17} &= T_1 & \mathcal{H}_{18} &= T_{1,2} - 2T_3 \\
 \mathcal{H}_{19} &= T_7 & \mathcal{H}_{20} &= T_{1,-2} - 2T_7 \\
 \mathcal{H}_{21} &= -T_{1,-5} + 2T_8 & \mathcal{H}_{22} &= T_{2,-3,-4,5,7,8} \\
 \mathcal{H}_{23} &= T_{7,-8} & \mathcal{H}_{24} &= -T_{1,-2,-5,6} + 2T_{7,8}, \quad (36)
 \end{aligned}$$

where for each DA's, we also have used  $X_{\pm i, \pm j, \dots} = \pm X_i \pm X_j \dots$



The explicit expressions for the nucleon DA's is given as:

$$\begin{aligned}
 V_1(x_i, \mu) &= 120x_1x_2x_3[\phi_3^0(\mu) + \phi_3^+(\mu)(1 - 3x_3)], \\
 V_2(x_i, \mu) &= 24x_1x_2[\phi_4^0(\mu) + \phi_3^+(\mu)(1 - 5x_3)], \\
 V_3(x_i, \mu) &= 12x_3\{\psi_4^0(\mu)(1 - x_3) + \psi_4^-(\mu)[x_1^2 + x_2^2 - x_3(1 - x_3)] \\
 &\quad + \psi_4^+(\mu)(1 - x_3 - 10x_1x_2)\}, \\
 V_4(x_i, \mu) &= 3\{\psi_5^0(\mu)(1 - x_3) + \psi_5^-(\mu)[2x_1x_2 - x_3(1 - x_3)] \\
 &\quad + \psi_5^+(\mu)[1 - x_3 - 2(x_1^2 + x_2^2)]\}, \\
 V_5(x_i, \mu) &= 6x_3[\phi_5^0(\mu) + \phi_5^+(\mu)(1 - 2x_3)], \\
 V_6(x_i, \mu) &= 2[\phi_6^0(\mu) + \phi_6^+(\mu)(1 - 3x_3)], \\
 A_1(x_i, \mu) &= 120x_1x_2x_3\phi_3^-(\mu)(x_2 - x_1), \\
 A_2(x_i, \mu) &= 24x_1x_2\phi_4^-(\mu)(x_2 - x_1), \\
 A_3(x_i, \mu) &= 12x_3(x_2 - x_1)\{\psi_4^0(\mu) + \psi_4^+(\mu) + \psi_4^-(\mu)(1 - 2x_3)\}, \\
 A_4(x_i, \mu) &= 3(x_2 - x_1)\{-\psi_5^0(\mu) + \psi_5^-(\mu)x_3 + \psi_5^+(\mu)(1 - 2x_3)\}, \\
 A_5(x_i, \mu) &= 6x_3(x_2 - x_1)\phi_5^-(\mu) \\
 A_6(x_i, \mu) &= 2(x_2 - x_1)\phi_6^-(\mu), \\
 T_1(x_i, \mu) &= 120x_1x_2x_3[\phi_3^0(\mu) + \frac{1}{2}(\phi_3^- - \phi_3^+)(\mu)(1 - 3x_3)], \\
 T_2(x_i, \mu) &= 24x_1x_2[\xi_4^0(\mu) + \xi_4^+(\mu)(1 - 5x_3)], \\
 T_3(x_i, \mu) &= 6x_3\{(\xi_4^0 + \phi_4^0 + \psi_4^0)(\mu)(1 - x_3) + (\xi_4^- + \phi_4^- - \psi_4^-)(\mu)[x_1^2 + x_2^2 - x_3(1 - x_3)] \\
 &\quad + (\xi_4^+ + \phi_4^+ + \psi_4^+)(\mu)(1 - x_3 - 10x_1x_2)\}, \\
 T_4(x_i, \mu) &= \frac{3}{2}\{(\xi_5^0 + \phi_5^0 + \psi_5^0)(\mu)(1 - x_3) + (\xi_5^- + \phi_5^- - \psi_5^-)(\mu)[2x_1x_2 - x_3(1 - x_3)] \\
 &\quad + (\xi_5^+ + \phi_5^+ + \psi_5^+)(\mu)(1 - x_3 - 2(x_1^2 + x_2^2))\}, \\
 T_5(x_i, \mu) &= 6x_3[\xi_5^0(\mu) + \xi_5^+(\mu)(1 - 2x_3)], \\
 T_6(x_i, \mu) &= 2[\phi_6^0(\mu) + \frac{1}{2}(\phi_6^- - \phi_6^+)(\mu)(1 - 3x_3)], \\
 T_7(x_i, \mu) &= 6x_3\{(-\xi_4^0 + \phi_4^0 + \psi_4^0)(\mu)(1 - x_3) + (-\xi_4^- + \phi_4^- - \psi_4^-)(\mu)[x_1^2 + x_2^2 - x_3(1 - x_3)] \\
 &\quad + (-\xi_4^+ + \phi_4^+ + \psi_4^+)(\mu)(1 - x_3 - 10x_1x_2)\}, \\
 T_8(x_i, \mu) &= \frac{3}{2}\{(-\xi_5^0 + \phi_5^0 + \psi_5^0)(\mu)(1 - x_3) + (-\xi_5^- + \phi_5^- - \psi_5^-)(\mu)[2x_1x_2 - x_3(1 - x_3)] \\
 &\quad + (-\xi_5^+ + \phi_5^+ + \psi_5^+)(\mu)(1 - x_3 - 2(x_1^2 + x_2^2))\}, \\
 S_1(x_i, \mu) &= 6x_3(x_2 - x_1) [(\xi_4^0 + \phi_4^0 + \psi_4^0 + \xi_4^+ + \phi_4^+ + \psi_4^+)(\mu) + (\xi_4^- + \phi_4^- - \psi_4^-)(\mu)(1 - 2x_3)] \\
 S_2(x_i, \mu) &= \frac{3}{2}(x_2 - x_1) [-(\psi_5^0 + \phi_5^0 + \xi_5^0)(\mu) + (\xi_5^- + \phi_5^- - \psi_5^-)(\mu)x_3 \\
 &\quad + (\xi_5^+ + \phi_5^+ + \psi_5^+)(\mu)(1 - 2x_3)] \\
 P_1(x_i, \mu) &= 6x_3(x_2 - x_1) [(\xi_4^0 - \phi_4^0 - \psi_4^0 + \xi_4^+ - \phi_4^+ - \psi_4^+)(\mu) + (\xi_4^- - \phi_4^- + \psi_4^-)(\mu)(1 - 2x_3)] \\
 P_2(x_i, \mu) &= \frac{3}{2}(x_2 - x_1) [(\psi_5^0 + \phi_5^0 - \xi_5^0)(\mu) + (\xi_5^- - \phi_5^- + \psi_5^-)(\mu)x_3 \\
 &\quad + (\xi_5^+ - \phi_5^+ - \psi_5^+)(\mu)(1 - 2x_3)] .
 \end{aligned} \tag{37}$$

The following functions are encountered to the above amplitudes and they can be defined in terms of the eight

independent parameters, namely  $f_N, \lambda_1, \lambda_2, V_1^d, A_1^u, f_1^d, f_2^d$  and  $f_u^1$ :

$$\begin{aligned}
 \phi_3^0 &= \phi_6^0 = f_N \\
 \phi_4^0 &= \phi_5^0 = \frac{1}{2}(\lambda_1 + f_N) \\
 \xi_4^0 &= \xi_5^0 = \frac{1}{6}\lambda_2 \\
 \psi_4^0 &= \psi_5^0 = \frac{1}{2}(f_N - \lambda_1) \\
 \phi_3^- &= \frac{21}{2}A_1^u, \\
 \phi_3^+ &= \frac{7}{2}(1 - 3V_1^d), \\
 \phi_4^- &= \frac{5}{4}(\lambda_1(1 - 2f_1^d - 4f_1^u) + f_N(2A_1^u - 1)) , \\
 \phi_4^+ &= \frac{1}{4}(\lambda_1(3 - 10f_1^d) - f_N(10V_1^d - 3)) , \\
 \psi_4^- &= -\frac{5}{4}(\lambda_1(2 - 7f_1^d + f_1^u) + f_N(A_1^u + 3V_1^d - 2)) , \\
 \psi_4^+ &= -\frac{1}{4}(\lambda_1(-2 + 5f_1^d + 5f_1^u) + f_N(2 + 5A_1^u - 5V_1^d)) , \\
 \xi_4^- &= \frac{5}{16}\lambda_2(4 - 15f_2^d), \\
 \xi_4^+ &= \frac{1}{16}\lambda_2(4 - 15f_2^d), \\
 \phi_5^- &= \frac{5}{3}(\lambda_1(f_1^d - f_1^u) + f_N(2A_1^u - 1)) , \\
 \phi_5^+ &= -\frac{5}{6}(\lambda_1(4f_1^d - 1) + f_N(3 + 4V_1^d)) , \\
 \psi_5^- &= \frac{5}{3}(\lambda_1(f_1^d - f_1^u) + f_N(2 - A_1^u - 3V_1^d)) , \\
 \psi_5^+ &= -\frac{5}{6}(\lambda_1(-1 + 2f_1^d + 2f_1^u) + f_N(5 + 2A_1^u - 2V_1^d)) , \\
 \xi_5^- &= -\frac{5}{4}\lambda_2 f_2^d, \\
 \xi_5^+ &= \frac{5}{36}\lambda_2(2 - 9f_2^d), \\
 \phi_6^- &= \frac{1}{2}(\lambda_1(1 - 4f_1^d - 2f_1^u) + f_N(1 + 4A_1^u)) , \\
 \phi_6^+ &= -\frac{1}{2}(\lambda_1(1 - 2f_1^d) + f_N(4V_1^d - 1))
 \end{aligned} \tag{38}$$