Scalar quarkonia at finite temperature

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Masses and decay constants of the scalar quarkonia, $\chi_{Q0}(Q = b, c)$ with quantum numbers $I^G(J^{PC}) = 0^+(0^{++})$, are calculated in the framework of the QCD sum rules approach both in vacuum and finite temperature. The masses and decay constants remain unchanged up to $T \simeq 100$ MeV but they start to diminish with increasing the temperature after this point. Near the critical or deconfinement temperature, the decay constants reach approximately to 25% of their values in vacuum, while the masses are decreased about 6% and 23% for bottom and charm cases, respectively. The results at zero temperature are consistent with the existing experimental values and predictions of the other nonperturbative approaches. Our predictions on the decay constants in vacuum, as well as the behavior of the masses and decay constants with respect to the temperature, can be checked in the future experiments.

I. INTRODUCTION

Understanding the internal structure of scalar mesons has been a prominent topic in the last 30–40 years. Although the scalar mesons have been investigated for several decades, many properties of them are not clear yet, and identifying the scalar mesons is difficult experimentally. Hence, the theoretical works can play a crucial role in this respect. In particle physics, the quarkonia refers to flavorless mesons containing a heavy b (c) quark and its own antiquark, i.e., $b\bar{b}$ (bottomonium) and $c\bar{c}$ (charmonium). These approximately nonrelativistic systems are the best candidates to investigate the hadronic dynamics and to study the perturbative and nonperturbative aspects of QCD. It was believed that the quarkonia can help us to extract the nature of quark-antiquark interaction at the hadronic scale and play the same role in probing the QCD as the hydrogen atom play in the atomic physics [1].

A large number of the beauty and charmed systems have been experimentally observed in the last few decades (see, for instance, [2–4]) and the theoretical calculations on the properties of these systems have been made mainly using potential model, where the quarkonia is described by a static potential, $V = -\frac{4}{3}\frac{\alpha_s}{r} + kr$ and its extensions like the Coulomb gauge model [5–9]. The first term in the potential is related to one gluon exchange, and the second term is called the confinement potential. The recent CLEO measurements on the two-photon decay rates of the even-parity, P-wave scalar 0^{++}, $\chi_{b(c)0}$ and tensor 2^{++}, $\chi_{b(c)2}$ states ([10,11], and references therein) were the motivation to investigate the properties of the quarkonia and their radiative decays from the quark-antiquark interaction point of view (see, for example, [12,13]).

In [14], which is a recent study on extraction of ground-state decay constant from both sum rules and potential models, it is stated that results obtained at each step of the extraction procedure both in QCD and in potential models follow the same pattern; hence all of our findings concerning the extraction of bound-state parameters from correlation functions obtained in potential model can apply also to QCD. It is also proven that in the QCD sum rules approach, by tuning the continuum threshold which is related to the energy of the first exited state specially with a Borel parameter-dependent threshold, we can get a more reliable and accurate determination of bound-state characteristics comparing the potential models. The QCD sum rules approach as a nonperturbative approach is one of the most powerful and applicable tools to studying the spectroscopy of hadrons and can play a crucial role in the investigation of the properties of the hadrons [15–18]. It has been used to calculate the masses and decay constants of mesons [19–26]. This approach was extended to contain the properties of the hadrons at finite temperature called thermal QCD sum rules [27–29] supposing that the operator product expansion (OPE) and the quark-hadron duality assumption remain valid; however, the quark-quark, quark-gluon, and gluon-gluon condensates are altered by their thermal versions. The main aspiration of this addendum was to explain the results obtained from the heavy ion collision experiments. It is presently believed that the hot and dense medium where the hadrons are formed modifies masses and decay widths of hadrons. It is shown that heavy mesons like J/ψ and also radial and orbital $c\bar{c}$ excitations have different behaviors when the temperature of the medium changes (see [30], and references therein). In [31], scalar mesons and scalar glueballs are investigated in holographic QCD at finite temperature. A flood of papers have also been dedicated generally to the

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determination of the condensates, mass and decay constant of mesons, and some properties of the nucleons at finite temperature [32–45].

In the present work, we calculate the mass and decay constant of the heavy scalar χ_{Q0} mesons with quantum numbers $I^G(J^{PC}) = 0^+(0^{++})$ using the thermal QCD sum rules approach. Here, we assume that with replacing the vacuum condensates and also the continuum threshold by their thermal version, the sum rules for the observables (masses and decay constants) remain valid. In calculations, we take into account the additional operators in the Wilson expansion at finite temperature [46] and modify spectral density in the QCD side. These operators are due to the breakdown of Lorentz invariance at finite temperature by the selection of the thermal rest frame, where matter is at rest at a definite temperature [38,47]. In this condition, the residual O(3) invariance brings these extra operators with the same mass dimension as the vacuum condensates. We also consider the interaction of the currents with the existing particles in the medium at finite temperature. Such interactions require modification of the hadron spectral density.

The outline of the paper is as follows: in the next section, sum rules for the mass and the decay constant of the heavy scalar χ_{Q0} mesons are obtained in the framework of the QCD sum rules at finite temperature. Section III encompasses our numerical predictions for the mass and decay constants as well as comparison of the results with the existing predictions of the other nonperturbative approaches and experimental values.

II. QCD SUM RULES FOR THE MASS AND DECAY CONSTANT

In this section, we obtain sum rules for the mass as well as the decay constant of the scalar quarkonia containing b or c quark in the framework of the thermal QCD sum rules. For this aim, we will evaluate the two-point thermal correlation function

$$\Pi(q, T) = i \int d^4 x e^{iq.x} \langle \mathcal{T}(J^S(x)\bar{J}^S(0)) \rangle \quad (1)$$

in two different ways: physical and theoretical representations. In the correlation function, T denotes the temperature, $\mathcal{T}$ is the time ordering product, and $J^S(x) = \bar{Q}(x)Q(x)$ is the interpolating current of the heavy scalar meson, $S = \chi_{Q0}(Q = b, c)$. The thermal average of any operator, A can be expressed as

$$\langle A \rangle = \frac{\text{Tr}(e^{-\beta H} A)}{\text{Tr}(e^{-\beta H})}, \quad (2)$$

where H is the QCD Hamiltonian, and $\beta = 1/T$ is the inverse of the temperature T and traces are carried out over any complete set of states.

The physical or phenomenological representation of the aforementioned two-point correlation function is obtained

in terms of the hadronic parameters saturating it with a tower of scalar mesons with the same quantum numbers as the interpolating current. The theoretical or QCD representation is gained via OPE in terms of the QCD parameters such as the quark's masses, and the vacuum condensates considering the internal structure of these mesons, i.e., quarks, gluons, and their interactions with each other as well as with the QCD vacuum. Sum rules for the physical observables such as the decay constant and mass are obtained equating these two different representations through dispersion relation. To suppress the contribution of the higher states and continuum, Borel transformation with respect to the $Q_0^2 = -q_0^2$ is applied to both sides of the sum rules for physical quantities.

To calculate the phenomenological part, we insert a complete set of intermediate states owing the same quantum numbers with current J^S between the currents in Eq. (1) and perform the integral over. As a result, at $T = 0$, we obtain

$$\Pi(q, 0) = \frac{\langle 0 \mid J(0) \mid S \rangle \langle S \mid J(0) \mid 0 \rangle}{m_S^2 - q^2} + \cdots, \quad (3)$$

where $\cdots$ represents the contributions of the higher states and continuum, and m_S is mass of the heavy scalar meson. The matrix element creating the scalar meson from the vacuum can be written in terms of the decay constant, f_S by the following manner:

$$\langle 0 \mid J(0) \mid S \rangle = f_S m_S. \quad (4)$$

Note that Eqs. (3) and (4) are valid also at finite temperature, hence, the final representation for the physical side can be written in terms of the temperature dependent mass and decay constant as

$$\Pi(q, T) = \frac{f_S^2(T)m_S^2(T)}{m_S^2(T) - q^2} + \cdots. \quad (5)$$

In the QCD side, the correlation function is calculated in deep Euclidean region, $q^2 \ll -\Lambda_{\text{QCD}}^2$ via OPE where the short or perturbative and long distance or nonperturbative effects are separated, i.e.,

$$\Pi^{\text{QCD}}(q, T) = \Pi^{\text{pert}}(q, T) + \Pi^{\text{nonpert}}(q, T). \quad (6)$$

The short distance contribution [bare loop diagram in Figure 1(a)] is calculated using the perturbation theory; whereas, the long distance contributions [diagrams shown in Figure 1(b)] are represented in terms of the thermal expectation values of some operators. To proceed, we write the perturbative part in terms of a dispersion integral,

$$\Pi^{\text{QCD}}(q, T) = \int \frac{ds \rho(s, T)}{s - q^2} + \Pi^{\text{nonpert}}, \quad (7)$$

where, $\rho(s, T)$ is called the spectral density at finite temperature. The thermal spectral density at fixed $\mid \mathbf{q} \mid$ can be expressed as

$$\rho(q, T) = \frac{1}{\pi} \text{Im}\Pi^{\text{pert}}(q, T) \tanh\left(\frac{\beta q_0}{2}\right). \qquad (8)$$

To proceed, we need to know the fermion propagator at finite temperature. The thermal fermion propagator at real time is given as

$$S(k) = (\gamma_\mu k^\mu + m_Q)$$
$$\times \left(\frac{i}{k^2 - m_Q^2 + i\varepsilon} - 2\pi n(|k_0|)\delta(k^2 - m_Q^2)\right), \qquad (9)$$

and here, $n(x)$ is Fermi distribution function,

$$n(x) = [\exp(\beta x) + 1]^{-1}. \qquad (10)$$

Using the above propagator, we find the following expression for the imaginary part of the correlation function at $|\mathbf{q}| = 0$ limit:

$$\text{Im}\Pi(q_0, T) = N_c \int \frac{d\mathbf{k}}{8\pi^2} \frac{1}{\omega^2} (q_0\omega - 2m_Q^2)$$
$$\times (1 - 2n(\omega) + 2n^2(\omega))\delta(q_0 - 2\omega), \qquad (11)$$

where, $\omega = \sqrt{m_Q^2 + \mathbf{k}^2}$. After some straightforward calculations, the thermal spectral density is obtained as

$$\rho(s) = \frac{3s}{8\pi^2}\left(1 - \frac{4m_Q^2}{s}\right)^{3/2}\left(1 - 2n\left(\frac{\sqrt{s}}{2}\right)\right). \qquad (12)$$

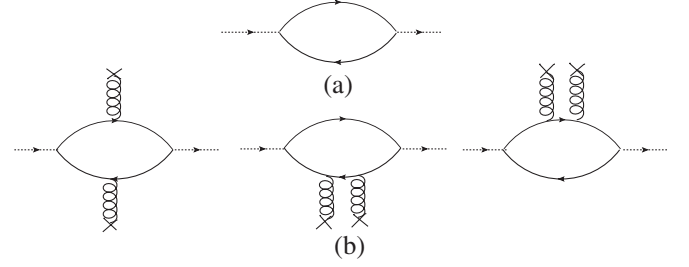In the nonperturbative part, the main contribution comes from the two gluon condensates since the heavy quark

condensates are suppressed by inverse powers of the heavy quark mass and can be safely removed. The gluon condensate diagrams are represented in part (b) of Fig. 1. In order to calculate nonperturbative contributions, we use Fock-Schwinger gauge, $x^\mu A_\mu^a(x) = 0$. In momentum space, the vacuum gluon field is expressed as

$$A_\mu^a(k') = -\frac{i}{2}(2\pi)^4 G_{\rho\mu}^a(0)\frac{\partial}{\partial k_\rho'}\delta^{(4)}(k'), \qquad (13)$$

and in calculations, we use the quark-gluon-quark vertex as

$$\Gamma_{ij\mu}^a = ig\gamma_\mu\left(\frac{\lambda^a}{2}\right)_{ij}, \qquad (14)$$

where k' is the gluon momentum.

After straightforward calculations, the nonperturbative part in momentum space is obtained as

FIG. 1. (a) Bare loop diagram; (b) Diagrams corresponding to gluon condensates.

$$\Pi^{\text{nonpert}} = \int_0^1 dx \frac{x^2}{288\pi(m_Q^2 + q^2x(-1 + x))^4}\{3\langle\alpha_s G^2\rangle[40m_Q^6 x^2 - 9q^6 x^2(-1 + x)^4(-1 - 2x + 2x^2)$$
$$- 12m_Q^2 q^4 x(-1 + x)^2(1 + 4x - 12x^2 + 6x^3) + m_Q^4 q^2(-15 + 156x - 441x^2 + 434x^3 - 134x^4)]$$
$$- \alpha_s\langle u^\alpha\Theta_{\alpha\beta}^g u^\beta\rangle[4q^2(-1 + x)(q^4(-1 + x)^2 x^2(9 + 11x - 14x^2 + 12x^3) + m_Q^4(-15 + 135x - 246x^2 + 176x^3)$$
$$+ 4m_Q^2 q^2 x(-3 - 8x + 28x^2 - 34x^3 + 17x^4)) - 16(-1 + x)(q.u)^2(q^4 x^2(-1 + x)^2(9 + 11x - 14x^2 + 12x^3)$$
$$+ m_Q^4(-15 + 135x - 246x^2 + 176x^3) + 4m_Q^2 q^2 x(-3 - 8x + 28x^2 - 34x^3 + 17x^4))]\}, \qquad (15)$$

where the 4-vector u^μ is the velocity of the heat bath and it is introduced to restore Lorentz invariance formally in the thermal field theory. In the rest frame of the heat bath, $u^\mu = (1, 0, 0, 0)$ and $u^2 = 1$. In deriving the above expression, we have used the following relation considering the Lorentz covariance [37]:

$$\langle\text{Tr}^c G_{\alpha\beta}G_{\lambda\sigma}\rangle = (g_{\alpha\lambda}g_{\beta\sigma} - g_{\alpha\sigma}g_{\beta\lambda})A - (u_\alpha u_\lambda g_{\beta\sigma}$$
$$- u_\alpha u_\sigma g_{\beta\lambda} - u_\beta u_\lambda g_{\alpha\sigma} + u_\beta u_\sigma g_{\alpha\lambda})B. \qquad (16)$$

Contracting indices on both sides, we obtain

$$A = \frac{1}{24}\langle G_{\alpha\beta}^a G^{a\alpha\beta}\rangle + \frac{1}{6}\langle u^\alpha\Theta_{\alpha\beta}^g u^\beta\rangle, \qquad (17)$$

$$B = \frac{1}{3}\langle u^\alpha\Theta_{\alpha\beta}^g u^\beta\rangle, \qquad (18)$$

where $\Theta_{\alpha\beta}^g$ is the traceless, gluonic part of the stress-tensor of the QCD, and it is defined as

$$\Theta_{\alpha\beta}^g = -G_{\alpha\lambda}^a G_\beta^{\lambda a} + \frac{1}{4}g_{\alpha\beta}G_{\lambda\sigma}^a G^{\lambda\sigma a}. \qquad (19)$$

Matching the phenomenological and QCD sides of the correlation function, the sum rules for the mass and decay constant of scalar meson are obtained. To suppress the contribution of the higher states and continuum, Borel transformation over q^2, as well as continuum subtraction, are performed. As a result of the above procedure, we obtain the following sum rule for the decay constant:

$$m_S^2(T)f_S^2(T)e^{(-m_S^2(T))/M^2}$$
$$= \left\{ \int_{4m_Q^2}^{s_0(T)} ds\rho(s)e^{-(s/M^2)} + \hat{B}\Pi^{\text{nonpert}} \right\}, \quad (20)$$

where M^2 is the Borel mass parameter and $s_0(T)$ is the temperature dependent continuum threshold. The sum rules for the mass is obtained applying derivative with respect to $-\frac{1}{M^2}$ to the both sides of the sum rule for the decay constant of the scalar meson in Eq. (20) and dividing by itself:

$$m_S^2(T) = \frac{\int_{4m_Q^2}^{s_0(T)} dss\rho(s)\exp(-\frac{s}{M^2}) + \Pi_1^{\text{nonpert}}(M^2,T)}{\int_{4m_Q^2}^{s_0(T)} ds\rho(s)\exp(-\frac{s}{M^2}) + \hat{B}\Pi^{\text{nonpert}}(M^2,T)}, \quad (21)$$

where

$$\Pi_1^{\text{nonpert}}(M^2,T) = -\frac{d}{d(1/M^2)}\hat{B}\Pi^{\text{nonpert}}(M^2,T), \quad (22)$$

and $\hat{B}\Pi^{\text{nonpert}}(M^2,T)$ shows contribution of the gluon condensates in Borel transformed scheme and is given by

$$\hat{B}\Pi^{\text{nonpert}} = \int_0^1 dx e^{m_Q^2/M^2x(x-1)} \frac{1}{96M^6\pi(x-1)^4x^3}\{[\langle\alpha_sG^2\rangle(m_Q^6(1-2x)^2(-3+5x)+9M^6(-1+x)^4x^3(-1-2x+2x^2)$$
$$- 3m_Q^2M^4(-1+x)^2x^2(-5+7x-12x^2+6x^3)+2m_Q^4M^2x(3-21x+48x^2-41x^3+11x^4))$$
$$- 4\alpha_s\langle\Theta^g\rangle(m_Q^6(1-2x)^2(-3+5x)+M^6(-1+x)^3x^3(9+11x-14x^2+12x^3)$$
$$+ 2m_Q^4M^2x(3-23x+58x^2-57x^3+19x^4)-m_Q^2M^4(-1+x)^2x^2(-15+11x-26x^2+32x^3))]\}. \quad (23)$$

We use the gluonic part of energy density both obtained from lattice QCD [48] and chiral perturbation theory [49]. In the rest frame of the heat bath, the results obtained in [48] at lattice QCD are reproduced well by the following fit parametrization for the thermal average of total energy density $\langle\Theta\rangle$:

$$\langle\Theta\rangle = 2\langle\Theta^g\rangle = 6\times10^{-6}\exp[80(T-0.1)]\,(\text{GeV}^4), \quad (24)$$

where temperature T is measured in units of GeV and this parametrization is valid in the interval $0.1\text{ GeV} \leq T \leq 0.17\text{ GeV}$. Note that the total energy density has been known for $T \gtrsim 0$ in the chiral perturbation theory, while this quantity has only been calculated for $T \geq 100$ MeV in lattice QCD (see [32,48]). In low temperature chiral perturbation limit, the results presented in [49] are better described by the expression

$$\langle\Theta\rangle = \langle\Theta_\mu^\mu\rangle + 3p, \quad (25)$$

where, p is pressure and $\langle\Theta_\mu^\mu\rangle$ is trace of the total energy momentum tensor. They are given as

$$\langle\Theta_\mu^\mu\rangle = \frac{\pi^2}{270}\frac{T^8}{F_\pi^4}\ln\left[\frac{\Lambda_p}{T}\right],$$

$$p = 3T\left(\frac{m_\pi T}{2\pi}\right)^{3/2}\left(1+\frac{15T}{8m_\pi}+\frac{105T^2}{128m_\pi^2}\right)\exp\left[-\frac{m_\pi}{T}\right], \quad (26)$$

where $\Lambda_p = 0.275$ GeV, $F_\pi = 0.093$ GeV and $m_\pi = 0.14$ GeV.

Our final task in this section is to introduce the temperature dependent continuum threshold, $s_0(T)$, gluon condensate, $\langle G^2\rangle$ and the strong coupling constant. The temperature dependent continuum threshold [50] and gluon condensate [32,48] are well described by the following fit parametrizations:

$$s(T) = s_0\left[1-\left(\frac{T}{T_c^*}\right)^8\right]+4m_Q^2\left(\frac{T}{T_c^*}\right)^8, \quad (27)$$

$$\langle G^2\rangle = \frac{\langle0|G^2|0\rangle}{\exp[12(\frac{T}{T_c}-1.05)]+1}, \quad (28)$$

where $T_c^* = 1.1\times T_c = 0.176$ GeV, and s_0 and $\langle0|G^2|0\rangle$ are the continuum threshold and the gluon condensate in vacuum, respectively. These parametrizations are valid only in the interval $0 \leq T \leq 170$ MeV. Here, we should stress that the continuum threshold presented above is equal to the continuum threshold in vacuum at $T = 0$, but it starts to diminish increasing the temperature such that at $T = T_c^*$ it reaches the perturbative QCD threshold, $4m_Q^2$. This parametrization belongs to heavy-heavy system and differ considerably with the case of light-light and heavy-light quark systems, where the continuum threshold is related to the thermal light quark condensate (for details, see [50]).

We also use temperature dependent strong coupling constant [51,52] as

$$g^{-2}(T) = \frac{11}{8\pi^2}\ln\left(\frac{2\pi T}{\Lambda_{\overline{MS}}}\right)+\frac{51}{88\pi^2}\ln\left[2\ln\left(\frac{2\pi T}{\Lambda_{\overline{MS}}}\right)\right], \quad (29)$$

where, $\Lambda_{\overline{MS}} \simeq T_c/1.14$ and in numerical calculations, instead of the α_s in front of $\langle\Theta^g\rangle$ in Eq. (23) the $\tilde{\alpha}(T) = 2.096\alpha^{\text{pert}}(T)$ has been used, where $\alpha^{\text{pert}}(T) = \frac{g^2(T)}{4\pi}$ (for details, see [51]).

III. NUMERICAL ANALYSIS

The present section is devoted to the numerical analysis of the sum rules for the mass and decay constant of the heavy scalar mesons. In further analysis, we use $m_c = (1.3 \pm 0.05)$ GeV, $m_b = (4.7 \pm 0.1)$ GeV and $\langle0|\frac{1}{\pi}\alpha_sG^2|0\rangle = (0.012 \pm 0.004)$ GeV^4. The sum rules for
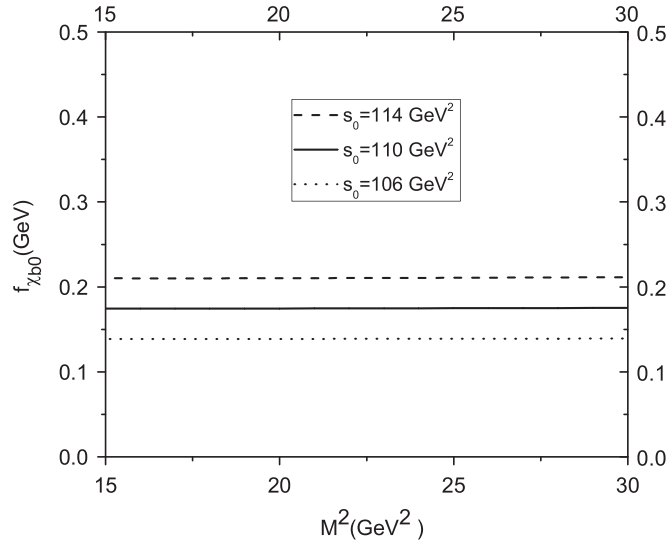
FIG. 2.   The dependence of the decay constant of scalar χ_{b0} meson on the Borel parameter, M^2 in vacuum at three fixed values of the continuum threshold.
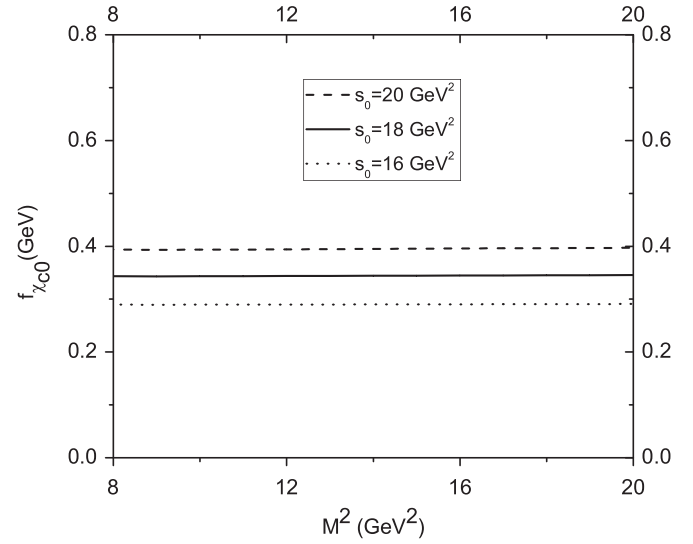

FIG. 4.   The dependence of the decay constant of scalar χ_{c0} meson on the Borel parameter, M^2 in vacuum at three fixed values of the continuum threshold.

the mass and decay constant also contain two auxiliary parameters, continuum threshold s_0, and Borel mass parameter M^2. The standard criteria in QCD sum rules is that the physical quantities should be independent of the auxiliary parameters. However, the continuum threshold s_0 is not completely arbitrary but is related to the energy of the first exited state with the same quantum numbers and can depend on the Borel mass parameter [53]. Therefore, the standard criteria does not render realistic errors, and in fact the existing error should be large. Hence, we will add also the systematic errors to the numerical results. We choose the values $s_0 = (110 \pm 4)$ GeV^2 and $s_0 = (18 \pm 2)$ GeV^2 for the continuum threshold at χ_{b0} and

χ_{c0} channels, respectively. The working region for the Borel mass parameter is determined requiring that not only the higher state and continuum contributions are suppressed, but also the contribution of the highest order operator should be small, i.e., the sum rules for the mass and decay constant should converge. As a result of the above procedure, the working region for the Borel parameter is found to be 8 GeV^2 $\leq M^2 \leq$ 20 GeV^2 for χ_{c0} and 15 GeV^2 $\leq M^2 \leq$ 30 GeV^2 for χ_{b0} mesons. The dependences of the masses and decay constants at $T = 0$ on Borel mass parameter are shown in Figs. 2–5. These figures depict that the observables depend very weakly on the Borel mass parameter, M^2 in the working regions.

FIG. 3.   The same as Fig. 2 but for mass of the scalar χ_{b0} meson.
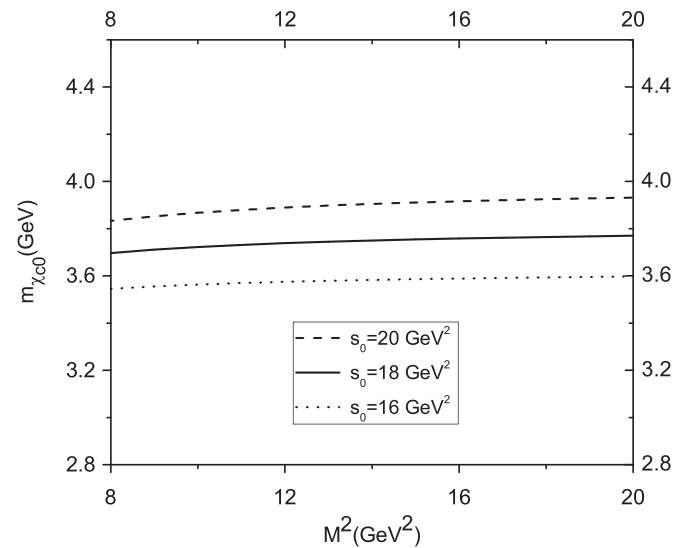

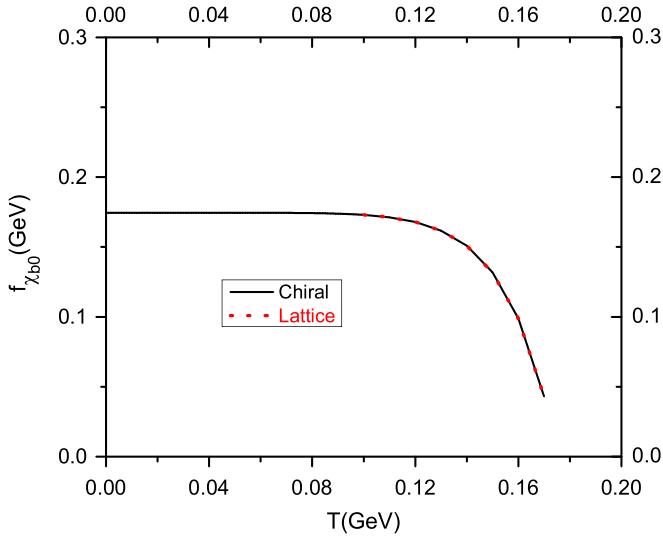FIG. 5.   The same as Fig. 4 but for mass of the scalar χ_{c0} meson.

FIG. 6 (color online). The dependence of the decay constant of scalar χ_{b0} meson on temperature obtained using both lattice QCD and chiral perturbation limit values for the gluonic part of energy density. The values $s_0 = 110$ GeV^2, and $M^2 = 17$ GeV^2 have been used for the continuum threshold and Borel mass parameter in vacuum, respectively.

FIG. 8 (color online). The dependence of the decay constant of scalar χ_{c0} meson on temperature obtained using both lattice QCD and chiral perturbation limit values for the gluonic part of energy density. The values $s_0 = 18$ GeV^2, and $M^2 = 9$ GeV^2 have been used for the continuum threshold and Borel mass parameter in vacuum, respectively.

The dependence of the mass and decay constant of the χ_{b0} and χ_{c0} mesons on temperature are presented in Figs. 6–9. In these figures, we show the results obtained using both lattice QCD and chiral perturbation limit values for the gluonic part of energy density. These figures depict that the results depend very weakly on the values of the gluonic part of energy density, i.e., both values obtained from lattice and chiral limit have approximately the same predictions in the interval, 0.1 GeV $\leq T \leq 0.17$ GeV at which the lattice results are valid. These figures also show that the masses and decay constants do not change up to
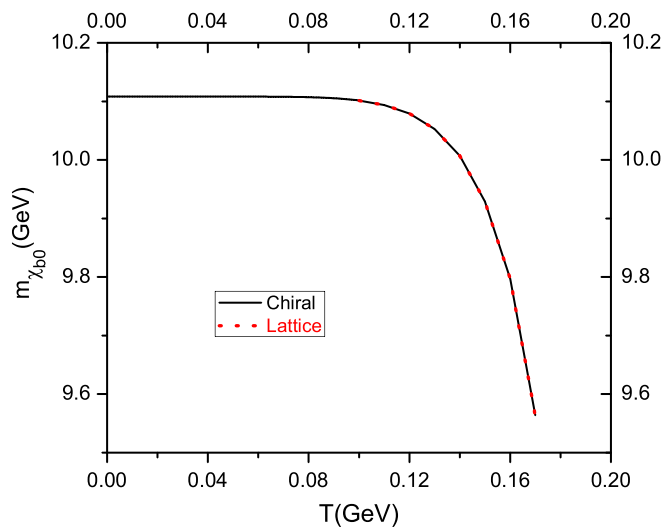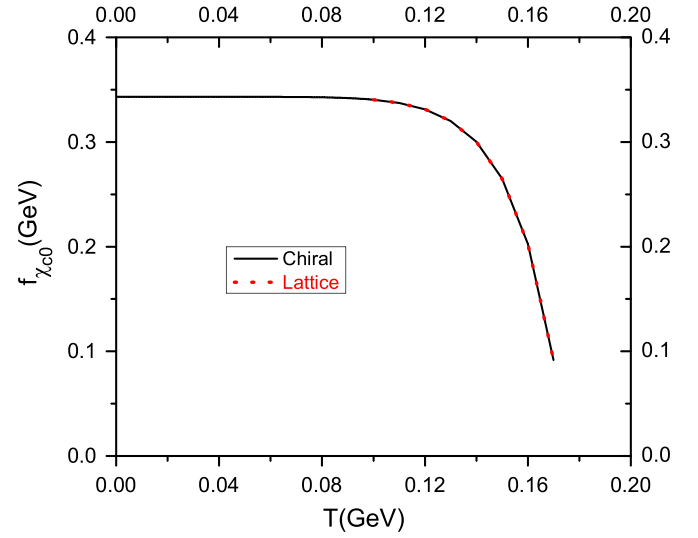
$T \simeq 100$ MeV, but they start to diminish with increasing the temperature after this point. Near the critical or deconfinement temperature, the decay constants reach approximately to 25% of their values in vacuum, while the masses are decreased about 6% and 23% for bottom and charm cases, respectively. From these figures, we deduce the results on the decay constant and mass in vacuum as presented in Tables I and II. The quoted errors in these Tables are due to the errors in variation of the continuum threshold, Borel mass parameter, and errors in other input parameters as well as the systematic uncertainties. Table I

FIG. 7 (color online). The same as Fig. 6 but for mass of the scalar χ_{b0} meson.
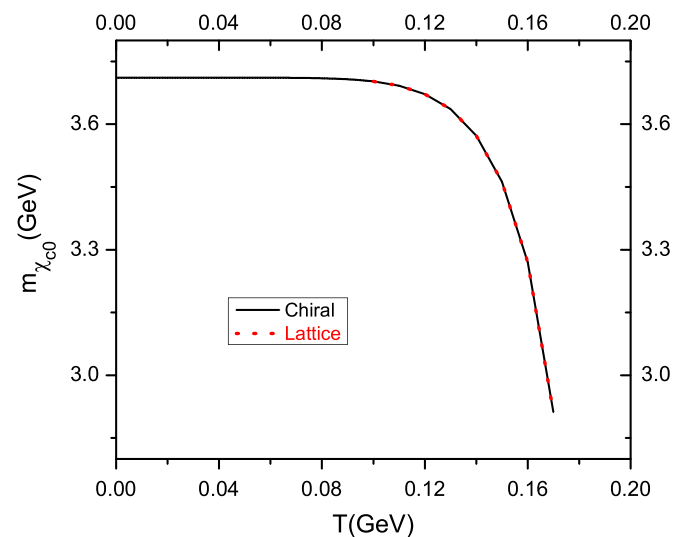

FIG. 9 (color online). The same as Fig. 8 but for mass of the scalar χ_{c0} meson.

TABLE I. Values of the leptonic decay constants of the heavy scalar, χ_{b0} and χ_{c0} mesons in vacuum. These results have been obtained using the values $s_0 = 110$ GeV^2 and $M^2 = 17$ GeV^2 for χ_{b0}, and $s_0 = 18$ GeV^2 and $M^2 = 9$ GeV^2 for χ_{c0} mesons.

	$f_{\chi_{b0}}$ (MeV)	$f_{\chi_{c0}}$ (MeV)
Present work	175 ± 55	343 ± 112
QCD sum rules [1]	$\cdots$	359
Cornell potential model [54]	$\cdots$	338
QCD sum rules [55]	$\cdots$	510 ± 40

TABLE II. Values of the mass of the heavy scalar, χ_{b0} and χ_{c0} mesons in vacuum.

	$m_{\chi_{b0}}$ (GeV)	$m_{\chi_{c0}}$ (GeV)
Present work	10.10 ± 1.75	3.71 ± 0.62
Experiment [11]	9.85944 ± 0.00042	3.41475 ± 0.00031

also include a comparison of the decay constant of charm case with the existing predictions of the same framework or other nonperturvative approaches. From this Table, we see that our predictions on the decay constant of the χ_{c0} at zero temperature are well consistent with the predictions of QCD sum rules [1] and Cornell potential model [54] predictions but differ considerably from the result obtained in [55] when the central values are considered. In Table II, we also compare our predictions on the masses of the heavy scalar mesons with the existing experimental data which are consistent. Our results for the leptonic decay constants, as well as their behavior with respect to the temperature, can be verified in the future experiments.

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