PHYSICAL REVIEW D **82,** 096006 (2010)

Strong coupling constants of decuplet baryons with vector mesons

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(Received 20 July 2010; published 10 November 2010)

We provide a comprehensive study of strong coupling constants of decuplet baryons with light nonet vector mesons in the framework of light cone QCD sum rules. Using the symmetry arguments, we argue that all coupling constants entering the calculations can be expressed in terms of only one invariant function even if the $SU(3)_f$ symmetry breaking effects are taken into account. We estimate the order of $SU(3)_f$ symmetry violations, which are automatically considered by the employed approach.

DOI: 10.1103/PhysRevD.82.096006 PACS numbers: 11.55.Hx, 13.30.−a, 13.30.Eg

I. INTRODUCTION

Theoretically, the baryon-baryon-meson coupling constants are fundamental objects as they can provide useful information on the low energy QCD, baryon-baryon interactions, and scattering of mesons from baryons. In other words, their values calculated in QCD can render important constraints in constructing baryon-baryon as well as baryon-meson potentials. They can help us to better analyze the results of existing experiments on the meson-nucleon, nucleon-hyperon, and hyperon-hyperon interactions held in different centers, such as MAMI, MIT, Bates, BNL, and Jefferson Laboratories.

Calculation of the baryon-meson coupling constants using the fundamental theory of QCD is highly desirable. However, such interactions occur in a region very far from the perturbative regime and the fundamental QCD Lagrangian is not suitable for calculation of these coupling constants. Therefore, we need some nonperturbative approaches. QCD sum rules [1] is one of the most powerful and applicable tools in this respect. It is based on the QCD Lagrangian, hence the problem of deriving the baryon-meson coupling from QCD sum rules is clearly of importance, both as a fundamental test of QCD and of the applied nonperturbative approach.

In the present work, we calculate the strong coupling constants of decuplet baryons with light nonet vector mesons in the framework of the light cone QCD sum rules [2]. Applying the symmetry arguments, we derive all related coupling constants in terms of only one universal function even if $SU(3)_f$ symmetry breaking effects are encountered. One of the main advantages of the approach used during this work is that it automatically includes the $SU(3)_f$ symmetry breaking effects. Calculation of these coupling constants is also very important for understanding the dynamics of light vector mesons and their electroproduction off the decuplet baryons. Note that the strong coupling constants of the octet and decuplet baryons with pseudoscalar mesons as well as octet baryons with vector mesons have been studied within the same framework in [3–7].

The layout of the paper is as follows. In Sec. II, using the symmetry relations, sum rules for the strong coupling constants of the light nonet vector mesons with decuplet baryons are obtained in the framework of light cone QCD sum rules (LCSR). In Sec. III, we numerically analyze the coupling constants of the light nonet vector mesons with decuplet baryons, estimate the order of $SU(3)_f$ symmetry violations, and discuss the obtained results.

II. SUM RULES FOR THE STRONG COUPLING CONSTANTS OF THE LIGHT NONET VECTOR MESONS WITH DECUPLET BARYONS

In this part, we derive LCSR for the coupling constants of the light nonet vector mesons with decuplet baryons and show how it is possible to express all couplings entering the calculations in terms of only one universal function. In $SU(3)_f$ symmetry, the interaction Lagrangian can be written as

$$\mathcal{L}_{\rm int} = g\varepsilon_{ijk}(\bar{\mathcal{D}}^j_{\ell m})^\mu(\mathcal{D}^{m\ell k})_\mu \partial^n V^i_n + \text{H.c.}, \qquad (1)$$

where the ε_{ijk} is the antisymmetric Levi-Civita tensor, $\mathcal{D}^{m\ell k}$ denote components of the decuplet baryons, the $\bar{\mathcal{D}}^j_{\ell m}$ is its Hermitian conjugation, V^i_n correspond to the components of octet vector mesons, and μ is the Rarita-Schwinger index for spin 3/2 particles. To obtain the sum rules for coupling constants, we start considering the following correlation function, which is the main building block in QCD sum rules:

$$\Pi_{\mu\nu} = i\int d^4x e^{ipx}\langle V(q)|\mathcal{T}\{\eta_\mu(x)\bar{\eta}_\nu(0)\}|0\rangle, \qquad (2)$$

where $V(q)$ corresponds to the light mesons with momentum q, η_μ is the interpolating currents for decuplet baryons, and $\mathcal{T}$ is the time ordering operator. To obtain sum

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rules for the coupling constants, we will calculate the correlation function in the following two different ways:

(i) in the phenomenological side, the correlation function is obtained in terms of hadronic parameters saturating it by a tower of hadrons with the same quantum numbers as the interpolating currents.

(ii) in the theoretical or the QCD side, the correlation function is calculated by means of operator product expansion (OPE) in the deep Euclidean region, where $-p^2 \to \infty$ and $-(p+q)^2 \to \infty$, in terms of quark and gluon degrees of freedom. With the help of the OPE, the short and large distance effects are separated. The short range effects are calculated using the perturbation theory, whereas the long distance contributions are parametrized in terms of distribution amplitudes (DA's) of the light nonet vector mesons.

Finally, to get the sum rules, we equate these two representations of the correlation functions through dispersion relation and apply Borel transformation with respect to the variables $(p+q)^2$ and p^2 to suppress the contribution of the higher states and continuum. Before starting calculations of the correlation function in physical or theoretical sides, let us introduce the interpolating currents of the decuplet baryons. The interpolating currents creating the decuplet baryons can be written in a compact form as

$$\eta_\mu = A\varepsilon^{abc}\{(q_1^{aT}C\gamma_\mu q_2^b)q_3^c + (q_2^{aT}C\gamma_\mu q_3^b)q_1^c + (q_3^{aT}C\gamma_5 q_1^b)q_2^c\}, \tag{3}$$

where a, b, and c are the color indices and C is the charge conjugation operator. The values of normalization constant A and the q_1, q_2, and q_3 quarks are represented in Table I.

As we already noted, the phenomenological side of the correlation function is obtained inserting a full set of hadrons with quantum numbers of η_μ and isolating the ground state baryons as

TABLE I. The values of A and the quark flavors q_1, q_2, and q_3 for decuplet baryons.

	A	q_1	q_2	q_3
Σ^{*0}	$\sqrt{2/3}$	u	d	s
Σ^{*+}	$\sqrt{1/3}$	u	u	s
Σ^{*-}	$\sqrt{1/3}$	d	d	s
Δ^{++}	$1/3$	u	u	u
Δ^{+}	$\sqrt{1/3}$	u	u	d
Δ^{0}	$\sqrt{1/3}$	d	d	u
Δ^{-}	$1/3$	d	d	d
Ξ^{*0}	$\sqrt{1/3}$	s	s	u
Ξ^{*-}	$\sqrt{1/3}$	s	s	d
Ω^{-}	$1/3$	s	s	s

$$\Pi_{\mu\nu}(p,q) = \frac{\langle 0|\eta_\mu|\mathcal{D}(p_2)\rangle\langle\mathcal{D}(p_2)V(q)|\mathcal{D}(p_1)\rangle\langle\mathcal{D}(p_1)|\bar{\eta}_\nu|0\rangle}{(p_2^2 - m_{\mathcal{D}2}^2)(p_1^2 - m_{\mathcal{D}1}^2)} + \cdots, \tag{4}$$

where $m_{\mathcal{D}1}$ and $m_{\mathcal{D}2}$ are masses of the initial and final state decuplet baryons with momentum $p_1 = p + q$ and $p_2 = p$, respectively, and $\cdots$ represents the contribution of the higher states and continuum.

To proceed, we need to know the matrix element of the interpolating current between the vacuum and the decuplet state as well as the transition matrix element. The $\langle\mathcal{D}(p_1)|\eta_\mu|0\rangle$ is defined in terms of the residue $\lambda_{\mathcal{D}}$ as

$$\langle 0|\eta_\mu|\mathcal{D}(p)\rangle = \lambda_{\mathcal{D}} u_\mu(p), \tag{5}$$

where u_μ is the Rarita-Schwinger spinor. The transition matrix element, $\langle\mathcal{D}(p_2)V(q)|\mathcal{D}(p_1)\rangle$, is parametrized in terms of coupling form factors g_1, g_2, g_3, and g_4 as

$$\langle\mathcal{D}(p_2)V(q)|\mathcal{D}(p_1)\rangle = \bar{u}_\alpha(p_2)\left\{g^{\alpha\beta}\left[\not{\varepsilon}g_1 + 2p.\varepsilon\frac{g_2}{(m_{\mathcal{D}1}+m_{\mathcal{D}2})}\right] + \frac{q^\alpha q^\beta}{(m_{\mathcal{D}1}+m_{\mathcal{D}2})^2}\left[\not{\varepsilon}g_3 + 2p.\varepsilon\frac{g_4}{(m_{\mathcal{D}1}+m_{\mathcal{D}2})}\right]\right\}u_\beta(p_1). \tag{6}$$

Using Eqs. (5) and (6) into (4) and performing a summation over spins of the decuplet baryons using

$$\sum_s u_\mu(p,s)\bar{u}_\nu(p,s) = (\not{p} + m_{\mathcal{D}})\left\{g_{\mu\nu} - \frac{\gamma_\mu\gamma_\nu}{3} - \frac{2p_\mu p_\nu}{3m_{\mathcal{D}}^2} + \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{3m_{\mathcal{D}}}\right\}, \tag{7}$$

in principle, one can obtain the final expression for the phenomenological side of the correlation function. However, there are two problems which we should overcome: all existing structures are not independent and the interpolating current for decuplet baryons couples also to unwanted spin-1/2 states, i.e.,

$$\langle 0|\eta_\mu|1/2(p)\rangle = (A\gamma_\mu + Bp_\mu)u(p) \tag{8}$$

exists and has nonzero value. Multiplying both sides of Eq. (8) with γ_μ and using $\eta_\mu\gamma^\mu = 0$, we get $B = -4A/m_{1/2}$. From this relation, we see that, to remove the contribution of the unwanted spin-1/2 states, we should eliminate the terms proportional to γ_μ at the left γ_ν at the right and also terms containing $p_{2\mu}$ and $p_{1\nu}$. For this aim and also to get independent structures, we order the Dirac matrices as $\gamma_\mu\not{p}\not{q}\not{\varepsilon}\gamma_\nu$ and set the terms containing the contribution of spin-1/2 particles to zero. After this procedure, we obtain the final expression for the phenomenological side as

$$\Pi_{\mu\nu} = \frac{\lambda_{\mathcal{D}1}\lambda_{\mathcal{D}2}}{[m_{\mathcal{D}1}^2 - (p+q)^2][m_{\mathcal{D}2}^2 - p^2]}\left\{2(\varepsilon.p)g_{\mu\nu}\not{q}\left[g_1 + g_2\frac{m_{\mathcal{D}2}}{(m_{\mathcal{D}1}+m_{\mathcal{D}2})}\right] - 2(\varepsilon.p)g_{\mu\nu}\not{p}\not{q}\frac{g_2}{(m_{\mathcal{D}1}+m_{\mathcal{D}2})} + q_\mu q_\nu \not{p}\not{q}\not{\varepsilon}\frac{g_3}{(m_{\mathcal{D}1}+m_{\mathcal{D}2})^2} - 2(\varepsilon.p)q_\mu q_\nu\not{p}\not{q}\frac{g_4}{(m_{\mathcal{D}1}+m_{\mathcal{D}2})^3} + \text{other structures}\right\}, \tag{9}$$

where, to obtain sum rules for coupling constants, we will choose the structures, $(\varepsilon.p)g_{\mu\nu}\not{q}$, $(\varepsilon.p)g_{\mu\nu}\not{p}\not{q}$, $q_\mu q_\nu\not{p}\not{q}\not{\varepsilon}$, and $(\varepsilon.p)q_\mu q_\nu\not{p}\not{q}$ for form factors $g_1 + g_2$, g_2, g_3, and g_4, respectively.

In this part, before calculation of the QCD side of the aforementioned correlation function, we would like to present the relations between invariant functions for the coefficients of the selected structures and show how we can express all coupling constants in terms of only one universal function. The main advantage of this approach used below is that it takes into account $SU(3)_f$ symmetry violating effects, automatically. Following the works [3–7], we start considering the transition, $\Sigma^{*0} \to \Sigma^{*0}\rho^0$, whose invariant function corresponding to each coupling g_1, g_2, g_3, and g_4 can formally be written as

$$\Pi^{\Sigma^{*0}\to\Sigma^{*0}\rho^0} = g_{\rho^0\bar{u}u}\Pi_1(u,d,s) + g_{\rho^0\bar{d}d}\Pi_1'(u,d,s) + g_{\rho^0\bar{s}s}\Pi_2(u,d,s), \tag{10}$$

where, from the interpolating current of the ρ^0 meson, we have $g_{\rho^0\bar{u}u} = -g_{\rho^0\bar{d}d} = 1/\sqrt{2}$, and $g_{\rho^0\bar{s}s} = 0$. In the above relation, the invariant functions Π_1, Π_1', and Π_2 refer to the radiation of the ρ^0 meson from u, d, and s quarks, respectively, and we formally define them as

$$\begin{aligned}\Pi_1(u,d,s) &= \langle\bar{u}u|\Sigma^{*0}\Sigma^{*0}|0\rangle,\\ \Pi_1'(u,d,s) &= \langle\bar{d}d|\Sigma^{*0}\Sigma^{*0}|0\rangle,\\ \Pi_2(u,d,s) &= \langle\bar{s}s|\Sigma^{*0}\Sigma^{*0}|0\rangle.\end{aligned} \tag{11}$$

The interpolating currents of the Σ^{*0} is symmetric under $u \leftrightarrow d$, hence $\Pi_1'(u,d,s) = \Pi_1(d,u,s)$ and Eq. (10) immediately yields

$$\Pi^{\Sigma^{*0}\to\Sigma^{*0}\rho^0} = \frac{1}{\sqrt{2}}[\Pi_1(u,d,s) - \Pi_1(d,u,s)], \tag{12}$$

where, in the $SU(2)_f$ symmetry limit, it vanishes. Now, we proceed considering the invariant function describing the transition, $\Sigma^{*+} \to \Sigma^{*+}\rho^0$. It can be obtained from Eq. (10) by replacing $d \to u$ and using the fact that $\Sigma^{*0}(d\to u) = \sqrt{2}\Sigma^{*+}$. As a result, we get

$$4\Pi_1(u,u,s) = 2\langle\bar{u}u|\Sigma^{*+}\Sigma^{*+}|0\rangle, \tag{13}$$

where the coefficient 4 in the left side comes from the fact that the Σ^{*+} contains two u quarks and there are four possibilities for the ρ^0 meson to be radiated from the u quark. Using Eq. (10) and considering the fact that Σ^{*+} does not contain the d quark, we obtain

$$\Pi^{\Sigma^{*+}\to\Sigma^{*+}\rho^0} = \sqrt{2}\Pi_1(u,u,s). \tag{14}$$

In a similar way, the invariant function describing $\Sigma^{*-} \to \Sigma^{*-}\rho^0$ is obtained from $\Sigma^{*0} \to \Sigma^{*-}\rho^0$ replacing $u \to d$ in Eq. (10) and taking into account $\Sigma^{*0}(u\to d) = \sqrt{2}\Sigma^{*-}$, i.e.,

$$\Pi^{\Sigma^{*-}\to\Sigma^{*-}\rho^0} = -\sqrt{2}\Pi_1(d,d,s). \tag{15}$$

Our next task is to expand the approach to include the Δ baryons. The invariant function for the $\Delta^+ \to \Delta^+\rho^0$ transition can be obtained from the $\Sigma^{*+} \to \Sigma^{*+}\rho^0$ transition. From the interpolating currents it is clear that $\eta_\mu^{\Delta^+} = \eta_\mu^{\Sigma^{*+}}(s \to d)$. Using this fact, we obtain

$$\begin{aligned}\Pi^{\Delta^+\to\Delta^+\rho^0} &= [g_{\rho^0\bar{u}u}\langle\bar{u}u|\Sigma^{*+}\Sigma^{*+}|0\rangle(s\to d) + g_{\rho^0\bar{s}s}\langle\bar{s}s|\Sigma^{*+}\Sigma^{*+}|0\rangle(s\to d)]\\ &= \sqrt{2}\Pi_1(u,u,d) - \frac{1}{\sqrt{2}}\Pi_2(u,u,d),\end{aligned} \tag{16}$$

but our calculations show that

$$\Pi_2(u,u,d) = \Pi_1(d,u,u), \tag{17}$$

hence,

$$\Pi^{\Delta^+\to\Delta^+\rho^0} = \sqrt{2}\Pi_1(u,u,d) - \frac{1}{\sqrt{2}}\Pi_1(d,u,u). \tag{18}$$

Similar to the above relations, our calculations lead also to the following relations for the couplings of the remaining decuplet baryons with a ρ^0 meson:

$$\Pi^{\Delta^{++}\to\Delta^{++}\rho^0} = \frac{3}{\sqrt{2}}\Pi_1(u,u,u), \tag{19}$$

$$\Pi^{\Delta^-\to\Delta^-\rho^0} = -\frac{3}{\sqrt{2}}\Pi_1(d,d,d), \tag{20}$$

$$\Pi^{\Delta^0\to\Delta^0\rho^0} = -\sqrt{2}\Pi_1(d,d,u) + \frac{1}{\sqrt{2}}\Pi_1(u,d,d), \tag{21}$$

$$\Pi^{\Xi^{*0}\to\Xi^{*0}\rho^0} = \frac{1}{\sqrt{2}}\Pi_1(u,s,s), \tag{22}$$

$$\Pi^{\Xi^{*-}\to\Xi^{*-}\rho^0} = \frac{-1}{\sqrt{2}}\Pi_1(d,s,s). \tag{23}$$

Up to here, we considered the neutral ρ meson case. Now, we go on considering the relations among the invariant functions corresponding to the charged ρ meson, for instance $\Sigma^{*0} \to \Sigma^{*+}\rho^-$. For this aim, we start considering the matrix element $\langle \bar{d}d|\Sigma^{*0}\Sigma^{*0}|0\rangle$, where the d quark from each Σ^{*0} constitutes the final $\bar{d}d$ state, and the remaining u and s are spectator quarks. In a similar way, in the matrix element $\langle \bar{u}d|\Sigma^{*+}\Sigma^{*0}|0\rangle$, the d quark from Σ^{*0} and the u quark from Σ^{*+} form the $\bar{u}d$ state and the remaining u and s quarks remain also as spectators. As a result, one expects that these two matrix elements should be proportional. Our calculations support this expectation and lead to the following relation:

$$\Pi^{\Sigma^{*0}\to\Sigma^{*+}\rho^-} = \langle \bar{u}d|\Sigma^{*+}\Sigma^{*0}|0\rangle = \sqrt{2}\langle \bar{d}d|\Sigma^{*0}\Sigma^{*0}|0\rangle = \sqrt{2}\Pi_1(d, u, s). \quad (24)$$

The $\Sigma^{*0} \to \Sigma^{*-}\rho^+$ invariant function is obtained exchanging the $u \leftrightarrow d$ in the above relation, i.e.,

$$\Pi^{\Sigma^{*0}\to\Sigma^{*-}\rho^+} = \langle \bar{d}u|\Sigma^{*-}\Sigma^{*0}|0\rangle = \sqrt{2}\langle \bar{u}u|\Sigma^{*0}\Sigma^{*0}|0\rangle = \sqrt{2}\Pi_1(u, d, s). \quad (25)$$

We obtain the following relations among other invariant functions involving the charged ρ meson using the similar arguments and calculations:

$$\Pi^{\Sigma^{*-}\to\Sigma^{0}\rho^-} = \sqrt{2}\Pi_1(u, d, s), \quad (26)$$

$$\Pi^{\Xi^{*-}\to\Xi^{0}\rho^-} = \Pi_1(d, s, s) = \Pi_1(u, s, s), \quad (27)$$

$$\Pi^{\Delta^{+}\to\Delta^{++}\rho^-} = \sqrt{3}\Pi_1(u, u, u), \quad (28)$$

$$\Pi^{\Delta^{0}\to\Delta^{+}\rho^-} = 2\Pi_1(u, u, d), \quad (29)$$

$$\Pi^{\Delta^{-}\to\Delta^{0}\rho^-} = \sqrt{3}\Pi_1(d, d, d), \quad (30)$$

$$\Pi^{\Sigma^{*+}\to\Sigma^{*0}\rho^+} = \sqrt{2}\Pi_1(d, u, s), \quad (31)$$

$$\Pi^{\Xi^{*0}\to\Xi^{*-}\rho^+} = \Pi_1(d, s, s), \quad (32)$$

$$\Pi^{\Delta^{+}\to\Delta^{0}\rho^+} = 2\Pi_1(d, d, u), \quad (33)$$

$$\Pi^{\Delta^{++}\to\Delta^{+}\rho^+} = \sqrt{3}\Pi_1(d, u, u), \quad (34)$$

$$\Pi^{\Delta^{0}\to\Delta^{-}\rho^+} = \sqrt{3}\Pi_1(u, d, d). \quad (35)$$

The remaining relations among the invariant functions involving other light nonet vector mesons, $K^{*0,\pm}$, $\bar{K}^{*0}$, ω, and ϕ, are represented in Appendix A. The above relations as well as those presented in the Appendix A show how we can express all strong coupling constants of the decuplet baryons to light vector mesons in terms of one universal function, Π_1.

Now, we focus our attention to calculate this invariant function in terms of the QCD degrees of freedom. As it is seen from the interpolating currents of the decuplet baryons previously shown, one can describe all transitions in terms of $\Sigma^{*0} \to \Sigma^{*0}\rho^0$, so we will calculate the invariant function Π_1 only for this transition. From QCD or the theoretical side, the correlation function can be calculated in the deep Euclidean region, where $-p^2 \to \infty$, $-(p+q)^2 \to \infty$, via operator product expansion (OPE) in terms of the DA's of the light vector mesons and light quark propagators. Therefore, to proceed, we need to know the expression of the light quark propagator as well as the matrix elements of the nonlocal operators $\bar{q}(x_1)\Gamma q'(x_2)$ and $\bar{q}(x_1)G_{\mu\nu}q'(x_2)$ between the vacuum and the vector meson states. Here, Γ refers to the Dirac matrices corresponding to the case under consideration and $G_{\mu\nu}$ is the gluon field strength tensor. Up to twist-4 accuracy, the matrix elements $\langle V(q)|\bar{q}(x)\Gamma q(0)|0\rangle$ and $\langle V(q)|\bar{q}(x)G_{\mu\nu}q(0)|0\rangle$ are determined in terms of the DA's of the vector mesons [8–10]. For simplicity, we present these nonlocal matrix elements in Appendix B. The expressions for DA's of the light vector mesons are also given in [8–10].

The light quark propagator used in our calculations is

$$\begin{aligned} S_q(x) = {} & \frac{i\not{x}}{2\pi^2x^4} - \frac{m_q}{4\pi^2x^2} - \frac{\langle\bar{q}q\rangle}{12}\left(1 - \frac{im_q}{4}\not{x}\right) \\ & - \frac{x^2}{192}m_0^2\langle\bar{q}q\rangle\left(1 - \frac{im_q}{6}\not{x}\right) \\ & - ig_s\int_0^1 du\left\{\frac{\not{x}}{16\pi^2x^2}G_{\mu\nu}(ux)\sigma^{\mu\nu}\right. \\ & - ux^\mu G_{\mu\nu}(ux)\gamma^\nu\frac{i}{4\pi^2x^2} \\ & \left. - \frac{im_q}{32\pi^2}G_{\mu\nu}(ux)\sigma^{\mu\nu}\left[\ln\left(\frac{-x^2\Lambda^2}{4}\right) + 2\gamma_E\right]\right\}, \end{aligned} \quad (36)$$

where γ_E is the Euler gamma and Λ is a scale parameter. Here, we should stress that, to achieve a factorization of the large and small scales in the OPE, all infrared logarithms should be removed from coefficient functions and absorbed in the matrix elements of operators. In our problem, this means that the $\ln\Lambda$ must be included in the condensates of different operators or distribution amplitudes. A more detailed discussion on this point can be found in [11]. For this reason, one can choose the scale parameter Λ as a factorization scale, i.e., $\Lambda = (0.5–1.0)$ GeV. We choose $\Lambda = 0.5$ GeV and our calculations show that the results of the coupling constants remain approximately unchanged in the interval, $\Lambda = (0.5–1.0)$ GeV.

Using the expression of the light quark propagator and the DA's of the light vector mesons, the theoretical or QCD side of the correlation function is obtained. Equating the coefficients of the structures, $(\varepsilon.p)g_{\mu\nu}\not{q}$, $(\varepsilon.p)g_{\mu\nu}\not{p}\not{q}$, $q_\mu q_\nu\not{p}\not{q}\not{\varepsilon}$, and $(\varepsilon.p)q_\mu q_\nu\not{p}\not{q}$ from both representations of the correlation function in phenomenological and theoretical sides and applying Borel transformation with respect to the variables p^2 and $(p+q)^2$ to suppress the contributions of the higher states and continuum, we get the sum rules for strong coupling constants of the vector mesons to decuplet baryons,

$$
\begin{aligned}
g_1 + \frac{g_2 m_{\mathcal{D}2}}{(m_{\mathcal{D}1} + m_{\mathcal{D}2})} &= \frac{1}{2\lambda_{\mathcal{D}1}\lambda_{\mathcal{D}2}} e^{(m_{\mathcal{D}1}^2/M_1^2)+(m_{\mathcal{D}2}^2/M_2^2)+(m_V^2/M_1^2+M_2^2)} \Pi_1^{(1)}, \\
g_2 &= -\frac{(m_{\mathcal{D}1} + m_{\mathcal{D}2})}{2\lambda_{\mathcal{D}1}\lambda_{\mathcal{D}2}} e^{(m_{\mathcal{D}1}^2/M_1^2)+(m_{\mathcal{D}2}^2/M_2^2)+(m_V^2/M_1^2+M_2^2)} \Pi_1^{(2)}, \\
g_3 &= \frac{(m_{\mathcal{D}1} + m_{\mathcal{D}2})^2}{\lambda_{\mathcal{D}1}\lambda_{\mathcal{D}2}} e^{(m_{\mathcal{D}1}^2/M_1^2)+(m_{\mathcal{D}2}^2/M_2^2)+(m_V^2/M_1^2+M_2^2)} \Pi_1^{(3)}, \\
g_4 &= -\frac{(m_{\mathcal{D}1} + m_{\mathcal{D}2})^3}{2\lambda_{\mathcal{D}1}\lambda_{\mathcal{D}2}} e^{(m_{\mathcal{D}1}^2/M_1^2)+(m_{\mathcal{D}2}^2/M_2^2)+(m_V^2/M_1^2+M_2^2)} \Pi_1^{(4)},
\end{aligned} \tag{37}
$$

where M_1^2 and M_2^2 are Borel parameters corresponding to the initial and final baryon channels, respectively, and the functions, $\Pi_1^{(i)}$ which are functions of the QCD degrees of freedom, continuum threshold as well as mass, decay constant, and DA's of the light vector mesons have very lengthy expressions and, for this reason, we do not present their explicit expressions here. It should be noted here that, the masses of the initial and final baryons are close to each other, so we will set $M_1^2 = M_2^2 = 2M^2$. From the sum rules for the strong couplings of the vector mesons to decuplet baryons in Eq. (37), it is clear that we also need the residues of decuplet baryons. These residues are obtained using the two-point correlation functions in [12–14] (see also [7]).

III. NUMERICAL ANALYSIS

In this section, we numerically analyze the sum rules of the strong coupling constants of the light nonet vector mesons with decuplet baryons and discuss our results. The sum rules for the couplings, g_1, g_2, g_3, and g_4, depict that the main input parameters are the vector meson DA's. The DA's of the vector mesons which are calculated in [8–10] include the leptonic constants, f_V and f_V^T, the twist-2 and twist-3 parameters, $a_i^{\parallel}$, $a_i^{\perp}$, $\zeta_{3V}^{\parallel}$, $\tilde{\lambda}_{3V}^{\parallel}$, $\tilde{\omega}_{3V}^{\parallel}$, $\kappa_{3V}^{\parallel}$, $\omega_{3V}^{\parallel}$, $\lambda_{3V}^{\parallel}$, $\kappa_{3V}^{\perp}$, $\omega_{3V}^{\perp}$, $\lambda_{3V}^{\perp}$, and twist-4 parameters $\zeta_4^{\parallel}$, $\tilde{\omega}_4^{\parallel}$, $\zeta_4^{\perp}$, $\tilde{\zeta}_4^{\perp}$, $\kappa_{4V}^{\parallel}$, $\kappa_{4V}^{\perp}$. The values of all these parameters are given in Tables I and II in [10]. The values of the remaining parameters entering the sum rules are $\langle 0|\frac{1}{\pi}\alpha_s G^2|0\rangle = (0.012 \pm 0.004)$ GeV^4 [15], $\langle \bar{u}u\rangle = \langle \bar{d}d\rangle = -(0.24 \pm 0.01)^3$ GeV^3, $\langle \bar{s}s\rangle = 0.8\langle \bar{u}u\rangle$ [15], $m_0^2 = (0.8 \pm 0.2)$ GeV^2 [12], and $m_s(2\ \text{GeV}) = (111 \pm 6)$ MeV at $\Lambda_{QCD} = 330$ MeV [16]. In numerical calculations, we set $m_u = m_d = 0$.

TABLE II. Coupling constant g_1 of light vector mesons with decuplet baryons.

Channel	g_1	$g_1(SU(3))$
$\Delta^+ \to \Delta^+\rho^0$	-4.4 ± 0.9	-4.4 ± 0.9
$\Omega^- \to \Xi^{*-}K^{*0}$	-23.5 ± 4.6	-13.2 ± 2.5
$\Sigma^{*0} \to \Sigma^{*0}\phi$	-8.0 ± 1.7	-7.3 ± 1.5
$\Sigma^{*0} \to \Xi^{*0}K^{*0}$	-18.5 ± 3.8	-10.8 ± 2.2
$\Sigma^{*-} \to \Sigma^{*-}\rho^0$	9.1 ± 2.0	8.8 ± 1.8
$\Xi^{*0} \to \Xi^{*0}\rho^0$	-4.8 ± 1.2	-4.4 ± 0.9
$\Xi^{*-} \to \Sigma^{*-}K^{*0}$	-26.0 ± 5.4	-15.2 ± 3.2

The sum rules for the coupling constants contain also two auxiliary parameters, Borel mass parameter M^2 and continuum threshold s_0. Therefore, we should find working regions of these parameters, where the results of coupling constants are reliable. In the reliable regions, the coupling constants are weakly depend on the auxiliary parameters. The upper limit of the Borel parameter, M^2, is found demanding that the contribution of the higher states and continuum should be less than say 40% of the total value of the same correlation function. The lower limit of M^2 is found requiring that the contribution of the highest term with the power of $1/M^2$ be 20%–25% less than that of the highest power of M^2. As a result, we obtain the working region, $1\ \text{GeV}^2 \leq M^2 \leq 1.5\ \text{GeV}^2$, for the Borel mass parameter. The continuum threshold is also not completely arbitrary but depends on the energy of the first excited state with the same quantum numbers. Our calculations lead to the working region, $(m_{\mathcal{D}} + 0.5)^2 \leq s_0 \leq (m_{\mathcal{D}} + 0.7)^2$, for the continuum threshold. In this region, the results of the coupling constants weakly depend on this parameter.

As an example, the dependence of the couplings g_1, g_2, g_3, and g_4 only for couplings of ρ^0 meson to Δ^+ baryon are shown in Figs. 1–4 at different values of the continuum threshold. From these figures, we observe that the couplings show good stability in the "working" region of M^2. Obviously, the coupling constants also weakly depend on the continuum threshold s_0. The results of the strong couplings g_1, g_2, g_3, and g_4 extracted from these figures and the similar analysis for the strong coupling of the other members of the light nonet vector mesons with decuplet baryons are presented in Tables I, II, III, and IV, respectively. Beside the general results, these tables also include the predictions of the $SU(3)_f$ symmetry on the strong coupling constants. The results of the $SU(3)_f$ symmetry are obtained setting $m_s = m_u = m_d = 0$, $\langle \bar{s}s\rangle = \langle \bar{u}u\rangle = \langle \bar{d}d\rangle$, $m_V = m_\rho$, and $m_{\mathcal{D}} = m_\Delta$. Note that, in these tables, we show only those couplings which could not be obtained by the $SU(2)$ symmetry rotations. The errors presented in

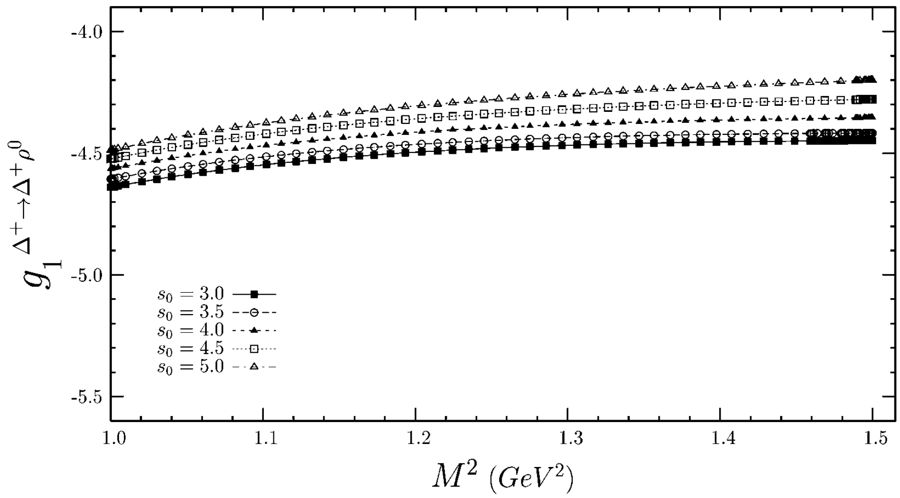

FIG. 1. The dependence of the strong coupling constant g_1 of the ρ^0 meson to the Δ^+ baryon on Borel mass M^2 for several fixed values of the continuum threshold s_0.

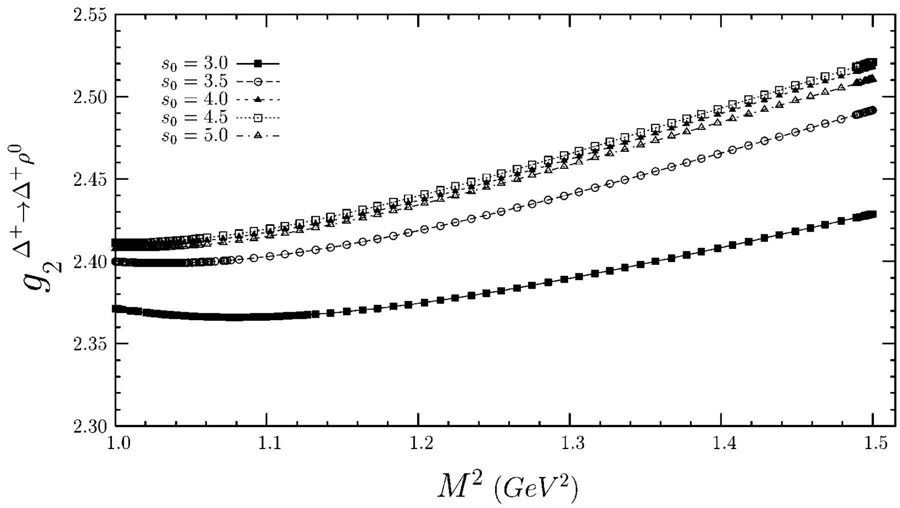

FIG. 2. The same as Fig. 1 but for g_2.

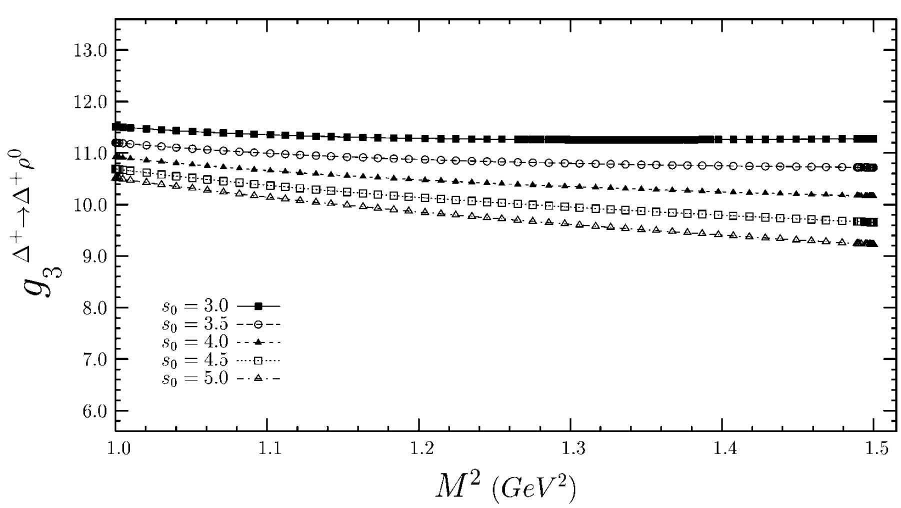

FIG. 3. The same as Fig. 1 but for g_3.

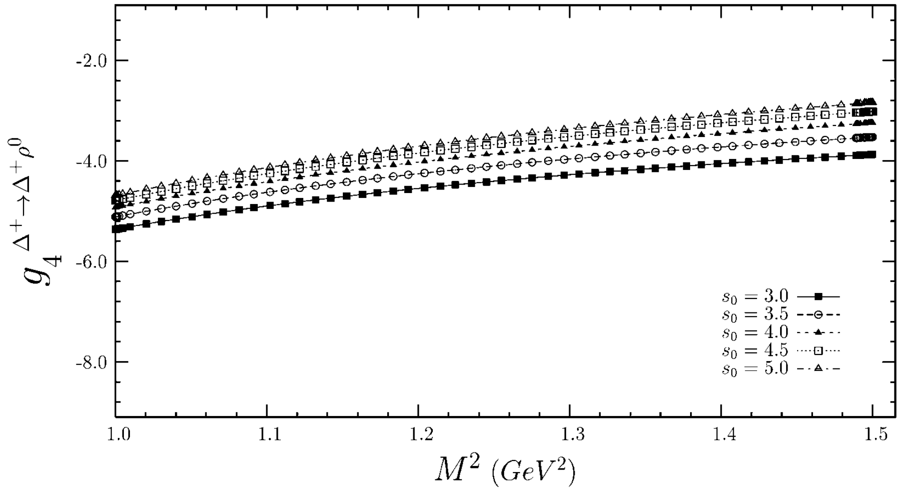

FIG. 4. The same as Fig. 1 but for g_4.

these tables include the uncertainties coming from the variation of auxiliary parameters M^2 and s_0 as well as uncertainties coming from the input parameters.

A quick running into Tables II, III, IV, and V resulted in

(i) For all strong couplings, g_1, g_2, g_3, and g_4, the channels having a large number of strange quarks show overall a large $SU(3)_f$ symmetry violation compared to those with a small number of s quarks. This is reasonable and is in agreement with our expectations.

(ii) The maximum $SU(3)_f$ symmetry violation for g_1 is 44% and belongs to the $\Omega^- \to \Xi^{*-} K^{*0}$ channel. The maximum violations of this symmetry for g_3 and g_4 which also belong to the same channel are 33% and 53%, respectively. However, the channel $\Sigma^{*0} \to \Sigma^{*0}\phi$ shows the maximum $SU(3)_f$ symmetry violation for g_2 with 30%.

(iii) The uncertainties on the values of the g_1, g_2, and g_3 are small compared with that of g_4. This is because of the fact that the g_1, g_2, and g_3 show a good stability with respect to the auxiliary parameters in working regions in comparison with g_4.

In conclusion, we studied the strong coupling constants of the decuplet baryons with light nonet vector mesons in the framework of light cone QCD sum rules. We expressed all coupling constants entering the calculations in terms of only one universal function even if the $SU(3)_f$ symmetry breaking effects are taken into account. We estimated the

TABLE III. Coupling constant g_2 of light vector mesons with decuplet baryons.

Channel	g_2	$g_2(SU(3))$
$\Delta^+ \to \Delta^+\rho^0$	2.45 ± 0.50	2.45 ± 0.50
$\Omega^- \to \Xi^{*-}K^{*0}$	7.7 ± 1.6	7.2 ± 1.4
$\Sigma^{*0} \to \Sigma^{*0}\phi$	2.5 ± 0.5	3.6 ± 0.8
$\Sigma^{*0} \to \Xi^{*0}K^{*0}$	5.4 ± 1.1	5.9 ± 1.2
$\Sigma^{*-} \to \Sigma^{*-}\rho^0$	-5.55 ± 1.20	-4.85 ± 0.95
$\Xi^{*0} \to \Xi^{*0}\rho^0$	3.21 ± 0.64	2.44 ± 0.48
$\Xi^{*-} \to \Sigma^{*-}K^{*0}$	7.7 ± 1.5	8.4 ± 1.8

TABLE IV. Coupling constant g_3 of light vector mesons with decuplet baryons.

Channel	g_3	$g_3(SU(3))$
$\Delta^+ \to \Delta^+\rho^0$	10.4 ± 2.4	10.4 ± 2.4
$\Omega^- \to \Xi^{*-}K^{*0}$	39.0 ± 8.0	26.0 ± 5.4
$\Sigma^{*0} \to \Sigma^{*0}\phi$	17.5 ± 3.6	14.0 ± 3.2
$\Sigma^{*0} \to \Xi^{*0}K^{*0}$	27.4 ± 5.6	21.0 ± 4.0
$\Sigma^{*-} \to \Sigma^{*-}\rho^0$	-24.0 ± 4.6	-21.0 ± 4.2
$\Xi^{*0} \to \Xi^{*0}\rho^0$	14.0 ± 2.8	10.5 ± 2.3
$\Xi^{*-} \to \Sigma^{*-}K^{*0}$	38.5 ± 7.6	29.6 ± 6.2

TABLE V. Coupling constant g_4 of light vector mesons with decuplet baryons.

Channel	g_4	$g_4(SU(3))$
$\Delta^+ \to \Delta^+\rho^0$	4.2 ± 1.6	-4.2 ± 1.6
$\Omega^- \to \Xi^{*-}K^{*0}$	-19.5 ± 6.5	-9.0 ± 3.0
$\Sigma^{*0} \to \Sigma^{*0}\phi$	-8.5 ± 2.8	-5.5 ± 1.8
$\Sigma^{*0} \to \Xi^{*0}K^{*0}$	-12.4 ± 4.2	-7.5 ± 2.4
$\Sigma^{*-} \to \Sigma^{*-}\rho^0$	10.5 ± 3.6	8.4 ± 2.7
$\Xi^{*0} \to \Xi^{*0}\rho^0$	-7.0 ± 2.4	-4.0 ± 1.5
$\Xi^{*-} \to \Sigma^{*-}K^{*0}$	-17.5 ± 5.6	-10.2 ± 3.2

order of $SU(3)_f$ symmetry violations. The main advantage of the approach used in the present work is that it takes into account the $SU(3)_f$ symmetry breaking effects automatically and we do not need to define another invariant function. The obtained results on the strong coupling constants of decuplet baryons with light nonet vector mesons can help us understand the dynamics of light vector mesons and their electroproduction off the decuplet baryons.

APPENDIX A

In this Appendix, we present the relations among the correlation functions involving K^*, ω, and ϕ mesons. We use corresponding quark contents for these mesons considering the ideal mixing:

(i) Vertices involving the K^{*+} meson:

$$\begin{aligned}
\Pi^{\Delta^+\to\Sigma^{*0}K^{*+}} &= \sqrt{2}\Pi_1(s,u,d),\\
\Pi^{\Delta^0\to\Sigma^{*-}K^{*+}} &= \Pi_1(s,d,d),\\
\Pi^{\Sigma^{*+}\to\Xi^{*0}K^{*+}} &= 2\Pi_1(s,s,u),\\
\Pi^{\Sigma^{*0}\to\Xi^{*-}K^{*+}} &= \sqrt{2}\Pi_1(u,d,s),\\
\Pi^{\Delta^{++}\to\Sigma^{*+}K^{*+}} &= \sqrt{3}\Pi_1(u,u,u),\\
\Pi^{\Xi^{*0}\to\Omega^{*-}K^{*+}} &= \sqrt{3}\Pi_1(s,s,s),
\end{aligned} \tag{A1}$$

(ii) Vertices involving the K^{*-} meson:

$$\begin{aligned}
\Pi^{\Sigma^{*0}\to\Delta^+K^{*-}} &= \sqrt{2}\Pi_1(s,u,d),\\
\Pi^{\Omega^-\to\Xi^{*0}K^{*-}} &= \sqrt{3}\Pi_1(s,s,s),\ \Pi^{\Sigma^{*-}\to\Delta^0K^{*-}}\\
&= \Pi_1(s,d,d),\\
\Pi^{\Xi^{*0}\to\Sigma^{*+}K^{*-}} &= 2\Pi_1(u,u,s),\\
\Pi^{\Xi^{*-}\to\Sigma^{*0}K^{*-}} &= \sqrt{2}\Pi_1(u,d,s),\\
\Pi^{\Sigma^{*+}\to\Delta^{++}K^{*-}} &= \sqrt{3}\Pi_1(u,u,u),
\end{aligned} \tag{A2}$$

(iii) Vertices involving the K^{*0} meson:

$$\begin{aligned}
\Pi^{\Xi^{*0}\to\Sigma^{*0}K^{*0}} &= \sqrt{2}\Pi_1(d,u,s),\\
\Pi^{\Xi^{*-}\to\Sigma^{*-}K^{*0}} &= 2\Pi_1(s,s,d),\\
\Pi^{\Sigma^{*0}\to\Delta^0K^{*0}} &= \sqrt{2}\Pi_1(s,d,u),\\
\Pi^{\Omega^-\to\Xi^{*-}K^{*0}} &= \sqrt{3}\Pi_1(s,s,s),\\
\Pi^{\Sigma^{*+}\to\Delta^+K^{*0}} &= \Pi_1(s,u,u),\\
\Pi^{\Sigma^{*-}\to\Delta^-K^{*0}} &= \sqrt{3}\Pi_1(s,d,d),
\end{aligned} \tag{A3}$$

(iv) Vertices involving the $\bar{K}^{*0}$ meson:

$$\begin{aligned}
\Pi^{\Sigma^{*0}\to\Xi^{*0}\bar{K}^{*0}} &= \sqrt{2}\Pi_1(d,u,s),\\
\Pi^{\Delta^-\to\Sigma^{*-}\bar{K}^{*0}} &= \sqrt{3}\Pi_1(s,d,d),\\
\Pi^{\Sigma^{*-}\to\Xi^{*-}\bar{K}^{*0}} &= 2\Pi_1(s,s,d),\\
\Pi^{\Delta^0\to\Sigma^{*0}\bar{K}^{*0}} &= \sqrt{2}\Pi_1(s,d,u),\\
\Pi^{\Delta^+\to\Sigma^{*+}\bar{K}^{*0}} &= \Pi_1(s,u,u),\\
\Pi^{\Omega^-\to\Xi^{*-}\bar{K}^{*0}} &= \sqrt{3}\Pi_1(s,s,s).
\end{aligned} \tag{A4}$$

(v) Vertices involving the ω meson:

$$\begin{aligned}
\Pi^{\Sigma^{*0}\to\Sigma^{*0}\omega} &= \frac{1}{\sqrt{2}}[\Pi_1(u,d,s)+\Pi_1(d,u,s)],\\
\Pi^{\Sigma^{*+}\to\Sigma^{*+}\omega} &= \sqrt{2}\Pi_1(u,u,s),\\
\Pi^{\Sigma^{*-}\to\Sigma^{*-}\omega} &= \sqrt{2}\Pi_1(d,d,s),\\
\Pi^{\Delta^+\to\Delta^+\omega} &= \frac{1}{\sqrt{2}}\Pi_1(d,u,u)+\sqrt{2}\Pi_1(u,u,d),\\
\Pi^{\Delta^{++}\to\Delta^{++}\omega} &= \frac{3\sqrt{2}}{2}\Pi_1(u,u,u),\\
\Pi^{\Delta^-\to\Delta^-\omega} &= \frac{3\sqrt{2}}{2}\Pi_1(d,d,d),\\
\Pi^{\Delta^0\to\Delta^0\omega} &= \sqrt{2}\Pi_1(d,d,u)+\frac{1}{2}\Pi_1(u,d,d)],\\
\Pi^{\Xi^{*0}\to\Xi^{*0}\omega} &= \frac{1}{\sqrt{2}}\Pi_1(u,s,s),\\
\Pi^{\Xi^{*-}\to\Xi^{*-}\omega} &= \frac{1}{\sqrt{2}}\Pi_1(d,s,s).
\end{aligned} \tag{A5}$$

(vi) Vertices involving the ϕ meson:

$$\begin{aligned}
\Pi^{\Sigma^{*0}\to\Sigma^{*0}\phi} &= [\Pi_1(s,d,u),\\
\Pi^{\Sigma^{*+}\to\Sigma^{*+}\phi} &= \Pi_1(s,u,u),\\
\Pi^{\Sigma^{*-}\to\Sigma^{*-}\phi} &= \Pi_1(s,d,d),\\
\Pi^{\Xi^{*0}\to\Xi^{*0}\phi} &= 2\Pi_1(s,s,u),\\
\Pi^{\Xi^{*-}\to\Xi^{*-}\phi} &= 2\Pi_1(s,s,d).
\end{aligned} \tag{A6}$$

APPENDIX B

In this Appendix we present the DA's of the vector mesons appearing in the matrix elements $\langle V(q)|\bar{q}(x)\Gamma q(0)|0\rangle$ and $\langle V(q)|\bar{q}(x)G_{\mu\nu}q(0)|0\rangle$, up to twist-4 accuracy [8–10]:

$$
\begin{aligned}
\langle V(q,\lambda)|\bar{q}_1(x)\gamma_\mu q_2(0)|0\rangle &= f_V m_V\bigg\{\frac{\varepsilon^\lambda\cdot x}{q\cdot x}q_\mu\int_0^1 due^{i\bar{u}q\cdot x}\bigg[\phi_\parallel(u)+\frac{m_V^2x^2}{16}A_\parallel(u)\bigg]+\bigg(\varepsilon_\mu^\lambda - q_\mu\frac{\varepsilon^\lambda\cdot x}{q\cdot x}\bigg)\\
&\quad\times\int_0^1 due^{i\bar{u}q\cdot x}g_\perp^v(u)-\frac{1}{2}x_\mu\frac{\varepsilon^\lambda\cdot x}{(q\cdot x)^2}m_V^2\int_0^1 due^{i\bar{u}q\cdot x}[g_3(u)\\
&\quad+\phi_\parallel(u)-2g_\perp^v(u)]\bigg\},\\
\langle V(q,\lambda)|\bar{q}_1(x)\gamma_\mu\gamma_5 q_2(0)|0\rangle &= -\frac{1}{4}\epsilon_\mu^{\nu\alpha\beta}\varepsilon^\lambda q_\alpha x_\beta f_V m_V\int_0^1 due^{i\bar{u}q\cdot x}g_\perp^a(u),\langle V(q,\lambda)|\bar{q}_1(x)\sigma_{\mu\nu}q_2(0)|0\rangle\\
&= -if_V^T\bigg\{(\varepsilon_\mu^\lambda q_\nu-\varepsilon_\nu^\lambda q_\mu)\int_0^1 due^{i\bar{u}q\cdot x}\bigg[\phi_\perp(u)+\frac{m_V^2x^2}{16}A_\perp(u)\bigg]\\
&\quad+\frac{\varepsilon^\lambda\cdot x}{(q\cdot x)^2}(q_\mu x_\nu-q_\nu x_\mu)\int_0^1 due^{i\bar{u}q\cdot x}\bigg[h_\parallel^t-\frac{1}{2}\phi_\perp-\frac{1}{2}h_3(u)\bigg]\\
&\quad+\frac{1}{2}(\varepsilon_\mu^\lambda x_\nu-\varepsilon_\nu^\lambda x_\mu)\frac{m_V^2}{q\cdot x}\int_0^1 due^{i\bar{u}q\cdot x}[h_3(u)-\phi_\perp(u)]\bigg\},\\
&\quad\langle V(q,\lambda)|\bar{q}_1(x)\sigma_{\alpha\beta}gG_{\mu\nu}(ux)q_2(0)|0\rangle\\
&= f_V^T m_V^2\frac{\varepsilon^\lambda\cdot x}{2q\cdot x}[q_\alpha q_\mu g_{\beta\nu}^\perp-q_\beta q_\mu g_{\alpha\nu}^\perp-q_\alpha q_\nu g_{\beta\mu}^\perp+q_\beta q_\nu g_{\alpha\mu}^\perp]\\
&\quad\times\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+u\alpha_g)q\cdot x}\mathcal{T}(\alpha_i)+f_V^Tm_V^2[q_\alpha\varepsilon_\mu^\lambda g_{\beta\nu}^\perp-q_\beta\varepsilon_\mu^\lambda g_{\alpha\nu}^\perp-q_\alpha\varepsilon_\nu^\lambda g_{\beta\mu}^\perp\\
&\quad+q_\beta\varepsilon_\nu^\lambda g_{\alpha\mu}^\perp]\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+u\alpha_g)q\cdot x}\mathcal{T}_1^{(4)}(\alpha_i)+f_V^Tm_V^2[q_\mu\varepsilon_\alpha^\lambda g_{\beta\nu}^\perp-q_\mu\varepsilon_\beta^\lambda g_{\alpha\nu}^\perp\\
&\quad-q_\nu\varepsilon_\alpha^\lambda g_{\beta\mu}^\perp+q_\nu\varepsilon_\beta^\lambda g_{\alpha\mu}^\perp]\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+u\alpha_g)q\cdot x}\mathcal{T}_2^{(4)}(\alpha_i)\\
&\quad+\frac{f_V^Tm_V^2}{q\cdot x}[q_\alpha q_\mu\varepsilon_\beta^\lambda x_\nu-q_\beta q_\mu\varepsilon_\alpha^\lambda x_\nu-q_\alpha q_\nu\varepsilon_\beta^\lambda x_\mu+q_\beta q_\nu\varepsilon_\alpha^\lambda x_\mu\\
&\quad\times\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+u\alpha_g)q\cdot x}\mathcal{T}_3^{(4)}(\alpha_i)+\frac{f_V^Tm_V^2}{q\cdot x}[q_\alpha q_\mu\varepsilon_\nu^\lambda x_\beta-q_\beta q_\mu\varepsilon_\nu^\lambda x_\alpha\\
&\quad-q_\alpha q_\nu\varepsilon_\mu^\lambda x_\beta+q_\beta q_\nu\varepsilon_\mu^\lambda x_\alpha\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+u\alpha_g)q\cdot x}\mathcal{T}_4^{(4)}(\alpha_i),\\
&\quad\langle V(q,\lambda)|\bar{q}_1(x)g_sG_{\mu\nu}(ux)q_2(0)|0\rangle\\
&= -if_V^Tm_V(\varepsilon_\mu^\lambda q_\nu-\varepsilon_\nu^\lambda q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+u\alpha_g)q\cdot x}\mathcal{S}(\alpha_i),\\
\langle V(q,\lambda)|\bar{q}_1(x)g_s\tilde{G}_{\mu\nu}(ux)\gamma_5q_2(0)|0\rangle &= -if_V^Tm_V(\varepsilon_\mu^\lambda q_\nu-\varepsilon_\nu^\lambda q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+u\alpha_g)q\cdot x}\tilde{\mathcal{S}}(\alpha_i),\\
\langle V(q,\lambda)|\bar{q}_1(x)g_s\tilde{G}_{\mu\nu}(ux)\gamma_\alpha\gamma_5q_2(0)|0\rangle &= f_Vm_Vq_\alpha(\varepsilon_\mu^\lambda q_\nu-\varepsilon_\nu^\lambda q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+u\alpha_g)q\cdot x}\mathcal{A}(\alpha_i),\\
\langle V(q,\lambda)|\bar{q}_1(x)g_sG_{\mu\nu}(ux)i\gamma_\alpha q_2(0)|0\rangle &= f_Vm_Vq_\alpha(\varepsilon_\mu^\lambda q_\nu-\varepsilon_\nu^\lambda q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+u\alpha_g)q\cdot x}\mathcal{V}(\alpha_i),
\end{aligned}
\tag{B1}
$$

where $\tilde{G}_{\mu\nu}=(1/2)\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$ is the dual gluon field strength tensor, and $\int\mathcal{D}\alpha_i=\int d\alpha_q d\alpha_{\bar{q}}d\alpha_g\delta(1-\alpha_q-\alpha_{\bar{q}}-\alpha_g)$.

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