Strong transitions of decuplet to octet baryons and pseudoscalar mesons

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Abstract

The strong coupling constants of light pseudoscalar π, K and η mesons with decuplet–octet baryons are studied within light cone QCD sum rules, where $SU(3)_f$ symmetry breaking effects are taken into account. It is shown that all coupling constants under the consideration are described by only one universal function even if $SU(3)_f$ symmetry breaking effects are switched into the game.

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1 Introduction

During recent years, intense studies have been made in the pion and kaon photo and electroproduction off the nucleon. Several exiting experimental programs exploiting these reactions have already been performed at electron beam facilities such as MIT–Bates, MAMI and Jefferson Laboratory. One main goal of these experiments is determination of the coupling constants of pion and kaon with baryons. Calculation of the coupling constants of pseudoscalar mesons with hadrons in the framework of QCD, is also very important for understanding the dynamics of pion and kaon photo and electroproduction reaction off the nucleon.

These coupling constants belong to the low energy sector of QCD, which is far from perturbative regime. Therefore, for calculation of these coupling constants some non-perturbative methods are needed. QCD sum rules method [1] is one of the most promising and predictive one among all existing non-perturbing methods in studying the properties of hadrons. In this work, we calculate the coupling constants of pseudoscalar mesons with decuplet–octet baryons within the light cone (LCSR) method (for more about this method, see [2]). In light cone QCD sum rules, the operator product expansion (OPE) is carried out near the light cone, $x^2 \simeq 0$, instead of the short distance, $x \simeq 0$ in traditional ones. In this approach, the OPE is also carried out over twist rather than dimension of operators in traditional sum rules. The main ingredient of LCSR are distribution amplitudes (DA's) which appear in matrix elements of nonlocal operators between the vacuum and the one-particle states.

Present work is an extension of our previous works, where coupling constants of pseudoscalar and vector mesons with octet baryons [3, 4], pseudoscalar mesons with decuplet baryons [5] and vector mesons with decuplet–octet baryons [6] are calculated. Here, using the DA's of the pseudoscalar mesons, we calculate the strong coupling constants of light pseudoscalar π, K and η mesons with decuplet–octet baryons in the framework of the light cone QCD sum rules both in full theory and when the $SU(3)_f$ symmetry breaking effects are taken into account. We would like especially to note that the main advantage of the approach presented in this work is that it takes into account the $SU(3)_f$ symmetry violation effects automatically.

The paper is organized as follows. In section 2, the strong coupling constants of pseudoscalar mesons with decuplet–octet baryons are calculated within LCSR method. In this section, we also obtain the relations between correlation functions describing various coupling constants. It is also shown that all coupling constants under the consideration are described by only one universal function even if $SU(3)_f$ symmetry breaking effects are considered. In section 3, we present our numerical analysis of the coupling constants of pseudoscalar mesons with decuplet–octet baryons. This section also includes comparison of our predictions on the coupling constants with the existing experimental data.

2 Sum rules for the coupling constants of the pseudoscalar mesons with decuplet–octet baryons

In this section, we obtain LCSR for the coupling constants of pseudoscalar mesons with decuplet–octet baryons. Before calculating these coupling constants within LCSR, it should be remembered that the within $SU(3)_f$ symmetry, the coupling of pseudoscalar mesons with decuplet–octet baryons is described by the single coupling constant whose interaction Lagrangian is given by

$$\mathcal{L}_{\rm int} = g_{\mathcal{DOP}} \varepsilon_{ijk} \bar{\mathcal{O}}_\ell^j (\mathcal{D}^{mk\ell})_\mu \partial^\mu \mathcal{P}_m^i + \text{h.c.} \ , \tag{1}$$

where $\mathcal{O}$, $\mathcal{D}$ and $\mathcal{P}$ correspond to octet, decuplet baryons and pseudoscalar mesons, respectively, and $g_{\mathcal{DOP}}$ is the coupling constant of the pseudoscalar mesons with decuplet–octet baryons. After this preliminary remark, we can proceed to derive the sum rules for the strong coupling constants of the pseudoscalar mesons with decuplet–octet baryons. For this purpose, we consider the correlation function

$$\Pi_\mu = i \int d^4x e^{ipx} \left\langle \mathcal{P}(q) \left| \mathcal{T} \left\{ \eta(x) \bar{\eta}_\mu(0) \right\} \right| 0 \right\rangle \ , \tag{2}$$

where $\mathcal{P}(q)$ is the pseudoscalar meson with momentum q, η_μ and η are the interpolating currents for decuplet and octet baryons, respectively, and $\mathcal{T}$ is the time ordering operator. The sum rules for the coupling constants can be obtained by calculating the correlation function in terms of hadrons and also in deep Euclidean region, where $-p^2 \to \infty$ and $-(p+q)^2 \to \infty$, in terms of quark and gluon degrees of freedom, and then equating these expressions using the dispersion relation. Note that in short-distance version of sum rules, where the operator product expansion is performed at $x \simeq 0$, the similar correlation function have been widely used in calculation of the pion-nucleon coupling constants in many works [7–10].

It follows from Eq. (2) that, in calculating the phenomenological and theoretical parts we need expressions of the interpolating currents for decuplet and octet baryons. The general form of the interpolating currents of the octets and decuplets are as follows [11–13]:

$$\begin{aligned} \eta &= A \varepsilon^{abc} \left\{ (q_1^{aT} C q_2^b) \gamma_5 q_3^c - (q_2^{aT} C q_3^b) \gamma_5 q_1^c + \beta (q_1^{aT} C \gamma_5 q_2^b) q_3^c - \beta (q_2^{aT} C \gamma_5 q_3^b) q_1^c \right\} \ , &(3) \\ \eta_\mu &= A' \varepsilon^{abc} \left\{ (q_1^{aT} C \gamma_\mu q_2^b) q_3^c + (q_2^{aT} C \gamma_\mu q_3^b) q_1^c + (q_3^{aT} C \gamma_5 q_1^b) q_2^c \right\} \ , &(4) \end{aligned}$$

where a, b, c are the color indices, β is an arbitrary parameter, C is the charge conjugation operator. The values of normalization constants A and A' and the q_1, q_2 and q_3 quarks for each octet and decuplet baryon are represented in Tables 1 and 2, respectively. Here, we would like to note that except the Λ current, all octet and decuplet currents can be obtained from Σ^0 and Σ^{*0} currents with the help of appropriate replacements among quark flavors. In [14], the following relations between the currents of the Λ and Σ^0 are obtained

$$\begin{aligned} 2\eta^{\Sigma^0}(d \to s) + \eta^{\Sigma^0} &= -\sqrt{3}\eta^\Lambda \ , \\ 2\eta^{\Sigma^0}(u \to s) + \eta^{\Sigma^0} &= \sqrt{3}\eta^\Lambda \ . \end{aligned} \tag{5}$$

	A	q_1	q_2	q_3
Σ^0	$-\sqrt{1/2}$	u	s	d
Σ^+	$1/2$	u	s	u
Σ^-	$1/2$	d	s	d
p	$-1/2$	u	d	u
n	$-1/2$	d	u	d
Ξ^0	$1/2$	s	u	s
Ξ^-	$1/2$	s	d	s

Table 1: The values of A and the quark flavors q_1, q_2 and q_3 for octet baryons.

	A'	q_1	q_2	q_3
Σ^{*0}	$\sqrt{2/3}$	u	d	s
Σ^{*+}	$\sqrt{1/3}$	u	u	s
Σ^{*-}	$\sqrt{1/3}$	d	d	s
Δ^{++}	$1/3$	u	u	u
Δ^{+}	$\sqrt{1/3}$	u	u	d
Δ^{0}	$\sqrt{1/3}$	d	d	u
Δ^{-}	$1/3$	d	d	d
Ξ^{*0}	$\sqrt{1/3}$	s	s	u
Ξ^{*-}	$\sqrt{1/3}$	s	s	d
Ω^{-}	$1/3$	s	s	s

Table 2: The values of A' and the quark flavors q_1, q_2 and q_3 for decuplet baryons.

We can now turn our attention to the calculation of theoretical and phenomenological part of the correlation function. In the case when pseudoscalar meson is on shell, $q^2 = m_{\mathcal{P}}^2$, the correlation function in Eq. (2) depends on two independent invariant variables, p^2 and $(p+q)^2$, the square of momenta in the two channels carried out by currents η and η_μ, respectively. Inserting the full set of hadrons with quantum numbers of currents η and η_μ and isolating the ground state octet and decuplet baryons by using narrow width approximation for phenomenological part of the correlation function, we obtain:

$$\Pi_\mu(p,q) = \frac{\langle 0\,|\eta|\,\mathcal{O}(p_2)\rangle\,\langle\mathcal{O}(p_2)\mathcal{P}(q)|\mathcal{D}(p_1)\rangle\,\langle\mathcal{D}(p_1)\,|\eta_\mu|\,0\rangle}{(p_2^2-m_{\mathcal{O}}^2)(p_1^2-m_{\mathcal{D}}^2)} + \cdots\ , \qquad (6)$$

where $\mathcal{O}(p_2)$ and $\mathcal{D}(p_1)$ denote the octet and decuplet baryons with momentum $p_2 = p$, $p_1 = p+q$, $m_{\mathcal{O}}$ and $m_{\mathcal{D}}$ are their masses, $\mathcal{P}(q)$ is the pseudoscalar meson with momentum q and $\cdots$ represents the higher states and the continuum contributions. Here, we would like to make the following remark about the Eq. (6). Since except the Ω baryon the widths of decuplet baryons are not small, hence the narrow width approximation which have been used in the Eq. (6) is questionable. In [15] it is obtained that the effect of width in calculation the mass of Δ-baryon changes the result about 10% compared to the narrow width approximation. For this reason, we shall neglect the width of decuplet baryons in our next discussions.

The matrix element of the interpolating current between vacuum and single octet (decuplet) baryon state is defined in standard way

$$\begin{aligned} \langle 0 \,|\eta|\, \mathcal{O}\rangle &= \lambda_{\mathcal{O}} u(p_2) \ , \\ \langle \mathcal{D}(p_1)\,|\eta_\mu|\, 0\rangle &= \lambda_{\mathcal{D}} \bar{u}_\mu(p_1) \ , \end{aligned} \tag{7}$$

where u_μ is the Rarita–Schwinger spinor. The remaining matrix element is defined as

$$\langle \mathcal{O}(p_2)\mathcal{P}(q)|\mathcal{D}(p_1)\rangle = g_{\mathcal{D}\mathcal{O}\mathcal{P}}\bar{u}(p_2)u_\mu(p_1)q^\mu \ , \tag{8}$$

where g is the coupling constant of pseudoscalar meson with octet and decuplet baryons. Putting Eqs. (7) and (8) into (6) and performing summation over spins of the octet and decuplet baryons using the formulas

$$\begin{aligned} \sum_s u(p_2,s)\bar{u}(p_2,s) &= (\not\!p_2 + m_{\mathcal{O}}) \ , \\ \sum_s u_\mu(p_1,s)\bar{u}_\nu(p_1,s) &= -(\not\!p_1 + m_{\mathcal{D}})\left\{ g_{\mu\nu} - \frac{\gamma_\mu\gamma_\nu}{3} - \frac{2p_{1\mu}p_{1\nu}}{3m_{\mathcal{D}}^2} + \frac{p_{1\mu}\gamma_\nu - p_{1\nu}\gamma_\mu}{3m_{\mathcal{D}}} \right\} \ , \end{aligned} \tag{9}$$

one can obtain the expression for the phenomenological part of the correlation function. But, unfortunately, we face with two drawbacks; one being that the interpolating current for decuplet baryons does also have nonzero matrix element between vacuum and spin–1/2 states (see [11, 13]),

$$\langle 0 \,|\eta_\mu|\, 1/2(p_1)\rangle = (A\gamma_\mu + Bp_{1\mu})u(p_1) \ , \tag{10}$$

where 1/2 stands for the spin–1/2 state. Multiplying both sides of Eq. (10) with γ_μ and using $\eta_\mu\gamma^\mu = 0$, we get $B = -4A/m_{1/2}$. In other words, we see that η_μ couples not only to spin–3/2, but also to unwanted spin–1/2 states. From Eqs. (10) and (6) we obtain that the structures proportional to γ_μ at the right end and $p_{1\mu}$ contain unwanted contribution from spin–1/2 states, which should be removed. The second drawback in obtaining the expression of the correlation function is related to the fact that not all structures appearing in Eq. (6) are independent of each other. In order to cure both these problems, we use ordering procedure of Dirac matrices as $\not\!q\not\!p\gamma_\mu$,. In this work, we choose the structure q_μ, which is free of the spin–1/2 contribution.

Using the ordering procedure, for the phenomenological part of the correlation function we obtain:

$$\Pi_\mu = \frac{g_{\mathcal{D}\mathcal{O}\mathcal{P}}(m_{\mathcal{O}}m_{\mathcal{D}} + m_{\mathcal{D}}^2 - m_{\mathcal{P}}^2)\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}}{[m_{\mathcal{D}}^2 - (p+q)^2][m_{\mathcal{O}}^2 - p^2]}\{q_\mu + \text{other structures}\} \ . \tag{11}$$

It follows from Eq. (11) that the interaction of pseudoscalar mesons with decuplet–octet baryons is described by a single coupling constant. In order to obtain the sum rule for the coupling constant g, the calculation of the correlation function from QCD side is needed. Before calculating it, we will present the relations between the correlation functions. In other words, we try to find relations between invariant functions for the coefficients of the structure q_μ. For establishing relations among invariant functions, we follow the approach presented in [3–6]. The main advantage of this approach presented below is that it takes into account $SU(3)_f$ symmetry violating effects automatically.

Similar to the works [3–6], we start our discussion by considering the transition, $\Sigma^{*0} \to \Sigma^0\pi^0$. The invariant function for this transition can formally be written in the following form

$$\Pi^{\Sigma^{*0}\to\Sigma^0\pi^0} = g_{\pi^0\bar{u}u}\Pi_1(u,d,s) + g_{\pi^0\bar{d}d}\Pi_1'(u,d,s) + g_{\pi^0\bar{s}s}\Pi_2(u,d,s) \ , \tag{12}$$

where the current for the π^0 meson is given by

$$\sum_{q=u,d,s} g_{\pi\bar{q}q}\bar{q}\gamma_5 q \ . \tag{13}$$

For the π^0 meson we have $g_{\pi^0\bar{u}u} = -g_{\pi^0\bar{d}d} = 1/\sqrt{2}$, and $g_{\pi^0\bar{s}s} = 0$. The invariant functions Π_1, Π_1' and Π_2 correspond to the radiation of π^0 meson from u, d and s quarks of the Σ^{*0} baryon, respectively, and they are formally defined as

$$\begin{aligned} \Pi_1(u,d,s) &= \left\langle \bar{u}u \left| \Sigma^{*0}\Sigma^0 \right| 0 \right\rangle \ , \\ \Pi_1'(u,d,s) &= \left\langle \bar{d}d \left| \Sigma^{*0}\Sigma^0 \right| 0 \right\rangle \ , \\ \Pi_2(u,d,s) &= \left\langle \bar{s}s \left| \Sigma^{*0}\Sigma^0 \right| 0 \right\rangle \ . \end{aligned} \tag{14}$$

Since the interpolating currents of the Σ^{*0} and Σ^0 baryons are symmetric under the replacement $u \leftrightarrow d$, it is obvious that $\Pi_1'(u,d,s) = \Pi_1(d,u,s)$. Using this relation we obtain from Eq. (12) immediately that

$$\Pi^{\Sigma^{*0}\to\Sigma^0\pi^0} = \frac{1}{\sqrt{2}}[\Pi_1(u,d,s) - \Pi_1(d,u,s)] \ . \tag{15}$$

Note that in the $SU(2)_f$ symmetry case, $\Pi^{\Sigma^{*0}\to\Sigma^0\pi^0} = 0$, obviously.

The invariant function describing the $\Sigma^{*+} \to \Sigma^+\pi^0$ transition can be obtained from Eq. (12) by making the replacement $d \to u$ and using the fact that $\Sigma^{*0}(d \to u) = \sqrt{2}\Sigma^{*+}$ and $\Sigma^0(d \to u) = -\sqrt{2}\Sigma^+$, which leads to the result

$$4\Pi_1(u,u,s) = -2\left\langle \bar{u}u \left| \Sigma^{*+}\Sigma^+ \right| 0 \right\rangle \ . \tag{16}$$

Since Σ^{*+} contains two u quarks there are 4 possible ways for π^0 meson to be radiated from the u quark. Using Eq. (12) and taking into account the fact that Σ^{*+} does not contain d quark, we get

$$\Pi^{\Sigma^{*+}\to\Sigma^+\pi^0} = -\sqrt{2}\Pi_1(u,u,s) \ . \tag{17}$$

The invariant function responsible for the $\Sigma^{*-} \to \Sigma^-\pi^0$ can be obtained from $\Sigma^{*0} \to \Sigma^0\pi^0$ transition simply by making the replacement $u \to d$ in Eq. (12) and taking into account $\Sigma^0(u \to d) = \sqrt{2}\Sigma^-$. As a result we obtain

$$\Pi^{\Sigma^{*-}\to\Sigma^-\pi^0} = -\sqrt{2}\Pi_1(d,d,s) \ . \tag{18}$$

Note here that, in $SU(2)_f$ symmetry case

$$\Pi^{\Sigma^{*+}\to\Sigma^+\pi^0} = \Pi^{\Sigma^{*-}\to\Sigma^-\pi^0} .$$

We now proceed by presenting the invariant functions involving Δ resonances. The invariant function for the $\Delta^+ \to p\pi^0$ transition can be obtained from the $\Sigma^{*+}\Sigma^+\pi^0$ transition by just using the identifications $\Delta^+ = \Sigma^{*+}(s \to d)$ and $p = -\Sigma^+(s \to d)$, as a result of which we get

$$\begin{aligned}
\Pi^{\Delta^+\to p\pi^0} &= -\Big[g_{\pi^0\bar{u}u} \left\langle \bar{u}u \left|\Sigma^{*+}\Sigma^+\right| 0\right\rangle (s \to d) + g_{\pi^0\bar{s}s} \left\langle \bar{s}s \left|\Sigma^{*+}\Sigma^+\right| 0\right\rangle (s \to d)\Big] \\
&= \sqrt{2}\Pi_1(u,u,d) - \frac{1}{\sqrt{2}}\Pi_2(u,u,d) . \qquad (19)
\end{aligned}$$

Using similar arguments, one can easily obtain the following relations

$$\begin{aligned}
\Pi^{\Delta^0\to n\pi^0} &= \sqrt{2}\Pi_1(d,d,u) - \frac{1}{\sqrt{2}}\Pi_2(d,d,u) , \\
\Pi^{\Xi^{*0}\to\Xi^0\pi^0} &= \frac{1}{\sqrt{2}}\Pi_2(s,s,u) , \\
\Pi^{\Xi^{*-}\to\Xi^-\pi^0} &= \frac{1}{\sqrt{2}}\Pi_2(s,s,d) . \qquad (20)
\end{aligned}$$

The relations presented in Eqs. (19) and (20) can be further simplified by using the relation

$$\Pi_2(u,d,s) = -\Pi_1(s,u,d) - \Pi_1(s,d,u) , \qquad (21)$$

which we obtain from our calculations.

We can now consider the transitions involving η meson. In this work, the mixing between η and η' is neglected and the interpolating current for η meson is chosen in the following form:

$$J_\eta = \frac{1}{\sqrt{6}}[\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s] . \qquad (22)$$

In order to find relations between invariant functions involving η meson, we choose $\Sigma^{*0} \to \Sigma^0\eta$ as the prototype. Similar to the π^0 case, the invariant function responsible for this transition can be written as:

$$\Pi^{\Sigma^{*0}\to\Sigma^0\eta} = g_{\eta\bar{u}u}\Pi_1(u,d,s) + g_{\eta\bar{d}d}\Pi_1'(u,d,s) + g_{\eta\bar{s}s}\Pi_2(u,d,s) . \qquad (23)$$

Using the relation given in Eq. (21) we get

$$\Pi^{\Sigma^{*0}\to\Sigma^0\eta} = \frac{1}{\sqrt{6}}[\Pi_1(u,d,s) + \Pi_1(d,u,s) + 2\Pi_1(s,u,d) + 2\Pi_1(s,d,u)] . \qquad (24)$$

The next step in our calculation is to obtain relations between invariant functions involving charged $\pi^\pm$ meson. Our starting point for this goal is considering the matrix element $\left\langle \bar{d}d \left|\Sigma^{*0}\Sigma^0\right| 0\right\rangle$, where d quarks from Σ^0 and Σ^{*0} form the final $\bar{d}d$ state, and u and s are the spectator quarks. In the matrix element $\langle \bar{u}d \,|\Sigma^{*+}\Sigma^0|\,0\rangle$, d quark from Σ^0 and u quark from Σ^{*0} form the $\bar{u}d$ state with the remaining u and s being the spectator quarks. For this reason one can expect that these matrix elements should have relations between each other. As a result of straightforward calculations we obtain that

$$\begin{aligned}
\Pi^{\Sigma^{*0}\to\Sigma^+\pi^-} &= \left\langle \bar{u}d \left|\Sigma^{*0}\Sigma^+\right| 0\right\rangle = -\sqrt{2}\left\langle \bar{d}d \left|\Sigma^{*0}\Sigma^0\right| 0\right\rangle \\
&= -\sqrt{2}\Pi_1(d,u,s) \ . \qquad (25)
\end{aligned}$$

Making the replacement $(u \leftrightarrow d)$ in Eq. (25), we get

$$\begin{aligned}
\Pi^{\Sigma^{*0}\to\Sigma^-\pi^+} &= \left\langle \bar{d}u \left|\Sigma^{*0}\Sigma^-\right| 0\right\rangle = \sqrt{2}\left\langle \bar{u}u \left|\Sigma^{*0}\Sigma^0\right| 0\right\rangle \\
&= \sqrt{2}\Pi_1(u,d,s) \ . \qquad (26)
\end{aligned}$$

Following similar line of reasoning and calculations one can find the remaining relations among the invariant functions involving charged pions, charged and neutral K mesons and η mesons, which are presented in appendix A.

It follows from above–mentioned relations that all couplings of the pseudoscalar mesons with decuplet–octet baryons are described with the help of only one invariant function even if $SU(3)_f$ symmetry is violated. In other words, this approach takes into account the $SU(3)_f$ symmetry violation effects, automatically. This observation constitute the principal result of the present work. Since the coupling constants of pseudoscalar mesons with decuplet–octet baryons are described by only one invariant function, we need its explicit expression in estimating their values.

As an example, we calculate the invariant function Π_1 responsible for the $\Sigma^{*0} \to \Sigma^0\pi^0$ transition. In deep Euclidean region, where $-p_1^2 \to \infty$ and $-p_2^2 \to \infty$, the correlation function can be calculated with the help of the operator product expansion. In obtaining the expression of the correlation function in LCSR from QCD side, the propagator of light quarks, as well as the matrix elements of nonlocal operators $\bar{q}(x_1)\Gamma q'(x_2)$ and $\bar{q}(x_1)G_{\mu\nu}q'(x_2)$ between vacuum and the pseudoscalar meson are needed, where Γ and $G_{\mu\nu}$ represents the Dirac matrices and the gluon field strength tensor, respectively.

Up to twist–4 accuracy, the matrix elements $\langle \mathcal{P}(q)\,|\bar{q}(x)\Gamma q(0)|\,0\rangle$ and $\langle \mathcal{P}(q)\,|\bar{q}(x)G_{\mu\nu}q(0)|\,0\rangle$ are parametrized in terms of the distribution amplitudes (DA's) as [16–18]:

$$\begin{aligned}
\langle \mathcal{P}(q)\,|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|\,0\rangle &= -if_{\mathcal{P}}q_\mu \int_0^1 du e^{i\bar{u}qx}\left(\varphi_{\mathcal{P}}(u) + \frac{1}{16}m_{\mathcal{P}}^2 x^2 \mathbb{A}(u)\right) \\
&- \frac{i}{2} f_{\mathcal{P}} m_{\mathcal{P}}^2 \frac{x_\mu}{qx}\int_0^1 du e^{i\bar{u}qx}\mathbb{B}(u) \ , \\
\langle \mathcal{P}(q)\,|\bar{q}(x)i\gamma_5 q(0)|\,0\rangle &= \mu_{\mathcal{P}}\int_0^1 du e^{i\bar{u}qx}\varphi_P(u) \ , \\
\langle \mathcal{P}(q)\,|\bar{q}(x)\sigma_{\alpha\beta}\gamma_5 q(0)|\,0\rangle &= \frac{i}{6}\mu_{\mathcal{P}}\left(1-\widetilde{\mu}_{\mathcal{P}}^2\right)(q_\alpha x_\beta - q_\beta x_\alpha)\int_0^1 du e^{i\bar{u}qx}\varphi_\sigma(u) \ , \\
\langle \mathcal{P}(q)\,|\bar{q}(x)\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta}(vx) q(0)|\,0\rangle &= i\mu_{\mathcal{P}}\left[q_\alpha q_\mu \left(g_{\nu\beta} - \frac{1}{qx}(q_\nu x_\beta + q_\beta x_\nu)\right)\right.
\end{aligned}$$

$$
\begin{aligned}
&- q_\alpha q_\nu \left(g_{\mu\beta} - \frac{1}{qx}(q_\mu x_\beta + q_\beta x_\mu) \right) \\
&- q_\beta q_\mu \left(g_{\nu\alpha} - \frac{1}{qx}(q_\nu x_\alpha + q_\alpha x_\nu) \right) \\
&+ q_\beta q_\nu \left(g_{\mu\alpha} - \frac{1}{qx}(q_\mu x_\alpha + q_\alpha x_\mu) \right) \Bigg] \\
&\times \int D\alpha e^{i(\alpha_{\bar q} + v\alpha_g)qx} \mathcal{T}(\alpha_i) \ , \\
\langle \mathcal{P}(q) | \bar q(x) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(vx) q(0) | 0 \rangle &= q_\mu (q_\alpha x_\beta - q_\beta x_\alpha) \frac{1}{qx} f_{\mathcal{P}} m_{\mathcal{P}}^2 \int D\alpha e^{i(\alpha_{\bar q} + v\alpha_g)qx} \mathcal{A}_\parallel(\alpha_i) \\
&+ \left[q_\beta \left(g_{\mu\alpha} - \frac{1}{qx}(q_\mu x_\alpha + q_\alpha x_\mu) \right) \right. \\
&- \left. q_\alpha \left(g_{\mu\beta} - \frac{1}{qx}(q_\mu x_\beta + q_\beta x_\mu) \right) \right] f_{\mathcal{P}} m_{\mathcal{P}}^2 \\
&\times \int D\alpha e^{i(\alpha_{\bar q} + v\alpha_g)qx} \mathcal{A}_\perp(\alpha_i) \ , \\
\langle \mathcal{P}(q) | \bar q(x) \gamma_\mu i g_s G_{\alpha\beta}(vx) q(0) | 0 \rangle &= q_\mu (q_\alpha x_\beta - q_\beta x_\alpha) \frac{1}{qx} f_{\mathcal{P}} m_{\mathcal{P}}^2 \int D\alpha e^{i(\alpha_{\bar q} + v\alpha_g)qx} \mathcal{V}_\parallel(\alpha_i) \\
&+ \left[q_\beta \left(g_{\mu\alpha} - \frac{1}{qx}(q_\mu x_\alpha + q_\alpha x_\mu) \right) \right. \\
&- \left. q_\alpha \left(g_{\mu\beta} - \frac{1}{qx}(q_\mu x_\beta + q_\beta x_\mu) \right) \right] f_{\mathcal{P}} m_{\mathcal{P}}^2 \\
&\times \int D\alpha e^{i(\alpha_{\bar q} + v\alpha_g)qx} \mathcal{V}_\perp(\alpha_i) \ . \qquad (27)
\end{aligned}
$$

In Eq. (27) we have,

$$
\mu_{\mathcal{P}} = f_{\mathcal{P}} \frac{m_{\mathcal{P}}^2}{m_{q_1} + m_{q_2}} \ , \qquad \widetilde{\mu}_{\mathcal{P}} = \frac{m_{q_1} + m_{q_2}}{m_{\mathcal{P}}} \ ,
$$

and $D\alpha = d\alpha_{\bar q} d\alpha_q d\alpha_g \delta(1 - \alpha_{\bar q} - \alpha_q - \alpha_g)$, and and the DA's $\varphi_{\mathcal{P}}(u)$, $\mathbb{A}(u)$, $\mathbb{B}(u)$, $\varphi_P(u)$, $\varphi_\sigma(u)$, $\mathcal{T}(\alpha_i)$, $\mathcal{A}_\perp(\alpha_i)$, $\mathcal{A}_\parallel(\alpha_i)$, $\mathcal{V}_\perp(\alpha_i)$ and $\mathcal{V}_\parallel(\alpha_i)$ are functions of definite twist whose explicit expressions can be found in [16–18].

Propagator of the light quark in an external field is calculated in [19, 20] having the form

$$
\begin{aligned}
S_q(x) = &\frac{i \not x}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar q q \rangle}{12} \left(1 - i \frac{m_q}{4} \not x \right) - \frac{x^2}{192} m_0^2 \langle \bar q q \rangle \left(1 - i \frac{m_q}{6} \not x \right) \\
&- i g_s \int_0^1 du \left[\frac{\not x}{16\pi^2 x^2} G_{\mu\nu}(ux) \sigma_{\mu\nu} - u x^\mu G_{\mu\nu}(ux) \gamma^\nu \frac{i}{4\pi^2 x^2} \right. \\
&- \left. i \frac{m_q}{32\pi^2} G_{\mu\nu} \sigma^{\mu\nu} \left(\ln \left(\frac{-x^2 \Lambda^2}{4} \right) + 2\gamma_E \right) \right] \ , \qquad (28)
\end{aligned}
$$

where $\gamma_E \simeq 0.577$ is the Euler Constant, and following the works [21, 22] $\Lambda = 0.5 \div 1.0\ GeV$ is used.

Using Eqs. (27) and (28) and choosing the coefficient of the structure q_μ, the expression of the correlation function from QCD side can be obtained. The sum rules for the coupling constants of the pseudoscalar mesons with decuplet–octet baryons are obtained by matching the coefficients of the structure q_μ from theoretical and phenomenological parts, and applying Borel transformation to both parts with respect to the parameters $p_2^2 = p^2$ and $p_1^2 = (p+q)^2$ in order to suppress the higher state and continuum contributions. As a result of these operations, we get the following sum rules for the pseudoscalar decuplet–octet coupling constants

$$g_{\mathcal{DOP}} = \frac{1}{m_{\mathcal{O}}\lambda_{\mathcal{D}}\lambda_{\mathcal{O}}} e^{m_{\mathcal{D}}^2/M_1^2 + m_{\mathcal{O}}^2/M_2^2 + m_{\mathcal{P}}^2/(M_1^2+M_2^2)} \Pi_{\mathcal{DOP}} \ . \qquad (29)$$

We only present the expression for the invariant function $\Pi_{\mathcal{DOP}}$ for the $\Sigma^{*+} \to \Sigma^+\pi^0$ transition in Appendix B, since invariant functions for other transitions can be obtained from it by appropriate replacements among quark flavors. Note that for the subtraction of continuum and higher states contribution we have used the procedure given in [6].

We observe from Eq. (29) that the residues $\lambda_{\mathcal{D}}$ and $\lambda_{\mathcal{O}}$ are needed for an estimation of the coupling constants $g_{\mathcal{DOP}}$. These residues are obtained in [12, 23, 24]. As has already been mentioned, the interpolating currents of decuplet and octet baryons (except Λ baryon) can be obtained from Σ^{*0} and Σ^0 currents with the help of the corresponding replacements. Therefore, we present the sum rules for the residues only for Σ^{*0} and Σ^0 baryons.

$$\begin{aligned}
\lambda_{\Sigma^0}^2 e^{-m_{\Sigma^0}^2/M^2} &= \frac{M^6}{1024\pi^2}(5+2\beta+5\beta^2)E_2(x) - \frac{m_0^2}{96M^2}(-1+\beta)^2 \langle\bar{u}u\rangle\langle\bar{d}d\rangle \\
&- \frac{m_0^2}{16M^2}(-1+\beta^2)\langle\bar{s}s\rangle\Big(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle\Big) \\
&+ \frac{3m_0^2}{128}(-1+\beta^2)\Big[m_s\Big(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle\Big) + (m_u+m_d)\langle\bar{s}s\rangle\Big] \\
&- \frac{1}{64\pi^2}(-1+\beta)^2 M^2\Big(m_d\langle\bar{u}u\rangle + m_u\langle\bar{d}d\rangle\Big)E_0(x) \\
&- \frac{3M^2}{64\pi^2}(-1+\beta^2)\Big[m_s\Big(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle\Big) + (m_u+m_d)\langle\bar{s}s\rangle\Big]E_0(x) \\
&+ \frac{1}{128\pi^2}(5+2\beta+5\beta^2)\Big(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle + m_s\langle\bar{s}s\rangle\Big) \\
&+ \frac{1}{24}\Big[3(-1+\beta^2)\langle\bar{s}s\rangle\Big(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle\Big) + (-1+\beta^2)\langle\bar{u}u\rangle\langle\bar{d}d\rangle\Big] \\
&+ \frac{m_0^2}{256\pi^2}(-1+\beta)^2\Big(m_u\langle\bar{d}d\rangle + m_d\langle\bar{u}u\rangle\Big) \\
&+ \frac{m_0^2}{26\pi^2}(-1+\beta^2)\Big[13m_s\Big(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle\Big) + 11(m_u+m_d)\langle\bar{s}s\rangle\Big] \\
&- \frac{m_0^2}{192\pi^2}(1+\beta+\beta^2)\Big(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle - 2m_s\langle\bar{s}s\rangle\Big) \\
&+ \frac{M^2}{2048\pi^4}\Big(5+2\beta+5\beta^2\Big)E_0(x)\langle g_s^2G^2\rangle \ ,
\end{aligned}$$

$$m_{\Sigma^{*0}}\lambda_{\Sigma^{*0}}^2 e^{-\frac{m_{\Sigma^{*0}}^2}{M^2}} = \Big(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle + \langle\bar{s}s\rangle\Big)\frac{M^4}{9\pi^2}E_1(x) - (m_u+m_d+m_s)\frac{M^6}{32\pi^4}E_2(x)$$

$$
\begin{aligned}
&- \left(\langle \bar{u}u\rangle + \langle \bar{d}d\rangle + \langle \bar{s}s\rangle\right) m_0^2 \frac{M^2}{18\pi^2} E_0(x) \\
&- \frac{2}{3}\left(1 + \frac{5m_0^2}{72M^2}\right)\left(m_u\langle \bar{d}d\rangle\langle \bar{s}s\rangle + m_d\langle \bar{s}s\rangle\langle \bar{u}u\rangle + m_s\langle \bar{d}d\rangle\langle \bar{u}u\rangle\right) \\
&+ \left(m_s\langle \bar{d}d\rangle\langle \bar{s}s\rangle + m_u\langle \bar{d}d\rangle\langle \bar{u}u\rangle + m_d\langle \bar{s}s\rangle\langle \bar{u}u\rangle\right)\frac{m_0^2}{12M^2} \\
&+ \frac{5M^2}{1152\pi^4} E_0(x)(m_s + m_u + m_d)\langle g_s^2 G^2\rangle \ , \qquad (30)
\end{aligned}
$$

where $x = s_0/M^2$, and

$$
E_n(x) = 1 - e^{-x}\sum_{i=0}^{n}\frac{x^i}{i!} \ .
$$

3 Numerical analysis

In the previous section, we obtained the sum rules for the coupling constants of the pseudoscalar mesons with decuplet–octet baryons. Here in this section, we shall present their numerical results. In further numerical analysis the DA's of the pseudoscalar mesons are needed, which are the main nonperturbative parameters. These DA's and other parameters entering into their expressions can be found in [16–18].

In the numerical analysis, $M_1^2 = M_2^2 = 2M^2$ is chosen since the masses of the initial and final baryons are close to each other. With this choice, we have $u_0 = 1/2$. The values of the remaining parameters entering the sum rules are: $\langle 0|\frac{1}{\pi}\alpha_s G^2|0\rangle = (0.012 \pm 0.004)\ GeV^4$ [25], $\langle \bar{u}u\rangle = \langle \bar{d}d\rangle = -(0.24\pm0.01)^3\ GeV^3$, $\langle \bar{s}s\rangle = 0.8\,\langle \bar{u}u\rangle$ [25], $m_0^2 = (0.8\pm0.2)\ GeV^2$ [12], $m_s(2\ GeV) = (111\pm6)\ MeV$ at $\Lambda_{QCD} = 330\ MeV$ [26], $m_u = 0$, $m_d = 0$, $m_\pi = 0.135\ GeV$, $m_\eta = 0.548\ GeV$, $m_K = 0.498\ GeV$, $f_\pi = 0.131\ GeV$, $f_K = 0.16\ GeV$ and $f_\eta = 0.13\ GeV$ [16]. Since LCSR method cannot predict the sign of $g_{\mathcal{DOP}}$, we shall present the absolute value of it.

The sum rules for the coupling of the pseudoscalar mesons with decuplet–octet baryons contain three auxiliary parameters, namely, Borel mass parameter M^2, continuum threshold s_0 and the parameter β in the interpolating current of octet baryons. Since physical quantities should be independent of these auxiliary parameters, it is necessary to find regions of these parameters where the coupling constant $g_{\mathcal{DOP}}$ is independent of them. The upper bound of M^2 is determined from the condition that the higher states and continuum contributions should be less than 40–50% of the total value of the correlation function. The lower bound of M^2 is obtained by requiring that the term with highest power in $1/M^2$ should be less than 20–25% of the highest power of M^2. Using these conditions, one can easily obtain the working region for the Borel parameter M^2. The value of the continuum threshold is varied in the region $2.5\ GeV^2 \le s_0 \le 4.0\ GeV^2$.

As an example, in Fig. (1), we present the dependence of the coupling constant for the $\Sigma^{*+} \to \Sigma^+\pi^0$ on M^2 at several fixed values of β and at $s_0 = 4\ GeV^2$. We see from this figure that the coupling constant g shows good stability to the variation in M^2, when M^2 varies in the "working region". As we already noted that the coupling constant $g_{\mathcal{DOP}}$ should be independent of auxiliary parameter β. For finding the working region of β, we present the dependence of $g_{\Sigma^{*+}\Sigma^+\pi^0}$ for the $\Sigma^{*+} \to \Sigma^+\pi^0$ transition on $\cos\theta$ as an example in Fig. (2), where θ is determined from $\tan\theta = \beta$. We obtain from this figure that when $\cos\theta$ varies in the region $-0.5 \le \cos\theta \le 0.5$ the coupling constant $g_{\Sigma^{*+}\Sigma^+\pi^0}$ is practically independent of it. The dependence of strong coupling constants of pion with baryons on auxiliary parameter β in the framework of operator product expansion at short distance for the coorelation function of time ordering product of two currents between the vacuum and pion states is also shown in [27]. The working region of β in our case and that of given in [27] overlap but this is accidental. Different problems may lead to different working regions for this auxiliary parameter. From Fig. (2), we see that the coupling constant for the $\Sigma^{*+} \to \Sigma^+\pi^0$ transition is $g_{\Sigma^{*+}\Sigma^+\pi^0} = 3.4 \pm 0.5$. The results for the coupling constants of the pseudoscalar mesons with decuplet–octet baryons are listed in Table 3. It should be emphasized that in this table we present only those results which cannot be obtained from each other by simple $SU(2)_f$ rotations. The results for strong coupling constant, $g_{\mathcal{DOP}}$, when the most general form of the interpolating currents for octet baryons have been used are presented under the category "general current". In the first column of this category, the results are given in full theory. In the second column, we present the predictions of $SU(3)_f$ symmetry case, where $m_s = m_u = m_d = 0$ and $\langle \bar{s}s \rangle = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$. In the next category containing the columns three and four, we present our result for the strong coupling constant, $g_{\mathcal{DOP}}$, when the Ioffe currents, $\beta = -1$ for the octet baryons have been used. The third column shows the predictions in full theory, while the presented results for the strong coupling constant in the last column have been obtained using the $SU(3)_f$ symmetry. The errors in the presented values in Table 3 are due to the variations in Borel mass parameter, M^2, continuum threshold, s_0, auxiliary parameter β as well as errors in input parameters entering the DA's, quark and gluon condensates, and mass of the strange quark.

A quick glance at Table 3 leads to the following conclusions.

- For all channels under consideration there is a good agreement between the predictions of the general form of the current and of the Ioffe current for the octet baryons.

- There seems to be a considerable discrepancy between these two predictions for the central values of $\Sigma^{*-} \to \Lambda\pi^-$, $\Omega^- \to \Xi^0 K^-$, $\Delta^0 \to p\pi^-$, $\Sigma^{*+} \to p\bar{K}^0$ and $\Sigma^{*+} \to \Sigma^+\eta$ channels.

- Maximum value of $SU(3)_f$ symmetry violation is about $(10 \div 15)\%$. Note that the approach presented in the present work takes into account the $SU(3)_f$ violation effects automatically, hence we can estimate order of $SU(3)_f$ violation. The essential point here is that the $SU(3)_f$ violating effects do not produce new invariant function compared to $SU(3)_f$ symmetry case.

Finally, let us compare our predictions on coupling constants in Table 3 with the existing experimental results. Using the explicit form of the interaction Lagrangian in Eq. (1), one can easily obtain expression for the decay width of the *decuplet* $\longrightarrow$ *octet+pseudoscalar meson* transition in terms of the strong coupling constant. Using the experimental values for the total widths of the Σ^* and Ξ^* baryons and the branching ratios of $\Sigma^* \longrightarrow \Sigma\pi$, $\Sigma^* \longrightarrow \Lambda\pi$ and $\Xi^* \longrightarrow \Xi\pi$ [28], we get the following results for the related coupling constants:

$$g_{\Sigma^{*+}\Sigma^+\pi^0} = 3.27 \pm 0.55, \qquad g_{\Sigma^{*-}\Lambda\pi^-} = 4.56 \pm 0.48, \qquad g_{\Xi^{*0}\Xi^0\pi^0} = 3.56 \pm 0.42 \tag{31}$$

Comparing these results with our predictions presented in Table 3, we see a good consistency between the values extracted from the experimental data and our predictions on the strong coupling constants related to the $\Sigma^{*+} \longrightarrow \Sigma^+\pi^0$, $\Sigma^{*-} \longrightarrow \Lambda\pi^-$ and $\Xi^{0*} \longrightarrow \Xi^0\pi^0$ channels. Our predictions on the coupling constants of channels which we have no experimental data can be verified in the future experiments.

Our concluding remarks on the present study can be summarized as follows. The strong coupling constants of pseudoscalar mesons with decuplet–octet baryons are investigated in LCSR by taking into account $SU(3)_f$ symmetry breaking effects. It is seen that all coupling constants of pseudoscalar π, K and η mesons with decuplet–octet baryons can be represented by only one invariant function. The order of the magnitude of $SU(3)_f$ symmetry breaking effects is also estimated.

$g_{\mathcal{DOP}}$	General current		Ioffe current	
	Result	$SU(3)_f$	Result	$SU(3)_f$
$g_{\Sigma^{*+}\Sigma^+\pi^0}$	3.4±0.5	3.3±0.3	2.8±0.3	2.5±0.2
$g_{\Xi^{*0}\Xi^0\pi^0}$	3.3±0.7	3.4±0.6	2.4±0.2	2.3±0.2
$g_{\Sigma^{*-}\Lambda\pi^-}$	7.0±1.5	6.5±1.0	4.7±0.3	4.2±0.4
$g_{\Delta^0 p\pi^-}$	5.5±1.5	5.0±1.0	4.0±0.5	4.2±0.5
$g_{\Delta^+\Sigma^0 K^+}$	7.0±1.0	6.5±0.5	6.0±1.0	5.0±1.0
$g_{\Sigma^{*+}\Xi^0 K^+}$	3.5±0.5	3.6±0.4	3.0±0.2	2.8±0.2
$g_{\Omega^-\Xi^0 K^-}$	8.0±2.0	7.0±1.5	6.5±1.0	6.0±1.0
$g_{\Xi^{*0}\Sigma^+ K^-}$	4.7±0.6	4.5±0.5	4.8±0.8	4.0±0.4
$g_{\Sigma^{*+} p\bar{K}^0}$	6.0±1.5	5.0±1.0	4.8±0.5	4.4±0.4
$g_{\Xi^{*0}\Lambda\bar{K}^0}$	6.4±1.0	5.5±1.0	5.0±0.6	4.8±0.4
$g_{\Sigma^{*+}\Sigma^+\eta}$	6.0±1.0	5.6±1.2	4.8±0.4	4.4±0.4
$g_{\Xi^{*0}\Xi^0\eta}$	5.6±0.8	5.0±1.0	4.8±0.4	4.0±0.4

Table 3: The values of the coupling constant g for various channels.

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Appendix A :

In this appendix, we give the representation of correlation functions in terms of invariant function Π_1 involving π, K and η mesons which are not presented in the main body of the text.

Correlation functions involving π mesons.

$$\begin{aligned}
\Pi^{\Sigma^{*0}\to\Lambda\pi^0} &= -\sqrt{1/6}[2\Pi_1(d,s,u)+\Pi_1(d,u,s)+\Pi_1(u,d,s)+2\Pi_1(u,s,d)] \ , \\
\Pi^{\Sigma^{*-}\to\Sigma^0\pi^-} &= \sqrt{2}\Pi_1(u,d,s) \ , \\
\Pi^{\Xi^{*-}\to\Xi^0\pi^-} &= -2\Pi_1(d,s,s) \ , \\
\Pi^{\Delta^-\to n\pi^-} &= 2\sqrt{3}\Pi_1(d,d,d) \ , \\
\Pi^{\Sigma^{*-}\to\Lambda\pi^-} &= -\sqrt{2/3}[2\Pi_1(u,s,d)+\Pi_1(u,d,s)] \ , \\
\Pi^{\Delta^0\to p\pi^-} &= 2\Pi_1(u,u,d) \ , \\
\Pi^{\Sigma^{*+}\to\Sigma^0\pi^+} &= \sqrt{2}\Pi_1(d,u,s) \ , \\
\Pi^{\Sigma^{*0}\to\Sigma^-\pi^+} &= \sqrt{2}\Pi_1(u,d,s) \ , \\
\Pi^{\Xi^{*0}\to\Xi^-\pi^+} &= 2\Pi_1(u,s,s) \ , \\
\Pi^{\Sigma^{*+}\to\Lambda\pi^+} &= \sqrt{2/3}[2\Pi_1(d,s,u)+\Pi_1(d,u,s)] \ , \\
\Pi^{\Delta^{++}\to p\pi^+} &= -2\sqrt{3}\Pi_1(u,u,u) \ , \\
\Pi^{\Delta^+\to n\pi^+} &= -2\Pi_1(d,d,u) \ .
\end{aligned}$$

Correlation functions involving K mesons.

$$\begin{aligned}
\Pi^{\Delta^+\to\Sigma^0 K^+} &= -\sqrt{2}[\Pi_1(s,d,u)+\Pi_1(s,u,d)] \ , \\
\Pi^{\Delta^+\to\Lambda K^+} &= \sqrt{2/3}[\Pi_1(s,d,u)-\Pi_1(s,u,d)] \ , \\
\Pi^{\Delta^0\to\Sigma^- K^+} &= -2\Pi_1(s,d,d) \ , \\
\Pi^{\Sigma^{*+}\to\Xi^0 K^+} &= 2\Pi_1(u,s,u) \ , \\
\Pi^{\Sigma^{*0}\to\Xi^- K^+} &= -\sqrt{2}\Pi_1(u,s,d) \ , \\
\Pi^{\Delta^{++}\to\Sigma^+ K^+} &= 2\sqrt{3}\Pi_1(u,u,u) \ , \\
\Pi^{\Sigma^{*0}\to p K^-} &= \sqrt{2}\Pi_1(u,u,d) \ , \\
\Pi^{\Omega^-\to\Xi^0 K^-} &= -2\sqrt{3}\Pi_1(s,s,s) \ , \\
\Pi^{\Sigma^{*-}\to n K^-} &= -2\Pi_1(s,d,d) \ , \\
\Pi^{\Xi^{*0}\to\Sigma^+ K^-} &= -2\Pi_1(u,u,s) \ , \\
\Pi^{\Xi^{*-}\to\Sigma^0 K^-} &= \sqrt{2}\Pi_1(u,d,s) \ , \\
\Pi^{\Xi^{*-}\to\Lambda K^-} &= -\sqrt{2/3}[2\Pi_1(u,s,d)+\Pi_1(u,d,s)] \ ,
\end{aligned}$$

$$\begin{aligned}
\Pi^{\Xi^{*0}\to\Sigma^0\bar{K}^0} &= \sqrt{2}\Pi_1(d,u,s)\ , \\
\Pi^{\Xi^{*0}\to\Lambda\bar{K}^0} &= \sqrt{2/3}[\Pi_1(d,s,u)+\Pi_1(d,u,s)]\ , \\
\Pi^{\Xi^{*-}\to\Sigma^-\bar{K}^0} &= 2\Pi_1(d,s,s)\ , \\
\Pi^{\Sigma^{*0}\to n\bar{K}^0} &= -\sqrt{2}\Pi_1(d,d,u)\ , \\
\Pi^{\Omega^-\to\Xi^-\bar{K}^0} &= 2\sqrt{3}\Pi_1(s,s,s)\ , \\
\Pi^{\Sigma^{*+}\to p\bar{K}^0} &= -2\Pi_1(s,u,u)\ , \\
\Pi^{\Sigma^{*0}\to\Xi^0 K^0} &= \sqrt{2}\Pi_1(d,s,u)\ , \\
\Pi^{\Delta^-\to\Sigma^- K^0} &= -2\sqrt{3}\Pi_1(d,d,d)\ , \\
\Pi^{\Sigma^{*-}\to\Xi^- K^0} &= -2\Pi_1(d,s,d)\ , \\
\Pi^{\Delta^0\to\Sigma^0 K^0} &= -\sqrt{2}[\Pi_1(s,d,u)+\Pi_1(s,u,d)]\ , \\
\Pi^{\Delta^+\to\Sigma^+ K^0} &= \sqrt{2}\Pi_1(s,u,u)\ .
\end{aligned}$$

Correlation functions involving η mesons.

$$\begin{aligned}
\Pi^{\Sigma^{*+}\to\Sigma^+\eta} &= -\sqrt{2/3}[\Pi_1(u,u,s)+2\Pi_1(s,u,u)]\ , \\
\Pi^{\Sigma^{*-}\to\Sigma^-\eta} &= \sqrt{2/3}[\Pi_1(d,d,s)+2\Pi_1(s,d,d)]\ , \\
\Pi^{\Delta^+\to p\eta} &= \sqrt{2/3}[\Pi_1(u,u,d)-\Pi_1(d,u,u)]\ , \\
\Pi^{\Delta^0\to n\eta} &= -\sqrt{2/3}[\Pi_1(d,d,u)-\Pi_1(u,d,d)]\ , \\
\Pi^{\Xi^{*0}\to\Xi^0\eta} &= -\sqrt{2/3}[\Pi_1(u,s,s)+2\Pi_1(s,s,u)]\ , \\
\Pi^{\Xi^{*-}\to\Xi^-\eta} &= \sqrt{2/3}[\Pi_1(d,s,s)+2\Pi_1(s,s,d)]\ , \\
\Pi^{\Sigma^{*0}\to\Lambda\eta} &= -(1/3\sqrt{2})[\Pi_1(u,d,s)+2\Pi_1(u,s,d)+2\Pi_1(s,d,u) \\
&- 2\Pi_1(s,u,d)-2\Pi_1(d,s,u)-\Pi_1(d,u,s)]\ .
\end{aligned}$$

In the derivation of these results, we have used Eq. (21).

Appendix B :

In this appendix, we present the expression for the invariant function Π responsible for the $\Sigma^{*+} \to \Sigma^+\pi^0$ transition,

$$
\begin{aligned}
\Pi_{\Sigma^{*+}\Sigma^+\pi^0} &= -\frac{M^4}{1152\sqrt{6}\pi^2}\Big\{48\mu_{\mathcal{P}}[4(1+\beta)i_2(\mathcal{T},1) - 8\beta i_2(\mathcal{T},v) - (1+3\beta)i_3(\mathcal{T},1) + 2(1+\beta)i_3(\mathcal{T},v)] \\
&+ 36 f_{\mathcal{P}}[(3+2\beta)m_d - (1-\beta)m_s]\phi_{\mathcal{P}}(u_0) + \beta\mu_{\mathcal{P}}[6\phi_P(u_0) + (1-\widetilde{\mu}_{\mathcal{P}}^2)(12\phi_\sigma(u_0) - \phi'_\sigma(u_0))]\Big\} \\
&+ \frac{M^2}{192\sqrt{6}\pi^2} f_{\mathcal{P}} m_{\mathcal{P}}^2\Big\{[(3+2\beta)m_d - (1-\beta)m_s][3\mathbb{A}(u_0) + 32 i_2(\mathcal{A}_\parallel, v)] + [m_d - (1+\beta)m_s] \\
&\times [64 i_1(\mathcal{A}_\parallel,1) + 64 i_1(\mathcal{A}_\perp,1) + 3\widetilde{i_4}(\mathbb{B})] + 64[\beta m_d - (1+\beta)m_s][i_1(\mathcal{V}_\parallel,1) + i_1(\mathcal{V}_\perp,1)] \\
&- 128[m_d - (1+\beta)m_s][i_1(\mathcal{A}_\parallel,v) + i_1(\mathcal{A}_\perp,v)] - 16[(2+4\beta)m_d + (1-3\beta)m_s]i_2(\mathcal{V}_\parallel,1) \\
&- 32(1+\beta)(2m_d + m_s)i_2(\mathcal{V}_\perp,1) + 64(3+2\beta)m_d i_2(\mathcal{V}_\perp,v) - 32[(1+\beta)m_d + \beta m_s]i_2(\mathcal{A}_\parallel,1) \\
&- 64[(1+2\beta)m_d - \beta m_s]i_2(\mathcal{A}_\perp,1) + 4\frac{\mu_{\mathcal{P}}}{f_{\mathcal{P}}}[(1+\beta)i_2(\mathcal{T},1) + 2\beta i_2(\mathcal{T},v)] \\
&+ \frac{16\pi^2}{m_{\mathcal{P}}^2}[(3+2\beta)\langle\bar{d}d\rangle - (1-\beta)\langle\bar{s}s\rangle]\phi_{\mathcal{P}}(u_0)\Big\} \\
&+ \frac{f_{\mathcal{P}} m_{\mathcal{P}}^2}{12\sqrt{6}\pi^2}\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)\Big\{\Big(m_{\mathcal{P}}^2[(1+\beta)m_d + \beta m_s] - 4M^2[(2+\beta)m_d - m_s]\Big) \\
&\times [i_1(\mathcal{A}_\parallel,1) + i_1(\mathcal{A}_\perp,1)] - \Big(m_{\mathcal{P}}^2[(1+\beta)m_d + \beta m_s] - 4M^2[(1+2\beta)m_d - \beta m_s]\Big) \\
&\times [i_1(\mathcal{V}_\parallel,1) + i_1(\mathcal{V}_\perp,1)] - \beta M^2(m_d - 2m_s)[2i_2(\mathcal{A}_\perp,1) + i_2(\mathcal{V}_\parallel,1)] \\
&+ \frac{1}{64M^2}[m_d - (1+\beta)m_s]\Big(64M^4[i_2(\mathcal{A}_\parallel,1) + 2i_2(\mathcal{V}_\perp,1)] + \langle g_s^2G^2\rangle\widetilde{i_4}(\mathbb{B})\Big) \\
&- \frac{\langle g_s^2G^2\rangle}{32m_{\mathcal{P}}^2}[(3+2\beta)m_d - (1-\beta)m_s]\phi_{\mathcal{P}}(u_0)\Big\} \\
&- \frac{\mu_{\mathcal{P}}\langle g_s^2G^2\rangle}{864\sqrt{6}M^6}(1-\beta)(m_0^2 + 2M^2)(1-\widetilde{\mu}_{\mathcal{P}}^2)(\langle\bar{d}d\rangle m_s + m_d\langle\bar{s}s\rangle)\phi_\sigma(u_0) \\
&- \frac{f_{\mathcal{P}}\langle g_s^2G^2\rangle m_{\mathcal{P}}^2}{4608\sqrt{6}\pi^2M^2}\Big\{[(3+2\beta)m_d - (1-\beta)m_s][3\mathbb{A}(u_0) - 16i_2(\mathcal{A}_\parallel,1) + 32i_2(\mathcal{A}_\parallel,v)] \\
&+ 64[m_d - (1+\beta)m_s][i_1(\mathcal{A}_\parallel,1) + i_1(\mathcal{A}_\perp,1) - 2i_1(\mathcal{A}_\parallel,v) - 2i_1(\mathcal{A}_\perp,v)] \\
&+ 64[\beta m_d - (1+\beta)m_s][i_1(\mathcal{V}_\parallel,1) + i_1(\mathcal{V}_\perp,1)] + 32m_d(2+3\beta)i_2(\mathcal{A}_\perp,1) \\
&+ 32m_d(3+2\beta)[i_2(\mathcal{V}_\perp,1) - 2i_2(\mathcal{V}_\perp,v)] + 16[(2+3\beta)m_d + (1-\beta)m_s]i_2(\mathcal{V}_\parallel,1) \\
&- 4[m_d - (1+\beta)m_s]\widetilde{i_4}(\mathbb{B})\Big\} \\
&+ \frac{\mu_{\mathcal{P}}}{3456\sqrt{6}M^2}\Big\{m_0^2(1+2\beta)(\langle\bar{d}d\rangle m_d + m_s\langle\bar{s}s\rangle)[6\phi_P(u_0) - (1-\widetilde{\mu}_{\mathcal{P}}^2)\phi'_\sigma(u_0)] \\
&+ 96[\beta\langle\bar{d}d\rangle m_{\mathcal{P}}^2 m_d + 4\langle\bar{d}d\rangle m_0^2 m_s - 4\beta\langle\bar{d}d\rangle m_0^2 m_s + m_{\mathcal{P}}^2 m_s\langle\bar{s}s\rangle]i_2(\mathcal{T},1) \\
&+ 192 m_{\mathcal{P}}^2[\langle\bar{d}d\rangle(1+2\beta)m_d - (1+\beta)m_s\langle\bar{s}s\rangle]i_2(\mathcal{T},v)
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{6 f_{\mathcal{P}} m_{\mathcal{P}}^2 m_0^2}{\mu_{\mathcal{P}}}[(1-2\beta)\langle \bar{d}d\rangle - 2(1+3\beta)\langle \bar{s}s\rangle]\widetilde{i}_4(\mathbb{B})\Big\} \\
&+ \frac{f_{\mathcal{P}}\langle g_s^2 G^2\rangle}{576\sqrt{6}\pi^2}[(3+2\beta)m_d - (1-\beta)m_s]\phi_{\mathcal{P}}(u_0) - \frac{f_{\mathcal{P}} m_0^2}{288\sqrt{6}}[3(5+4\beta)\langle \bar{d}d\rangle - 2(1-4\beta)\langle \bar{s}s\rangle]\phi_{\mathcal{P}}(u_0) \\
&+ \frac{f_{\mathcal{P}} m_{\mathcal{P}}^4}{12\sqrt{6}\pi^2}[(1+\beta)m_d + \beta m_s][i_1(\mathcal{A}_\parallel,1) + i_1(\mathcal{A}_\perp,1) - i_1(\mathcal{V}_\parallel,1) - i_1(\mathcal{V}_\perp,1)] \\
&- \frac{f_{\mathcal{P}} m_{\mathcal{P}}^2}{144\sqrt{6}}\Big\{[(3+2\beta)\langle \bar{d}d\rangle - (1-\beta)\langle \bar{s}s\rangle][3\mathbb{A}(u_0) - 16 i_2(\mathcal{A}_\parallel,1) + 32 i_2(\mathcal{A}_\parallel,v)] \\
&- 32\langle \bar{d}d\rangle(2+3\beta)i_2(\mathcal{A}_\perp,1) - 32\langle \bar{d}d\rangle(3+2\beta)[i_2(\mathcal{V}_\perp,1) - 2 i_2(\mathcal{V}_\perp,v)] \\
&- 16[(2+3\beta)\langle \bar{d}d\rangle + (1-\beta)\langle \bar{s}s\rangle] i_2(\mathcal{V}_\parallel,1) + 6[\langle \bar{d}d\rangle - (1+\beta)\langle \bar{s}s\rangle]\widetilde{i}_4(\mathbb{B})\Big\} \\
&+ \frac{\mu_{\mathcal{P}}}{288\sqrt{6}}\Big\{\beta(1-\widetilde{\mu}_{\mathcal{P}}^2)(\langle \bar{d}d\rangle m_d + m_s\langle \bar{s}s\rangle)\phi_\sigma'(u_0) - 16(1-\widetilde{\mu}_{\mathcal{P}}^2)(\langle \bar{d}d\rangle m_s + m_d\langle \bar{s}s\rangle)\phi_\sigma(u_0) \\
&- 4\beta(1-\widetilde{\mu}_{\mathcal{P}}^2)[\langle \bar{d}d\rangle(m_d - 4m_s) - (4m_d - m_s)\langle \bar{s}s\rangle]\phi_\sigma(u_0) + 128\langle \bar{d}d\rangle(1-\beta)m_s i_2(\mathcal{T},1) \\
&+ 16[2\beta m_d\langle \bar{d}d\rangle + (1+\beta)m_s\langle \bar{s}s\rangle] i_3(\mathcal{T},1) - 32(\beta m_d\langle \bar{d}d\rangle + m_s\langle \bar{s}s\rangle) i_3(\mathcal{T},v) \\
&- 6\beta(\langle \bar{d}d\rangle m_d + m_s\langle \bar{s}s\rangle)\phi_P(u_0)\Big\}\ ,
\end{aligned}
$$

and the functions i_n and $\widetilde{i}_4$ are defined as

$$
\begin{aligned}
i_1(\phi, f(v)) &= \int D\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\theta(k-u_0)\ , \\
i_2(\phi, f(v)) &= \int D\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\delta(k-u_0)\ , \\
i_3(\phi, f(v)) &= \int D\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\delta'(k-u_0)\ , \\
\widetilde{i}_4(f(u)) &= \int_{u_0}^1 du f(u)\ ,
\end{aligned}
$$

where

$$
k = \alpha_q + \alpha_g \bar{v}\ , \qquad u_0 = \frac{M_1^2}{M_1^2 + M_2^2}\ , \qquad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}\ .
$$

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Figure captions

Fig. (1) The dependence of the strong coupling constant of π^0 meson with Σ^{*+} and Σ^+ baryons on Borel mass M^2 for several fixed values of the parameter β and at $s_0 = 4.0\ GeV^2$.

Fig. (2) The dependence of the same coupling constant as in Fig. (1), on $\cos\theta$ for several fixed values of the continuum threshold s_0 and at $M^2 = 1.1\ GeV^2$.

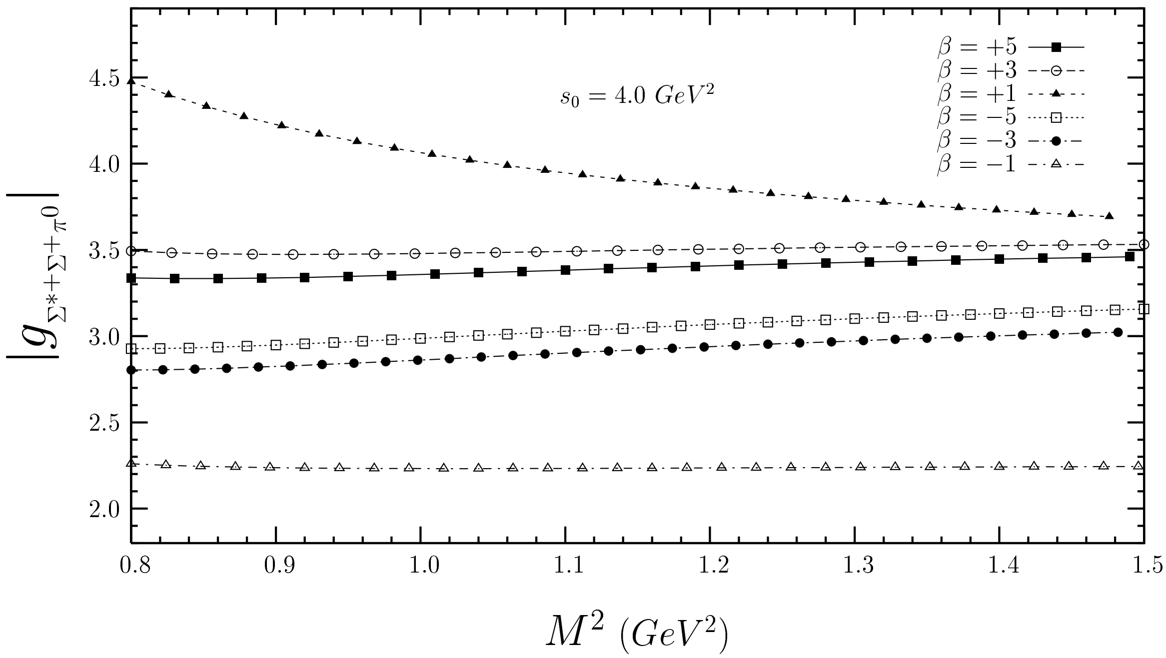

Figure 1:

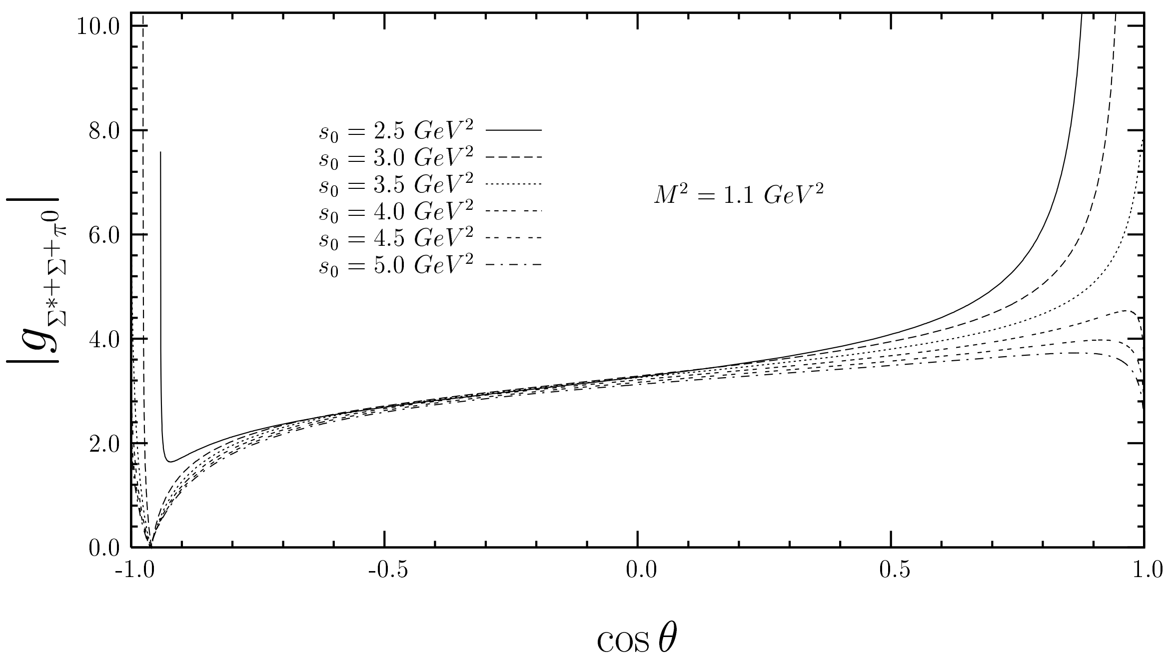

Figure 2: