$g_{D_s^* D K^*(892)}$ and $g_{B_s^* B K^*(892)}$ coupling constants in QCD sum rules

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The coupling constants $g_{D_s^* D K^*(892)}$ and $g_{B_s^* B K^*(892)}$ are calculated in the framework of three-point QCD sum rules. The correlation functions responsible for these coupling constants are evaluated considering contributions of both $D(B)$ and $K^*(892)$ mesons as off-shell states, but in the absence of radiative corrections. The results, $g_{D_s^* D K^*(892)} = (3.74 \pm 1.38)\ GeV^{-1}$ and $g_{B_s^* B K^*(892)} = (3.24 \pm 1.08)\ GeV^{-1}$ are obtained for the considered strong coupling constants.

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I. INTRODUCTION

The heavy-heavy-light mesons coupling constants are fundamental objects as they can provide essential information on the low energy QCD. Their numerical values obtained in QCD can bring important constraints in constructing the meson-meson potentials and strong interactions among them. In the recent years, both theoretical and experimental studies on heavy mesons have received considerable attention. In this connection, excited experimental results obtained in BABAR, FERMILAB, CLEO, CDF, D0, etc. [1–9] and some physical properties of these mesons have been studied using various theoretical models (see for instance [10–15]).

In this article, we calculate the strong coupling constants, $g_{D_s^* D K^*(892)}$ and $g_{B_s^* B K^*(892)}$ in the framework of three-point QCD sum rules considering contributions of both $D(B)$ and $K^*(892)$ mesons as off-shell states, but in the absence of radiative corrections. The result of these coupling constants can help us to better analyze the results of existing experiments hold at different centers. For instance, consider the B_c meson or the newly discovered charmonium states, X, Y and Z by BABAR and BELLE collaborations. These states decay to a J/ψ or ψ' and a light meson in the final state. However, it is supposed that first these states decay into an intermediate two body states containing D_q or D_q^* with $q = u,\ d$ or s quarks, then these intermediate states decay into final stats with the exchange of one or more virtual mesons. The similar procedure may happen in decays of heavy bottonium. Hence, to get precise information about such transitions, we need to have information about the coupling constants between participating particles.

Calculation of the heavy-heavy-light mesons coupling constants via the fundamental theory of QCD is highly valuable. However, such interactions lie in a region very far from the perturbative regime, hence the fundamental QCD Lagrangian can not be responsible in this respect. Therefore, we need some non-perturbative approaches like QCD sum rules [16] as one of the most powerful and applicable tools to hadron physics. Note that, the coupling constants, $D^* D_s K$, $D_s^* D K$ [17, 18], $D_0 D_s K$, $D_{s0} D K$ [18], $D^* D \rho$ [19], $D^* D \pi$ [20, 21], $D D \rho$ [22], $D D J/\psi$ [23], $D^* D J/\psi$ [24], $D^* D^* \pi$ [25, 26], $D^* D^* J/\psi$ [27], $D_s D^* K$, $D_s^* D K$ [28], $D D \omega$ [29], $D^* D^* \rho$ [30], and $B_{s0} B K$, $B_{s1} B^* K$ [31] have been investigated using different approaches.

This paper is organized as follows. In section 2, we give the details of QCD sum rules for the considered coupling constants when both $D(B)$ and $K^*(892)$ mesons in the final state are off-shell. The next section is devoted to the numerical analysis and discussion.

II. QCD SUM RULES FOR THE COUPLING CONSTANTS

In this section, we derive QCD sum rules for coupling constants. For this aim, we will evaluate the three-point correlation functions,

$$\Pi_{\mu\nu}^{D(B)}(p', q) = i^2 \int d^4x\ d^4y\ e^{ip'\cdot x}\ e^{iq\cdot y} \langle 0| \mathcal{T} \left(\eta_\nu^{K^*}(x)\ \eta^{D(B)}(y)\ \eta_\mu^{D_s^*(B_s^*)\dagger}(0) \right) |0\rangle \tag{1}$$

for $D(B)$ off-shell, and

$$\Pi_{\mu\nu}^{K^*(892)}(p', q) = i^2 \int d^4x\ d^4y\ e^{ip'\cdot x}\ e^{iq\cdot y} \langle 0| \mathcal{T} \left(\eta^{D(B)}(x)\ \eta_\nu^{K^*}(y)\ \eta_\mu^{D_s^*(B_s^*)\dagger}(0) \right) |0\rangle \tag{2}$$

for $K^*(892)$ off-shell. Here $\mathcal{T}$ is the time ordering operator and $q = p - p'$ is transferred momentum. Each meson interpolating field can be written in terms of the quark field operators as following form:

$$\begin{aligned} \eta_\nu^{K^*}(x) &= \overline{s}(x)\gamma_\nu u(x) \\ \eta^{D[B]}(y) &= \overline{u}(y)\gamma_5 c[b](y) \\ \eta_\mu^{D_s^*[B_s^*]}(0) &= \overline{s}(0)\gamma_\mu c[b](0) \end{aligned} \tag{3}$$

The correlation functions are calculated in two different ways. In phenomenological or physical side, they are obtained in terms of hadronic parameters. In theoretical or QCD side, they are evaluated in terms of quark and gluon degrees of freedom by the help of the operator product expansion (OPE) in deep Euclidean region. The sum rules for the coupling constants are obtained equating the coefficient of a sufficient structure from both sides of the same correlation functions. To suppress contribution of the higher states and continuum, double Borel transformation with respect to the variables, p^2 and p'^2 is applied.

First, let us focus on the calculation of the physical side of the first correlation function (Eq.(1)) for an off-shell $D(B)$ meson. The physical part is obtained by saturating Eq. (1) with the complete sets of appropriate D^0, D_s^* and $K^*(892)$ states with the same quantum numbers as the corresponding interpolating currents. After performing four-integrals over x and y, we obtain:

$$\Pi^{D(B)}_{\mu\nu}(p',q) = \frac{\langle 0|\eta^{K^*}_\nu|K^*(p',\epsilon)\rangle\langle 0|\eta^{D(B)}|D(B)(q)\rangle\langle K^*(p',\epsilon)D(B)(q)|D^*_s(B^*_s)(p,\epsilon')\rangle\langle D^*_s(B^*_s)(p,\epsilon')|\eta^{D^*_s(B^*_s)}_\mu|0\rangle}{(q^2-m^2_{D(B)})(p^2-m^2_{D^*_s(B^*_s)})(p'^2-m^2_{K^*})} + ... \tag{4}$$

where .... represents the contribution of the higher states and continuum. The matrix elements appearing in the above equation are defined in terms of hadronic parameters such as masses, leptonic decay constants and coupling constant, i.e.,

$$\begin{aligned}
\langle 0|\eta^{K^*}_\nu|K^*(q,\epsilon)\rangle &= m_{K^*}f_{K^*}\epsilon_\nu \\
\langle 0|\eta^{D(B)}|D(B)(p')\rangle &= i\frac{m^2_{D(B)}\,f_{D(B)}}{m_{c(b)}+m_u} \\
\langle D^*_s(B^*_s)(p,\epsilon')|\eta^{D^*_s(B^*_s)}_\mu|0\rangle &= m_{D^*_s(B^*_s)}f_{D^*_s(B^*_s)}{\epsilon^*}'_\mu \\
\langle K^*(q,\epsilon)D(B)(p')|D^*_s(B^*_s)(p,\epsilon')\rangle &= ig^{D(B)}_{D^*_sDK^*(B^*_sBK^*)}\varepsilon^{\alpha\beta\eta\theta}\epsilon^*_\theta\epsilon'_\eta p'_\beta p_\alpha
\end{aligned} \tag{5}$$

where $g^{D(B)}_{D^*_sDK^*(B^*_sBK^*)}$ is coupling constant when $D(B)$ is off-shell and ϵ and ϵ' are the polarization vectors associated with the K^* and $D^*_s(B^*_s)$, respectively. Using Eq. (5) in Eq. (4) and summing over polarization vectors via,

$$\begin{aligned}
\epsilon_\nu\epsilon^*_\theta &= -g_{\nu\theta} + \frac{q_\nu q_\theta}{m^2_{K^*}}, \\
\epsilon'_\eta{\epsilon'}^*_\mu &= -g_{\eta\mu} + \frac{p_\eta p_\mu}{m^2_{D^*_s(B^*_s)}},
\end{aligned} \tag{6}$$

the physical side of the correlation function for $D(B)$ off-shell is obtained as:

$$\Pi^{D(B)}_{\mu\nu}(p',p) = -g^{D(B)}_{D^*_sDK^*(B^*_sBK^*)}(q^2)\frac{f_{D^*_s(B^*_s)}f_{D(B)}f_{K^*}\frac{m^2_{D(B)}}{m_{c(b)+m_u}}m_{D^*_s(B^*_s)}m_{K^*}}{(q^2-m^2_{D(B)})(p^2-m^2_{D^*_s(B^*_s)})(p'^2-m^2_{K^*})}\varepsilon^{\alpha\beta\mu\nu}p'_\beta p_\alpha + ... \tag{7}$$

To calculate the coupling constant, we will choose the structure, $\varepsilon^{\alpha\beta\mu\nu}p'_\beta p_\alpha$ from both sides of the correlation functions. From a similar way, we obtain the final expression of the physical side of the correlation function for an off-shell K^* meson as:

$$\Pi^{K^*}_{\mu\nu}(p',p) = -g^{K^*}_{D^*_sDK^*(B^*_sBK^*)}(q^2)\frac{f_{D^*_s(B^*_s)}f_{D(B)}f_{K^*}\frac{m^2_{D(B)}}{m_{c(b)+m_u}}m_{D^*_s(B^*_s)}m_{K^*}}{(p'^2-m^2_{D(B)})(p^2-m^2_{D^*_s(B^*_s)})(q^2-m^2_{K^*})}\varepsilon^{\alpha\beta\mu\nu}p'_\beta p_\alpha + ... \tag{8}$$

Now, we concentrate our attention to calculate the QCD or theoretical side of the correlation functions in deep Euclidean space, where $p^2 \to -\infty$ and $p'^2 \to -\infty$. For this aim, each correlation function, $\Pi^i_{\mu\nu}(p',p)$, where i stands for $D(B)$ or K^*, can be written in terms of perturbative and non-perturbative parts as:

$$\Pi^i_{\mu\nu}(p',p) = (\Pi_{per} + \Pi_{nonper})\,\varepsilon^{\alpha\beta\mu\nu}p'_\beta p_\alpha, \tag{9}$$

where the perturbative part is defined in terms of double dispersion integral as:

$$\Pi_{per} = -\frac{1}{4\pi^2}\int ds'\int ds\frac{\rho(s,s',q^2)}{(s-p^2)(s'-p'^2)} + \text{subtraction terms}, \tag{10}$$

here, $\rho(s,s',q^2)$ is called spectral density. In order to obtain the spectral density, we need to calculate the bare loop diagrams (a) and (b) in Fig.(1) for $D(B)$ and K^* off-shell, respectively. We calculate these diagrams in terms of the usual Feynman integral by the help of Cutkosky rules, i.e., by replacing the quark propagators with Dirac delta functions: $\frac{1}{q^2-m^2} \to (-2\pi i)\delta(q^2-m^2)$. After some straightforward calculations, we obtain the spectral densities as following:

$$\begin{aligned}
\rho^D(s,s',q^2) = \frac{N_c}{\lambda^{3/2}(s,s',q^2)}&\left\{2m_s^3q^2 + m_us\left(2m_u^2-q^2+s-s'\right) - m_s^2m_u\left(q^2+s-s'\right) + 2m_c^3s' + m_c^2\left[m_s\left(-q^2+s-s'\right)\right.\right. \\
&- \left. m_u\left(-q^2+s+s'\right)\right] + m_c\left[m_s^2\left(-q^2+s-s'\right) - \left(q^2+s-s'\right)s' - m_u^2\left(-q^2+s+s'\right)\right] \\
&- \left. m_s\left(m_u^2\left(q^2+s-s'\right) + q^2\left[-q^2+s+s'\right]\right)\right\},
\end{aligned} \tag{11}$$

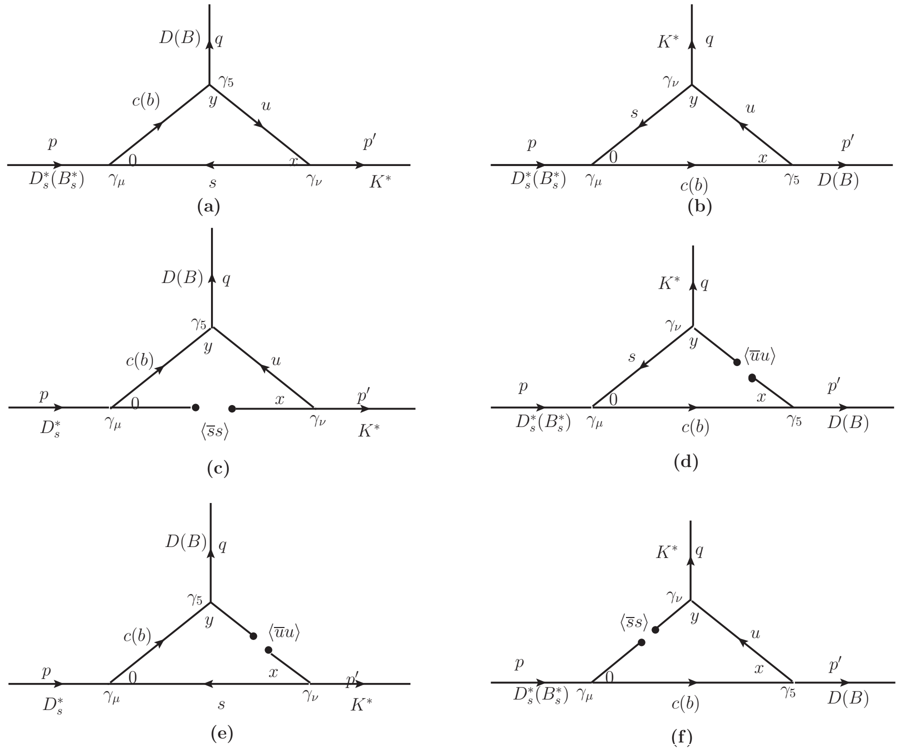

FIG. 1. (a) and (b): Bare loop diagram for the $D(B)$ and K^* off-shell, respectively; (c) and (e): Diagrams corresponding to quark condensate for the $D(B)$ off-shell; (d) and (f): Diagrams corresponding to quark condensate for the K^* off-shell.

$$\rho_1^{K^*}(s,s',q^2) = \frac{N_c}{\lambda^{3/2}(s,s',q^2)}\left\{2m_c^3q^2 + m_u s\left(2m_u^2 - q^2 + s - s'\right) - m_c^2 m_u\left(q^2+s-s'\right) + 2m_s^3 s' + m_s^2\left(m_c\left(-q^2+s-s'\right) - m_u\left(-q^2+s+s'\right)\right) + m_s\left[m_c^2\left(-q^2+s-s'\right) - \left(q^2+s-s'\right)s' - m_u^2\left(-q^2+s+s'\right)\right] - m_c\left[m_u^2\left(q^2+s-s'\right) + q^2\left(-q^2+s+s'\right)\right]\right\}, \tag{12}$$

for the $D_s^*DK^*(892)$ vertex associated with the off-shell D and $K^*(892)$ meson, respectively, and

$$\rho^{B}(s,s',q^2) = \frac{N_c}{\lambda^{3/2}(s,s',q^2)}\left\{2m_s^3q^2 + m_u s\left(2m_u^2 - q^2 + s - s'\right) - m_s^2 m_u\left(q^2+s-s'\right) + 2m_b^3 s' + m_b^2\left[m_s\left(-q^2+s-s'\right) - m_u\left(-q^2+s+s'\right)\right] + m_b\left[m_s^2\left(-q^2+s-s'\right) - \left(q^2+s-s'\right)s' - m_u^2\left(-q^2+s+s'\right)\right] - m_s\left(m_u^2\left(q^2+s-s'\right) + q^2\left[-q^2+s+s'\right]\right)\right\}, \tag{13}$$

$$\rho_2^{K^*}(s,s',q^2) = \frac{N_c}{\lambda^{3/2}(s,s',q^2)}\left\{2m_b^3q^2 + m_u s\left(2m_u^2 - q^2 + s - s'\right) - m_b^2 m_u\left(q^2+s-s'\right) + 2m_s^3 s' + m_s^2\left(m_b\left(-q^2+s-s'\right) - m_u\left(-q^2+s+s'\right)\right) + m_s\left[m_b^2\left(-q^2+s-s'\right) - \left(q^2+s-s'\right)s' - m_u^2\left(-q^2+s+s'\right)\right] - m_b\left[m_u^2\left(q^2+s-s'\right) + q^2\left(-q^2+s+s'\right)\right]\right\}, \tag{14}$$

for the $B_s^*BK^*(892)$ vertex associated with the off-shell B and $K^*(892)$ meson, respectively. Here $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ac - 2bc - 2ab$ and $N_c = 3$ is the color number.

To calculate the nonperturbative contributions in QCD side, we consider the quark condensate diagrams presented in (c), (d), (e) and (f) parts of Fig. (1). It should be reminded that the heavy quark condensates contributions are suppressed by inverse of the heavy quark mass, so they can be safely neglected. Therefore, as nonperturbative part, we only encounter contributions coming from light quark condensates. Contributions of the diagrams (d), (e) and (f) in Fig.(1) are zero since applying double Borel transformation with respect to the both variables p^2 and p'^2 will kill them because of appearing only one variable in the denominator in these cases. Hence, we calculate the diagram (c) in Fig.(1) for the off-shell $D(B)$ meson. As a result, we obtain:

$$\Pi^{D}_{nonper}=-\frac{\langle\overline{s}s\rangle}{(p^2-m_c^2)(p'^2-m_u^2)}, \tag{15}$$

for the off-shell D meson and

$$\Pi^{B}_{nonper}=-\frac{\langle\overline{s}s\rangle}{(p^2-m_b^2)(p'^2-m_u^2)}, \tag{16}$$

for the off-shell B meson.

Now, it is time to apply the double Borel transformations with respect to the $p^2(p^2\rightarrow M^2)$ and $p'^2\rightarrow(p'^2\rightarrow M'^2)$ to the physical as well as the QCD sides and equate the coefficient of the selected structure from two representations. Finally, we get the following sum rules for the corresponding coupling constant form factors:

$$g^{D}_{D_s^*DK^*}(q^2)=\frac{(q^2-m_D^2)}{f_{D_s^*}f_Df_{K^*}\frac{m_D^2}{m_c+m_u}m_{D_s^*}m_{K^*}}e^{\frac{m_{D^*_s}^2}{M^2}}e^{\frac{m_{K^*}^2}{M'^2}}\Bigg[\frac{1}{4\,\pi^2}\int_{(m_c+m_s)^2}^{s_0}ds\int_{(m_s+m_u)^2}^{s'_0}ds'\rho^D(s,s',q^2)$$
$$\theta[1-(f^D(s,s'))^2]e^{\frac{-s}{M^2}}e^{\frac{-s'}{M'^2}}+\langle\overline{s}s\rangle e^{\frac{m_c^2}{M^2}}e^{\frac{m_u^2}{M'^2}}\Bigg], \tag{17}$$

$$g^{K^*}_{D_s^*DK^*}(q^2)=\frac{(q^2-m_{K^*}{}^2)}{f_{D_s^*}f_Df_{K^*}\frac{m_D^2}{m_c+m_u}m_{D_s^*}m_{K^*}}e^{\frac{m_{D^*_s}^2}{M^2}}e^{\frac{m_D^2}{M'^2}}\Bigg[\frac{1}{4\,\pi^2}\int_{(m_c+m_s)^2}^{s_0}ds\int_{(m_c+m_u)^2}^{s'_0}ds'\rho_1^{K^*}(s,s',q^2)$$
$$\theta[1-(f_1^{K^*}(s,s'))^2]e^{\frac{-s}{M^2}}e^{\frac{-s'}{M'^2}}\Bigg], \tag{18}$$

for the off-shell D and $K^*(892)$ meson associated with the $D_s^*DK^*(892)$ vertex, respectively, and

$$g^{B}_{B_s^*BK^*}(q^2)=\frac{(q^2-m_B^2)}{f_{B_s^*}f_Bf_{K^*}\frac{m_B^2}{m_b+m_u}m_{B_s^*}m_{K^*}}e^{\frac{m_{B^*_s}^2}{M^2}}e^{\frac{m_{K^*}^2}{M'^2}}\Bigg[\frac{1}{4\,\pi^2}\int_{(m_b+m_s)^2}^{s_0}ds\int_{(m_s+m_u)^2}^{s'_0}ds'\rho^B(s,s',q^2)$$
$$\theta[1-(f^B(s,s'))^2]e^{\frac{-s}{M^2}}e^{\frac{-s'}{M'^2}}+\langle\overline{s}s\rangle e^{\frac{m_b^2}{M^2}}e^{\frac{m_u^2}{M'^2}}\Bigg], \tag{19}$$

$$g^{K^*}_{B_s^*BK^*}(q^2)=\frac{(q^2-m_{K^*}{}^2)}{f_{B_s^*}f_Bf_{K^*}\frac{m_B^2}{m_b+m_u}m_{B_s^*}m_{K^*}}e^{\frac{m_{B^*_s}^2}{M^2}}e^{\frac{m_B^2}{M'^2}}\Bigg[\frac{1}{4\,\pi^2}\int_{(m_b+m_s)^2}^{s_0}ds\int_{(m_b+m_u)^2}^{s_0{}'}ds'\rho_2^{K^*}(s,s',q^2)$$
$$\theta[1-(f_2^{K^*}(s,s'))^2]e^{\frac{-s}{M^2}}e^{\frac{-s'}{M'^2}}\Bigg], \tag{20}$$

for the off-shell B and $K^*(892)$ meson associated with the $B_s^*BK^*(892)$ vertex, respectively. The integration regions in the perturbative part in Eqs. (17)-(20) are determined requiring that the arguments of the three δ functions coming from Cutkosky rule vanish simultaneously. So, the physical regions in the s - s' plane are described by the following non-equalities:

$$-1\leq f^D(s,s')=\frac{2\,s\,(m_s^2-m_u^2+s')+(m_c^2-m_s^2-s)(-q^2+s+s')}{\lambda^{1/2}(m_c^2,m_s^2,s)\lambda^{1/2}(s,s',q^2)}\leq 1, \tag{21}$$

$$-1\leq f_1^{K^*}(s,s')=\frac{2\,s\,(-m_c^2+m_u^2-s')+(m_c^2-m_s^2+s)(-q^2+s+s')}{\lambda^{1/2}(m_c^2,m_s^2,s)\lambda^{1/2}(s,s',q^2)}\leq 1, \tag{22}$$

for the off-shell D and $K^*(892)$ meson associated with the $D_s^*DK^*(892)$ vertex, respectively, and

$$-1\leq f^B(s,s')=\frac{2\,s\,(m_s^2-m_u^2+s')+(m_b^2-m_s^2-s)(-q^2+s+s')}{\lambda^{1/2}(m_b^2,m_s^2,s)\lambda^{1/2}(s,s',q^2)}\leq 1, \tag{23}$$

$$-1\leq f_2^{K^*}(s,s')=\frac{2\,s\,(-m_b^2+m_u^2-s')+(m_b^2-m_s^2+s)(-q^2+s+s')}{\lambda^{1/2}(m_b^2,m_s^2,s)\lambda^{1/2}(s,s',q^2)}\leq 1, \tag{24}$$

for the off-shell B and $K^*(892)$ meson associated with the $B_s^* B K^*(892)$ vertex, respectively. These physical regions are imposed by the limits on the integrals and step functions in the integrands of the sum rules. In order to subtract the contributions of the higher states and continuum, the quark-hadron duality assumption is used, i.e., it is assumed that,

$$\rho^{higherstates}(s,s') = \rho^{OPE}(s,s')\theta(s-s_0)\theta(s'-s'_0). \tag{25}$$

Note that, the double Borel transformation used in calculations is defined as:

$$\hat{B}\frac{1}{(p^2-m_1^2)^m}\frac{1}{(p'^2-m_2^2)^n} \to (-1)^{m+n}\frac{1}{\Gamma(m)}\frac{1}{\Gamma(n)}e^{-m_1^2/M^2}e^{-m_2^2/M'^2}\frac{1}{(M^2)^{m-1}(M'^2)^{n-1}}. \tag{26}$$

III. NUMERICAL ANALYSIS

Present section is devoted to the numerical analysis of the sum rules for the coupling constants. In further analysis, we use, $m_{K^*}(892) = (0.89166 \pm 0.00026)\ GeV$, $m_{D^0} = (1.8648 \pm 0.00014)\ GeV$, $m_{D_s^*} = (2.1123 \pm 0.0005)\ GeV$, $m_{B^\pm} = (5.2792 \pm 0.0003)\ GeV$, $m_{B_s^*} = (5.4154 \pm 0.0014)\ GeV$ [32], $m_c = 1.3\ GeV$, $m_b = 4.7\ GeV$, $m_s = 0.14\ GeV$[33], $m_u = 0$, $f_{K^*} = 225\ MeV$[34], $f_{D_s^*} = (272 \pm 16^{0}_{-20})\ MeV$, $f_{B_s^*} = (229 \pm 20^{31}_{-16})\ MeV$ [35], $f_B = (190 \pm 13)\ MeV$ [36], $f_D = (206.7 \pm 8.9)\ MeV$ [37] and $\langle \bar{s}s \rangle = -0.8(0.24 \pm 0.01)^3\ GeV^3$ [33].

The sum rules for the strong coupling constants contain also four auxiliary parameters, namely the Borel mass parameters, M^2 and M'^2 and the continuum thresholds, s_0 and s'_0. Since these parameters are not physical quantities, our results should be independent of them. Therefore, we look for working regions at which the dependence of coupling constants on these auxiliary parameters are weak. The working regions for the Borel mass parameters M^2 and M'^2 are determined requiring that both the contributions of the higher states and continuum are sufficiently suppressed and the contributions coming from higher dimensions are small. As a result, we obtain, $8\ GeV^2 \le M^2 \le 25\ GeV^2$ and $3\ GeV^2 \le M'^2 \le 15\ GeV^2$ for D off-shell, and $4\ GeV^2 \le M^2 \le 10\ GeV^2$ and $3\ GeV^2 \le M'^2 \le 9\ GeV^2$ for K^* off-shell associated with the $D_s^* D K^*(892)$ vertex. Similarly, the regions, $14\ GeV^2 \le M^2 \le 30\ GeV^2$ and $5\ GeV^2 \le M'^2 \le 20\ GeV^2$ for B off-shell, and $5\ GeV^2 \le M^2 \le 20\ GeV^2$ and $5\ GeV^2 \le M'^2 \le 15\ GeV^2$ for K^* off-shell are obtained for the $B_s^* B K^*(892)$ vertex. The dependence of considered coupling constants on Borel parameters for different cases are shown in Figs.(2-5) and (7-10). From these figures, we see a good stability of the results with respect to the Borel mass parameters in the working regions. The continuum thresholds, s_0 and s'_0 are not completely arbitrary but they are correlated to the energy of the first excited states with the same quantum numbers. Our numerical calculations lead to the following regions for the continuum thresholds in s and s' channels for different cases: $(m_{D_s^*(B_s^*)} + 0.3)^2 \le s_0 \le (m_{D_s^*(B_s^*)} + 0.5)^2$ in s channel for both off-shell cases and two vertexes, and $(m_{D(B)} + 0.3)^2 \le s'_0 \le (m_{D(B)} + 0.7)^2$ and $(m_{K^*} + 0.3)^2 \le s'_0 \le (m_{K^*} + 0.7)^2$ for K^* and $D(B)$ off-shell cases, respectively in s' channel. Here, we should stress that the analysis of sum rules in our work is based on, so called the standard procedure in QCD sum rules, i.e., the continuum thresholds are independent of Borel mass parameters and q^2. However, recently it is believed that the standard procedure does not render realistic errors and the continuum thresholds depend on Borel parameters and q^2 and this leads to some uncertainties (see for instance [38]).

Now, using the working region for auxiliary parameters and other input parameters, we would like to discuss the behavior of the strong coupling constant form factors in terms of q^2. In the case of off-shell D meson related to the $D_s^* D K^*$ vertex, our numerical result is described well by the following mono-polar fit parametrization shown by the dashed line in Fig. (6):

$$g^{(D)}_{D_s^* D K^*}(Q^2) = \frac{-103.34}{Q^2 - 28.57}, \tag{27}$$

where $Q^2 = -q^2$. The coupling constants are defined as the values of the form factors at $Q^2 = -m^2_{meson}$ (see also [19]), where m_{meson} is the mass of the on shell meson. Using $Q^2 = -m_D^2$ in Eq. (27), the coupling constant for off-shell D is obtained as: $g^D_{D_s^* D K^*} = 3.23\ GeV^{-1}$. The result for an off-shell K^* meson can be well fitted by the exponential parametrization presented by solid line in Fig. (6) ,

$$g^{(K^*)}_{D_s^* D K^*}(Q^2) = 4.44\ e^{\frac{-Q^2}{7.24}} - 0.70. \tag{28}$$

Using $Q^2 = -m^2_{K^*}$ in Eq. (28), the $g^{K^*}_{D_s^* D K^*} = 4.25\ GeV^{-1}$ is obtained. Taking the average of two above obtained values, finally we get the value of the $g_{D^* D K^*}$ coupling constant as:

$$g_{D_s^* D K^*} = (3.74 \pm 1.38)\ GeV^{-1}. \tag{29}$$

From figure (6) it is also clear that the form factor, $g^{D}_{D_s^* DK^*}$ is more stable comparing to $g^{K^*}_{D_s^* DK^*}$ with respect to the Q^2. The similar observation has also obtained in [19] in analysis of the $D^* D\rho$ vertex. In our case, the two form factors coincide at $Q^2 = 0.1612\ GeV^2$ and have the value $3.64\ GeV^{-1}$ very close to the value obtained taking average of the coupling constants for two off-shell cases at $Q^2 = -m^2_{meson}$.

Similarly, for $B_s^* BK^*$ vertex, our result for B off-shell is better extrapolated by the mono-polar fit parametrization,

$$g^{(B)}_{B_s^* BK^*}(Q^2) = \frac{-354.37}{Q^2 - 98.14}, \tag{30}$$

presented by dashed line in Fig. (11) and for K^* off-shell case, the parametrization

$$g^{(K^*)}_{B_s^* BK^*}(Q^2) = 3.02\ e^{\frac{-Q^2}{2.90}} - 0.28, \tag{31}$$

shown by the solid line in Fig. (11), describes better the results in terms of Q^2. Using $Q^2 = -m_B^2$ in Eq. (30), the coupling constant is obtained as $g^{B}_{B_s^* BK^*} = 2.78\ GeV^{-1}$. Also, $g^{K^*}_{B_s^* BK^*} = 3.69\ GeV^{-1}$ is obtained at $Q^2 = -m^2_{K^*}$ in Eq. (31). Taking the average of these results, we get,

$$g_{B_s^* BK^*} = (3.24 \pm 1.08)\ GeV^{-1}. \tag{32}$$

The errors in the results are due to the uncertainties in determination of the working regions for the auxiliary parameters as well as the errors in the input parameters. From the figure Fig. (11), we also deduce that the heavier is the off-shell meson, the more stable is its coupling form factor in terms of Q^2. From this figure, we also see that the two form factors related to the $B_s^* BK^*$ vertex coincide at $Q^2 = -0.7152\ GeV^2$ and have the value $3.58\ GeV^{-1}$ also close to the value obtained taking average of the corresponding coupling constants for two off-shell cases at $Q^2 = -m^2_{meson}$.

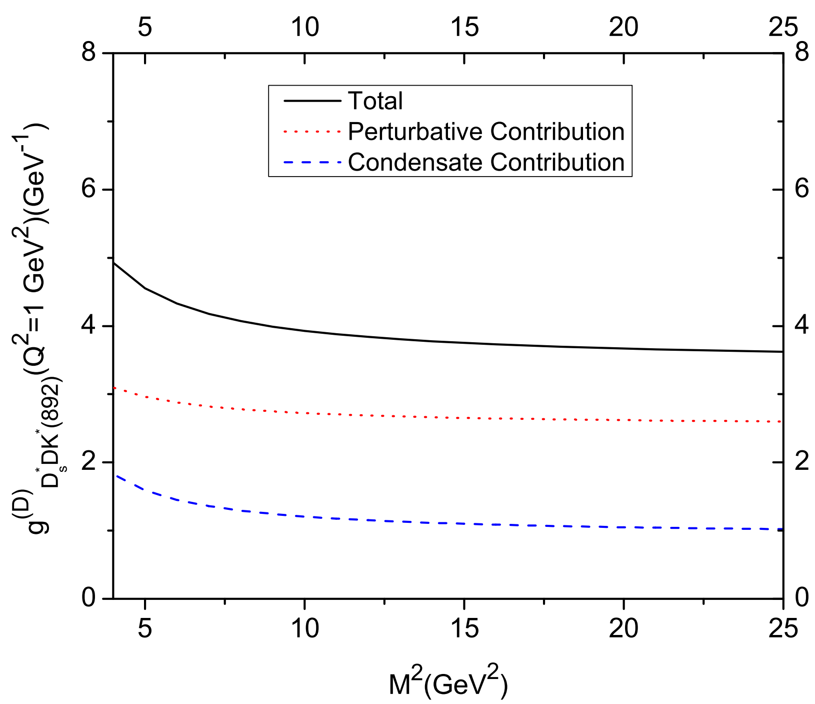

FIG. 2. $g^{D}_{D_s^* DK^*}(Q^2 = 1\ GeV^2)$ as a function of the Borel mass M^2. The continuum thresholds, $s_0 = 6.83\ GeV^2$, $s_0' = 2.54\ GeV^2$ and $M'^2 = 5\ GeV^2$ have been used.

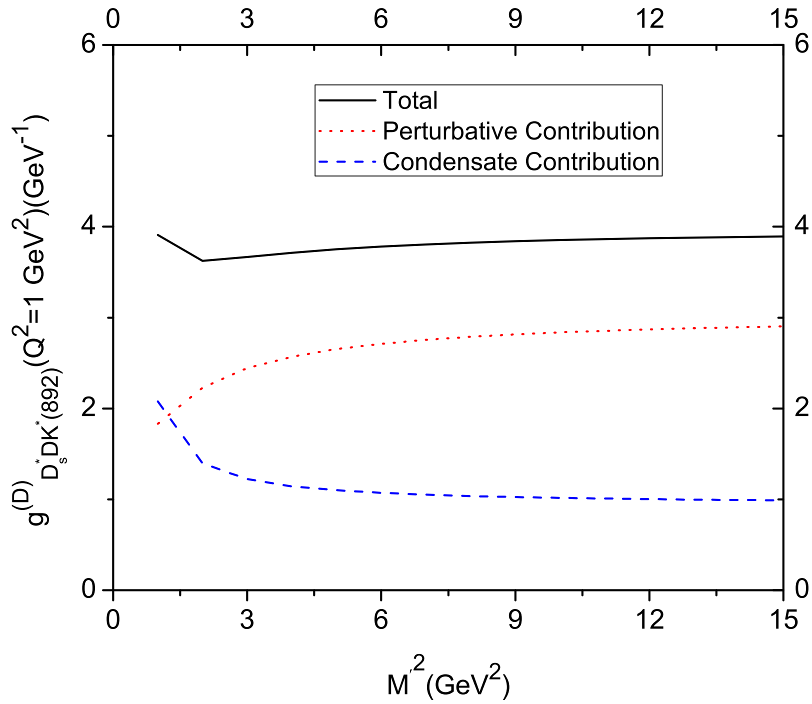

FIG. 3. $g^D_{D_s^* DK^*}(Q^2 = 1\ GeV^2)$ as a function of the Borel mass M'^2. The continuum thresholds, $s_0 = 6.83\ GeV^2$, $s'_0 = 2.54\ GeV^2$ and $M^2 = 15\ GeV^2$ have been used.

IV. ACKNOWLEDGEMENT

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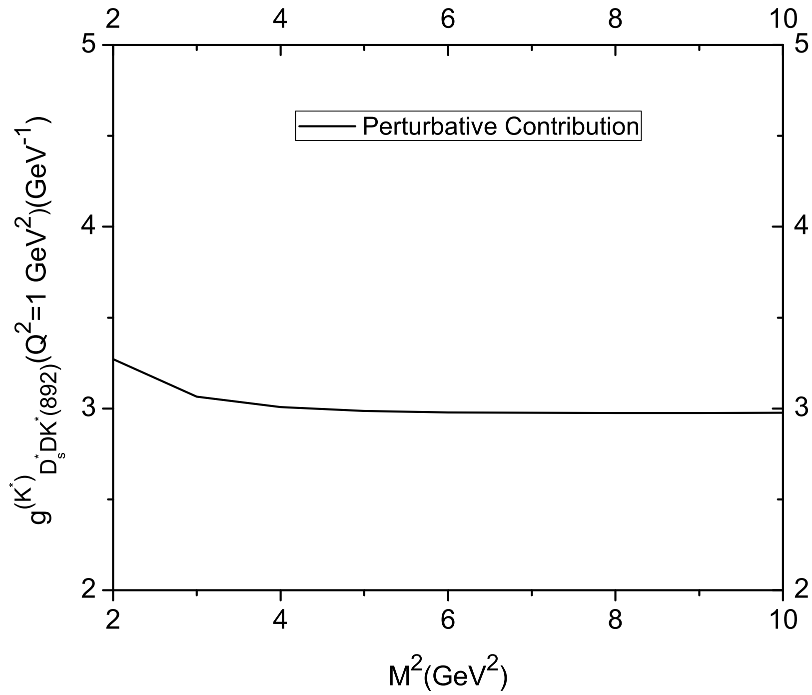

FIG. 4. $g^{K^*}_{D_s^* D K^*}(Q^2 = 1\,GeV^2)$ as a function of the Borel mass M^2. The continuum thresholds, $s_0 = 6.83\,GeV^2$, $s_0' = 6.57\,GeV^2$ and ${M'}^2 = 5\,GeV^2$ have been used.

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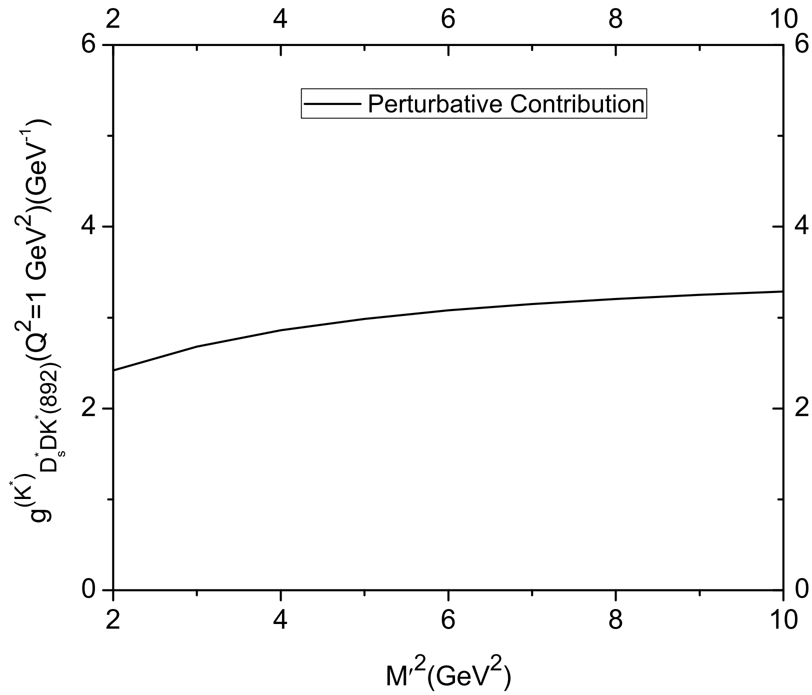

FIG. 5. $g^{K^*}_{D_s^* DK^*}(Q^2 = 1\ GeV^2)$ as a function of the Borel mass M'^2. The continuum thresholds, $s_0 = 6.83\ GeV^2$, $s'_0 = 6.57\ GeV^2$ and $M^2 = 5\ GeV^2$ have been used.

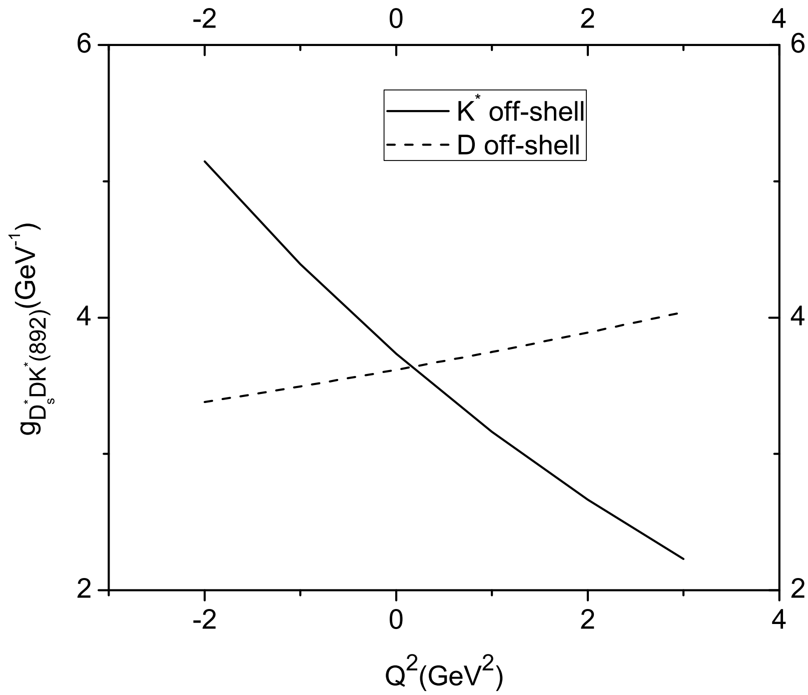

FIG. 6. $g_{D_s^* DK^*}$ as a function of Q^2.

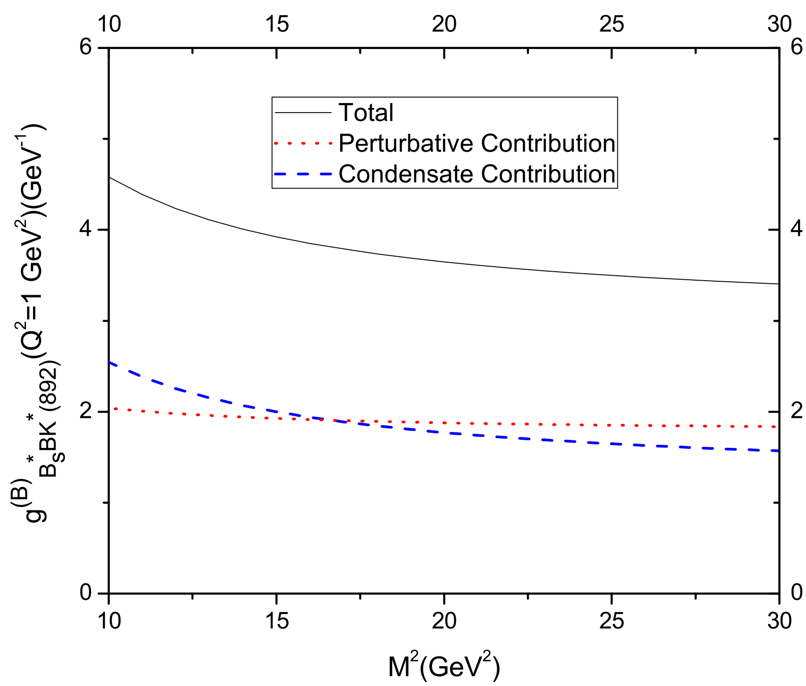

FIG. 7. $g^B_{B_s^* BK^*}(Q^2 = 1\ GeV^2)$ as a function of the Borel mass M^2. The continuum thresholds, $s_0 = 34.99\ GeV^2$, $s_0' = 2.54\ GeV^2$ and $M'^2 = 10\ GeV^2$ have been used.

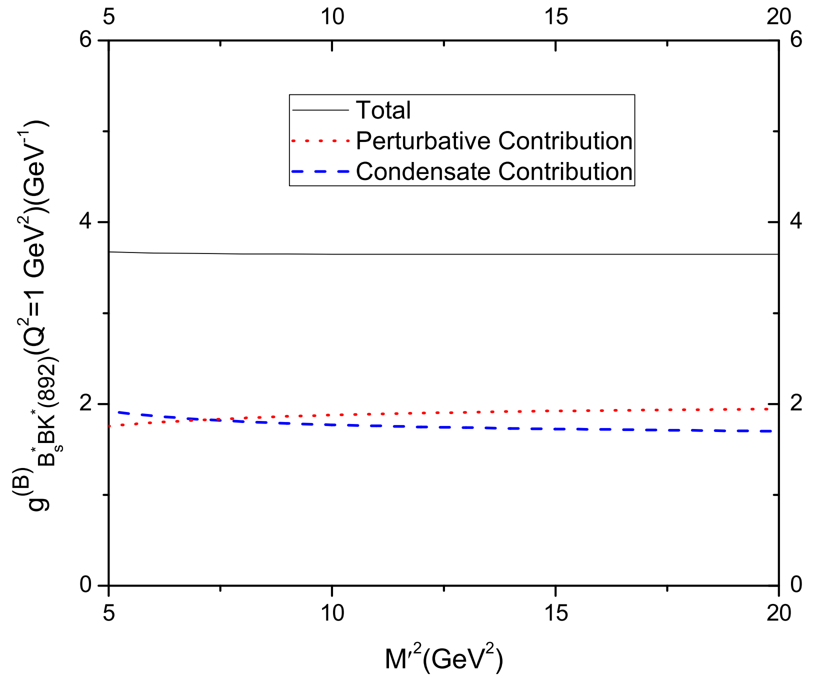

FIG. 8. $g^{K^*}_{B_s^* BK^*}(Q^2 = 1\ GeV^2)$ as a function of the Borel mass M'^2. The continuum thresholds, $s_0 = 34.99\ GeV^2$, $s_0' = 2.54\ GeV^2$ and $M^2 = 20\ GeV^2$ have been used.

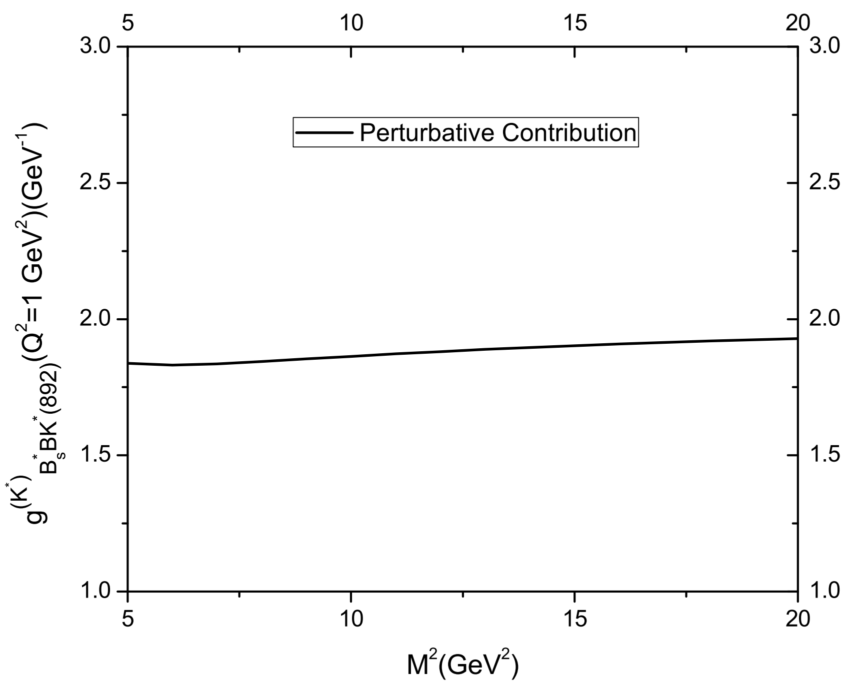

FIG. 9. $g^{K^*}_{B_s^* BK^*}(Q^2 = 1\ GeV^2)$ as a function of the Borel mass M^2. The continuum thresholds, $s_0 = 34.99\ GeV^2$, $s_0' = 35.75\ GeV^2$ and $M'^2 = 8\ GeV^2$ have been used.

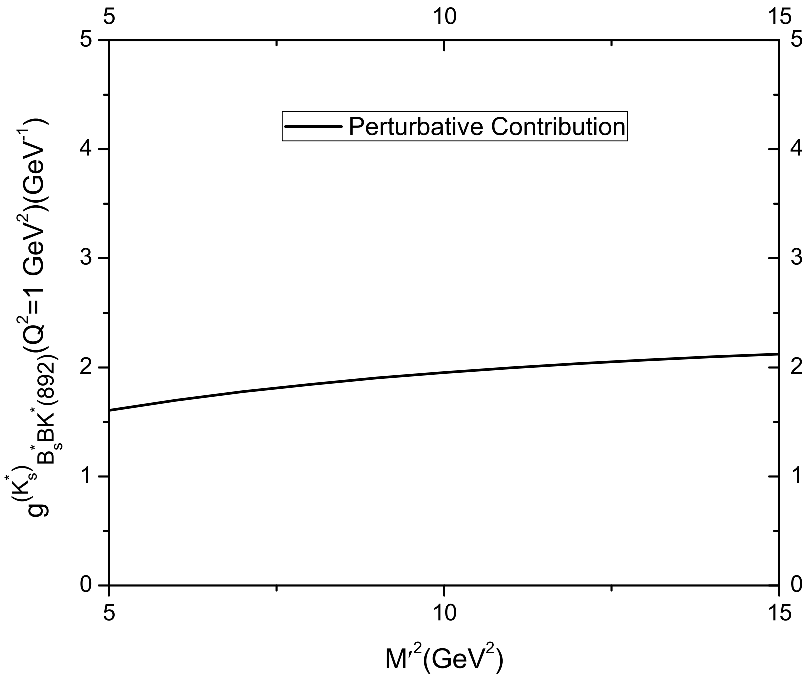

FIG. 10. $g^{K^*}_{B^*_s BK^*}(Q^2 = 1\ GeV^2)$ as a function of the Borel mass M'^2. The continuum thresholds, $s_0 = 34.99\ GeV^2$, $s'_0 = 35.75\ GeV^2$ and $M^2 = 8\ GeV^2$ have been used.

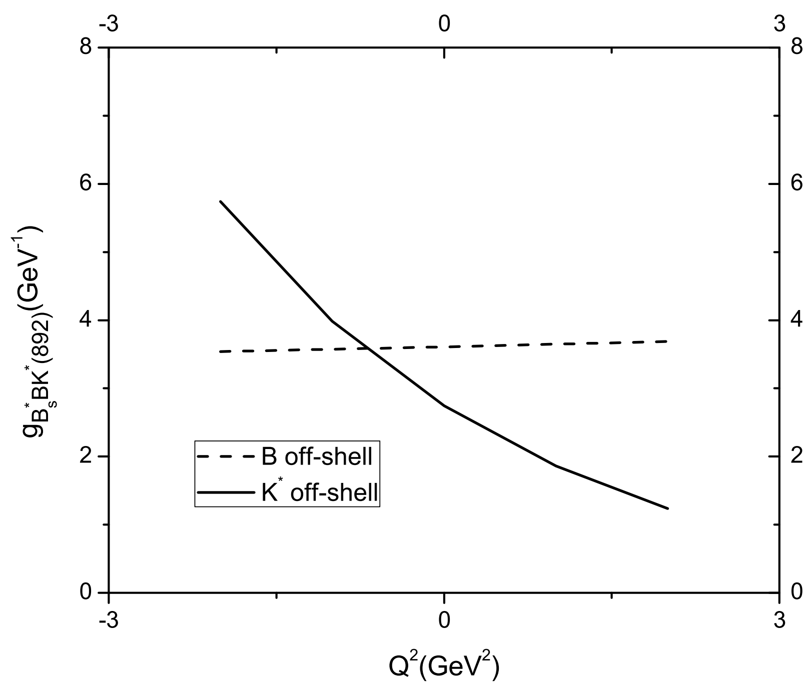

FIG. 11. $g_{B^*_s BK^*}$ as a function of Q^2.