Masses and decay constants of bound states containing fourth family quarks from QCD sum rules

V. Bashiry[†1], K. Azizi[‡2], S. Sultansoy[*3]

[†]Engineering Faculty, Cyprus International University, Via Mersin 10, Turkey
[‡]Physics Division, Faculty of Arts and Sciences, Doğuş University, Acıbadem-Kadıköy, 34722 Istanbul, Turkey
[*] Physics Division, TOBB University of Economics and Technology, Ankara, Turkey
and
Institute of Physics, National Academy of Sciences, Baku, Azerbaijan
[1]bashiry@ciu.edu.tr
[2]kazizi@dogus.edu.tr
[3]ssultansoy@etu.edu.tr

The heavy fourth generation of quarks that have sufficiently small mixing with the three known SM families form hadrons. In the present work, we calculate the masses and decay constants of mesons containing either both quarks from the fourth generation or one from fourth family and the other from known third family SM quarks in the framework of the QCD sum rules. In the calculations, we take into account two gluon condensate diagrams as nonperturbative contributions. The obtained results reduce to the known masses and decay constants of the $\bar{b}b$ and $\bar{c}c$ quarkonia when the fourth family quark is replaced by the bottom or charm quark.

PACS numbers: 11.55.Hx, 12.60.-i

I. INTRODUCTION

In the standard model (SM), we have three generation of quarks experimentally observed. Among these quarks, the top (t) quark does not form bound states (hadrons) as a consequence of the high value of its mass. The top quark immediately decays to the bottom quark giving a W boson and this transition has full strength. The number of quark and lepton generations is one of the mysteries of nature and can not be addressed by the SM. There are flavor democracy arguments that predict the existence of the fourth generation of quarks [1–3]. It is expected that the masses of the fourth generation quarks are in the interval $(300 - 700)$ GeV [4], in which the upper limit coincides with the one obtained from partial-wave unitarity at high energies [5]. Within the flavor democracy approach, the Dirac masses of the fourth family fermions are almost equal, whereas masses of the first three family fermions as well as the CKM and PMNS mixings are obtained via small violations of democracy [6, 7]. For the recent status of the SM with fourth generation (SM$_4$), see e.g. [8–10] and references therein.

Although the masses of fourth generation quarks are larger than the top quark mass (the last analysis of the Tevatron data implies $m_{d_4} > 372$ GeV [11] and $m_{u_4} > 358$ GeV [12]), they can form bound states as a result of the smallness of the mixing between these quarks and ordinary SM quarks [13–19]. As the mass difference between these two quarks is small, we will refer to both members of the fourth family by u_4. The condition for formation of new hadrons containing ultra-heavy quarks (Q) is given by [20]:

$$|V_{Qq}| \leq \left(\frac{100 \ GeV}{m_Q} \right)^{3/2} . \tag{1}$$

For t-quark with $m_t = 172$ GeV, Eq. (1) leads to $V_{tq} < 0.44$, whereas the single top production at the Tevatron gives $V_{tb} > 0.74$ [21]. When the fourth family quarks have sufficiently small mixing with the ordinary quarks, the hadrons made up from these quarks can live longer enough, and the bound state $\bar{u}_4 u_4$ decays through its annihilation and not via u_4 decays to a lower family quark plus a W boson [19]. Concerning the flavor democracy approach, this situation is realized for parameterizations proposed in [7] and [22], whereas parameterization in [6] predicts $V_{u_4q} \sim 0.2$ which does not allow formation of the fourth family quarkonia for $m_{u_4} > 300GeV$.

Considering the above discussions, the production of such bound states if they exist will be possible at LHC. The conditions for observation of the fourth SM family quarks at the LHC has been discussed in [13, 23–30]. As there is a possibility to observe the bound states which consist of fourth family quarks at the LHC, it is reasonable to investigate their properties, theoretically and phenomenologically.

In the present work, we calculate the masses and decay constants of the bound state mesons containing two heavy quarks either both from the SM$_4$ or one from heavy fourth family and the other from ordinary heavy b or c quark. Here, we consider the ground state mesons with different quantum numbers, namely scalars ($\bar{u}_4 u_4$, $\bar{u}_4 b$ and $\bar{u}_4 c$), pseudoscalars ($\bar{u}_4 \gamma_5 u_4$, $\bar{u}_4 \gamma_5 b$ and $\bar{u}_4 \gamma_5 c$), vectors ($\bar{u}_4 \gamma_\mu u_4$, $\bar{u}_4 \gamma_\mu b$ and $\bar{u}_4 \gamma_\mu c$) and axial vector ($\bar{u}_4 \gamma_\mu \gamma_5 u_4$, $\bar{u}_4 \gamma_\mu \gamma_5 b$ and $\bar{u}_4 \gamma_\mu \gamma_5 c$) mesons. These mesons, similar to the ordinary hadrons, are formed in low energies very far from the asymptotic region. Therefore, to calculate their hadronic parameters such as their masses and leptonic decay constants, we need to consult some nonperturbative approaches. Among the nonperturbative methods, the QCD sum rules [31], which is based on QCD Lagrangian and is free of model dependent parameters, is one

of the most applicable and predictive approaches to hadron physics. This method has been successfully used to calculate the masses and decay constants of mesons both in vacuum and at finite temperature (see for instance [32–41]). Now, we extend the application of this method to calculate the masses and decay constants of the considered mesons containing fourth family quarkonia. The heavy quark condensates are suppressed by the inverse powers of the heavy quark mass. Therefore, as the first nonperturbative contributions, we take into account the two-gluon condensate diagrams.

The outline of the paper is as follows. In the next section, QCD sum rules for masses and decay constants of the considered bound states are obtained. Section III encompasses our numerical analysis on the masses and decay constants of the ground state ultra heavy scalar, pseudoscalar, vector and axial vector mesons as well as our discussions.

II. QCD SUM RULES FOR MASSES AND DECAY CONSTANTS OF THE BOUND STATES (MESONS) CONTAINING HEAVY FOURTH FAMILY QUARKS

We start to this section considering sufficient correlation functions responsible for calculation of the masses and decay constants of the bound states containing heavy fourth generation quarks in the framework of QCD sum rules. The two point correlation function corresponding to the scalar (S) and pseudoscalar (PS) cases is written as:

$$\Pi^{S(PS)} = i \int d^4x e^{ip.x} \langle 0 \mid \mathcal{T} \left(J^{S(PS)}(x) \bar{J}^{S(PS)}(0) \right) \mid 0 \rangle, \tag{2}$$

where $\mathcal{T}$ is the time ordering product and $J^S(x) = \overline{u}_4(x)q(x)$ and $J^{PS}(x) = \overline{u}_4(x)\gamma_5 q(x)$ are the interpolating currents of the heavy scalar and pseudoscalar bound states, respectively. Here, the q can be either fourth family u_4 quark or ordinary heavy b or c quark. Similarly, the correlation function for the vector (V) and axial vector (AV) is written as:

$$\Pi_{\mu\nu}^{V(AV)} = i \int d^4x e^{ip.x} \langle 0 \mid \mathcal{T} \left(J_\mu^{V(AV)}(x) \bar{J}_\nu^{V(AV)}(0) \right) \mid 0 \rangle, \tag{3}$$

where, the currents $J_\mu^V = \overline{u}_4(x)\gamma_\mu q(x)$ and $J_\mu^{AV} = \overline{u}_4(x)\gamma_\mu\gamma_5 q(x)$ are responsible for creating the vector and axial vector quarkonia from vacuum with the same quantum numbers as the interpolating currents.

From the general philosophy of the QCD sum rules, we calculate the aforesaid correlation functions in two alternative ways. From the physical or phenomenological side, we calculate them in terms of hadronic parameters such as masses and decay constants. In QCD or theoretical side, they are calculated in terms of QCD degrees of freedom such as quark masses and gluon condensates by the help of operator product expansion (OPE) in deep Euclidean region. Equating these two representations of the correlation functions through dispersion relations, we acquire the QCD sum rules for the masses and decay constants. These sum rules relate the hadronic parameters to the fundamental QCD parameters. To suppress contribution of the higher states and continuum, Borel transformation with respect to the momentum squared is applied to both sides of the correlation functions.

First, to calculate the phenomenological part, we insert a complete set of intermediate states having the same quantum numbers as the interpolating currents to the correlation functions. Performing the integral over x and isolating the ground state, we obtain

$$\Pi^{S(PS)} = \frac{\langle 0 \mid J^{S(PS)}(0) \mid S(PS) \rangle \langle S(PS) \mid J^{S(PS)}(0) \mid 0 \rangle}{m_{S(PS)}^2 - p^2} + \cdots, \tag{4}$$

where $\cdots$ represents contributions of the higher states and continuum and $m_{S(PS)}$ is mass of the heavy scalar(pseudoscalar) meson. From a similar manner, for the vector (axial vector) case, we obtain

$$\Pi_{\mu\nu}^{V(AV)} = \frac{\langle 0 \mid J_\mu^{V(AV)}(0) \mid V(AV) \rangle \langle V(AV) \mid J_\nu^{V(AV)}(0) \mid 0 \rangle}{m_{V(AV)}^2 - p^2} + \cdots, \tag{5}$$

To proceed, we need to know the matrix elements of the interpolating currents between the vacuum and mesonic states. These matrix elements are parametrized in terms of leptonic decay constants as:

$$\langle 0 \mid J(0) \mid S \rangle = f_S m_S,$$

$$\langle 0 \mid J(0) \mid PS \rangle = f_{PS} \frac{m_{PS}^2}{m_{u4} + m_q},$$

$$\langle 0 \mid J(0) \mid V(AV) \rangle = f_{V(AV)} m_{V(AV)} \varepsilon_\mu, \tag{6}$$

where f_i are the leptonic decay constants of the considered bound state mesons. Using summation over polarization vectors in the $V(AV)$ case as

$$\epsilon_\mu \epsilon_\nu^* = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_{V(AV)}^2}, \tag{7}$$

we get, the final expressions of the physical sides of the correlation functions as:

$$\Pi^S = \frac{f_S^2 m_S^2}{m_S^2 - p^2} + \cdots$$

$$\Pi^{PS} = \frac{f_{PS}^2 (\frac{m_{PS}^2}{m_{u4} + m_q})^2}{m_{PS}^2 - p^2} + \cdots$$

$$\Pi_{\mu\nu}^{V(AV)} = \frac{f_{V(AV)}^2 m_{V(AV)}^2}{m_{V(AV)}^2 - p^2} \left[-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_{V(AV)}^2} \right] + \cdots, \tag{8}$$

where to calculate the mass and decay constant in the $V(AV)$ channel, we choose the structure $g_{\mu\nu}$.

In QCD side, the correlation functions are calculated in deep Euclidean region, $p^2 \ll -\Lambda_{QCD}^2$ via OPE, where short or perturbative and long distance or non-perturbative effects are separated. For each correlation function in $S(PS)$ case and coefficient of the selected structure in $V(AV)$ channel, we write

$$\Pi^{QCD} = \Pi_{pert} + \Pi_{nonpert}. \tag{9}$$

The short distance contribution (bare loop diagram in figure (1) part (a)) in each case is calculated using the perturbation theory, whereas the long distance contributions (diagrams shown in figure (1) part (b)) are parameterized in terms of gluon condensates. To proceed, we write the perturbative part in terms of a dispersion integral,

$$\Pi^{QCD} = \int \frac{ds \rho(s)}{s - p^2} + \Pi_{nonpert}, \tag{10}$$
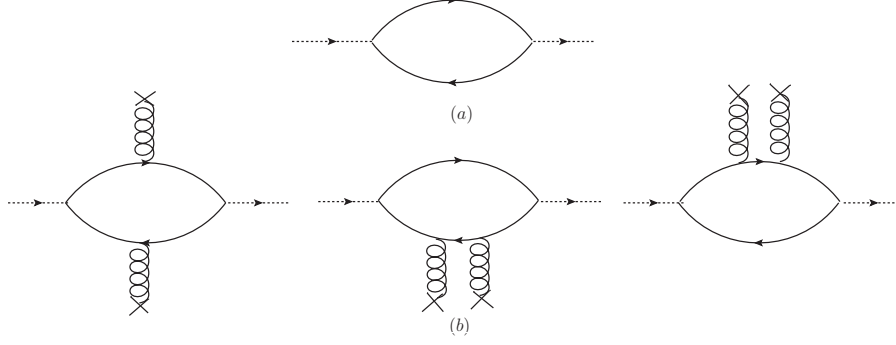
FIG. 1. (a): Bare loop diagram (b): Diagrams corresponding to gluon condensates.

where, $\rho(s)$ is called the spectral density. To calculate the spectral density, we calculate the Feynman amplitude of the bare loop diagram by the help of Cutkosky rules, where the quark propagators are replaced by Dirac delta function, i.e., $\frac{1}{p^2 - m^2} \rightarrow (-2\pi i)\delta(p^2 - m^2)$. As a result, the spectral density is obtained as follows:

$$\rho(s) = \frac{3s}{8\pi^2}(1 - \frac{(m_1 \pm m_2)^2}{s})\sqrt{1 - 2\frac{m_1^2 + m_2^2}{s} + \frac{(m_1^2 - m_2^2)^2}{s^2}} \qquad (11)$$

where $+$ sign in $(m_1 \pm m_2)$ is chosen for scalar and axial vector cases and $-$ sign is for psoduscalar and vector channels. Here, $m_1 = m_{u_4}$ and m_2 is either m_{u_4} or $m_{c(b)}$.

To obtain the non-perturbative part, we calculate the gluon condensate diagrams represented in part (b) of figure (1). For this aim, we use Fock-Schwinger gauge, $x^\mu A_\mu^a(x) = 0$. In momentum space, the vacuum gluon field is expressed as:

$$A_\mu^a(k') = -\frac{i}{2}(2\pi)^4 G_{\rho\mu}^a(0)\frac{\partial}{\partial k'_\rho}\delta^{(4)}(k'), \qquad (12)$$

where k' is the gluon momentum. In the calculations, we also use the quark-gluon-quark vertex as:

$$\Gamma_{ij\mu}^a = ig\gamma_\mu\left(\frac{\lambda^a}{2}\right)_{ij}, \qquad (13)$$

After straightforward but lengthy calculations, the non-perturbative part for each channel in momentum space is obtained as:

$$\Pi_{nonpert}^i = \int_0^1 \langle \alpha_s G^2 \rangle \frac{\Theta^i + \Theta^i(m_1 \leftrightarrow m_2)}{96\pi(m_2^2 + m_1^2 x - m_2^2 x - p^2 x + p^2 x^2)^4}dx \qquad (14)$$

where $\Theta^i(m_1 \leftrightarrow m_2)$ means that in Θ^i, we exchange m_1 and m_2. The explicit expressions for Θ^i are given as:

$$\Theta^S = \frac{1}{2}x^2\Big\{3m_1^4 x(m_2^2(x(17 - 2x(2x(9x - 26) + 47)) + 8)$$

$$+ p^2 x(x(27x - 25) - 7)(x - 1)^2) + 2m_2 m_1^3(m_2^2(x(x(x(21x - 58) + 39)$$

$$+12) - 15) - p^2(x-1)x(x(x(7x-13)-3)+12))$$
$$+m_1^2(-m_2^2p^2(x-1)x(x(x(2x(81x-242)+455)-96)-33)$$
$$+m_2^4(x(x(x(3x(36x-145)+652)-414)+72)+15)+3p^4(x-1)^3$$
$$x^2(24x^2-22x-5)) - m_2m_1(x-1)(-m_2^2p^2(x^2-2)(x(14x-27)+15)$$
$$+m_2^4(3x-5)(x(7x-12)+6)+p^4(x-1)x(x(2x(7x-13)+3)+12))$$
$$+(x-1)(-m_2^2p^4(x-1)x(2x(x(2x(18x-55)+109)-30)-9)$$
$$+m_2^4p^2(x(x(x(x(81x-328)+490)-299)+42)+15)$$
$$-m_2^6(2x-3)(x(6x(3x-8)+47)-15)+3p^6(x-1)^3x^2(6(x-1)x-1))$$
$$+9m_1^6(x-1)^2x^2(4x+1)+3m_2m_1^5x(x((8-7x)x+2)-4)\Big\},$$

$$\Theta^{PS} = -\frac{1}{2}x^2\Big\{-3m_1^4x(m_2^2(36x^4-104x^3+94x^2-17x-8)$$

$$-p^2(x-1)^2x(27x^2-25x-7))-2m_2m_1^3(m_2^2(21x^4-58x^3+39x^2+12x-15)$$
$$+p^2x(-7x^4+20x^3-10x^2-15x+12))+m_2m_1(x-1)(m_2^2p^2(-14x^4$$
$$+27x^3+13x^2-54x+30)+m_2^4(21x^3-71x^2+78x-30)$$
$$+p^4x(14x^4-40x^3+29x^2+9x-12))+m_1^2(-m_2^2p^2x(162x^5-646x^4+939x^3$$
$$-551x^2+63x+33)+m_2^4(108x^5-435x^4+652x^3-414x^2+72x+15)$$
$$+3p^4(x-1)^3x^2(24x^2-22x-5))+(x-1)(-m_2^2p^4x(72x^5-292x^4+438x^3$$
$$-278x^2+51x+9)+m_2^4p^2(81x^5-328x^4+490x^3-299x^2+42x+15)$$
$$+m_2^6(-36x^4+150x^3-238x^2+171x-45)+3p^6(x-1)^3x^2(6x^2-6x-1))$$

$$+9m_1^6(x-1)^2x^2(4x+1)+3m_2m_1^5x(7x^3-8x^2-2x+4)\Big\},$$

$$\Theta^V = -\frac{1}{2}(x-1)^2\Big\{m_1^4x^2(m_2^2(2x(1-18(x-1)x)+3)$$

$$+p^2(x(27x-25)-7)x^2)+2m_2m_1^3(x-1)^2x(m_2^2(3x-4)$$
$$-p^2(x-3)x)-m_2m_1(x-1)^2(m_2^2p^2x((7-2x)x-8)+m_2^4(x-1)(3x-5)$$
$$+p^4x^2(2(x-1)x+3))+m_1^2(x-1)x(m_2^2p^2x(x(-54x^2+56x+5)+4)$$
$$+m_2^4(9(x-1)x(4x-1)-8)+p^4x^3(24x^2-22x-5))+(x-1)^2$$
$$(m_2^2p^4x^2(4(7-6x)x^2+1)+m_2^4p^2x(x^2(27x-31)-3)$$
$$+m_2^6(5-2x(6x^2-9x+4))+p^6x^4(6(x-1)x-1))$$

$$+3m_1^6x^4(4x+1)-3m_2m_1^5(x-1)^2x^2\Big\},$$

$$\Theta^{AV} = -\frac{1}{2}x^2\Big\{2m_2m_1^3x^3(m_2^2(4-3x)+p^2(x^2+x-2))$$

$$+m_1^4x(m_2^2(x(17-2x(18(x-3)x+47))+8)+p^2x(x(27x-25)-7)(x-1)^2)$$
$$+m_1^2(-m_2^2p^2(x-1)x(x(x(2x(27x-82)+149)-32)-11)$$
$$+m_2^4(3x(x(x(3x(4x-17)+76)-46)+8)+5)+p^4(x-1)^3x^2(24x^2-22x-5))$$
$$+m_2m_1(x-1)x^2(m_2^2p^2(7-x(2x+3))+m_2^4(3x-5)$$
$$+p^4(x-1)(2(x-1)x+3))+(x-1)(-m_2^2p^4(x-1)x(2x(x(2x(6x-19)$$

$$+37) - 10) - 3) + m_2^4 p^2 (x(x(x(x(x(27x - 112) + 162) - 97) + 14) + 5)$$
$$+ m_2^6 (x(57 - 2x(3x(2x - 9) + 43)) - 15) + p^6 (x - 1)^3 x^2 (6(x - 1)x - 1))$$
$$+ 3m_2 m_1^5 x^4 + 3m_1^6 (x - 1)^2 x^2 (4x + 1) \Big\}. \tag{15}$$

The next step is to match the phenomenological and QCD sides of the correlation functions to get sum rules for the masses and decay constants of the bound states. To suppress contribution of the higher states and continuum, Borel transformation over p^2 as well as continuum subtraction are performed. As a result of this procedure, we obtain the following sum rules:

$$m_{S(V)(AV)}^2 f_{S(V)(AV)}^2 e^{\frac{-m_{S(V)(AV)}^2}{M^2}} = \int_{(m_1+m_2)^2}^{s_0} ds \, \rho^{S(V)(AV)}(s) \, e^{-\frac{s}{M^2}} + \hat{B}\Pi_{nonpert}^{S(V)(AV)},$$

$$\frac{m_{PS}^4 f_{PS}^2}{(m_{u_4} + m_q)^2} e^{\frac{-m_{PS}^2}{M^2}} = \int_{(m_1+m_2)^2}^{s_0} ds \, \rho^{PS}(s) \, e^{-\frac{s}{M^2}} + \hat{B}\Pi_{nonpert}^{PS}, \tag{16}$$

where M^2 is the Borel mass parameter and s_0 is the continuum threshold. The sum rules for the masses are obtained applying derivative with respect to $-\frac{1}{M^2}$ to the both sides of the above sum rules and dividing by themselves. i.e.,

$$m_{S(PS)(V)(AV)}^2 = \frac{-\frac{d}{d(\frac{1}{M^2})} \left[\int_{(m_1+m_2)^2}^{s_0} ds \, \rho^{S(PS)(V)(AV)}(s) \, e^{-\frac{s}{M^2}} + \hat{B}\Pi_{nonpert}^{S(PS)(V)(AV)} \right]}{\int_{(m_1+m_2)^2}^{s_0} ds \, \rho^{S(PS)(V)(AV)}(s) \, e^{-\frac{s}{M^2}} + \hat{B}\Pi_{nonpert}^{S(PS)(V)(AV)}}, \tag{17}$$

where

$$\hat{B}\Pi_{nonpert}^i = \int_0^1 e^{\frac{m_2^2 + x(m_1^2 - m_2^2)}{M^2 x(x-1)}} \frac{\Delta^i + \Delta^i(m_1 \leftrightarrow m_2)}{\pi 96 M^6 (x-1)^4 x^3} \langle \alpha_s G^2 \rangle dx, \tag{18}$$

and

$$\begin{aligned}
\Delta^S =& -m_2 m_1^3 (x - 1) x^2 (m_2^2 (14x^2 - 29x + 14) \\
&+ 2M^2 x (7x^2 - 13x + 6)) + m_1^4 (x - 1) x^3 (m_2^2 (9x^2 - 14x + 6) \\
&+ 3M^2 x (3x^2 - 4x + 1)) + m_2 m_1 (x - 1)(m_2^2 M^2 x \\
&(14x^4 - 53x^3 + 71x^2 - 36x + 6) + m_2^4 (7x^4 - 28x^3 + 40x^2 - 25x + 6) \\
&+ 2M^4 x^2 (14x^4 - 40x^3 + 29x^2 + 9x - 12)) + m_1^2 x (m_2^2 M^2 x \\
&(-18x^5 + 70x^4 - 105x^3 + 77x^2 - 27x + 3) + m_2^4 (-9x^5 + 37x^4 \\
&-61x^3 + 52x^2 - 21x + 3) - 12M^4 x^2 (3x + 1)(x - 1)^4) - (x - 1) \\
&(-2m_2^2 M^4 x^3 (18x^4 - 76x^3 + 123x^2 - 89x + 24) \\
&+ m_2^4 M^2 x (-9x^5 + 40x^4 - 71x^3 + 68x^2 - 33x + 6) + m_2^6 (-3x^5 + 14x^4 \\
&-27x^3 + 29x^2 - 15x + 3) + 6M^6 (x - 1)^3 x^3 (6x^2 - 6x - 1)) \\
&-3m_1^6 (x - 1) x^5 + m_2 m_1^5 x^3 (7x^2 - 8x + 1), \\
\Delta^{PS} =& -m_2 m_1^3 (x - 1) x^2 (m_2^2 (14x^2 - 29x + 14) \\
&+ 2M^2 x (7x^2 - 13x + 6)) - m_1^4 (x - 1) x^3 (m_2^2 (9x^2 - 14x + 6) \\
&+ 3M^2 x (3x^2 - 4x + 1)) + m_2 m_1 (x - 1)(m_2^2 M^2 x (14x^4 - 53x^3 + 71x^2 - 36x + 6) \\
&+ m_2^4 (7x^4 - 28x^3 + 40x^2 - 25x + 6) + 2M^4 x^2 (14x^4 - 40x^3 + 29x^2 + 9x - 12))
\end{aligned}$$

$$
\begin{aligned}
&+m_1^2 x(m_2^2 M^2 x(18x^5 - 70x^4 + 105x^3 - 77x^2 + 27x - 3) + m_2^4(9x^5 - 37x^4 \\
&+61x^3 - 52x^2 + 21x - 3) + 12M^4 x^2(3x+1)(x-1)^4) \\
&+(x-1)(-2m_2^2 M^4 x^3(18x^4 - 76x^3 + 123x^2 - 89x + 24) \\
&+m_2^4 M^2 x(-9x^5 + 40x^4 - 71x^3 + 68x^2 - 33x + 6) \\
&+m_2^6(-3x^5 + 14x^4 - 27x^3 + 29x^2 - 15x + 3) \\
&+6M^6(x-1)^3 x^3(6x^2 - 6x - 1)) + 3m_1^6(x-1)x^5 + m_2 m_1^5 x^3(7x^2 - 8x + 1),
\end{aligned}
$$

$$
\begin{aligned}
\Delta^V =\ & m_2 m_1^3(x-1)^2 x^2(m_2^2(2x-1) + 2M^2 x(x+2)) \\
&-m_1^4(x-1)x^3(m_2^2(3x^2 - 3x + 1) + M^2(3x-1)x^2) - m_2 m_1(x-1)^3 x \\
&(m_2^2 M^2(2x^2 + 3x - 2) + m_2^4(x-1) + 2M^4 x(2x^2 - 2x + 3)) \\
&+m_1^2(x-1)^2 x(m_2^2 M^2 x(6x^3 - 8x^2 + x + 2) + m_2^4(3x^3 - 6x^2 + 4x - 1) \\
&+4M^4 x^3(3x^2 - 2x - 1)) + (x-1)^3(2m_2^2 M^4 x^2(-6x^3 + 10x^2 - 3x + 1) \\
&-m_2^4 M^2 x(3x^3 - 7x^2 + 3x + 1) - m_2^6(x-1)^3 + 2M^6 x^4(6x^2 - 6x - 1)) \\
&+m_1^6 x^6 - m_2 m_1^5(x-1)x^4,
\end{aligned}
$$

$$
\begin{aligned}
\Delta^{AV} =\ & -m_2 m_1^3(x-1)x^2(m_2^2(2x^2 - 5x + 2) \\
&+2M^2 x(x^2 - 4x + 3)) - m_1^4(x-1)x^3(m_2^2(3x^2 - 6x + 2) \\
&+M^2 x(3x^2 - 4x + 1)) + m_2 m_1(x-1)x(m_2^2 M^2 x(2x^3 - 11x^2 + 17x - 6) \\
&+m_2^4(x^3 - 4x^2 + 4x - 1) + 2M^4 x^2(2x^3 - 4x^2 + 5x - 3)) + m_1^2 x(m_2^2 M^2 x(6x^5 \\
&-26x^4 + 43x^3 - 31x^2 + 9x - 1) + m_2^4(3x^5 - 15x^4 + 27x^3 - 20x^2 + 7x - 1) \\
&+4M^4 x^2(3x+1)(x-1)^4) + (x-1)(-2m_2^2 M^4 x^3(6x^4 - 28x^3 + 45x^2 - 31x + 8) \\
&+m_2^4 M^2 x(-3x^5 + 16x^4 - 33x^3 + 28x^2 - 11x + 2) - m_2^6(x^5 - 6x^4 + 13x^3 - 11x^2 \\
&+5x - 1) + 2M^6(x-1)^3 x^3(6x^2 - 6x - 1)) + m_1^6(x-1)x^5 + m_2 m_1^5(x-1)^2 x^3.
\end{aligned}
\tag{19}
$$

III. NUMERICAL RESULTS

To obtain numerical values for the masses and decay constants of the considered bound states containing heavy fourth family from the obtained QCD sum rules, we take the mass of the u_4 in the interval $m_{u_4} = (450 - 550)\ GeV$, $m_b = 4.8\ GeV$, $m_c = 1.3\ GeV$ and $\langle 0 \mid \frac{1}{\pi}\alpha_s G^2 \mid 0 \rangle = 0.012\ GeV^4$. The sum rules for the masses and decay constants contain also two auxiliary parameters, namely Borel mass parameter M^2 and continuum threshold s_0. The standard criteria in QCD sum rules is that the physical quantities should be independent of the auxiliary parameters. Therefor, we should look for working regions of these parameters such that our results are approximately insensitive to their variations. The working region for the Borel mass parameter is determined demanding that not only the higher states and continuum contributions are suppressed but contributions of the highest order operators should also be small, i.e., the sum rules for the masses and decay constants should converge. As a result of the above procedure, the working region for the Borel parameter is found to be $500\ GeV^2 \leq M^2 \leq 900\ GeV^2$ for $\bar{u}_4 b$ and $\bar{u}_4 c$, and $1200\ GeV^2 \leq M^2 \leq 2000\ GeV^2$ for $\bar{u}_4 u_4$ heavy SM$_4$ mesons. The continuum threshold s_0 is not completely arbitrary but correlated to the energy of the first exited state with the same quantum number as the interpolating current. We have no information about the energy of the first excitation of the bound states containing fourth family quarks. Hence, the only way to determine the working region is to choose a region such that not only the results depend weakly on this

parameter but the dependence of the physical observables on the Borel parameter M^2 is also minimal. Our numerical calculations lead to the interval $(m_1 + m_2 + 3.3)^2 \ GeV^2 \leq s_0 \leq (m_1 + m_2 + 3.7)^2 \ GeV^2$ for the continuum threshold.
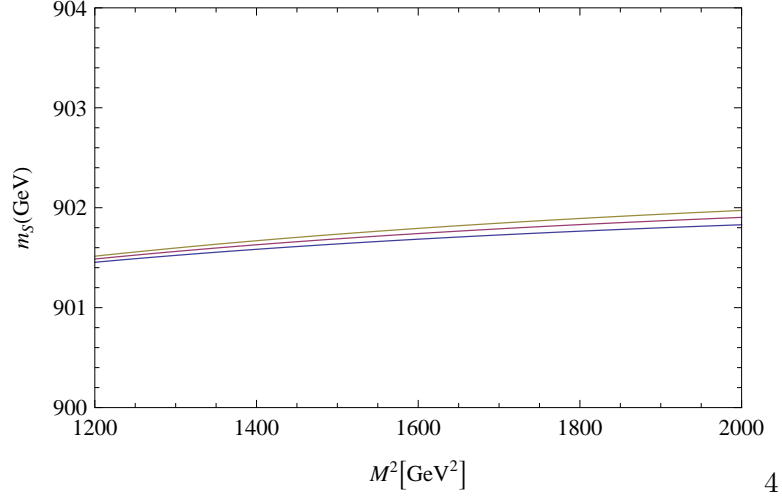

4

FIG. 2. Dependence of mass of the scalar $\bar{u}_4 u_4$ on the Borel parameter, M^2 at three fixed values of the continuum threshold. The upper, middle and lower lines belong to the values $s_0 = (m_1 + m_2 + 3.7)^2 \ GeV^2$, $s_0 = (m_1 + m_2 + 3.5)^2 \ GeV^2$ and $s_0 = (m_1 + m_2 + 3.3)^2 \ GeV^2$, respectively.
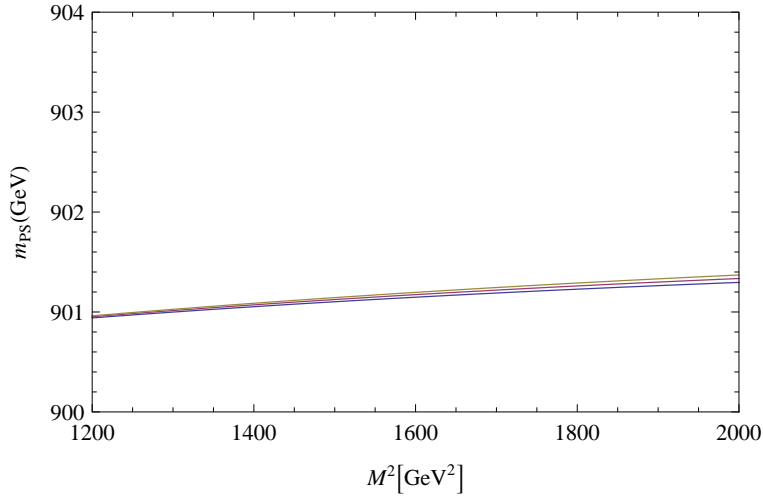

FIG. 3. The same as Fig. 2 but for pseudoscaler $\bar{u}_4 \gamma_5 u_4$.

As an example, let us consider the case of the bound state $\bar{u}_4 u_4$. The dependence of the masses of scalar $\bar{u}_4 u_4$, pseudoscalar $\bar{u}_4 \gamma_5 u_4$, vector $\bar{u}_4 \gamma_\mu u_4$ and axial vector $\bar{u}_4 \gamma_5 \gamma_\mu u_4$ are presented in figures (2-5) at three different fixed values from the considered working region for the continuum threshold. From these figures, we see a good stability of the masses with respect to the Borel mass parameter M^2. From these figures, it is also clear that the results do not depend on the continuum threshold in its working region. The dependence of
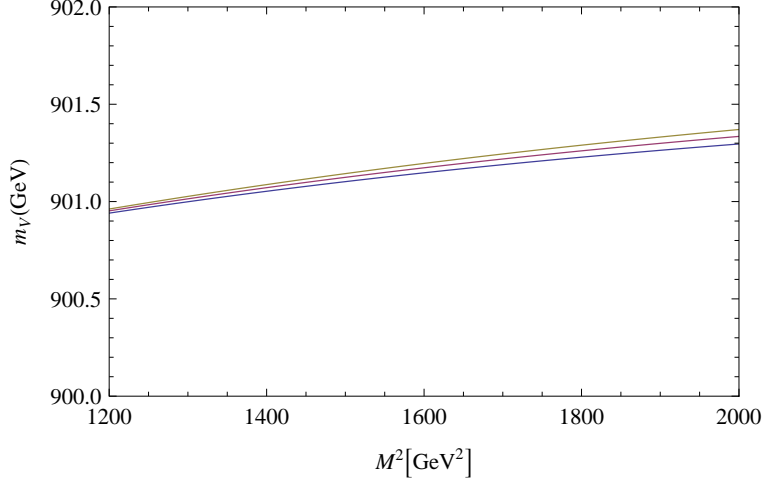
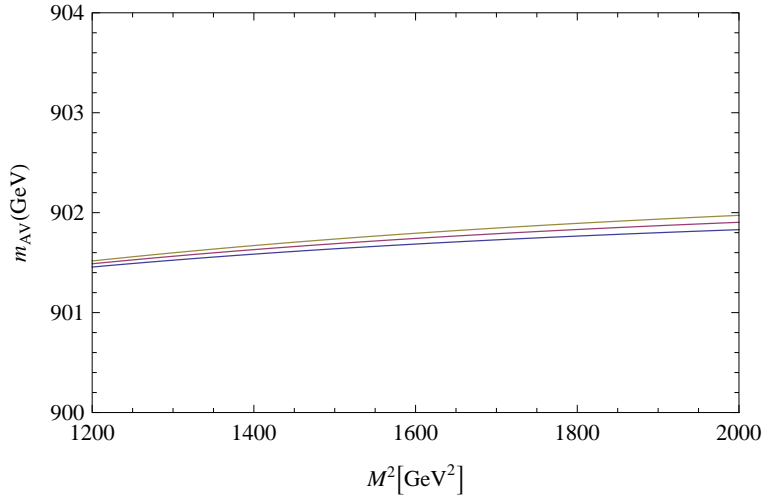FIG. 4. The same as Fig. 2 but for vector $\bar{u}_4\gamma_\mu u_4$.

FIG. 5. The same as Fig. 2 but for axial vector $\bar{u}_4\gamma_5\gamma_\mu u_4$.

the decay constants of the scalar $\bar{u}_4 u_4$, pseudoscalar $\bar{u}_4\gamma_5 u_4$, vector $\bar{u}_4\gamma_\mu u_4$ and axial vector $\bar{u}_4\gamma_5\gamma_\mu u_4$ are presented in figures (6-9) also at three different fixed values of the continuum threshold. These figures also depict approximately insensitivity of the results under variation of the Borel mass parameter in its working region. The results of decay constants also show very weak dependency on the continuum threshold in its working region. From a similar way, we analyze the mass and decay constants of the cases when one of the quarks belong to the heavy fourth generation and the other is ordinary bottom or charm quark. The numerical results deduced from the figures are collected in Tables I-VI for three different values of the m_{u_4}, namely $m_{u_4} = 450\,GeV$, $m_{u_4} = 500\,GeV$ and $m_{u_4} = 550\,GeV$. The errors presented in these tables are only due to the uncertainties coming from determination of the working regions for the auxiliary parameters. Here, we should stress that the obtained results in Tables I-VI are within QCD and do not include contributions coming from the
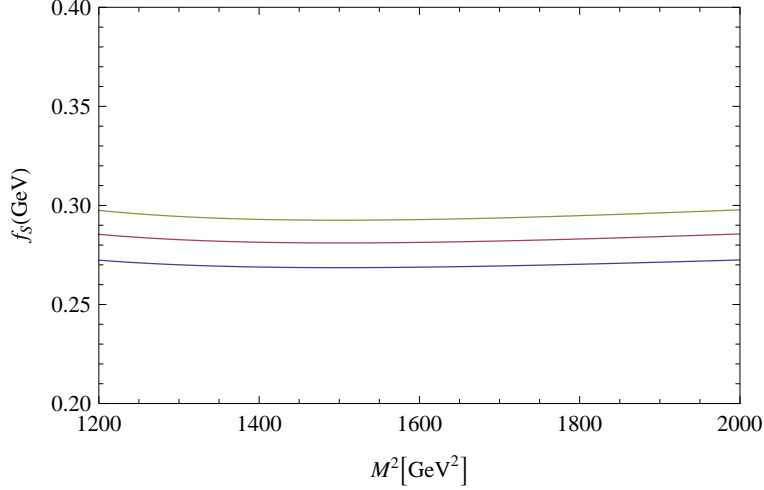
FIG. 6. Dependence of the decay constant of the scalar $\bar{u}_4 u_4$ on the Borel parameter, M^2 at three fixed values of the continuum threshold. The upper, middle and lower lines belong to the values $s_0 = (m_1 + m_2 + 3.7)^2 \ GeV^2$, $s_0 = (m_1 + m_2 + 3.5)^2 \ GeV^2$ and $s_0 = (m_1 + m_2 + 3.3)^2 \ GeV^2$, respectively.
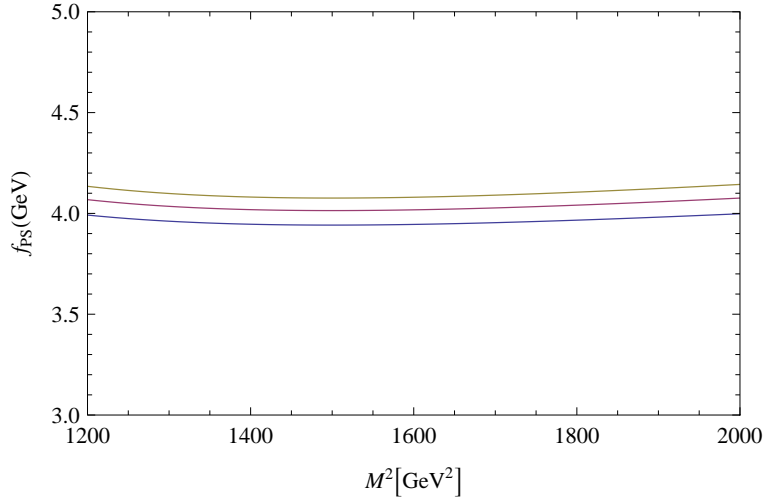

FIG. 7. The same as Fig. 6 but for the decay constant of pseudoscalar $\bar{u}_4 \gamma_5 u_4$.

Higgs couplings to the ultra heavy quarks. Such contributions to the binding energy have been calculated in [19], where it is shown that these contributions are more than several GeV in the case when both quarks belong to the fourth family. The Higgs contribution calculated in [19] is proportional to the product of two quark masses. When we replace one of the ultra heavy quarks by b or c quark, the binding energy obtained in [19] reduces to a value which is less than the QCD sum rules predictions in the present work. However, when both quarks belong to the fourth family, the binding energy obtained in the present work is very small comparing to the Higgs corrections in [19].

At the end of this part, we would like to mention that the obtained QCD sum rules in the
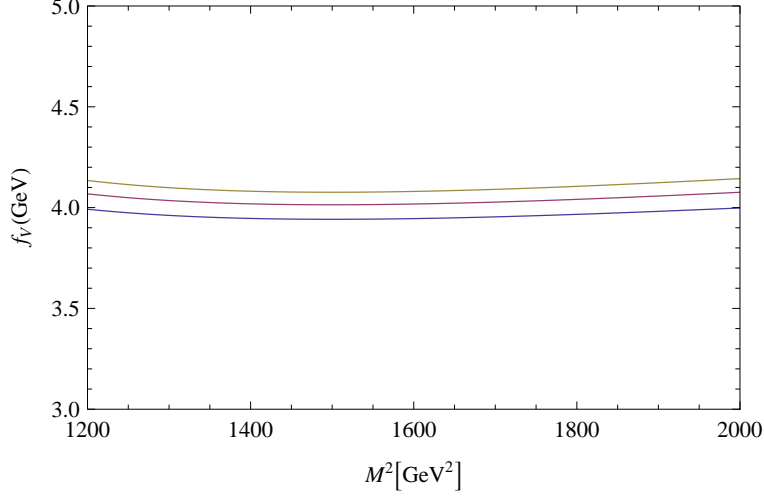
FIG. 8. The same as Fig. 6 but for the decay constant of vector $\bar{u}_4\gamma_\mu u_4$.
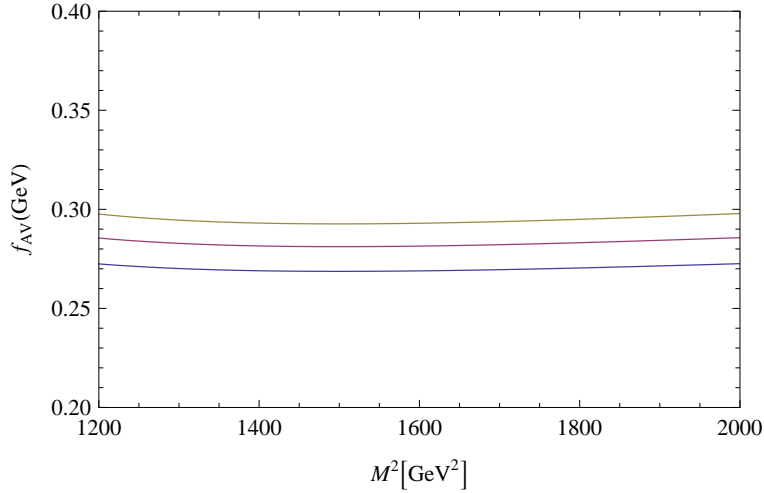
4

FIG. 9. The same as Fig. 6 but for the decay constant of axial vector $\bar{u}_4\gamma_5\gamma_\mu u_4$.

present work reproduce the masses and decay constants of the ordinary $\bar{b}b(\bar{c}c)$ states when we set $u_4 \to b(c)$. The obtained numerical values in this limit are in a good consistency with the existing experimental data [42] and QCD sum rules predictions [40, 41].

To sum up, against the top quark, the heavy fourth generation of quarks that have sufficiently small mixing with the three known SM families form hadrons. Considering the arguments mentioned in the text, the production of such bound states will be possible at LHC. Hoping for this possibility, we calculated the masses and decay constants of the bound state objects containing two quarks either both from the SM_4 or one from heavy fourth generation and the other from observed SM bottom or charm quarks in the framework of the QCD sum rules. The obtained numerical results approach to the known masses and decay constants of the $\bar{b}b$ and $\bar{c}c$ heavy quarkonia, when the fourth family quark is replaced

4

mass (GeV)	$u_4\bar{c}$	$u_4\bar{b}$	$u_4\bar{u}_4$
Scalar	453.01 ± 0.25	456.45 ± 0.25	901.68 ± 0.50
Pseudoscalar	452.62 ± 0.15	455.95 ± 0.15	901.12 ± 0.30
axial vector	453.00 ± 0.25	456.44 ± 0.25	901.70 ± 0.50
vector	452.62 ± 0.15	455.94 ± 0.15	901.13 ± 0.30

TABLE I. The values of masses of different bound states obtained using $m_{u_4} = 450 \ GeV$.

mass (GeV)	$u_4\bar{c}$	$u_4\bar{b}$	$u_4\bar{u}_4$
Scalar	502.91 ± 0.28	506.36 ± 0.28	1001.61 ± 0.55
Pseudoscalar	502.52 ± 0.17	505.86 ± 0.17	1001.04 ± 0.33
Axial Vector	502.91 ± 0.28	506.35 ± 0.28	1001.60 ± 0.55
Vector	502.57 ± 0.17	505.85 ± 0.17	1001.04 ± 0.33

TABLE II. The values of masses of different bound states obtained using $m_{u_4} = 500 \ GeV$.

by the bottom or charm quark.

[1] H. Fritzsch, Phys. Lett. B 289, 92 (1992).
[2] A. Datta, Pramana 40, L503 (1993).
[3] A. Celikel, A. K. Ciftci, S. Sultansoy, Phys. Lett. B 342, 257 (1995).
[4] S. Sultansoy, Contributed paper to ICHEP 2000, arXiv:hep-ph/0004271.
[5] M. S. Chanowitz, M. A. Furman, I. Hinchliffe, Phys. Lett. B 78, 285 (1978); Nucl. Phys. B

mass (GeV)	$u_4\bar{c}$	$u_4\bar{b}$	$u_4\bar{u}_4$
Scalar	552.82 ± 0.31	556.27 ± 0.31	1101.67 ± 0.60
Pseudoscalar	552.43 ± 0.18	555.78 ± 0.18	1101.11 ± 0.36
Axial Vector	552.81 ± 0.31	556.25 ± 0.31	1101.68 ± 0.60
Vector	552.42 ± 0.18	555.77 ± 0.18	1101.12 ± 0.36

TABLE III. The values of masses of different bound states obtained using $m_{u_4} = 550 \ GeV$.

Leptonic decay constant f (GeV)	$u_4\bar{c}$	$u_4\bar{b}$	$u_4\bar{u}_4$
Scalar	0.12 ± 0.01	0.15 ± 0.02	0.28 ± 0.03
Pseudoscalar	0.17 ± 0.01	0.34 ± 0.02	4.01 ± 0.20
Axial Vector	0.12 ± 0.01	0.15 ± 0.02	0.28 ± 0.03
Vector	0.17 ± 0.01	0.34 ± 0.02	4.01 ± 0.20

TABLE IV. The values of decay constants of different bound states obtained using $m_{u_4} = 450\, GeV$.

Leptonic decay constant f (GeV)	$u_4\bar{c}$	$u_4\bar{b}$	$u_4\bar{u}_4$
Scalar	0.11 ± 0.01	0.13 ± 0.01	0.26 ± 0.03
Pseudoscalar	0.15 ± 0.01	0.30 ± 0.02	3.91 ± 0.19
Axial Vector	0.11 ± 0.01	0.13 ± 0.01	0.26 ± 0.03
Vector	0.15 ± 0.01	0.29 ± 0.02	3.91 ± 0.19

TABLE V. The values of decay constants of different bound states obtained using $m_{u_4} = 500\, GeV$.

153, 402 (1979).
 [6] A. Datta, S. Raychaudhuri, Phys. Rev. D 49, 4762 (1994).
 [7] S. Atag et al., Phys. Rev. D 54 (1996) 5745.
 [8] B. Holdom et al., PMC Phys. A 3, 4 (2009).
 [9] O. Eberhardt, A. Lenz, J. Rohrwild, Phys. Rev. D 82, 095006 (2010).
[10] M. Sahin, S. Sultansoy, S. Turkoz, Phys. Rev. D 83, 054022 (2011).
[11] T. Altonen et al., (CDF Collaboration), arXiv:1101.5782 [hep-ex].
[12] J. Convay et al., CDF public conference note CDF/PUB/TOP/PUBLIC/10395.
[13] ATLAS Detector and Physics Performance TDR, CERN/LHCC/99-15 (1999); P. Jenni et al. (ATLAS Collaboration), Report No CERN-LHCC-99-14/15, 1999, Sect. 18.2.

Leptonic decay constant f (GeV)	$u_4\bar{c}$	$u_4\bar{b}$	$u_4\bar{u}_4$
Scalar	0.10 ± 0.01	0.12 ± 0.01	0.26 ± 0.03
Pseudoscalar	0.14 ± 0.01	0.27 ± 0.01	4.19 ± 0.20
Axial Vector	0.10 ± 0.01	0.12 ± 0.01	0.26 ± 0.03
Vector	0.14 ± 0.01	0.27 ± 0.01	4.18 ± 0.20

TABLE VI. The values of decay constants of different bound states obtained using $m_{u_4} = 550\, GeV$.

[14] A. K. Ciftci, R. Ciftci, S. Sultansoy, Phys. Rev. D 65, 055001 (2002).

[15] E. Arik et al., Phys. Rev. D 66, 116006 (2002).

[16] H. Ciftci, S. Sultansoy, Modern Physics Letters A 18, 859 (2003).

[17] R. Ciftci et al., Turk. J. Phys. 27, 179 (2003).

[18] E. Accomando et al,, arXiv:hep-ph/0412251.

[19] K. Ishiwata, M. B. Wise, arXiv:1103.0611 [hep-ph].

[20] I. Bigi et al., Phys. Lett. B 181 (1986) 157.

[21] T. Altonen et al. (CDF Collaboration), Phys. Rev. Lett. 103, 092002 (2009).

[22] A. K. Ciftci, R. Ciftci, S. Sultansoy, Phys. Rev. D 72, 053006 (2005).

[23] E. Arik et al., Phys. Rev. D 58, 117701 (1998) .

[24] B. Holdom, J. High Energy Phys. 03, 063 (2007).

[25] B. Holdom, J. High Energy Phys. 08, 069 (2007).

[26] O. Cakir et al., Eur. Phys. J. C 56, 537 (2008).

[27] Q. F. del Agulia et al., Eur. Phys. J. C 57, 183 (2008).

[28] V. E. Ozcan, S. Sultansoy, G. Unel, Eur. Phys. J. C 57, 621 (2008).

[29] R. Ciftci, Phys. Rev. D 78, 075018 (2008).

[30] I. T. Cakir et al., Phys. Rev. D 80, 095009 (2009).

[31] M. A. Shifman, A. I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 147, 385 (1979).

[32] A. I. Vainshtein, M. B. Voloshin, V. I. Zakharov, M. A. Shifman, Sov. J. Nucl. Phys. 28, 237 (1978) (Yad.Fiz.28(1978)465).

[33] L. J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rep. 127, No1 (1985) 1.

[34] S. Narison, QCD Spectral Sum Rules (World Scientific, Singapore, 1989).

[35] M. Jamin, B. O. Lange, Phys. Rev. D 65, 056005 (2002).

[36] A. A. Penin, M. Steinhauser, Phys. Rev. D 65 054006 (2002).

[37] Dong-Sheng Du, Jing-Wu Li, Mao-Zhi Yang, Phys. Lett. B619 105-114 (2005).

[38] T. M. Aliev, K. Azizi, V. Bashiry, J. Phys. G 37, 025001 (2010).

[39] T. M. Aliev, K. Azizi, M. Savci, Phys. Lett B 690, 164 (2010).

[40] E. V. Veliev, H. Sundu, K. Azizi, M. Bayar, Phys. Rev.D 82, 056012 (2010).

[41] E. Veli Veliev, K. Azizi, H. Sundu, N. Aksit, arXiv:1010.3110 [hep-ph].

[42] K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010).