Thermal QCD Sum Rules Study of Vector Charmonium and Bottomonium States

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We calculate the masses and leptonic decay constants of the heavy vector quarkonia, J/ψ and Υ mesons at finite temperature. In particular, considering the thermal spectral density as well as additional operators coming up at finite temperature, the thermal QCD sum rules are acquired. Our numerical calculations demonstrate that the masses and decay constants are insensitive to the variation of temperature up to $T \cong 100\ MeV$, however after this point, they start to fall altering the temperature. At deconfinement temperature, the decay constants attain roughly to 45% of their vacuum values, while the masses are diminished about 12%, and 2.5% for J/ψ and Υ states, respectively. The obtained results at zero temperature are in good consistency with the existing experimental data as well as predictions of the other nonperturbative models. Considerable decreasing in the values of the decay constants can be considered as a sign of the quark gluon plasma phase transition.

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I. INTRODUCTION

Investigation of the heavy mesons can play essential role in understanding the vacuum properties of the nonperturbative QCD [1]. In particular, analysis of the variation of the parameters of the heavy quarkonia, namely bottonium ($\bar{b}b$) and charmonium ($\bar{c}c$) in hadronic medium with respect to the temperature can give valuable information about the QCD vacuum and transition to the quark gluon plasma (QGP) phase. Determination of the hadronic properties of the vector mesons in hot and dense QCD medium has become one of the most important research subject in the last twenty years both theoretically and experimentally. J/ψ suppression effect due to color screening can be considered as an important evidence for QGP [2]. This suppression effect has been observed experimentally in heavy ion collision experiments held in super proton synchrotron (SPS) at CERN and relativistic heavy ion collider (RHIC) at BNL.

Properties of the heavy mesons in vacuum have been probed widely in the literature using the nonperturbative approaches like QCD sum rules, nonrelativistic potential models, lattice theory, heavy quark effective theory and chiral perturbation theory. However, in expansion of most of these models to finite temperature we are face to face with some difficulties. QCD sum rules which is based on the operator product expansion (OPE), QCD Lagrangian and quark-hadron duality, is one of the most informative, applicable and predictive models in hadron physics [3, 4]. The thermal version of this model proposed by Bochkarev and Shaposhnikov [5] has some new features at $T \neq 0$ [6–8]. One of the new feature is the interaction of the particles existing in the medium with the currents which demands the modification of the hadronic spectral function. The other new picture of the thermal QCD is breakdown of the Lorentz invariance via the choice of reference frame. Due to residual O(3) symmetry at finite temperature, more operators with the same dimensions appear in the OPE comparing to the QCD sum rules in vacuum. Thermal version of QCD sum rules has been successfully used to study the thermal properties of light [9–11], heavy-light[12–14] and heavy-heavy [15–18] mesons as a trusty and well-established approach.

In the present work, we calculate the mases and decay constants of the heavy vector quarkonia J/ψ ($\bar{c}c$) and Υ ($\bar{b}b$) in the framework of the thermal QCD sum rules. Using the thermal quark propagator, we calculate the expression for the spectral density in one loop approach. Taking into account also the two loop perturbative contributions in α_s order [1, 3] as well as new nonperturbative contributions arising in thermal QCD in addition to vacuum version, we acquire thermal QCD sum rules for the masses and decay constants. Using the results of the energy density for the interval $T = (0 - 170)$ MeV obtained via Chiral perturbation theory [19] as well as the values of the energy density and gluon condensates obtained in the region $T = (100 - 170)$ MeV via lattice QCD [20, 21], we present the sensitivity of the masses and decay constants of the J/ψ and Υ heavy vector mesons on the temperature. In our calculations, we also use the temperature dependent two loop expression for the strong coupling constant obtained using the perturbation theory and improved by the lattice results [16, 22]. We see that the values of the decay constants decrease considerably near to the critical or deconfinement temperature comparing to their values in vacuum. This can be considered as a sign of the QGP phase transition.

The rest of the paper is organized as follows. In the next two sections, we derive thermal QCD sum rules for the considered observables. The last section is devoted to the numerical analysis of the observables and present their temperature dependency as well as our discussion.

II. OPE OF THERMAL CORRELATION FUNCTION FOR HEAVY-HEAVY VECTOR MESONS

To obtain the thermal QCD sum rules for physical quantities, we need to calculate the convenient thermal correlation function in two different ways: in terms of QCD degrees of freedom and in terms of hadronic parameters. In QCD side, the correlation function is evaluated via OPE which helps us expand the time ordering product of currents in terms of operators with different dimensions. In the present section, we obtain the OPE for the considered quantities. We begin by considering the following two point thermal correlation function:

$$\Pi_{\mu\nu}\big(q, T\big) = i \int d^4x \ e^{iq \cdot x} Tr\Big(\rho \ \mathcal{T}\big(J_\mu(x) J_\nu^\dagger(0)\big)\Big), \tag{1}$$

where $J_\mu(x) =: \overline{Q}(x)\gamma_\mu Q(x) :$ with $Q = b$ or c is the vector current, $\mathcal{T}$ indicates the time ordered product and $\rho = e^{-\beta H}/Tre^{-\beta H}$ is the thermal density matrix of QCD at temperature $T = 1/\beta$. As we previously mentioned, the Lorentz invariance breaks down via the choice of reference frame at which the matter is at rest. However, using the four velocity vector u_μ of the matter, we can define Lorentz invariant quantities such as $\omega = u \cdot q$ and $\overline{q}^2 = \omega^2 - q^2$. By the help of these quantities, the aforesaid thermal correlation function can be expressed in terms of two independent tensors $P_{\mu\nu}$ and $Q_{\mu\nu}$ at finite temperature [9], i.e.,

$$\Pi_{\mu\nu}\big(q, T\big) = Q_{\mu\nu}\Pi_l(q^2, \omega) + P_{\mu\nu}\Pi_t(q^2, \omega), \tag{2}$$

where

$$P_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} - \frac{q^2}{\overline{q}^2}\widetilde{u}_\mu\widetilde{u}_\nu,$$

$$Q_{\mu\nu} = \frac{q^4}{\overline{q}^2}\widetilde{u}_\mu\widetilde{u}_\nu, \tag{3}$$

and $\widetilde{u}_\mu = u_\mu - \omega \, q_\mu/q^2$. The functions Π_l and Π_t are the following Lorentz invariant functions:

$$\Pi_l\big(q^2, \omega\big) = \frac{1}{\overline{q}^2}u^\mu\Pi_{\mu\nu}u^\nu, \tag{4}$$

$$\Pi_t\big(q^2, \omega\big) = -\frac{1}{2}\Big(g^{\mu\nu}\Pi_{\mu\nu} + \frac{q^2}{\overline{q}^2}u^\mu\Pi_{\mu\nu}u^\nu\Big). \tag{5}$$

It can be shown that in the limit $|\mathbf{q}| \to 0$, the Π_t function can be expressed as $\Pi_t = -\frac{1}{3}g^{\mu\nu}\Pi_{\mu\nu}$ [10] and one can easily find the $\Pi_t\big(q_0, |\mathbf{q}| = 0\big) = q_0^2 \ \Pi_l\big(q_0, |\mathbf{q}| = 0\big)$ relation between two Π_l and Π_t functions. In real time thermal field theory, the function $\Pi_l(q^2, \omega)$ or $\Pi_t(q^2, \omega)$ can be written in 2×2 matrix form and elements of this matrix depend on only one analytic function [23]. Therefore, calculation of the 11-component of this matrix is sufficient to determine completely the dynamics of the corresponding two-point function. It can also be shown that in the fixed value of $|\mathbf{q}|$, the spectral representation of the thermal correlation function can be written as [9]:

$$\Pi_{l,t}\big(q_0^2, T\big) = \int_0^\infty dq_0'^2 \ \frac{\rho_{l,t}\big(q_0'^2, T\big)}{q_0'^2 + Q_0^2}, \tag{6}$$

where $Q_0^2 = -q_0^2$, and

$$\rho_{l,t}\left(q_0^2, T\right) = \frac{1}{\pi} Im\Pi_{l,t}\left(q_0^2, T\right) \tanh\frac{\beta q_0}{2}. \tag{7}$$

The thermal correlation function of Eq. (1) can be written in momentum space as:

$$\Pi_{\mu\nu}\left(q, T\right) = i \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma_\mu S(k)\gamma_\nu S(k-q)\right], \tag{8}$$

where, we consider the 11-component of the $S(k)$ (thermal quark propagator) which is expressed as a sum of its vacuum expression and a term depending on the Fermi distribution function [24]

$$S(k) = (\gamma^\mu k_\mu + m)\left(\frac{1}{k^2 - m^2 + i\varepsilon} + 2\pi i n(|k_0|)\delta(k^2 - m^2)\right), \tag{9}$$

where $n(x) = [\exp(\beta x) + 1]^{-1}$ is the Fermi distribution function. Now, we insert the propagator of Eq. (9) in Eq. (8) and consider $\Pi_1(q, T) = g^{\mu\nu}\Pi_{\mu\nu}(q, T)$ function. Carrying out the integral over k_0, we obtain the imaginary part of the $\Pi_1(q, T)$ in the following form:

$$Im\Pi_1(q, T) = L(q_0) + L(-q_0), \tag{10}$$

where

$$L(q_0) = N_c \int \frac{d\mathbf{k}}{4\pi^2} \frac{\omega_1^2 - \mathbf{k}^2 + \mathbf{k} \cdot \mathbf{q} - \omega_1 q_0 - 2m^2}{\omega_1\omega_2}$$
$$\times \left([(1-n_1)(1-n_2) + n_1 n_2]\delta(q_0 - \omega_1 - \omega_2) - [(1-n_1)n_2 + (1-n_2)n_1]\delta(q_0 - \omega_1 + \omega_2)\right), \tag{11}$$

and $n_1 = n(\omega_1)$, $n_2 = n(\omega_2)$, $\omega_1 = \sqrt{\mathbf{k}^2 + m^2}$ and $\omega_2 = \sqrt{(\mathbf{k} - \mathbf{q})^2 + m^2}$. The terms without the Fermi distribution functions show the vacuum contributions but those including the Fermi distribution functions depict medium contributions. The delta-functions in the different terms of Eq. (11) control the regions of non-vanishing imaginary parts of $\Pi_1(q, T)$, which define the position of branch cuts [5]. After straightforward calculations, the annihilation and scattering parts of $\rho_1\left(q_0^2, T\right) = \frac{1}{\pi} Im\Pi_1\left(q_0^2, T\right) \tanh\frac{\beta q_0}{2}$ at nonzero momentum can be written as:

$$\rho_{1,a} = \frac{-3q^2}{8\pi^2}(3 - \nu^2)\left[\nu - \int_{-\nu}^{\nu} dx \, n_+(x)\right] \qquad \text{for} \qquad 4m^2 + \mathbf{q}^2 \le q_0^2 \le \infty, \tag{12}$$

$$\rho_{1,s} = \frac{3q^2}{16\pi^2}(3 - \nu^2)\int_{\nu}^{\infty} dx\left[n_-(x) - n_+(x)\right] \qquad \text{for} \qquad q_0^2 \le \mathbf{q}^2, \tag{13}$$

where $\nu(q_0^2) = \sqrt{1 - 4m^2/q_0^2}$, $n_+(x) = n\left[\frac{1}{2}(q_0 + |\mathbf{q}|x)\right]$ and $n_-(x) = n\left[\frac{1}{2}(|\mathbf{q}|x - q_0)\right]$. From the similar manner, one can calculate also the function $\Pi_2(q, T) = u^\mu \Pi_{\mu\nu}(q, T)u^\nu$. Using the obtained results in Eqs. (4) and (5), the annihilation and scattering parts of ρ_t at nonzero momentum is obtained as:

$$\rho_{t,a} = \frac{3q^2}{32\pi^2}\int_{-\nu}^{\nu} dx(2 - \nu^2 + x^2)[1 - 2n_+(x)], \tag{14}$$

$$\rho_{t,s} = -\frac{3q^2}{32\pi^2} \int_\nu^\infty dx (2 - \nu^2 + x^2)[n_-(x) - n_+(x)]. \tag{15}$$

The annihilation part of ρ_l, i.e., $\rho_{l,a}$ and its scattering part $\rho_{l,s}$ also at nonzero momentum can be found from Eqs. (14) and (15) replacing the coefficient $(2 - \nu^2 + x^2)$ by $2(1 - x^2)$.

In our calculations, we also take into account the perturbative two-loop order α_s correction to the spectral density. This correction at zero temperature can be written as [1, 3]:

$$\rho_{\alpha_s}(s) = \alpha_s \frac{s}{6\pi^2} \nu(s) \left(3 - \nu^2(s)\right) \left[\frac{\pi}{2\nu(s)} - \frac{1}{4}\left(3 + \nu(s)\right)\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right], \tag{16}$$

where, we replace the strong coupling α_s in Eq. (16) with its temperature dependent lattice improved expression $\alpha(T) = 2.095(82)\frac{g^2(T)}{4\pi}$ [16, 22], here

$$g^{-2}(T) = \frac{11}{8\pi^2} \ln\left(\frac{2\pi T}{\Lambda_{\overline{MS}}}\right) + \frac{51}{88\pi^2} \ln\left[2\ln\left(\frac{2\pi T}{\Lambda_{\overline{MS}}}\right)\right]. \tag{17}$$

where $\Lambda_{\overline{MS}} = T_c/1.14(4)$ and $T_c = 0.160 GeV$.

Now, we proceed to calculate the nonperturbative part in QCD side. For this aim, we use the nonperturbative part of the quark propagator in an external gluon field, $A_\mu^a(x)$ in the Fock-Schwinger gauge, $x^\mu A_\mu^a(x) = 0$. Taking into account one and two gluon lines attached to the quark line, the massive quark propagator can be written in momentum space as [3]:

$$S^{aa' nonpert}(k) = -\frac{i}{4} g(t^c)^{aa'} G_{\kappa\lambda}^c(0) \frac{1}{(k^2 - m^2)^2} \left[\sigma_{\kappa\lambda}(\not{k} + m) + (\not{k} + m)\sigma_{\kappa\lambda}\right]$$
$$- \frac{i}{4} g^2 (t^c t^d)^{aa'} G_{\alpha\beta}^c(0) G_{\mu\nu}^d(0) \frac{\not{k} + m}{(k^2 - m^2)^5} (f_{\alpha\beta\mu\nu} + f_{\alpha\mu\beta\nu} + f_{\alpha\mu\nu\beta})(\not{k} + m),$$
$$\tag{18}$$

where,

$$f_{\alpha\beta\mu\nu} = \gamma_\alpha(\not{k} + m)\gamma_\beta(\not{k} + m)\gamma_\mu(\not{k} + m)\gamma_\nu. \tag{19}$$

To go on, we also need to know the expectation value $\langle Tr G_{\alpha\beta} G_{\mu\nu}\rangle$. The Lorentz covariance at finite temperature permits us to write the general structure of this expectation value in the following manner:

$$\langle Tr^c G_{\alpha\beta} G_{\mu\nu}\rangle = \frac{1}{24}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu})\langle G_{\lambda\sigma}^a G^{a\lambda\sigma}\rangle$$
$$+ \frac{1}{6}\left[g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu} - 2(u_\alpha u_\mu g_{\beta\nu} - u_\alpha u_\nu g_{\beta\mu} - u_\beta u_\mu g_{\alpha\nu} + u_\beta u_\nu g_{\alpha\mu})\right]\langle u^\lambda \Theta_{\lambda\sigma}^g u^\sigma\rangle,$$
$$\tag{20}$$

where, u^μ as we also previously mentioned is the four-velocity of the heat bath and it is introduced to restore Lorentz invariance formally in the thermal field theory. In the rest frame of the heat bath $u^\mu = (1, 0, 0, 0)$ and $u^2 = 1$. Furthermore, $\Theta_{\lambda\sigma}^g$ is the traceless gluonic

part of the stress-tensor of the QCD. Up to terms required for our calculations, the non perturbative part of massive quark propagator at finite temperature is obtained as:

$$
S^{aa'\,nonpert}(k) = -\frac{i}{4}g(t^c)^{aa'}G^c_{\kappa\lambda}\frac{1}{(k^2-m^2)^2}\Big[\sigma_{\kappa\lambda}(\slashed{k}+m)+(\slashed{k}+m)\sigma_{\kappa\lambda}\Big]
$$
$$
+\frac{i\,g^2\,\delta^{aa'}}{3\,(k^2-m^2)^4}\Big\{\frac{m(k^2+m\,\slashed{k})}{4}\langle G^c_{\alpha\beta}G^{c\alpha\beta}\rangle+\frac{1}{3(k^2-m^2)}\Big[m(k^2-m^2)\big(k^2-4(k\cdot u)^2\big)
$$
$$
+(m^2-k^2)\big(-m^2+4(k\cdot u)^2\big)\,\slashed{k}+4(k\cdot u)(m^2-k^2)^2\,\slashed{u}\Big]\langle u^\alpha\Theta^g_{\alpha\beta}u^\beta\rangle\Big\}. \tag{21}
$$

Using the above expression and after straightforward but lengthy calculations, the nonperturbative part in QCD side is obtained as:

$$
\Pi^{nonpert}_t = \int_0^1 dx\Big\{-\frac{\langle\alpha_s G^2\rangle}{72\pi\big[m^2+q^2(-1+x)x\big]^4}\Big[6q^6(-1+x)^4x^4+6m^2q^4x^2(-1+x)^2(1-6x+6x^2)
$$
$$
+m^6(5-32x+42x^2-20x^3+10x^4)+m^4q^2x\big(-14+95x-140x^2+65^3+6x^4-2x^5\big)\Big]
$$
$$
-\frac{\alpha_s\langle u^\alpha\Theta^g_{\alpha\beta}u^\beta\rangle}{54\pi\big[m^2+q^2(-1+x)x\big]^4}\Big[x(-1+x)\big(4q^4x^2(1-3x+2x^2)^2+m^4(12-35x+21x^2+28x^3
$$
$$
-14x^4)+m^2q^2x(-13+55x-82x^2+36x^3+6x^4-2x^5)\big)\big(q^2-4(q\cdot u)^2\big)\Big]\Big\}, \tag{22}
$$

where, $\langle G^2\rangle = \langle G^c_{\alpha\beta}G^{c\alpha\beta}\rangle$.

III.   PHENOMENOLOGICAL PART AND THERMAL SUM RULES

Now, we turn our attention to calculate the physical or phenomenological side of the correlation function. For this aim, we insert a complete set of physical intermediate state to Eq. (1) and perform integral over x. Isolating the ground state, we get

$$
\Pi_{\mu\nu}(q) = \sum_\lambda\frac{\langle 0|J_\mu|V(q,\lambda)\rangle\langle V(q,\lambda)|J^\dagger_\nu|0\rangle}{m_V^2-q^2}+\ldots, \tag{23}
$$

where the hadronic states $\{|V(q,\lambda)\rangle\}$ form a complete set and ..... indicate the contributions of excited vector mesons and continuum states.

In order to obtain thermal sum rules, now we equate the spectral representation and results of operator product expansion for amplitudes $\Pi_l(q^2,\omega)$ or $\Pi_t(q^2,\omega)$ at sufficiently high Q_0^2. When performing numerical results, we should exchange our reference to one at which the particle is at rest, i.e., we shall set $|\mathbf{q}|\to 0$. In this limit since the functions Π_l and Π_t are related to each other, it is enough to use one of them to acquire thermal sum rules. Here, we use the function Π_t. When we use the standard spectral representation, if the spectral density at $s\to\infty$ does not approach to zero, in this case the correlation function is expressed in terms of a diverge integral. In such a case, to overcome this problem, we subtract first few terms of its Taylor expansion at $q^2=0$ from $\Pi_t(q^2,\omega)$,

$$
\Pi_t(q_0^2,|\mathbf{q}|) = \Pi_t(0)+\Big(\frac{d\Pi_t}{dQ_0^2}\Big)_{Q_0^2=0}Q_0^2+\frac{Q_0^4}{\pi}\int_0^\infty\frac{\rho_t(s)}{s^2(s+Q_0^2)}ds. \tag{24}
$$

Equating the OPE and hadronic representations of the correlation function and applying quark-hadron duality, our sum-rule takes the form:

$$\frac{f_V^2 Q_0^4}{(m_V^2 + Q_0^2)\, m_V^2} = Q_0^4 \int_{4m^2}^{s_0} \frac{[\rho_{t,a}(s) + \rho_{\alpha_s}(s)]}{s^2(s + Q_0^2)} ds + \int_0^{|\mathbf{q}|^2} \frac{\rho_{t,s}}{s + Q_0^2} ds + \Pi_t^{nonpert}, \qquad (25)$$

where, for simplicity, the total decay width of meson has been neglected. The decay constant f_V is defined by the matrix element of the current J_μ between the vacuum and the vector-meson state, i.e.,

$$\langle 0|J_\mu|V(q,\lambda)\rangle = f_V m_V \varepsilon_\mu^{(\lambda)}. \qquad (26)$$

In derivation of Eq. (25) we have also used summation over polarization states, $\sum_\lambda \varepsilon_\mu^{(\lambda)*} \varepsilon_\nu^{(\lambda)} = -(g^{\mu\nu} - q_\mu q_\nu/m_V^2)$. The Borel transformation removes subtraction terms in the dispersion relation and also exponentially suppresses the contributions coming from the excited resonances and continuum states heavier than considered vector ground states. Applying Borel transformation with respect to Q_0^2 to both sides of Eq.(25), we obtain

$$f_V^2 m_V^2 \exp\left(-\frac{m_V^2}{M^2}\right) = \int_{4m^2}^{s_0} ds\, [\rho_{t,a}(s) + \rho_{\alpha_s}(s)] e^{-\frac{s}{M^2}} + \int_0^{|\mathbf{q}|^2} ds\, \rho_{t,s}(s) e^{-\frac{s}{M^2}} + \hat{B}\Pi_t^{nonpert}. \qquad (27)$$

As we also previously mentioned, when doing numerical analysis, we will set $|\mathbf{q}| \to 0$ representing the rest frame of the particle. In this case, the scattering cut shrinks to a point and the spectral density becomes a singular function. Hence, the second term in the right side of Eq. (27) must be detailed analyzed. Similar analysis has been also performed in [5, 9]. Detailed analysis shows that

$$\lim_{|\mathbf{q}|\to 0} \int_0^{|\mathbf{q}|^2} ds\rho_{t,s}(s) exp\left(-\frac{s}{M^2}\right) = 0. \qquad (28)$$

In Eq. (27), $\hat{B}\Pi_t^{nonpert}$ shows the nonperturbative part of QCD side in Borel transformed scheme, which is given by:

$$\hat{B}\Pi_t^{nonpert} = \int_0^1 dx\, \frac{1}{144\, \pi\, M^6\, x^4\, (-1+x)^4} \exp\left[\frac{m^2}{M^2\, x\, (-1+x)}\right]\left\{\langle\alpha_s G^2\rangle\left[12\, M^6\, x^4\, (-1+x)^4\right.\right.$$
$$- m^6\, (1-2x)^2(-1-x+x^2) - 12\, m^2\, M^4\, x^2\, (-1+x)^2\, (1-3x+3x^2) + m^4\, M^2\, x\, (-2+19x$$
$$\left. - 32x^2 + 11x^3 + 6x^4 - 2x^5)\right] + 4\, \alpha_s\langle\Theta^g\rangle\left[-8\, M^6\, x^3\, (1-2x)^2(-1+x)^3 + m^6\, (1-2x)^2(-1-x\right.$$
$$+ x^2) - 2\, m^2\, M^4\, x^2\, (-1+x)^2(-1-6x+8x^2-4x^3+2x^4) + m^4\, M^2\, x\, (-2+3x-12x^2+31x^3$$
$$\left.\left. - 30x^4 + 10x^5)\right]\right\}, \qquad (29)$$

where, $\Theta^g = \Theta_{00}^g$.

IV.   NUMERICAL ANALYSIS

In this section, we discuss the sensitivity of the masses and leptonic decay constants of the J/ψ and Υ vector mesons to temperature and obtain the numerical results for these

quantities in vacuum. Taking into account the Eqs. (28) and (29) and applying derivative with respect to $1/M^2$ to both sides of the Eq. (27) and dividing by themselves, we obtain

$$m_V^2(T) = \frac{\int_{4m^2}^{s_0(T)} ds \; s \; [\rho_{t,a}(s) + \rho_{\alpha_s}(s)] \exp\left(-\frac{s}{M^2}\right) + \Pi_1^{nonpert}(M^2, T)}{\int_{4m^2}^{s_0(T)} ds [\rho_{t,a}(s) + \rho_{\alpha_s}(s)] \exp\left(-\frac{s}{M^2}\right) + \hat{B}\Pi_t^{nonpert}}, \tag{30}$$

where

$$\Pi_1^{nonpert}(M^2, T) = M^4 \frac{d}{dM^2} \hat{B}\Pi_t^{nonpert}, \tag{31}$$

and

$$\rho_{t,a}(s) = \frac{1}{8\pi^2} s\nu(s)(3 - \nu^2(s)) \left[1 - 2n\left(\frac{\sqrt{s}}{2T}\right)\right]. \tag{32}$$

As we did also in [18], we use the gluonic part of the energy density both obtained from lattice QCD [20, 21] and chiral perturbation theory [19]. In the rest frame of the heat bath, the results of some quantities obtained using lattice QCD in [20] are well fitted by the help of the following parametrization for the thermal average of total energy density $\langle\Theta\rangle$:

$$\langle\Theta\rangle = 2\langle\Theta^g\rangle = 6 \times 10^{-6} exp[80(T - 0.1)](GeV^4), \tag{33}$$

where temperature T is measured in units of GeV and this parametrization is valid only in the region $0.1 \; GeV \leq T \leq 0.17 \; GeV$. Here, we should stress that the total energy density has been calculated for $T \geq 0$ in chiral perturbation theory, while this quantity has only been obtained for $T \geq 100 \; MeV$ in lattice QCD (see [20, 21] for more details). In low temperature chiral perturbation limit, the thermal average of the energy density is expressed as [19]:

$$\langle\Theta\rangle = \langle\Theta_\mu^\mu\rangle + 3 \; p, \tag{34}$$

where $\langle\Theta_\mu^\mu\rangle$ is trace of the total energy momentum tensor and p is pressure. These quantities are given by:

$$\langle\Theta_\mu^\mu\rangle = \frac{\pi^2}{270} \frac{T^8}{F_\pi^4} \ln\left(\frac{\Lambda_p}{T}\right),$$
$$p = 3T\left(\frac{m_\pi \; T}{2 \; \pi}\right)^{\frac{3}{2}}\left(1 + \frac{15 \; T}{8 \; m_\pi} + \frac{105 \; T^2}{128 \; m_\pi^2}\right)exp\left(-\frac{m_\pi}{T}\right), \tag{35}$$

where $\Lambda_p = 0.275 \; GeV$, $F_\pi = 0.093 \; GeV$ and $m_\pi = 0.14 \; GeV$.

The next step is to present the temperature dependent continuum threshold and gluon condensate. In the present work, we use the $s_0(T)$ [13] and $\langle G^2\rangle$ [20, 21] as:

$$s_0(T) = s_0\left[1 - \left(\frac{T}{T_c^*}\right)^8\right] + 4 \; m_Q^2 \; \left(\frac{T}{T_c^*}\right)^8, \tag{36}$$

where $T_c^* = 1.1 \; T_c = 0.176 \; GeV$.

$$\langle G^2\rangle = \frac{\langle 0|G^2|0\rangle}{exp\left[12\left(\frac{T}{T_c} - 1.05\right)\right] + 1}. \tag{37}$$
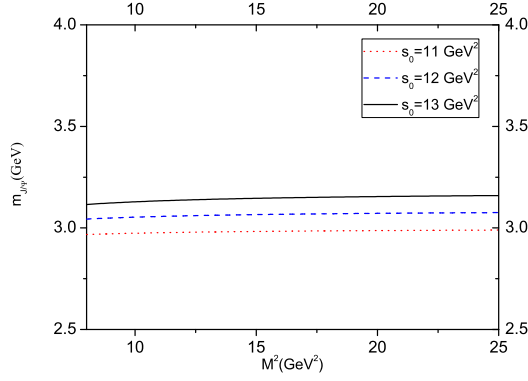
FIG. 1. The dependence of the mass of J/ψ meson in vacuum on the Borel parameter M^2.
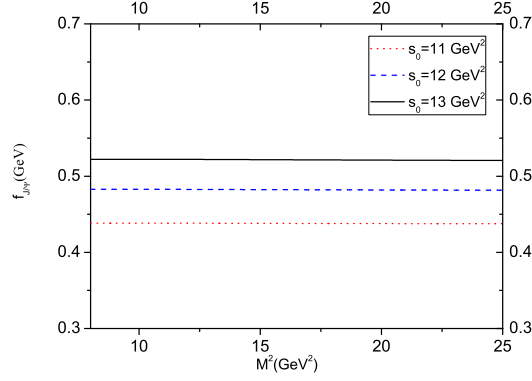

FIG. 2. The dependence of the leptonic decay constant of J/ψ meson in vacuum on the Borel parameter M^2.

In further analysis, we use the values, $m_c = (1.3 \pm 0.05) \ GeV$, $m_b = (4.7 \pm 0.1) \ GeV$ and $\langle 0 \mid \frac{1}{\pi} \alpha_s G^2 \mid 0 \rangle = (0.012 \pm 0.004) \ GeV^4$ for quarks masses and gluon condensate at zero temperature. The sum rules for the masses and decay constants also include two more auxiliary parameters: continuum threshold s_0 and Borel mass parameter M^2. These are not physical quantities, hence the physical observables should be approximately insensitive to these parameters. Therefore, we look for working regions of these parameters such that the dependences of the masses and decay constants on these parameters are weak. The continuum threshold, s_0 is not completely arbitrary, but it is related to the energy of the first exited state with the same quantum numbers as the interpolating currents. Our numerical analysis show that in the intervals $s_0 = (11 - 13) \ GeV^2$ and $s_0 = (98 - 102) \ GeV^2$, respectively for the J/ψ and Υ channels, the results weakly depend on this parameter. The working region for the Borel mass parameter, M^2 is determined demanding that both the contributions of the higher states and continuum are sufficiently suppressed and the contributions coming from the higher dimensional operators are small. As a result, the working region for the Borel parameter is found to be $8 \ GeV^2 \le M^2 \le 25 \ GeV^2$ and $12 \ GeV^2 \le M^2 \le 35 \ GeV^2$
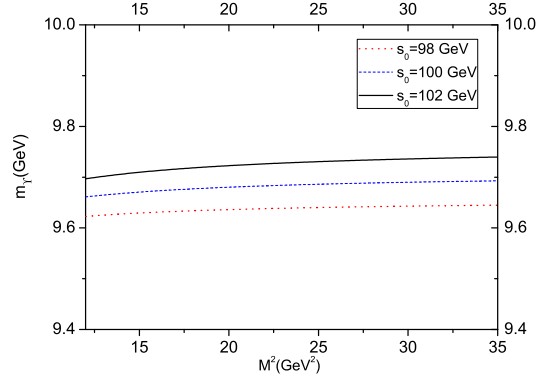
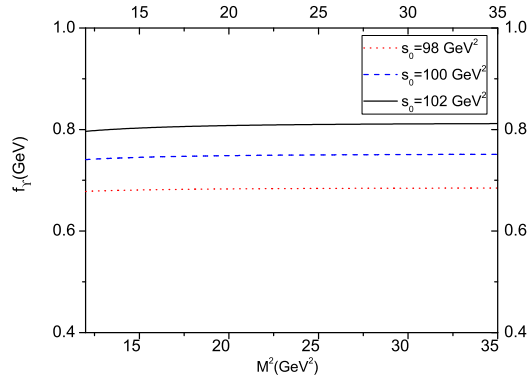FIG. 3. The dependence of the mass of Υ meson in vacuum on the Borel parameter M^2.

FIG. 4. The dependence of the leptonic decay constant of Υ meson in vacuum on the Borel parameter M^2.

in J/ψ and Υ channels, respectively.

Using the working regions for auxiliary parameters as well as other input parameters, we depict the dependence of the masses and leptonic decay constants of the heavy J/ψ and Υ vector quarkonia in Figs. (1-4) at $T = 0$ (vacuum). By a quick glance in these figures, we see that the masses and decay constants represent good stability with respect to the variation of the Borel parameter in its working region. Also, we see a weak dependence of the results on the continuum threshold, s_0. From these figures, we deduce the numerical values of these parameters as shown in the Tables (I) and (II). The uncertainties presented in these Tables are due to the uncertainties in calculation of the working regions for the auxiliary parameters as well as errors in the values of the other input parameters. In these Tables, we also present the existing numerical predictions from the other approaches such as lattice QCD, potential model and nonrelativistic quark model as well as existing experimental data. As far as the leptonic decay constants are concerned, our predictions for the central values are a bit bigger than the predictions of the other approaches and experiment, but when taking into account the uncertainties, our results become comparable especially with the potential

	$f_{J/\psi}(MeV)$	$f_{\Upsilon}(MeV)$
Present Work	481 ± 36	746 ± 62
Lattice[25, 26]	399 ± 4	–
Experimental [25, 26]	409 ± 15	708 ± 8
Potential Model [25]	400 ± 45	685 ± 30
Nonrelativistic Quark Model [26]	423	716

TABLE I. Values of the leptonic decay constants of the heavy-heavy J/ψ and Υ vector mesons in vacuum.

	$m_{J/\psi}$ (GeV)	m_{Υ} (GeV)
Present Work	3.05 ± 0.08	9.68 ± 0.25
Experimental [27]	3.096916 ± 0.000011	9.46030 ± 0.00026

TABLE II. Values of the masses of the heavy-heavy J/ψ and Υ vector mesons in vacuum.
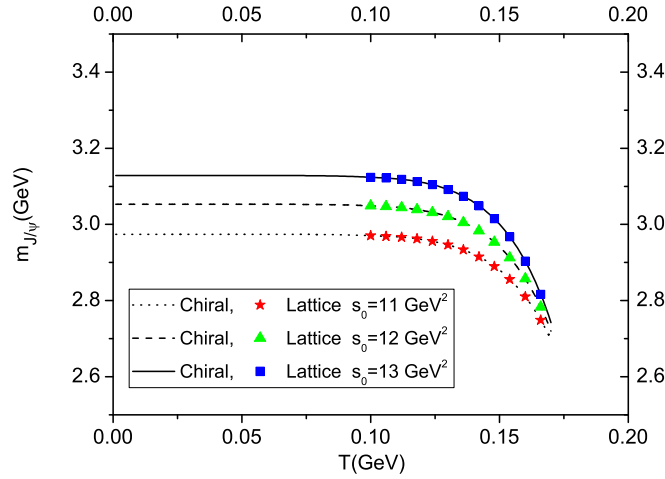

FIG. 5. The dependence of the mass of J/ψ vector meson in GeV on temperature at $M^2 = 10\,GeV^2$.

and nonrelativistic quark models predictions as well as experimental data. However, our predictions on the masses are in good consistency with the experimental values.

Our final task is to discuss the temperature dependence of the leptonic decay constant and masses of the considered particles. For this aim, we plot these quantities in terms of temperature in figures (5-8) using the total energy density from both chiral perturbation
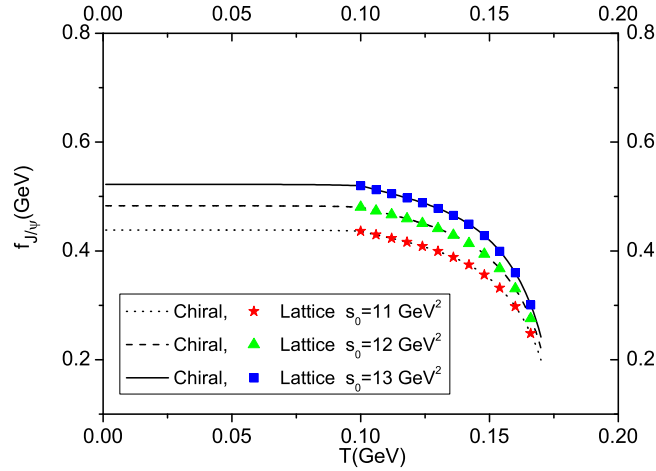
FIG. 6. The dependence of the leptonic decay constant of J/ψ vector meson in GeV on temperature at $M^2 = 10 \ GeV^2$.
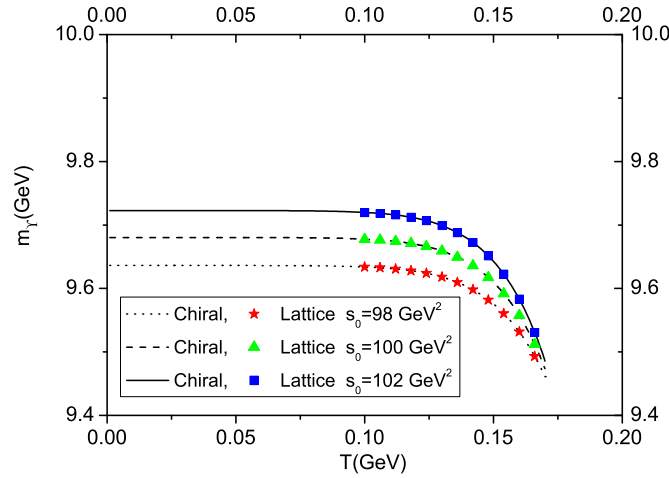

FIG. 7. The dependence of the mass of Υ vector meson in GeV on temperature at $M^2 = 20 \ GeV^2$.

theory and lattice QCD (valid only for $T \geq 100 \ MeV$) and at different fixed values of the s_0 but a fixed value of the Borel mass parameter. From these figures, we observe that the masses and decay constants remain insensitive to the variation of the temperature up to $T \cong 100 \ MeV$, however after this point, they start to diminish increasing the temperature. At deconfinement or critical temperature, the decay constants approach roughly to 45% of their values at zero temperature, while the masses are decreased about 12%, and 2.5% for J/ψ and Υ states, respectively. Considerable decreasing in the values of the decay constants near the deconfinement temperature can be judged as a sign of the quark gluon plasma phase transition.
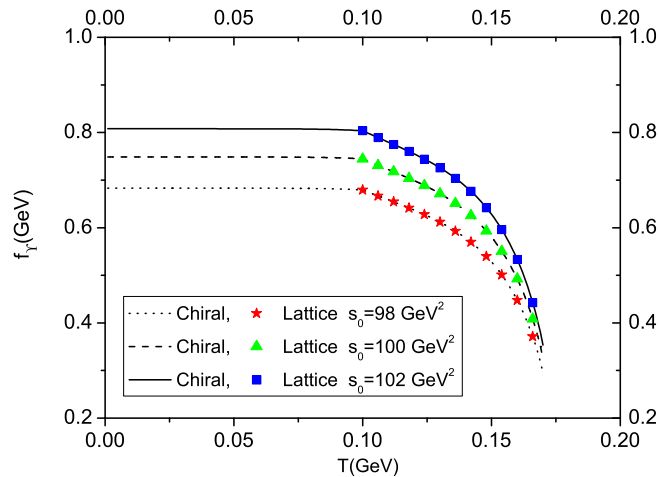
FIG. 8. The dependence of the leptonic decay constant of Υ vector meson in GeV on temperature at $M^2 = 20\ GeV^2$.

V.   ACKNOWLEDGEMENT

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