# Strong coupling constants of heavy spin–3/2 baryons with light pseudoscalar mesons

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## Abstract

The strong coupling constants among members of the heavy spin–3/2 baryons containing single heavy quark with light pseudoscalar mesons are calculated in the framework of the light cone QCD sum rules. Using symmetry arguments, some structure independent relations among different correlation functions are obtained. It is shown that all possible transitions can be described in terms of one universal invariant function whose explicit expression is Lorenz structure dependent.

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# 1   Introduction

The heavy baryons have been at the focus of much attention both theoretically and experimentally during the last decade. The heavy quarks inside these baryons provide windows helping us see somewhat further under the skin of the nonperturbative QCD as compared to the light baryons. Since a part of polarization of heavy quark is transferred to the baryon, so investigation of polarization effects of these baryons can give information about the heavy quark spin. Moreover, these baryons provide a possibility to study the predictions of heavy quark effective theory (HQET). Besides the spectroscopy of heavy baryons, which have been discussed widely in the literature, their electromagnetic, weak and strong decays are very promising tools to get knowledge on their internal structure. In this decade, essential experimental results have been obtained in the spectroscopy of heavy baryons. The $\frac{1}{2}^{+}$ [ $\frac{1}{2}^{-}$] antitriplet states $\Lambda_c^{+}$, $\Xi_c^{+}$, $\Xi_c^{0}$ [ $\Lambda_c^{+}(2593)$, $\Xi_c^{+}(2790)$, $\Xi_c^{0}(2790)$] as well as the $\frac{1}{2}^{+}$ [ $\frac{3}{2}^{+}$] sextet states $\Omega_c, \Sigma_c, \Xi_c'$ [$\Omega_c^{*}, \Sigma_c^{*}, \Xi_c^{*}$] have been observed [1]. Among the S–wave bottom baryons, only the $\Lambda_b$, $\Sigma_b$, $\Sigma_b^{*}$, $\Xi_b$ and $\Omega_b$ have been discovered. It is expected that the LHC will open new horizons in the discovery of the excited bottom baryon sates [2] and provide possibility to study the electromagnetic properties of heavy baryons as well as their weak and strong transitions. The experimental progress in this area stimulates intensive theoretical studies (for a review see for instance [3, 4] and references therein). Theoretical calculations of parameters characterizing the decay of the heavy baryons will help us better understand the experimental results.

The strong coupling constants are the main ingredients for strong decays of heavy baryons. These couplings occur in a low energy scale far from the asymptotic freedom region, where the strong coupling constant between quarks and gluons is large and perturbation theory is invalid. Therefore, to calculate such coupling constants a nonperturbative approach is needed. One of the most reliable and attractive nonperturbative methods is QCD sum rules [5]. This method is based on QCD Lagrangian and does not contain any model dependent parameter. The light cone QCD sum rules (LCSR) method [6] is an extended version of the traditional QCD sum rules in which the operator product expansion (OPE) is carried over twists rather than the dimensions of the operators as in the case of traditional sum rules. In the present work, we calculate the strong coupling constants among sextet of the heavy spin–3/2 baryons containing single heavy quark with the light pseudoscalar mesons in the framework of the LCSR. Using symmetry arguments, we show that all allowed strong transitions among members of these baryons in the presence of light pseudoscalar mesons can be expressed in terms of only one universal invariant function. Note that the coupling constants of heavy spin–1/2 baryons with pseudoscalar and vector mesons have been recently calculated in [7, 8]. The heavy spin 3/2–heavy spin 1/2 baryon–pseudoscalar meson and heavy spin 3/2–heavy spin 1/2 baryon–vector meson coupling constants have also been calculated in the same framework in [9, 10]. It should be mentioned here that the couplings of the heavy baryons with mesons is first calculated in [11] within the framework of HQET.

Rest of the article is organized as follows. In section 2, we derive some structure independent relations among the corresponding correlation functions, and demonstrate how the considered coupling constants can be calculated in terms of only one universal function. In this section, we also derive the LCSR for the heavy spin–3/2 baryon–light pseudoscalar

meson coupling constants using the distribution amplitudes (DA's) of the pseudoscalar mesons. Section 3 is devoted to the numerical analysis of the related coupling constants and discussion.

## 2 Light cone QCD sum rules for the coupling constants of pseudoscalar mesons with heavy spin–3/2 baryons

In this section, the LCSR for the coupling constants among heavy spin–3/2 baryons and light pseudoscalar mesons are derived. We start our discussion by considering the following correlation function:

$$\Pi_{\mu\nu} = i \int d^4 x e^{ipx} \langle \mathcal{P}(q) | \mathcal{T} \{ \eta_\mu(x) \bar{\eta}_\nu(0) \} | 0 \rangle \ , \tag{1}$$

where $\mathcal{P}(q)$ is the pseudoscalar–meson with momentum $q$, $\eta_\mu$ is the interpolating current for the heavy spin–3/2 baryons and $\mathcal{T}$ denotes the time ordering operator. The correlation function in Eq. (1) is calculated in two different ways:

- in terms of hadronic parameters called the physical or phenomenological representation,

- in terms of QCD degrees of freedom by the help of OPE called theoretical or QCD representation.

Matching then these two representations of the same correlation function, we obtain the QCD sum rules for strong coupling constants. To suppress contributions of the higher states and continuum, we apply the Borel transformation with respect to the momentum squared of the initial and final states to both sides of the sum rules.

We start our calculations by considering the physical side. Inserting complete sets of hadrons with the same quantum numbers as the interpolating currents and isolating the ground states, we obtain

$$\Pi_{\mu\nu} = \frac{\langle 0 | \eta_\mu(0) | B_2(p) \rangle \langle B_2(p) \mathcal{P}(q) | B_1(p+q) \rangle \langle B_1(p+q) | \bar{\eta}_\nu(0) | 0 \rangle}{(p^2 - m_2^2) [(p+q)^2 - m_1^2]} + \cdots \ , \tag{2}$$

where $|B_1(p+q)\rangle$ and $|B_2(p)\rangle$ are the initial and final spin–3/2 states, and $m_1$ and $m_2$ are their masses, respectively. The dots in Eq. (2) represent contributions of the higher states and continuum. It follows from Eq. (2) that, in order to calculate the phenomenological part of the correlation function, the following matrix elements are needed:

$$\langle 0 | \eta_\mu(0) | B_2(p) \rangle = \lambda_{B_2} u_\mu(p) \ ,$$
$$\langle B_1(p+q) | \bar{\eta}_\nu(0) | 0 \rangle = \lambda_{B_1} \bar{u}_\nu(p+q) \ ,$$
$$\langle B_2(p) \mathcal{P}(q) | B_1(p+q) \rangle = g_{B_1 B_2 \mathcal{P}} \bar{u}_\alpha(p) \gamma_5 u^\alpha(p+q) \ , \tag{3}$$

where $\lambda_{B_1}$ and $\lambda_{B_2}$ are the residues of the initial and final spin–3/2 heavy baryons, $g_{B_1 B_2 \mathcal{P}}$ is the strong coupling constant of pseudoscalar mesons with heavy spin–3/2 baryons and

$u_\mu$ is the the Rarita-Schwinger spinor. Using the above matrix elements and performing summation over the spins of the Rarita–Schwinger spinors defined as

$$\sum_s u_\mu(p,s)\bar{u}_\nu(p,s) = -(\not{p}+m)\left(-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{2p_\mu p_\nu}{3m^2} - \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{3m}\right), \qquad (4)$$

in principle, one can find the final expression of the correlation function in phenomenological side. However, the following two principal problems are unavoidable: 1) all Lorentz structures are not independent; 2) not only the heavy spin–3/2, but also the heavy spin–1/2 states contribute to the physical side, i.e., the matrix element of the current $\eta_\mu$, sandwiched between the vacuum and the heavy spin–1/2 states, is nonzero and determined in the following way:

$$\langle 0 \left|\eta_\mu\right| B(p, s=1/2)\rangle = A(4p_\mu - m\gamma_\mu)u(p, s=1/2), \qquad (5)$$

where the condition $\gamma_\mu\eta^\mu = 0$ is imposed. To remove the contribution of the unwanted heavy spin–1/2 baryons and obtain only independent structures, we order the Dirac matrices in a specific way and eliminate the ones that receive contributions from spin–1/2 states. Here, we choose the $\gamma_\mu\not{p}\not{q}\gamma_\nu\gamma_5$ ordering of the Dirac matrices and obtain the final expression

$$\Pi_{\mu\nu} = \frac{\lambda_{B_1}\lambda_{B_2}g_{B_1B_2\mathcal{P}}}{(p^2-m_2^2)[(p+q)^2-m_1^2]}\left(g_{\mu\nu}\not{p}\not{q}\gamma_5 + \text{other structures with } \gamma_\mu \text{ at the beginning and}\right.$$

$$\left.\gamma_\nu\gamma_5 \text{ at the end, or terms that are proportional to } (p+q)_\nu \text{ or } p_\mu\right), \qquad (6)$$

and to calculate the strong coupling constant $g_{B_1B_2\mathcal{P}}$, we choose the structure $g_{\mu\nu}\not{p}\not{q}\gamma_5$, which is free of the unwanted heavy spin–1/2 states.

Now, we proceed to calculate the correlation function from QCD side. For this aim, we need to know the explicit expression for the interpolating current of the heavy spin–3/2 baryons. In constructing the interpolating current for these baryons, we use the fact that the interpolating current for this case should be symmetric with respect to the light quarks. Using this condition, the interpolating current for the heavy baryons with $J = 3/2$ containing single heavy quark is written as

$$\eta_\mu = A\epsilon^{abc}\left\{(q_1^a C\gamma_\mu q_2^b)Q^c + (q_2^a C\gamma_\mu Q^b)q_1^c + (Q^a C\gamma_\mu q_1^b)q_2^c\right\}, \qquad (7)$$

where $A$ is the normalization factor, and $a$, $b$ and $c$ are the color indices. In Table 1, we present the values of $A$ and light quark content of heavy spin-3/2 baryons.

Before obtaining the explicit form of the correlation functions on the QCD side, we first try to obtain relations among the correlation functions of the different transitions using some symmetry arguments. Next, we show that all possible transitions can be described in terms of only one universal invariant function. We follow the approach given in [7–10], where the coupling constants of heavy spin–1/2 baryons with light mesons as well as spin 3/2 baryons–spin 1/2 baryons– light mesons vertices are calculated (see also [12–16] for the couplings among light baryons and light mesons). Here, we should stress that the relations which are presented below are independent of the choice of Lorentz structures. We start our

3

| | $\Sigma_{b(c)}^{*+(++)}$ | $\Sigma_{b(c)}^{*0(+)}$ | $\Sigma_{b(c)}^{*-(0)}$ | $\Xi_{b(c)}^{*0(+)}$ | $\Xi_{b(c)}^{*-(0)}$ | $\Omega_{b(c)}^{*-(0)}$ |
|---|---|---|---|---|---|---|
| $q_1$ | $u$ | $u$ | $d$ | $u$ | $d$ | $s$ |
| $q_2$ | $u$ | $d$ | $d$ | $s$ | $s$ | $s$ |
| $A$ | $\sqrt{1/3}$ | $\sqrt{2/3}$ | $\sqrt{1/3}$ | $\sqrt{2/3}$ | $\sqrt{2/3}$ | $\sqrt{1/3}$ |

Table 1: The light quark content $q_1$ and $q_2$ for the heavy baryons with spin–3/2

discussion by considering the $\Sigma_b^{*0} \to \Sigma_b^{*0} \pi^0$ transition. The invariant function describing this strong transition can be written in the following general form:

$$\Pi^{\Sigma_b^{*0} \to \Sigma_b^{*0} \pi^0} = g_{\pi^0 \bar{u}u} \Pi_1(u,d,b) + g_{\pi^0 \bar{d}d} \Pi_1'(u,d,b) + g_{\pi^0 \bar{b}b} \Pi_2(u,d,b) , \tag{8}$$

where $g_{\pi^0 \bar{u}u}$, $g_{\pi^0 \bar{d}d}$ and $g_{\pi^0 \bar{b}b}$ show coupling of the $\pi^0$ meson to the $\bar{u}u$, $\bar{d}d$ and $\bar{b}b$ states, respectively. The interpolating current of $\pi^0$ meson is written as

$$J_{\pi^0} = \sum_{u,d} g_{\pi^0 \bar{q}q} \bar{q} \gamma_5 q , \tag{9}$$

where $g_{\pi^0 \bar{u}u} = -g_{\pi^0 \bar{d}d} = \frac{1}{\sqrt{2}}$, $g_{\pi^0 \bar{b}b} = 0$. The invariant functions $\Pi_1$, $\Pi_1'$ and $\Pi_2$ describe the radiation of $\pi^0$ meson from $u$, $d$ and $b$ quarks of heavy $\Sigma_b^{*0}$ baryon, respectively, and they can formally be defined as:

$$\begin{aligned}
\Pi_1(u,d,b) &= \langle \bar{u}u | \Sigma_b^{*0} \bar{\Sigma}_b^{*0} | 0 \rangle , \\
\Pi_1'(u,d,b) &= \langle \bar{d}d | \Sigma_b^{*0} \bar{\Sigma}_b^{*0} | 0 \rangle , \\
\Pi_2(u,d,b) &= \langle \bar{b}b | \Sigma_b^{*0} \bar{\Sigma}_b^{*0} | 0 \rangle .
\end{aligned} \tag{10}$$

From the interpolating current of $\Sigma_b^*$ baryon, we see that it is symmetric under the exchange $u \leftrightarrow d$, so $\Pi_1'(u,d,b) = \Pi_1(d,u,b)$ and we immediately obtain

$$\Pi^{\Sigma_b^{*0} \to \Sigma_b^{*0} \pi^0} = \frac{1}{\sqrt{2}} \Big[ \Pi_1(u,d,b) - \Pi_1(d,u,b) \Big] , \tag{11}$$

where under the $SU_2(2)_f$ limit, $\Pi^{\Sigma_b^{*0} \to \Sigma_b^{*0} \pi^0} = 0$. Now, we proceed to obtain the invariant function responsible for other transitions containing the $\pi^0$ meson. The invariant function for $\Sigma_b^{*+} \to \Sigma_b^{*+} \pi^0$ transition can be obtained from the $\Sigma_b^{*0} \to \Sigma_b^{*0} \pi^0$ case by making the replacement $d \to u$, and using the fact that $\eta_\mu^{\Sigma_b^{*0}} = \sqrt{2} \eta_\mu^{\Sigma_b^{*+}}$, from which we get,

$$4\Pi_1(u,u,b) = 2 \langle \bar{u}u | \Sigma_b^{*+} \bar{\Sigma}_b^{*+} | 0 \rangle . \tag{12}$$

The factor 4 on the left hand side appears due to the fact that each $\Sigma_b^{*+}$ contains two $u$ quark, hence there are 4 possible ways for radiating $\pi^0$ from the $u$ quark. From Eq. (11), we get

$$\Pi^{\Sigma_b^{*+} \to \Sigma_b^{*+} \pi^0} = \sqrt{2} \Pi_1(u,u,b). \tag{13}$$

4

Similar arguments lead to the following result for the $\Sigma_b^{*-} \to \Sigma_b^{*-}\pi^0$ transition:

$$\Pi^{\Sigma_b^{*-} \to \Sigma_b^{*-}\pi^0} = -\sqrt{2}\Pi_1(d, d, b). \tag{14}$$

Consider the strong transition $\Xi_b^{*-(0)} \to \Xi_b^{*-(0)}\pi^0$. The invariant function for this decay can be obtained from the $\Sigma_b^{*0} \to \Sigma_b^{*0}\pi^0$ case using the fact that $\eta_\mu^{\Xi_b^{*0}} = \eta_\mu^{\Sigma_b^{*0}}(d \to s)$ and $\eta_\mu^{\Xi_b^{*-}} = \eta_\mu^{\Sigma_b^{*0}}(u \to s)$. As a result, we get

$$\Pi^{\Xi_b^{*0} \to \Xi_b^{*0}\pi^0} = \frac{1}{\sqrt{2}}\Pi_1(u, s, b) \ ,$$

$$\Pi^{\Xi_b^{*-} \to \Xi_b^{*-}\pi^0} = -\frac{1}{\sqrt{2}}\Pi_1(d, s, b) \ . \tag{15}$$

Now, we proceed to find relations among the invariant functions involving charged $\pi^\pm$ mesons. We start by considering the matrix element $\langle \bar{d}d | \Sigma_b^{*0}\bar{\Sigma}_b^{*0} | 0 \rangle$, where $d$ quarks from the $\Sigma_b^{*0}$ and $\bar{\Sigma}_b^{*0}$ form the final $\bar{d}d$ state, and $u$ and $b$ quarks are the spectators. The matrix element $\langle \bar{u}d | \Sigma_b^{*+}\bar{\Sigma}_b^{*0} | 0 \rangle$ explains the case where $d$ quark from $\bar{\Sigma}_b^{*0}$ and $u$ quark from $\Sigma_b^{*+}$ form the $\bar{u}d$ state and the remaining $u$ and $b$ are being again the spectators. From this observations, one expects that these matrix elements be proportional to each other. Our calculations support this expectation. Hence,

$$\Pi^{\Sigma_b^{*0} \to \Sigma_b^{*+}\pi^-} = \langle \bar{u}d | \Sigma_b^{*+}\bar{\Sigma}_b^{*0} | 0 \rangle = \sqrt{2}\langle \bar{d}d | \Sigma_b^{*0}\bar{\Sigma}_b^{*0} | 0 \rangle = \sqrt{2}\Pi_1(d, u, b) \ . \tag{16}$$

Making the exchange $u \leftrightarrow d$ in Eq. (16), we obtain

$$\Pi^{\Sigma_b^{*0} \to \Sigma_b^{*-}\pi^+} = \langle \bar{d}u | \Sigma_b^{*-}\bar{\Sigma}_b^{*0} | 0 \rangle = \sqrt{2}\langle \bar{u}u | \Sigma_b^{*0}\bar{\Sigma}_b^{*0} | 0 \rangle = \sqrt{2}\Pi_1(u, d, b) \ . \tag{17}$$

Similarly, one can easily show that

$$\Pi^{\Sigma_b^{*+} \to \Sigma_b^{*0}\pi^+} = \sqrt{2}\Pi_1(d, u, b) \ ,$$
$$\Pi^{\Sigma_b^{*-} \to \Sigma_b^{*0}\pi^-} = \sqrt{2}\Pi_1(u, d, b) \ ,$$
$$\Pi^{\Xi_b^{*0} \to \Xi_b^{*-}\pi^+} = \Pi_1(d, s, b) \ ,$$
$$\Pi^{\Xi_b^{*-} \to \Xi_b^{*0}\pi^-} = \Pi_1(u, s, b) \ . \tag{18}$$

Calculation of the coupling constants of the members of heavy spin–3/2 baryons to other pseudoscalar mesons can be done in a similar way as we did for the $\pi^0$ meson. Here, we shall say that in our calculations, we ignore the mixing between $\eta$ and $\eta'$ mesons and only consider $\eta_8$ instead of physical $\eta$ meson. The interpolating current for $\eta_8$ meson has the following form

$$J_{\eta_8} = \frac{1}{\sqrt{6}}[\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s] \ , \tag{19}$$

we see that

$$g_{\eta_8 \bar{u}u} = g_{\eta_8 \bar{d}d} = \frac{1}{\sqrt{6}} \ , \quad \text{and} \quad g_{\eta_8 \bar{s}s} = -\frac{2}{\sqrt{6}} \ . \tag{20}$$

5

For instance, consider the $\Sigma_b^{*0} \to \Sigma_b^{*0} \eta_8$ transition. Following the same lines of calculations as in the $\pi^0$ meson case, we immediately obtain,

$$\Pi^{\Sigma_b^{*0} \to \Sigma_b^{*0} \eta_8} = \frac{1}{\sqrt{6}} [\Pi_1(u,d,b) + \Pi_1(d,u,b)] . \tag{21}$$

The invariant function responsible for the $\Xi_b^{*0} \to \Xi_b^{0*} \eta_8$ transition can be written as:

$$\begin{aligned}
\Pi^{\Xi_b^{*0} \to \Xi_b^{*0} \eta_8} &= g_{\eta_8 \bar{u} u} \Pi_1(u,s,b) + g_{\eta_8 \bar{s} s} \Pi_1'(u,s,b) + g_{\eta_8 \bar{b} b} \Pi_2(u,s,b) \\
&= \frac{1}{\sqrt{6}} [\Pi_1(u,s,b) - 2\Pi_1'(u,s,b)] \\
&= \frac{1}{\sqrt{6}} [\Pi_1(u,s,b) - 2\Pi_1(s,u,b)] .
\end{aligned} \tag{22}$$

For the remaining transitions containing the $\eta_8$ meson we obtain

$$\Pi^{\Sigma_b^{*+} \to \Sigma_b^{*+} \eta_8} = \frac{2}{\sqrt{6}} \Pi_1(u,u,b) ,$$

$$\Pi^{\Sigma_b^{*-} \to \Sigma_b^{*-} \eta_8} = \frac{2}{\sqrt{6}} \Pi_1(d,d,b) ,$$

$$\Pi^{\Xi_b^{*-} \to \Xi_b^{*-} \eta_8} = \frac{1}{\sqrt{6}} [\Pi_1(d,s,b) - 2\Pi_1(s,d,b)] ,$$

$$\Pi^{\Omega_b^{*-} \to \Omega_b^{*-} \eta_8} = -\frac{4}{\sqrt{6}} \Pi_1(s,s,b) . \tag{23}$$

We also find the following relations for transitions involving $K$ mesons:

$$\begin{aligned}
\Pi^{\Xi_b^{*0} \to \Sigma_b^{*0} \bar{K}^0} &= \Pi^{\Sigma_b^{*0} \to \Xi_b^{*0} K^0} = \Pi_1(d,u,b) , \\
\Pi^{\Xi_b^{*-} \to \Sigma_b^{*0} K^-} &= \Pi^{\Sigma_b^{*0} \to \Xi_b^{*-} K^+} = \Pi_1(u,d,b) , \\
\Pi^{\Xi_b^{*0} \to \Sigma_b^{*+} K^-} &= \Pi^{\Sigma_b^{*+} \to \Xi_b^{*0} K^+} = \sqrt{2} \Pi_1(u,u,b) , \\
\Pi^{\Omega_b^{*-} \to \Xi_b^{*0} K^-} &= \Pi^{\Xi_b^{*0} \to \Omega_b^{*-} K^+} = \Pi^{\Omega_b^{*-} \to \Xi_b^{*-} \bar{K}^0} \\
&= \Pi^{\Xi_b^{*-} \to \Omega_b^{*-} K^0} = \sqrt{2} \Pi_1(s,s,b) , \\
\Pi^{\Xi_b^{*-} \to \Sigma_b^{*-} \bar{K}^0} &= \Pi^{\Sigma_b^{*-} \to \Xi_b^{*-} K^0} = \sqrt{2} \Pi_1(d,d,b) ,
\end{aligned} \tag{24}$$

The expressions for the charmed baryons can easily be obtained by making the replacement $b \to c$ and adding to charge of each baryon a positive unit charge.

So far we have obtained all possible strong transitions among the heavy spin–3/2 baryons with pseudoscalar mesons that are described in terms of only one invariant function $\Pi_1$. This function can be calculated in deep Euclidean region, where $-p^2 \to +\infty$ and $-(p+q)^2 \to +\infty$ using the OPE in terms of DA's of the pseudoscalar mesons as well as light and heavy quark propagators. In obtaining the expression of $\Pi_1$ in QCD side, the nonlocal matrix elements of types $\langle P(q) | \bar{q}(x) \Gamma q(0) | 0 \rangle$ and $\langle P(q) | \bar{q}(x) G_{\mu\nu} q(0) | 0 \rangle$ appear, where $\Gamma$ is any arbitrary Dirac matrix. Up to twist–4 accuracy, these matrix elements are determined in terms of the DA's of the pseudoscalar mesons. These matrix elements as well as the explicit expressions of DA's are given in [17–19].

In calculation of the invariant function $\Pi_1$, we also need to know the expressions of the light and heavy quark propagators. The light quark propagator, in the presence of an external gluon field, is calculated in [20]:

$$
\begin{aligned}
S_q(x) ={}& \frac{i\slashed{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12}\left(1 - i\frac{m_q}{4}\slashed{x}\right) - \frac{x^2}{192}m_0^2\langle \bar{q}q \rangle\left(1 - i\frac{m_q}{6}\slashed{x}\right) \\
& -ig_s\int_0^1 du\left[\frac{\slashed{x}}{16\pi^2 x^2}G_{\mu\nu}(ux)\sigma_{\mu\nu} - \frac{i}{4\pi^2 x^2}ux^\mu G_{\mu\nu}(ux)\gamma^\nu \right. \\
& \left. -i\frac{m_q}{32\pi^2}G_{\mu\nu}\sigma^{\mu\nu}\left(\ln\left(\frac{-x^2\Lambda^2}{4}\right) + 2\gamma_E\right)\right] ,
\end{aligned}
\tag{25}
$$

where $\gamma_E \simeq 0.577$ is the Euler constant, and $\Lambda$ is the scale parameter. In further numerical calculations, we choose it as $\Lambda = (0.5 \div 1)\ GeV$ (see [21, 22]). The heavy quark propagator in an external gluon field is given as:

$$
S_Q(x) = S_Q^{free}(x) - ig_s\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 du\left[\frac{\slashed{k} + m_Q}{2(m_Q^2 - k^2)^2}G^{\mu\nu}(ux)\sigma_{\mu\nu} + \frac{u}{m_Q^2 - k^2}x_\mu G^{\mu\nu}\gamma_\nu\right] ,
\tag{26}
$$

where $S_Q^{free}(x)$ is the free heavy quark operator in coordinate space and it is given by

$$
S_Q^{free}(x) = \frac{m_Q^2}{4\pi^2}\frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} - i\frac{m_Q^2\slashed{x}}{4\pi^2 x^2}K_2(m_Q\sqrt{-x^2}) ,
\tag{27}
$$

where $K_1$ and $K_2$ are the modified Bessel function of the second kind.

Using the explicit expressions of the heavy and light quark propagators and definitions of DA's for the pseudoscalar mesons, we calculate the correlation function from the QCD side. Equating the coefficients of the structure $g_{\mu\nu}\slashed{p}\slashed{q}\gamma_5$ from both sides of the correlation function and applying the Borel transformation with respect to the variables $p^2$ and $(p+q)^2$ in order to suppress the contributions of the higher states and continuum, we get the following sum rules for the strong coupling constants of the pseudoscalar mesons with heavy spin–3/2 baryons:

$$
g_{B_1 B_2 \mathcal{P}} = \frac{1}{\lambda_{B_1}\lambda_{B_2}}e^{\frac{m_1^2}{M_1^2} + \frac{m_2^2}{M_2^2}}\ \Pi_1 ,
\tag{28}
$$

where $M_1^2$ and $M_2^2$ are the Borel mass parameters correspond to the initial and final heavy baryons, respectively. The contributions of the higher states and continuum are obtained by invoking the duality condition, which means that above the thresholds $s_1$ and $s_2$ the double spectral density $\rho^h(s_1, s_2)$ coincides with the spectral density derived from QCD side of the correlation function. The procedure for obtaining double spectral density from QCD side and subtraction of higher states and continuum contributions are explained in detail in [23], which we use in the present work.

The masses of the initial and final baryons are equal to each other, so we take $M_1^2 = M_2^2 = 2M^2$ and the residues $\lambda_{B_1}$ and $\lambda_{B_2}$ are calculated in [24]. The explicit expression for $\Pi_1$ function is quite lengthy and we do not present its explicit form here.

# 3 Numerical results

This section is devoted to the numerical analysis of the sum rules for strong coupling constants of pseudoscalar mesons with heavy spin–$3/2$ baryons. The main input parameters of these LCSRs are DA's of the pseudoscalar mesons which are given in [17–19]. Some other input parameters entering to the sum rules are $\langle \bar{q}q\,(2\ GeV) \rangle = -(274^{+15}_{-17}\ MeV)^3$ and $\mu_\pi(2\,GeV) = f_\pi m_\pi^2/(m_u + m_d) = (2.43 \pm 0.42)\,GeV$ [25], $\langle \bar{s}s \rangle = 0.8 \langle \bar{u}u \rangle$, $\langle 0 \mid \frac{1}{\pi}\alpha_s G^2 \mid 0 \rangle = (0.012 \pm 0.004)\ GeV^4$, $m_0^2 = (0.8 \pm 0.2)\ GeV^2$ [26], $f_\pi = 0.131\ GeV$, $f_K = 0.16\ GeV$ and $f_{\eta_8} = 0.13\ GeV$ [17].



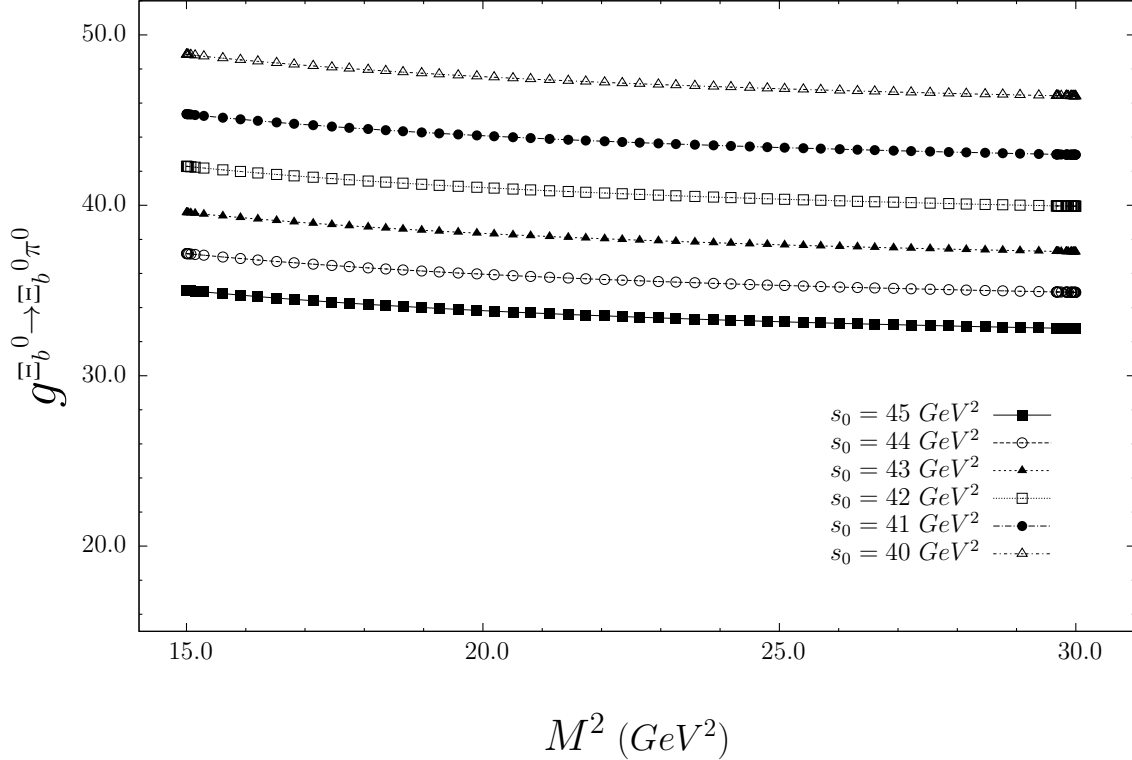Figure 1: The dependence of the strong coupling constant for the $\Xi_b^{*0} \to \Xi_b^{*0}\pi^0$ transition at several fixed values of $s_0$.

The sum rules for the strong coupling constants include also two auxiliary parameters: Borel mass parameter $M^2$ and continuum threshold $s_0$. These are not physical quantities, hence the result for coupling constants should be independent of them. Therefore, we should look for working regions of these parameters, where coupling constants remain approximately unchanged. The upper limit of $M^2$ is obtained requiring that the contribution of the higher states and continuum is small and constitutes only few percent of the total dispersion integral. The lower bound of $M^2$ is obtained demanding that the series of the light cone expansion with increasing twist should be convergent. These conditions lead to the working region $15\ GeV^2 \leq M^2 \leq 30\ GeV^2$ for the bottom heavy spin–$3/2$ baryons and $4\ GeV^2 \leq M^2 \leq 10\ GeV^2$ for the charmed cases. In these intervals, the twist–4 contributions does not exceed $(4$–$6)\%$ of the total result. Our analysis also shows that contribution

8

| | Bottom Baryons | | Charmed Baryons |
|---|---|---|---|
| $g^{\Xi_b^{*0}\to\Xi_b^{*0}\pi^0}$ | 41±9 (4.0 ± 0.5) | $g^{\Xi_c^{*+}\to\Xi_c^{*+}\pi^0}$ | 22±6 (5.3 ± 0.6) |
| $g^{\Sigma_b^{*0}\to\Sigma_b^{*-}\pi^+}$ | 75±15 (8.0 ± 0.7) | $g^{\Sigma_c^{*+}\to\Sigma_c^{*0}\pi^+}$ | 42±10 (11.0 ± 0.7) |
| $g^{\Xi_b^{*0}\to\Sigma_b^{*+}K^-}$ | 65±14 (7.8 ± 0.8) | $g^{\Xi_c^{*+}\to\Sigma_c^{*++}K^-}$ | 37±8 (10.2 ± 0.6) |
| $g^{\Omega_b^{*-}\to\Xi_b^{*0}K^-}$ | 73±16 (8.0 ± 0.8) | $g^{\Omega_c^{*0}\to\Xi_c^{*+}K^-}$ | 37±9 (9.8 ± 0.8) |
| $g^{\Sigma_b^{*+}\to\Sigma_b^{*+}\eta_8}$ | 45±10 (5.8 ± 0.6) | $g^{\Sigma_c^{*++}\to\Sigma_c^{*++}\eta_8}$ | 25±6 (7.8 ± 0.6) |
| $g^{\Xi_b^{*0}\to\Xi_b^{*0}\eta_8}$ | 19±4 (2.7 ± 0.3) | $g^{\Xi_c^{*+}\to\Xi_c^{*+}\eta_8}$ | 11.5±1.8 (3.2 ± 0.4) |
| $g^{\Omega_b^{*-}\to\Omega_b^{*-}\eta_8}$ | 95±22 (11.5 ± 1.8) | $g^{\Omega_c^{*0}\to\Omega_c^{*0}\eta_8}$ | 52±10 (13.6 ± 0.7) |

Table 2: The values of the strong coupling constants for the transitions among the heavy spin–3/2 baryons with pseudoscalar mesons.

of the higher states and continuum is less than 25%. The continuum threshold $s_0$ is not totally arbitrary but it is correlated to the energy of the first excited state with the same quantum numbers as the interpolating current. Our calculations show that in the interval $(m_B + 0.4)^2 \, GeV^2 \leq s_0 \leq (m_B + 0.8)^2 \, GeV^2$, the strong coupling constants weakly depend on this parameter.

As an example, let us consider the $\Xi_b^{*0} \to \Xi_b^{*0}\pi^0$ transition. The dependence of the strong coupling constant for the $\Xi_b^{*0} \to \Xi_b^{*0}\pi^0$ transition on $M^2$ at different fixed values of the $s_0$ is depicted in Fig. (1). From this figure, we see that the strong coupling constant for $\Xi_b^{*0} \to \Xi_b^{*0}\pi^0$ shows a good stability in the "working region" of $M^2$. This figure also depicts that the result of strong coupling constant has weak dependency on the continuum threshold in its working region. From this figure, we deduce $g_{\Xi_b^{*0}\Xi_b^{*0}\pi^0} = 41 \pm 7$. From the same manner, we analyze all considered strong vertices and obtain the numerical values as presented in Table (2). Note that, in this Table, we show only those couplings which could not be obtained by the SU(2) symmetry rotations. The errors in the values of the coupling constants presented in the Table include uncertainties coming from the variations of the $s_0$ and $M^2$ as well as those coming from the other input parameters. Here, we should stress that in the Table (2), we only present the modules of the strong coupling constants, since the sum rules approach can not predict the sign of the residues of the heavy baryons. Our numerical calculations show that the HQET are violated approximately 5% (16%) for the coupling constants of the heavy baryons containing $b(c)$ quark. Finally, we also check the $SU(3)_f$ symmetry violating effects and see that they change the results maximally about 8%.

At the end of this section, it should be mentioned that the predictions of the sum rules on light baryon-meson couplings strongly depend on the choice of the structure (for more detail see [27]). In connection with this point, here immediately arises the question whether or not a similar situation occurs for the case of the heavy baryon-light meson couplings. In order to answer this question, we also analyze the coupling constants predicted by the $g_{\mu\nu}\gamma_5$ structure, which are presented in Table (2) (see the values in the brackets). From these results, it follows that the values of the strong coupling constants of the heavy hadrons containing $b(c)$ quark with light pseudoscalar mesons decrease by a factor of about 8(4)

times for the $g_{\mu\nu}\gamma_5$ structure. However, sticking on the same criteria as is used in [27], we find that the $g_{\mu\nu}\not{p}\not{q}\gamma_5$ is a more pertinent Dirac structure.

In conclusion, the strong coupling constants of light pseudoscalar mesons with heavy spin–3/2 baryons have been studied within LCSR. Using symmetry arguments, the Lorenz structure independent relations among different correlation functions have been obtained. It has been shown that all possible transitions can be written in terms of one universal invariant function. Furthermore, it has been observed that the values of the coupling constants are strongly structure dependent similar to the case of light baryon-meson couplings. The numerical values of those strong coupling constants which could not be obtained via the SU(2) symmetry rotations have been also presented.

# Appendix :

In this appendix, we present some details of our calculations, i.e. how we perform the Fourier and Borel transformations as well as continuum subtraction. For this aim, let us consider the following generic term:

$$T = \int d^4x \; e^{ipx} \int_0^1 du \; e^{iuqx} f(u) \frac{K_\nu(m_Q\sqrt{-x^2})}{(\sqrt{-x^2})^n}, \tag{.1}$$

where $K_\nu$ is the modified Bessel function of order $\nu$ appearing in the propagator of heavy quark. Using the integral representation of the modified Bessel function

$$K_\nu(m_Q\sqrt{-x^2}) = \frac{\Gamma(\nu + 1/2)2^\nu}{\sqrt{\pi}m_Q^\nu} \int_0^\infty dt \; cos(m_Qt) \frac{(\sqrt{-x^2})^\nu}{(t^2 - x^2)^{\nu+1/2}}, \tag{.2}$$

we get

$$T = \int d^4x \int_0^1 du \; e^{iPx} f(u) \frac{\Gamma(\nu + 1/2)2^\nu}{\sqrt{\pi}m_Q^\nu} \int_0^\infty dt \; cos(m_Qt) \frac{1}{(\sqrt{-x^2})^{n-\nu}(t^2 - x^2)^{\nu+1/2}}, \tag{.3}$$

where $P = p + uq$. For further calculations we go to the Euclidean space. Using the identity

$$\frac{1}{Z^n} = \frac{1}{\Gamma(n)} \int_0^\infty d\alpha \; \alpha^{n-1}e^{-\alpha Z}, \tag{.4}$$

we have

$$T = \frac{-i2^\nu}{\sqrt{\pi}m_Q^\nu \Gamma(\frac{n-\nu}{2})} \int_0^1 du f(u) \int_0^\infty dt \; e^{im_Qt} \int_0^\infty dy \; y^{\frac{n-\nu}{2}-1} \int_0^\infty dv \; v^{\nu-\frac{1}{2}}e^{-vt^2} \int d^4\tilde{x}e^{-i\tilde{P}\tilde{x}-y\tilde{x}^2-v\tilde{x}^2}, \tag{.5}$$

where $\tilde{}$ means vectors in Euclidean space and we will consider only the real part of the complex exponential function, $e^{im_Qt}$. After performing Gaussian integral over $\tilde{x}$, we obtain

$$T = \frac{-i2^\nu \pi^2}{\sqrt{\pi}m_Q^\nu \Gamma(\frac{n-\nu}{2})} \int_0^1 du f(u) \int_0^\infty dt \; e^{im_Qt} \int_0^\infty dy \; y^{\frac{n-\nu}{2}-1} \int_0^\infty dv \; v^{\nu-\frac{1}{2}}e^{-vt^2} \frac{e^{-\frac{\tilde{P}^2}{4(y+v)}}}{(y+v)^2}. \tag{.6}$$

The next step is to perform the integration over $t$. As a result we obtain

$$T = \frac{-i2^\nu \pi^2}{m_Q^\nu \Gamma(\frac{n-\nu}{2})} \int_0^1 du f(u) \int_0^\infty dy \; y^{\frac{n-\nu}{2}-1} \int_0^\infty dv \; v^{\nu-1}e^{-\frac{m_Q^2}{4v}} \frac{e^{-\frac{\tilde{P}^2}{4(y+v)}}}{(y+v)^2}. \tag{.7}$$

Let us define new variables,

$$\lambda = v + y, \qquad\qquad \tau = \frac{y}{v + y}, \tag{.8}$$

11

hence

$$T = \frac{-i2^\nu \pi^2}{m_Q^\nu \Gamma(\frac{n-\nu}{2})} \int_0^1 du f(u) \int d\lambda \int d\tau\, \lambda^{\frac{n+\nu}{2}-3}\tau^{\frac{n-\nu}{2}-1}(1-\tau)^{\nu-1}e^{-\frac{m_Q^2}{4\lambda(1-\tau)}}e^{-\frac{\tilde{P}^2}{4\lambda}} \tag{.9}$$

Performing Double Borel transformation with respect to the $\tilde{p}^2$ and $(\tilde{p}+\tilde{p})^2$ using the

$$\mathcal{B}(M^2)e^{-\alpha p^2} = \delta(1/M^2 - \alpha), \tag{.10}$$

we get

$$\begin{aligned}
\mathcal{B}(M_1^2)\mathcal{B}(M_2^2)T &= \frac{-i2^\nu \pi^2}{m_Q^\nu \Gamma(\frac{n-\nu}{2})} \int_0^1 du f(u) \int d\lambda \int d\tau\, \lambda^{\frac{n+\nu}{2}-3}\tau^{\frac{n-\nu}{2}-1}(1-\tau)^{\nu-1}e^{-\frac{m_Q^2}{4\lambda(1-\tau)}}e^{-\frac{u(u-1)\tilde{q}^2}{4\lambda}} \\
&\times\ \delta(\frac{1}{M_1^2}-\frac{u}{4\lambda})\delta(\frac{1}{M_2^2}-\frac{1-u}{4\lambda}).
\end{aligned} \tag{.11}$$

Performing integrals over $u$ and $\lambda$, we obtain

$$\mathcal{B}(M_1^2)\mathcal{B}(M_2^2)T = \frac{-i2^\nu 4^2 \pi^2}{m_Q^\nu \Gamma(\frac{n-\nu}{2})} \int d\tau f(u_0)\left(\frac{M^2}{4}\right)^{\frac{n+\nu}{2}}\tau^{\frac{n-\nu}{2}-1}(1-\tau)^{\nu-1}e^{-\frac{m_Q^2}{M^2(1-\tau)}}e^{\frac{\tilde{q}^2}{M_1^2+M_2^2}}, \tag{.12}$$

where, $u_0 = \frac{M_2^2}{M_1^2+M_2^2}$ and $M^2 = \frac{M_1^2 M_2^2}{M_1^2+M_2^2}$. Replacing $\tau = x^2$, we will have

$$\mathcal{B}(M_1^2)\mathcal{B}(M_2^2)T = \frac{-i2^{\nu+1}4^2\pi^2}{m_Q^\nu \Gamma(\frac{n-\nu}{2})} \int_0^1 dx f(u_0)\left(\frac{M^2}{4}\right)^{\frac{n+\nu}{2}}x^{n-\nu-1}(1-x^2)^{\nu-1}e^{-\frac{m_Q^2}{M^2(1-x^2)}}e^{\frac{\tilde{q}^2}{M_1^2+M_2^2}}. \tag{.13}$$

Finally, after changing the variable $\eta = \frac{1}{1-x^2}$ and using $q^2 = m_\mathcal{P}^2$, we get

$$\mathcal{B}(M_1^2)\mathcal{B}(M_2^2)T = \frac{-i2^{\nu+1}4^2\pi^2}{m_Q^\nu \Gamma(\frac{n-\nu}{2})} f(u_0)\left(\frac{M^2}{4}\right)^{\frac{n+\nu}{2}}e^{-\frac{m_\mathcal{P}^2}{M_1^2+M_2^2}}\Psi\left(\alpha,\beta,\frac{m_Q^2}{M^2}\right), \tag{.14}$$

where

$$\Psi\left(\alpha,\beta,\frac{m_Q^2}{M^2}\right) = \frac{1}{\Gamma(\alpha)}\int_1^\infty d\eta\, e^{-\eta\frac{m_Q^2}{M^2}}\eta^{\beta-\alpha-1}(\eta-1)^{\alpha-1}, \tag{.15}$$

with $\alpha = \frac{n-\nu}{2}$ and $\beta = 1 - \nu$.

Now, let us discuss how contribution of the continuum and higher states are subtracted. For this aim we consider a generic term of the form

$$A = (M^2)^n f(u_0)\Psi\left(\alpha,\beta,\frac{m_Q^2}{M^2}\right). \tag{.16}$$

We should find the spectral density corresponding to this term (see also [28]). The first step is to expand $f(u_0)$ as

$$f(u_0) = \Sigma a_k u_0^k. \tag{.17}$$

12

As a result we get

$$A = \left(\frac{M_1^2 M_2^2}{M_1^2 + M_2^2}\right)^n \Sigma a_k \left(\frac{M_2^2}{M_1^2 + M_2^2}\right)^k \frac{1}{\Gamma(\alpha)} \int_1^\infty d\eta e^{-\eta \frac{m_Q^2}{M^2}} \eta^{\beta-\alpha-1}(\eta-1)^{\alpha-1}. \qquad (.18)$$

Introducing new variables, $\sigma_1 = \frac{1}{M_1^2}$ and $\sigma_2 = \frac{1}{M_2^2}$, we have

$$
\begin{aligned}
A &= \Sigma a_k \frac{\sigma_1^k}{(\sigma_1+\sigma_2)^{n+k}} \frac{1}{\Gamma(\alpha)} \int_1^\infty d\eta e^{-\eta m_Q^2(\sigma_1+\sigma_2)} \eta^{\beta-\alpha-1}(\eta-1)^{\alpha-1} \\
&= \Sigma a_k \frac{\sigma_1^k}{\Gamma(n+k)\Gamma(\alpha)} \int_1^\infty d\eta e^{-\eta m_Q^2(\sigma_1+\sigma_2)} \eta^{\beta-\alpha-1}(\eta-1)^{\alpha-1} \int_0^\infty d\xi e^{-\xi(\sigma_1+\sigma_2)} \xi^{n+k-1} \\
&= \Sigma a_k \frac{\sigma_1^k}{\Gamma(n+k)\Gamma(\alpha)} \int_1^\infty d\eta \eta^{\beta-\alpha-1}(\eta-1)^{\alpha-1} \int_0^\infty d\xi \xi^{n+k-1} e^{-(\xi+\eta m_Q^2)(\sigma_1+\sigma_2)} \\
&= \Sigma a_k \frac{(-1)^k}{\Gamma(n+k)\Gamma(\alpha)} \int_1^\infty d\eta \eta^{\beta-\alpha-1}(\eta-1)^{\alpha-1} \int_0^\infty d\xi \xi^{n+k-1} \left((\frac{d}{d\xi})^k e^{-(\xi+\eta m_Q^2)\sigma_1}\right) e^{-(\xi+\eta m_Q^2)\sigma_2}.
\end{aligned}
$$

$$(.19)$$

Applying double Borel transformation with respect to $\sigma_1 \to \frac{1}{s_1}$ and $\sigma_2 \to \frac{1}{s_2}$, we obtain the spectral density

$$
\begin{aligned}
\rho(s_1, s_2) &= \Sigma a_k \frac{(-1)^k}{\Gamma(n+k)\Gamma(\alpha)} \int_1^\infty d\eta \eta^{\beta-\alpha-1}(\eta-1)^{\alpha-1} \int_0^\infty d\xi \xi^{n+k-1} \left((\frac{d}{d\xi})^k \delta(s_1 - (\xi + \eta m_Q^2))\right) \\
&\times \delta(s_2 - (\xi + \eta m_Q^2)).
\end{aligned}
$$

$$(.20)$$

Performing integration over $\xi$, finally we obtain the following expression for the double spectral density:

$$
\begin{aligned}
\rho(s_1, s_2) &= \Sigma a_k \frac{(-1)^k}{\Gamma(n+k)\Gamma(\alpha)} \int_1^\infty d\eta \eta^{\beta-\alpha-1}(\eta-1)^{\alpha-1}(s_1 - \eta m_Q^2)^{n+k-1} \left((\frac{d}{ds_1})^k \delta(s_2 - s_1)\right) \\
&\times \theta(s_1 - \eta m_Q^2),
\end{aligned}
$$

$$(.21)$$

or

$$\rho(s_1, s_2) = \Sigma a_k \frac{(-1)^k}{\Gamma(n+k)\Gamma(\alpha)} \int_1^{s_1/m_Q^2} d\eta \eta^{\beta-\alpha-1}(\eta-1)^{\alpha-1}(s_1 - \eta m_Q^2)^{n+k-1} \left((\frac{d}{ds_1})^k \delta(s_2 - s_1)\right).$$

$$(.22)$$

Using this spectral density, the continuum subtracted correlation function in the Borel scheme corresponding to the considered term can be written as:

$$\Pi^{sub} = \int_{m_Q^2}^{s_0} ds_1 \int_{m_Q^2}^{s_0} ds_2\, \rho(s_1, s_2) e^{-s_1/M_1^2} e^{-s_2/M_2^2}. \qquad (.23)$$

Defining new variables, $s_1 = 2sv$ and $s_2 = 2s(1-v)$, we get

$$\Pi^{sub} = \int_{m_Q^2}^{s_0} ds \int dv\, \rho(s_1, s_2)(4s) e^{-2sv/M_1^2} e^{-2s(1-v)/M_2^2}. \qquad (.24)$$

13

Using the expression for the spectral density, one can get

$$
\begin{aligned}
\Pi^{sub} &= \Sigma a_k \frac{(-1)^k}{\Gamma(n+k)\Gamma(\alpha)} \int_{m_Q^2}^{s_0} ds \int dv \frac{1}{2^k s^k} \left( (\frac{d}{dv})^k \delta(v-1/2) \right) \\
&\times \int_1^{2sv/m_Q^2} d\eta \; \eta^{\beta-\alpha-1}(\eta-1)^{\alpha-1}(2sv-\eta m_Q^2)^{n+k-1} e^{-2sv/M_1^2} e^{-2s(1-v)/M_2^2}. \quad (.25)
\end{aligned}
$$

Integrating over $v$, finally we obtain

$$
\begin{aligned}
\Pi^{sub} &= \Sigma a_k \frac{(-1)^k(-1)^k}{\Gamma(n+k)\Gamma(\alpha)} \int_{m_Q^2}^{s_0} ds \frac{1}{2^k s^k} \\
&\times \left[ (\frac{d}{dv})^k \int_1^{2sv/m_Q^2} d\eta \; \eta^{\beta-\alpha-1}(\eta-1)^{\alpha-1}(2sv-\eta m_Q^2)^{n+k-1} e^{-2sv/M_1^2} e^{-2s(1-v)/M_2^2} \right]_{v=1/2}.
\end{aligned}
$$
$$(.26)$$

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