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# Properties of $D_{s2}^{*}(2573)$ charmed-strange tensor meson

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The mass and current coupling constant of the  $D_{s2}^*(2573)$  charmed-strange meson is calculated in the framework of the two-point QCD sum rule approach. Although the quantum numbers of this meson are not exactly known, its width and decay modes are consistent with  $I(J^P) = 0(2^+)$ , which we consider to write the interpolating current used in our calculations. Replacing the light strange quark with the up or down quark, we also compare the results with those of the  $D_2^*$  charmed tensor meson and estimate the order of SU(3) flavor symmetry violation.

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#### **I. INTRODUCTION**

During last few years many new particles have been discovered in different experiments. With the increased running energies of colliders and improved sensitivity of detectors, more hadrons are expected to be observed. To better understand and analyze the experimental results, parallel theoretical and phenomenological studies on the spectroscopy and decay properties of newly discovered particles are needed. The LHCb Collaboration at CERN reported the first observation of the  $D_{s2}^*(2573)$  particle through the semileptonic  $\bar{B}_s^0 \rightarrow D_{s2}^{*+} X \mu^- \bar{\nu}$  transition in 2011 [1]. This decay makes an important contribution to the total branching ratio of the semileptonic  $\bar{B}_s^0$  decays, so its analysis helps us get more information about the semileptonic  $\bar{B}_s^0$  decays, which are less known experimentally than the lighter *B* mesons.

Although the quantum numbers of the observed  $D_{s2}^{*}(2573)$  particle are not exactly known, its width and decay modes favor the quantum number  $I(J^P) = 0(2^+)$  [2]. In this paper, we calculate the mass and current coupling constant of the  $D_{s2}^{*}(2573)$  in the framework of two-point QCD sum rules, considering it as a charmed-strange tensor meson. The interpolating currents of the tensor mesons contain derivatives, so we calculate the two-point correlation function first in coordinate space then transform calculations to the momentum space to apply Borel transformation and continuum subtraction in order to isolate the ground state particle from the higher states and continuum. For some experimental and theoretical reviews of the properties, structure, and decay channels of charmedstrange mesons, see for instance [3-9] and references therein.

The outline of the paper is as follows. Starting from an appreciate two-point correlation function, we derive QCD

sum rules for the mass and current coupling constant of the  $D_{s2}^*(2573)$  charmed-strange tensor meson in the next section. In Sec. III, we numerically analyze the sum rules obtained in Sec. II and obtain working regions for the auxiliary Borel parameter and continuum threshold entered in the calculations. Making use of the working regions for auxiliary parameters, we obtain the numerical values of the mass and decay constant of the tensor meson under consideration. Replacing the strange quark with the up or down quark, we also find the masses and decay constant of the corresponding  $\overline{d}(\overline{u})c$  system, by comparison of which we estimate the order of SU(3) flavor symmetry violation in the charmed tensor system.

### II. MASS AND CURRENT COUPLING OF $D_{s2}^*(2573)$ CHARMED-STRANGE TENSOR MESON

Hadrons are formed in a range of energy much lower than the perturbative or asymptotic region, so to investigate their properties some nonperturbative approaches are required. Among the nonperturbative methods, the QCD sum rule [10] is one of the most attractive and applicable tools in hadron physics since it is free of any model-dependent parameters and is based on the QCD Lagrangian. According to the philosophy of this model, to calculate the masses and current coupling constant, we start with a two-point correlation function and calculate it once in terms of hadronic parameters, called the physical or phenomenological side, and another time in terms of QCD parameters in the deep Euclidean region via operator product expansion, which is called the QCD or theoretical side. The QCD sum rules for the mass and current coupling constant are obtained matching both sides of the two-point correlation function under consideration. To stamp down the contribution of the higher states and continuum, we apply Borel transformation to both sides of the acquired sum rules and use the quark-hadron duality assumption.

To derive the QCD sum rules for physical quantities under consideration, we start with the following two-point correlation function:

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AZIZI et al.

$$\Pi_{\mu\nu,\alpha\beta} = i \int d^4x e^{iq(x-y)} \langle 0 \mid \mathcal{T}[j_{\mu\nu}(x)\bar{j}_{\alpha\beta}(y)] \mid 0 \rangle, \quad (1)$$

where  $\mathcal{T}$  is the time-ordering operator and  $j_{\mu\nu}$  is the interpolating current of the  $D_{s2}^*(2573)$  charmed-strange tensor meson. Considering the quantum numbers of  $D_{s2}^*(2573)$  meson, its interpolating current in terms of quark fields can be written as

$$j_{\mu\nu}(x) = \frac{i}{2} [\bar{s}(x)\gamma_{\mu} \overleftarrow{\mathcal{D}}_{\nu}(x)c(x) + \bar{s}(x)\gamma_{\nu} \overleftarrow{\mathcal{D}}_{\mu}(x)c(x)], \quad (2)$$

where the two-side covariant derivative  $\bar{\mathcal{D}}_{\mu}(x)$  is defined as

$$\vec{\mathcal{D}}_{\mu}(x) = \frac{1}{2} [\vec{\mathcal{D}}_{\mu}(x) - \hat{\mathcal{D}}_{\mu}(x)], \qquad (3)$$

and

$$\begin{split} \vec{\mathcal{D}}_{\mu}(x) &= \vec{\partial}_{\mu}(x) - i\frac{g}{2}\lambda^{a}A_{\mu}^{a}(x), \\ \tilde{\mathcal{D}}_{\mu}(x) &= \tilde{\partial}_{\mu}(x) + i\frac{g}{2}\lambda^{a}A_{\mu}^{a}(x). \end{split}$$
(4)

Here  $\lambda^a$  are the Gell-Mann matrices and  $A^a_{\mu}(x)$  denote the external gluon fields. In the Fock-Schwinger gauge, where  $x^{\mu}A^a_{\mu}(x) = 0$ , the external gluon fields are expanded in terms of the gluon field strength tensor as

$$A^{a}_{\mu}(x) = \int_{0}^{1} d\alpha \alpha x_{\beta} G^{a}_{\beta\mu}(\alpha x)$$
  
=  $\frac{1}{2} x_{\beta} G^{a}_{\beta\mu}(0) + \frac{1}{3} x_{\eta} x_{\beta} \mathcal{D}_{\eta} G^{a}_{\beta\mu}(0) + \cdots$  (5)

Note that we consider the currents in the aforementioned correlation function at points x and y; however, we have only an integral over four x. The interpolating current of the tensor meson contains derivatives with respect to the space-time. Hence, after applying derivatives, we will set y = 0 then perform an integral over four x.

On the physical side, the correlation function in Eq. (1) is calculated by saturating it via a complete set of states with the quantum numbers of  $D_{s2}^*$ (2573). After isolating the ground state and performing the four-integral, we get

$$\Pi_{\mu\nu,\alpha\beta} = \frac{\langle 0 \mid j_{\mu\nu}(0) \mid D_{s2}^{*}(2573) \rangle \langle D_{s2}^{*}(2573) \mid \overline{j}_{\alpha\beta}(0) \mid 0 \rangle}{(m_{D_{s2}^{*}(2573)}^{2} - q^{2})} + \cdots,$$
(6)

where  $\cdots$  symbolizes the contribution of higher states and the continuum. To proceed, we need to define the matrix element  $\langle 0 | j_{\mu\nu}(0) | D_{s2}^*(2573) \rangle$  in terms of current coupling constant  $f_{D_{s2}^*(2573)}$  and polarization tensor  $\varepsilon_{\mu\nu}$ ,

$$\langle 0 \mid j_{\mu\nu}(0) \mid D_{s2}^{*}(2573) \rangle = f_{D_{s2}^{*}(2573)} m_{D_{s2}^{*}(2573)}^{3} \varepsilon_{\mu\nu}.$$
 (7)

Using Eq. (7) in Eq. (6) requires performing summation over the polarization tensor, which is given as

$$\varepsilon_{\mu\nu}\varepsilon^*_{\alpha\beta} = \frac{1}{2}\eta_{\mu\alpha}\eta_{\nu\beta} + \frac{1}{2}\eta_{\mu\beta}\eta_{\nu\alpha} - \frac{1}{3}\eta_{\mu\nu}\eta_{\alpha\beta}, \quad (8)$$

where

$$\eta_{\mu\nu} = -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{m_{D^*_{2}(2573)}^2}.$$
(9)

As a result, for the final expression on the physical side, we get

$$\Pi_{\mu\nu,\alpha\beta} = \frac{f_{D_{s2}^*(2573)}^2 m_{D_{s2}^*(2573)}^6}{(m_{D_{s2}^*(2573)}^2 - q^2)} \left\{ \frac{1}{2} (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) \right\}$$
  
+ other structures + \dots, (10)

where we will choose the explicitly written structure to extract the QCD sum rules for the mass and current coupling constant of the tensor meson.

On the QCD side, the correlation function in Eq. (1) is calculated in the deep Euclidean region where  $q^2 \ll 0$ , with the help of operator product expansion where the short- (perturbative) and long- distance (nonperturbative) contributions are separated. The perturbative part is calculated using the perturbation theory, while the nonperturbative part is parametrized in terms of QCD parameters such as quarks masses, quarks, and gluon condensates, etc. Therefore, any coefficient of the structure  $\{\frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})\}$  in QCD side can be written as a dispersion integral plus a nonperturbative part,

$$\Pi(q^2) = \int ds \frac{\rho^{\text{pert}}(s)}{(s-q^2)} + \Pi^{\text{nonpert}}(q^2), \qquad (11)$$

where the spectral density  $\rho^{\text{pert}}(s)$  is obtained from the imaginary of the perturbative contribution, i.e.,  $\rho^{\text{pert}}(s) = \frac{1}{\pi} \text{Im}[\Pi^{\text{pert}}(s)].$ 

Our main goal in the following is to calculate the spectral density  $\rho^{\text{pert}}(s)$  and the nonperturbative part  $\Pi^{\text{nonpert}}(q^2)$ . Using the tensor current presented in Eq. (2) in the correlation function in Eq. (1) and contracting out all quark pairs via the Wick's theorem, we obtain

$$\Pi_{\mu\nu,\alpha\beta} = \frac{i}{4} \int d^4x e^{iq(x-y)} \{ \operatorname{Tr}[S_s(y-x)\gamma_{\mu} \vec{\mathcal{D}}_{\nu}(x) \\ \times \vec{\mathcal{D}}_{\beta}(y)S_c(x-y)\gamma_{\alpha}] + [\beta \leftrightarrow \alpha] + [\nu \leftrightarrow \mu] \\ + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu] \}.$$
(12)

To proceed, we need to know the expressions of the heavy and light quark propagators, which are calculated in [11]. By ignoring from the gluon fields, which have very small contributions, to the mass and current coupling of the tensor meson (see also [12–14]), the explicit expressions of the heavy and light quark propagators are given by

$$S_c^{ij}(x-y) = \frac{i}{(2\pi)^4} \int d^4k e^{-ik \cdot (x-y)} \left\{ \frac{(\not\!k + m_c)}{(k^2 - m_c^2)} \delta_{ij} + \cdots \right\},$$
(13)

and

$$S_{s}^{ij}(x-y) = i \frac{(\not{x}-\not{y})}{2\pi^{2}(x-y)^{4}} \delta_{ij} - \frac{m_{s}}{4\pi^{2}(x-y)^{2}} \delta_{ij} - \frac{\langle \bar{s}s \rangle}{12} \Big[ 1 - i \frac{m_{s}}{4} (\not{x}-\not{y}) \Big] \delta_{ij} - \frac{(x-y)^{2}}{192} m_{0}^{2} \langle \bar{s}s \rangle \Big[ 1 - i \frac{m_{s}}{6} (\not{x}-\not{y}) \Big] \delta_{ij} + \cdots.$$
(14)

The next step is to use the expressions of the quark propagators and apply derivatives with respect to x and y

in Eq. (12). As a result, after setting y = 0, for the QCD side of the correlation function in coordinate space, we get

$$\Pi_{\mu\nu,\alpha\beta} = \frac{N_c}{16} \int \frac{d^4k}{(2\pi)^4} \int d^4x e^{iq\cdot x} \frac{e^{-ik\cdot x}}{(k^2 - m_c^2)} \{ [\mathrm{Tr}\Gamma_{\mu\nu,\alpha\beta}] + [\beta \leftrightarrow \alpha] + [\nu \leftrightarrow \mu] + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu] \},$$
(15)

where  $N_c = 3$  is the color factor, and the function  $\Gamma_{\mu\nu,\alpha\beta}$  is given by

$$\Gamma_{\mu\nu,\alpha\beta} = k_{\nu}k_{\beta} \left[ \frac{i\not t}{2\pi^{2}x^{4}} + \frac{m_{s}}{4\pi^{2}x^{2}} + \left( \frac{1}{12} + \frac{im_{s}\not t}{48} + \frac{x^{2}m_{0}^{2}}{192} + \frac{ix^{2}m_{s}m_{0}^{2}\not t}{1152} \right) \langle \bar{s}s \rangle \right] \gamma_{\mu}(\not t + m_{c})\gamma_{\alpha} + ik_{\beta} \left[ \frac{i}{2\pi^{2}} \left( \frac{4x_{\nu}\not t}{x^{6}} - \frac{\gamma_{\nu}}{x^{4}} \right) \right. \\ \left. + \frac{m_{s}x_{\nu}}{2\pi^{2}x^{4}} + \left( \frac{im_{s}\gamma_{\nu}}{48} - \frac{x_{\nu}m_{0}^{2}}{96} - \frac{im_{0}^{2}m_{s}(2x_{\nu}\not t + x^{2}\gamma_{\nu})}{1152} \right) \langle \bar{s}s \rangle \right] \gamma_{\mu}(\not t + m_{c})\gamma_{\alpha} + \left[ \frac{4i}{2\pi^{2}x^{6}} \left( \frac{6x_{\beta}x_{\nu}\not t}{x^{2}} + g_{\nu\beta}\not t \right) \right] \\ \left. - \gamma_{\nu}x_{\beta} + \gamma_{\beta}x_{\nu} \right) + \frac{m_{s}}{2\pi^{2}} \left( -\frac{4x_{\beta}x_{\nu}}{x^{6}} + \frac{g_{\nu\beta}}{x^{4}} \right) + \left( \frac{g_{\nu\beta}m_{0}^{2}}{96} + \frac{im_{s}m_{0}^{2}}{576} (g_{\nu\beta}\not t + x_{\beta}\gamma_{\nu} + x_{\nu}\gamma_{\beta}) \right) \langle \bar{s}s \rangle \right] \gamma_{\mu}(\not t + m_{c})\gamma_{\alpha} \\ \left. - ik_{\nu} \left[ \frac{i}{2\pi^{2}} \left( \frac{\gamma_{\beta}}{x^{4}} - \frac{4x_{\beta}\not t}{x^{6}} \right) - \frac{m_{s}x_{\beta}}{2\pi^{2}x^{4}} + \left( \frac{im_{s}\gamma_{\beta}}{48} + \frac{m_{0}^{2}x_{\beta}}{96} + \frac{im_{s}m_{0}^{2}}{1152} \left( 2x_{\beta}\not t + x^{2}\gamma_{\beta} \right) \langle \bar{s}s \rangle \right] \gamma_{\mu}(\not t + m_{c})\gamma_{\alpha} \\ \left. + \left[ \beta \leftrightarrow \alpha \right] + \left[ \nu \leftrightarrow \mu \right] + \left[ \beta \leftrightarrow \alpha, \nu \leftrightarrow \mu \right]. \right]$$

After performing all traces in Eq. (15), in order to calculate the integrals, we first transform the terms containing  $\frac{1}{(x^2)^n}$  to the momentum space and replace  $x_{\mu} \rightarrow -i \frac{\partial}{\partial q_{\mu}}$ . The integral over four x gives us a Dirac delta function, which we make use of to perform the integral over four k. To perform the final integral over four p, we use the Feynman parametrization method and the relation

$$\int d^4p \, \frac{(p^2)^{\beta}}{(p^2+L)^{\alpha}} = \frac{i\pi^2(-1)^{\beta-\alpha}\Gamma(\beta+2)\Gamma(\alpha-\beta-2)}{\Gamma(2)\Gamma(\alpha)[-L]^{\alpha-\beta-2}}.$$
(17)

After lengthy calculations for the spectral density, we get

$$\rho^{\text{pert}}(s) = N_c \frac{(m_c^2 - s)^3 (2m_c^4 + m_c^2 s + 10m_c m_s s - 3s^2)}{960\pi^2 s^3}.$$
(18)

For the nonperturbative part, we also obtain

$$\Pi^{\text{nonpert}}(q^2) = m_0^2 \langle \bar{s}s \rangle \frac{(24m_c^3 - m_c^2 m_s - 24m_c q^2 - 5m_s q^2)}{1152(m_c^2 - q^2)^2}.$$
(19)

After acquiring the correlation function on both the phenomenological and QCD sides, by the procedures mentioned in the beginning of this section, we obtain the following sum rule for the mass and current coupling of the  $D_{s2}^*(2573)$  tensor meson:

$$f_{D_{s2}^{*}(2573)}^{2}e^{-m_{D_{s2}^{*}(2573)}^{2}/M^{2}} = \frac{1}{m_{D_{s2}^{*}(2573)}^{6}}\int_{(m_{c}+m_{s})^{2}}^{s_{0}}ds\rho(s)e^{-s/M^{2}} + \hat{\mathbf{B}}\Pi^{\text{nonpert}}(q^{2}),$$
(20)

where  $s_0$  is the continuum threshold and  $M^2$  is the Borel mass parameter. The function  $\hat{\mathbf{B}}\Pi^{\text{nonpert}}(q^2)$  in the Borel scheme is obtained as

$$\hat{\mathbf{B}}\Pi^{\text{nonpert}}(q^2) = m_0^2 \langle \bar{s}s \rangle \frac{(-24M^2m_c - 5M^2m_s - 6m_c^2m_s)}{1152M^2} e^{-m_c^2/M^2}.$$
(21)

The mass of the  $D_{s2}^{*}(2573)$  tensor meson alone is obtained from

$$m_{D_{2}^{*}(2573)}^{2} = \frac{\int_{(m_{c}+m_{s})^{2}}^{s_{0}} ds s \rho(s) e^{-s/M^{2}} + \frac{\partial}{\partial(-1/M^{2})} \hat{\mathbf{B}} \Pi^{\text{nonpert}}(q^{2})}{\int_{(m_{c}+m_{s})^{2}}^{s_{0}} ds \rho(s) e^{-s/M^{2}} + \hat{\mathbf{B}} \Pi^{\text{nonpert}}(q^{2})}$$
(22)

## **III. NUMERICAL RESULTS**

In this section, we numerically analyze the sum rules obtained for the mass and current coupling constant of the  $D_{s2}^*(2573)$  tensor meson in the previous section. For this we use the following input parameters:  $m_c = (1.275 \pm 0.025) \text{ GeV}$  [2],  $\langle \bar{s}s(1 \text{ GeV}) \rangle = -0.8(0.24 \pm 0.01)^3 \text{ GeV}^3$  [15], and  $m_0^2(1 \text{ GeV}) = (0.8 \pm 0.2) \text{ GeV}^2$  [16].

The sum rules for the above-mentioned physical quantities also contain two auxiliary parameters: the Borel parameter  $M^2$  and the continuum threshold  $s_0$  coming from the Borel transformation and the continuum subtraction, respectively. In the following, we shall find working regions of these parameters such that the results of the mass and current coupling show weak dependence on these auxiliary parameters according to the general criteria of the method. The continuum threshold  $s_0$  is not completely capricious, but it is correlated with the energy of the first excited state with the same quantum numbers. As a result, we choose  $s_0 = (10.0 \pm 0.5) \text{ GeV}^2$  for the continuum threshold.

The working region for the Borel mass parameter is calculated demanding not only that the contributions of the higher state and continuum are stamped down but also that the contribution of the higher order operators is very small. The latter means that the series of sum rules for physical quantities is convergent and the perturbative part constitutes an important part of the whole contribution. In other words, the upper bound on the Borel parameter is found by demanding, that

$$\frac{\int_{s_{\min}}^{s_0} \rho(s) e^{-s/M^2}}{\int_{s_{\min}}^{\infty} \rho(s) e^{-s/M^2}} > 1/2,$$
(23)

which leads to

$$M_{\rm max}^2 = 3 {\rm ~GeV^2}.$$
 (24)

The lower bound on this parameter is obtained requiring that the contribution of the perturbative part exceeds the nonperturbative contributions. From this restriction we get



FIG. 1 (color online). The dependence of current coupling  $f_{D_{s2}^*(2573)}$  on Borel mass parameter  $M^2$  at  $s_0 = 10 \text{ GeV}^2$ .



FIG. 2 (color online). The dependence of the mass of  $D_{s2}^*(2573)$  on Borel mass parameter  $M^2$  at  $s_0 = 10 \text{ GeV}^2$ .

$$M_{\rm min}^2 = 1.5 \ {\rm GeV^2}.$$
 (25)

We depict in Figs. 1 and 2 the dependence of the current coupling constant and mass of the tensor meson under consideration on the Borel mass parameter at a fixed value of continuum threshold. From these figures we see that the results weakly depend on the Borel mass parameter in its working region. Here, we would like to make the following comment. The above analyses have been based on the so-called standard procedure in the QCD sum rule technique, such that the quark-hadron duality assumption as a systematic error has been used and the continuum threshold has been taken independent of the Borel mass parameter. However, as also stated in [17], the continuum threshold can depend on  $M^2$ . Hence, the standard procedure does not render the realistic errors and, in fact, the actual errors should be large. Our numerical calculations show that taking the continuum threshold dependent on the Borel mass parameter brings an extra systematic error of 15%, which we will add to our numerical values.

Making use of the working regions for auxiliary parameters and taking into account all systematic uncertainties, we obtain the numerical results of the mass and current coupling constant for  $D_{s2}^*(2573)$  tensor meson as presented in Table I. We also compare our result of the mass with the existing experimental data, which shows a good consistency. The errors quoted in our predictions belong to the uncertainties in the determination of the working

TABLE I. Values for the mass and current coupling constant of the  $D_{s2}^*(2573)$  tensor meson.

	Present Work	Experiment [2]
$\frac{m_{D_{s2}^{*}(2573)}}{f_{D_{s2}^{*}(2573)}}$	$(2549 \pm 440) \text{ MeV}$ $0.023 \pm 0.011$	$(2571.9 \pm 0.8) \text{ MeV}$

PROPERTIES OF  $D_{s2}^*(2573)$  CHARMED-STRANGE ...

regions for both auxiliary parameters, those existing in other inputs, and systematic errors as well. Our result on the current coupling constant of the charmed-strange  $D_{s2}^*(2573)$  tensor meson can be checked in future experiments.

Our final goal is to replace the strange quark with the up or down quark and estimate the order of SU(3) flavor

violation. Our calculations show that this violation is maximally 7% in the case of the charmed tensor meson.

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