Properties of triply heavy spin–3/2 baryons

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Abstract

The masses and residues of the triply heavy spin–3/2 baryons are calculated in framework of the QCD sum rule approach. The obtained results are compared with the existing theoretical predictions in the literature.

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1 Introduction

The quark model predicts heavy baryons containing single, doubly or triply heavy charm or bottom quarks having either spin–1/2 or spin–3/2. So far, all heavy baryons with single charm quark have been discovered in the experiments. Some of the heavy baryons with single bottom quark like Λ_b, Σ_b, Ξ_b and Ω_b baryons with spin–1/2 as well as Σ_b^* baryon with spin–3/2 have been observed in the experiments (for a review see for instance [1]). In 2012, the CMS Collaboration reported the observation of the Ξ_b^* state with spin–3/2 [2]. At present, among all possible doubly heavy baryon states only the spin–1/2 Ξ_{cc}^+ charmed baryon have experimentally been observed by the SELEX Collaboration [3–5]. The experimental attempts, especially at LHCb, have still been continuing to complete the remaining members of the heavy baryons with one, two or three heavy quarks predicted by the quark model.

From the theoretical side, there are a lot of works in the literature devoted to the spectroscopy and decay properties of the heavy baryons containing single heavy quark. There are also dozens of works dedicated to the study of the properties of the doubly heavy baryons. However, there are limited numbers of works devoted to the investigation of the properties of the triply heavy baryons. The masses of the triply heavy baryons have been studied in framework of the various approaches such as effective field theory, lattice QCD, bag model, various quark models, variational approach, hyper central model, potential model and Regge trajectory ansatz [6–19]. The masses and residues of the triply heavy spin–1/2 baryons for the Ioffe current, as well as the masses of the triply heavy spin–3/2 baryons, are also calculated in [20, 21] in framework of the QCD sum rule approach. In the present work, we extend our previous work on the spectroscopy of the triply heavy spin–1/2 baryons for the general form of the interpolating current [22] to calculate the masses and residues of both positive and negative parity triply heavy spin–3/2 baryons in framework of the QCD sum rules. We compare our results on the masses and residues of these baryons with the predictions of the existing approaches in literature [8–15, 20, 21]. Information on the masses of the triply heavy baryons can play essential role in understanding the heavy quark dynamics.

The paper is organized as follows. In the following section, we derive QCD sum rules for the masses and residues of both negative and positive parity triply heavy spin–3/2 baryons. Section 3 is devoted to the analysis of the sum rules for the masses and residues of the triple heavy baryons. This section contains also a comparison of the obtained results with the predictions of other approaches existing in literature.

2 QCD sum rules for the masses and residues of the triply heavy spin–3/2 baryons

In order to calculate the masses and residues of the triply heavy spin–3/2 baryons we start by the following two–point correlation function as the main object of the method:

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iqx} \langle 0 | \mathcal{T}\{\eta_\mu(x)\bar{\eta}_\nu(0)\} | 0 \rangle \ , \tag{1}$$

where $\mathcal{T}$ is the time ordering operator, q is the four–momentum of the corresponding triply heavy baryon and η_μ stands for its interpolating current, whose general form can written as

$$\eta_\mu = \frac{1}{\sqrt{3}}\epsilon^{abc}\left\{2(Q^{aT}C\gamma_\mu Q'^b)Q^c + (Q^{aT}C\gamma_\mu Q^b)Q'^c\right\} , \qquad (2)$$

where Q and Q' are heavy quarks. The quark contents for all members of the triply heavy spin–3/2 baryons are given in Table 1.

Baryon	Q	Q'
Ω^*_{bbc}	b	c
Ω^*_{ccb}	c	b
Ω^*_{bbb}	b	b
Ω^*_{ccc}	c	c

Table 1: The quark contents of the triply heavy spin–3/2 baryons.

Having constructed the correlation function, our next task is construction of the sum rule for the masses of the triply heavy baryons. In order to construct the sum rules, this correlation function should be calculated in two different ways: in terms of hadronic parameters (physical side), and in terms of QCD degrees of freedom (QCD side). Equating these two representations of the correlation function gives us the sum rules for the masses of the triply heavy baryons in terms of quark and gluon degrees of freedom.

Before calculating the correlation function from the physical side, we should mention that the interpolating current η_μ of the triply heavy baryons can interact not only with the positive and negative parity spin–3/2 baryons, but also it couples to the triply heavy spin–1/2 baryons with both parities. In order to obtain reliable results we should eliminate the unwanted spin–1/2 baryons' contributions.

Let us discuss the elimination of the contributions coming from spin–1/2 states. Using the parity and Lorentz covariance considerations, the matrix elements of the interpolating current η_μ for the masses of the spin–3/2 triply heavy baryons between the vacuum and the baryonic states, are defined as:

$$\langle 0\left|\eta_\mu\right| B_{(3/2)^+}(q)\rangle = \lambda_{(3/2)^+}u_\mu(q) ,$$
$$\langle 0\left|\eta_\mu\right| B_{(3/2)^-}(q)\rangle = \lambda_{(3/2)^-}\gamma_5 u_\mu(q) ,$$
$$\langle 0\left|\eta_\mu\right| B_{(1/2)^+}(q)\rangle = \lambda_{(1/2)^+}\left(\frac{4q_\mu}{m_{B_{(1/2)^+}}} + \gamma_\mu\right)\gamma_5 u(q),$$
$$\langle 0\left|\eta_\mu\right| B_{(1/2)^-}(q)\rangle = \lambda_{(1/2)^-}\left(\frac{-4q_\mu}{m_{B_{(1/2)^-}}} + \gamma_\mu\right)u(q) , \qquad (3)$$

where $u(q)$ and $u_\mu(q)$ are the Dirac and Rarita–Schwinger spinors for the spin–1/2 and spin–3/2 baryons, respectively, and λ_i are the corresponding residues. Obviously, we see from Eq. (3) that the contributions coming from the spin–1/2 states are proportional to

q_μ or γ_μ. The physical part of the correlation function can be calculated by saturating the correlation function with the ground state baryons as follows:

$$\Pi_{\mu\nu} = \frac{\langle 0\,|\eta_\mu|\,B(q)\rangle\,\langle B(q)\,|\bar\eta_\nu|\,0\rangle}{q^2 - m_B^2} + \cdots \,, \tag{4}$$

where dots represent the contributions coming from the higher states and continuum. Using Eqs. (3) and (4), for the physical part of the correlation function we get

$$\begin{aligned}
\Pi_{\mu\nu}(q) = {} & \frac{\lambda^2_{(3/2)^+}}{m^2_{(3/2)^+} - q^2}(\not{q} + m_{(3/2)^+})\left(g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2q_\mu q_\nu}{m^2_{(3/2)^+}} + \frac{q_\mu\gamma_\nu - q_\nu\gamma_\mu}{3m_{(3/2)^+}}\right), \\
& - \frac{\lambda^2_{(3/2)^-}}{m^2_{(3/2)^-} - q^2}\gamma_5(\not{q} + m_{(3/2)^-})\left(g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2q_\mu q_\nu}{m^2_{(3/2)^-}} + \frac{q_\mu\gamma_\nu - q_\nu\gamma_\mu}{3m_{(3/2)^-}}\right)\gamma_5, \\
& - \frac{\lambda^2_{(1/2)^+}}{m^2_{(1/2)^+} - q^2}\left(\frac{4q_\mu}{m_{(1/2)^+}} + \gamma_\mu\right)\gamma_5(\not{q} + m_{(1/2)^+})\left(\frac{4q_\nu}{m_{(1/2)^+}} + \gamma_\nu\right)\gamma_5 \\
& + \frac{\lambda^2_{(1/2)^-}}{m^2_{(1/2)^-} - q^2}\left(-\frac{4q_\mu}{m_{(1/2)^-}} + \gamma_\mu\right)(\not{q} + m_{(1/2)^-})\left(\frac{-4q_\nu}{m_{(1/2)^-}} + \gamma_\nu\right),
\end{aligned} \tag{5}$$

where summation over spins of the Dirac and Rarita–Schwinger spinors is performed using

$$\sum u(q,s)\bar u(q,s) = (\not{q} + m_B),$$
$$\sum u_\mu(q,s)\bar u_\nu(q,s) = (\not{q} + m_B)\left(g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2q_\mu q_\nu}{3m_B^2} + \frac{q_\mu\gamma_\nu - q_\nu\gamma_\mu}{3m_B}\right). \tag{6}$$

If we now take into account the fact that the contributions of the spin–1/2 states are proportional to $\gamma_\mu(\gamma_\nu)$ and $q_\mu(q_\nu)$. It follows from Eq. (5) that only the structures $\not{q}g_{\mu\nu}$ and $g_{\mu\nu}$ contain contributions solely coming from the spin–3/2 baryons, which we shall consider in further discussion. As a result, for the physical part of the correlator containing contributions only of the positive and negative parity heavy spin–3/2 baryons, we get

$$\Pi_{\mu\nu}(q) = \frac{\lambda^2_{(3/2)^+}}{m^2_{(3/2)^+} - q^2}(\not{q} + m_{(3/2)^+})g_{\mu\nu} + \frac{\lambda^2_{(3/2)^-}}{m^2_{(3/2)^-} - q^2}(\not{q} - m_{(3/2)^-})g_{\mu\nu} + \cdots \tag{7}$$

On the QCD side, the correlation function in Eq. (1) is calculated in terms of the quark and gluon degrees of freedom using the operator product expansion in deep Euclidean region, where the large and short distance effects are separated. After some simple calculation for the correlation function, we obtain

$$\begin{aligned}
\Pi_{\mu\nu}(q) = {} & \frac{1}{3}\epsilon^{abc}\epsilon^{a'b'c'}\int d^4x\, e^{iqx}\langle 0|\Big\{4S_Q^{cb'}\gamma_\nu\widetilde{S}_{Q'}^{ba'}\gamma_\mu S_Q^{ac'} + 2S_Q^{ca'}\gamma_\nu\widetilde{S}_{Q'}^{ab'}\gamma_\mu S_{Q'}^{bc'} - 2S_Q^{cb'}\gamma_\nu\widetilde{S}_Q^{aa'}\gamma_\mu S_{Q'}^{bc'} \\
& + 2S_{Q'}^{ca'}\gamma_\nu\widetilde{S}_Q^{ab'}\gamma_\mu S_Q^{bc'} - 2S_{Q'}^{ca'}\gamma_\nu\widetilde{S}_Q^{bb'}\gamma_\mu S_Q^{ac'} - S_{Q'}^{cc'}\mathrm{Tr}\Big[S_Q^{ba'}\gamma_\nu\widetilde{S}_Q^{ab'}\gamma_\mu\Big] \\
& + S_{Q'}^{cc'}\mathrm{Tr}\Big[S_Q^{bb'}\gamma_\nu\widetilde{S}_Q^{aa'}\gamma_\mu\Big] - 4S_Q^{cc'}\mathrm{Tr}\Big[S_{Q'}^{ba'}\gamma_\nu\widetilde{S}_Q^{ab'}\gamma_\mu\Big]\Big\}|0\rangle\,,
\end{aligned} \tag{8}$$

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where S_Q is the heavy quark operator; and $\widetilde{S} = CS^T C$. The expression for the heavy quark propagator in x-representation is given by

$$
\begin{aligned}
S_Q(x) &= \frac{m_Q^2}{4\pi^2}\frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} - i\frac{m_Q^2 \slashed{x}}{4\pi^2 x^2}K_2(m_Q\sqrt{-x^2}) \\
&\quad - ig_s \int \frac{d^4 k}{(2\pi)^4}e^{-ikx}\int_0^1 du\left[\frac{\slashed{k}+m_Q}{2(m_Q^2-k^2)^2}G^{\mu\nu}(ux)\sigma_{\mu\nu} + \frac{u}{m_Q^2-k^2}x_\mu G^{\mu\nu}\gamma_\nu\right] + \cdots ,
\end{aligned}
\tag{9}
$$

with K_1 and K_2 being the modified Bessel functions of the second kind. The invariant functions of the structures $\slashed{q}g_{\mu\nu}$ or $g_{\mu\nu}$ in QCD side can be written in terms of the dispersion relations Π_i as

$$
\Pi_i(q) = \int ds\frac{\rho_i(s)}{s-q^2} ,
\tag{10}
$$

where $i = 1(2)$ corresponds to the structure $\slashed{q}g_{\mu\nu}$ $(g_{\mu\nu})$, and the spectral density ρ_i is given by the imaginary part of the invariant function as

$$
\rho_i(s) = \frac{1}{\pi}\text{Im}\Pi_i(s) .
$$

In order to calculate the invariant function we need to know the spectral densities $\rho_1(s)$ and $\rho_2(s)$. Using Eq. (9) in Eq. (8), and after lengthy calculations we get

$$
\begin{aligned}
\rho_1(s) &= \frac{1}{8\pi^4}\int_{\psi_{min}}^{\psi_{max}}\int_{\eta_{min}}^{\eta_{max}}d\psi d\eta\Big\{\mu_{QQQ'}\Big[2m_Q^2\psi - 4m_Q m_{Q'}(-1+\psi+\eta) \\
&\quad + 3\eta\psi(-1+\psi+\eta)(\mu_{QQQ'}-s)\Big]\Big\} \\
&\quad + \frac{\langle g_s^2 GG\rangle}{288\pi^4 m_Q m_{Q'}}\int_{\psi_{min}}^{\psi_{max}}\int_{\eta_{min}}^{\eta_{max}}d\psi d\eta\Big\{-6(-3+4\eta)(-1+\psi+\eta)m_{Q'}^2 \\
&\quad - 6(-1+\psi+\eta)m_Q^2(-3+4\psi) + m_Q m_{Q'}\Big[10 - 12\eta^2 + \eta(2-60\psi) + \\
&\quad + (25-48\psi)\psi\Big]\Big\} ,
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
\rho_2(s) &= \frac{1}{8\pi^4}\int_{\psi_{min}}^{\psi_{max}}\int_{\eta_{min}}^{\eta_{max}}d\psi d\eta\Big\{\mu_{QQQ'}\Big[3m_Q^2 m_{Q'} + \eta m_{Q'}(-1+\psi+\eta)(\mu_{QQQ'}-2s) \\
&\quad + 2m_Q\psi(-1+\psi+\eta)(\mu_{QQQ'}-2s)\Big]\Big\} \\
&\quad + \frac{\langle g_s^2 GG\rangle}{288\pi^4 m_Q m_{Q'}}\int_{\psi_{min}}^{\psi_{max}}\int_{\eta_{min}}^{\eta_{max}}\frac{d\psi d\eta}{\eta\psi}\Big\{2\psi m_Q^2 m_{Q'} + \eta\Big[10m_Q^2 m_{Q'}\psi \\
&\quad + 9m_Q^3(-1+\psi)\psi + m_Q m_{Q'}^2\Big[2+(7-24\psi)\psi\Big] + 6\psi^2(-1+\psi)m_{Q'}(-3\mu_{QQQ'}+5s)\Big]
\end{aligned}
$$

4

$$
+ \eta^3 \psi \Big[8 m_{Q'} \psi (3\mu_{QQQ'} - 7s) + m_Q (-9\mu_{QQQ'} + 12\mu_{QQQ'}\psi + 15s - 28\psi s) \Big]
$$

$$
- \eta^2 \Big[6 m_Q^2 m_{Q'} \psi + 2 m_{Q'} \psi^2 \Big[3\mu_{QQQ'}(7 - 4\psi) + (-43 + 28\psi)s \Big]
$$

$$
+ m_Q \Big[m_{Q'}^2 (2 + 24\psi) - (-1 + \psi)\psi(-9\mu_{QQQ'} + 12\mu_{QQQ'}\psi + 15s - 28\psi s) \Big] \Big] \Big\} , \quad (12)
$$

where

$$
\mu_{QQQ'} = \frac{m_Q^2}{1 - \psi - \eta} + \frac{m_Q^2}{\eta} + \frac{m_{Q'}^2}{\psi} - s ,
$$

$$
\eta_{min} = \frac{1}{2}\left[1 - \psi - \sqrt{(1 - \psi)\left(1 - \psi - \frac{4\psi m_Q^2}{\psi s - m_{Q'}^2}\right)} \ \right] ,
$$

$$
\eta_{max} = \frac{1}{2}\left[1 - \psi + \sqrt{(1 - \psi)\left(1 - \psi - \frac{4\psi m_Q^2}{\psi s - m_{Q'}^2}\right)} \ \right] ,
$$

$$
\psi_{min} = \frac{1}{2s}\left[s + m_{Q'}^2 - 4m_Q^2 - \sqrt{(s + m_{Q'}^2 - 4m_Q^2)^2 - 4m_{Q'}^2 s} \ \right] ,
$$

$$
\psi_{max} = \frac{1}{2s}\left[s + m_{Q'}^2 - 4m_Q^2 + \sqrt{(s + m_{Q'}^2 - 4m_Q^2)^2 - 4m_{Q'}^2 s} \ \right] . \quad (13)
$$

It should be noted here that, these expressions for the spectral densities do not coincide with the results presented in [20, 21].

Having calculated the correlation function for both physical and QCD sides we now equate the coefficients of the structures $\slashed{q}g_{\mu\nu}$ and $g_{\mu\nu}$ from both sides and perform Borel transformation with respect to q^2. The continuum subtraction is done using the quark–hadron duality ansatz. Finally, we get the following results for the sum rules:

$$
\lambda_{(3/2)+}^2 e^{-m_{(3/2)+}^2/M^2} + \lambda_{(3/2)-}^2 e^{-m_{(3/2)-}^2/M^2} = \int_{(2m_Q+m_Q')^2}^{s_0} ds \rho_1(s) e^{-s/M^2} ,
$$

$$(14)$$

$$
\lambda_{(3/2)+}^2 m_{(3/2)+} e^{-m_{(3/2)+}^2/M^2} - \lambda_{(3/2)-}^2 m_{(3/2)-} e^{-m_{(3/2)-}^2/M^2} = \int_{(2m_Q+m_Q')^2}^{s_0} ds \rho_2(s) e^{-s/M^2} ,
$$

$$(15)$$

where M^2 and s_0 are Borel mass parameter and continuum threshold, respectively. These equations contain four unknowns: $\lambda_{(3/2)+}$, $m_{(3/2)+}$, $\lambda_{(3/2)-}$ and $m_{(3/2)-}$. Hence we need two more equations in order to solve for these quantities. Two more equations can be found by taking derivatives of both sides of the above equations with respect to $-1/M^2$, which gives:

$$
\lambda_{(3/2)+}^2 m_{(3/2)+}^2 e^{-m_{(3/2)+}^2/M^2} + \lambda_{(3/2)-}^2 m_{(3/2)-}^2 e^{-m_{(3/2)-}^2/M^2} = \int_{(2m_Q+m_Q')^2}^{s_0} ds \, s\rho_1(s) e^{-s/M^2} ,
$$

$$(16)$$

$$
\lambda_{(3/2)+}^2 m_{(3/2)+}^3 e^{-m_{(3/2)+}^2/M^2} - \lambda_{(3/2)-}^2 m_{(3/2)-}^3 e^{-m_{(3/2)-}^2/M^2} = \int_{(2m_Q+m_Q')^2}^{s_0} ds \, s\rho_2(s) e^{-s/M^2} .
$$

$$(17)$$

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Solving equations (14), (15), (16) and (17) simultaneously, one can find the four unknowns $\lambda_{(3/2)^+}$, $m_{(3/2)^+}$, $\lambda_{(3/2)^-}$ and $m_{(3/2)^-}$.

At the end of this section we would like to make the following remark about the radiative $\mathcal{O}(\alpha_s)$ corrections to the spectral densities. These corrections modify the perturbative parts by the factor of $1 + \frac{\alpha_s}{\pi} f(m_Q, m'_Q, s)$, where $f(m_Q, m'_Q, s)$ is a function of quark masses and s. The mass of baryons from sum rules is determined by the ratio of the two corresponding spectral densities. Therefore, even if the radiative corrections are large, they can not change the values of the masses, considerably. Because these two large corrections are practically cancel each other. Formally, these corrections can be absorbed by the pole residues.

3   Numerical results

In performing the numerical analysis of the sum rules for the masses and residues of the triply heavy spin–3/2 baryons, we need the values of the input parameters entering into the sum rules. For the heavy quark masses we use their pole values, $m_b = (4.8 \pm 0.1)\ GeV$ and $m_c = (1.46 \pm 0.05)\ GeV$ [23]. The numerical value of the gluon condensate is taken to be $\langle g_s^2 GG \rangle = 4\pi^2 (0.012 \pm 0.004)\ GeV^4$ [23]. It should be noted here that, if instead of the pole mass values of the heavy quarks their $\overline{MS}$ [24] values are used, our analysis shows that the results for the masses do not change considerably.

The sum rules for the masses and residues contain two auxiliary parameters, namely continuum threshold s_0 and Borel mass parameter M^2. Obviously, the physical quantities should be independent of the variations of these auxiliary parameters. The continuum threshold is not totally arbitrary but it is correlated with the energy of the first excited state in each channel. It is usually chosen as $\sqrt{s_0} = (m_{ground} + 0.5)\ GeV$, and the domain of those values of s_0 is searched which reproduces this relation. As the result of using this requirement we have obtained the intervals of s_0 for each baryon, and presented them in Table 2. Numerical analysis shows that our results on the masses are weakly dependent on the variations in s_0 in the considered interval.

The working region for the Borel mass parameter M^2 is found as follows. The upper bound on this parameter is obtained by requiring that the pole contribution to the sum rules exceeds the contributions of the higher states and continuum, i.e., the condition,

$$\frac{\int_{s_0}^{\infty} ds \rho(s) e^{-s/M^2}}{\int_{s_{min}}^{\infty} ds \rho(s) e^{-s/M^2}} < 1/3 \ , \tag{18}$$

should be satisfied. The lower bound on M^2 is obtained by demanding that the contribution of the perturbative part exceeds the non-perturbative contributions. From these restrictions we obtain the working regions for the Borel mass parameter for all members of the triply heavy baryons, which are also presented in Table 2.

Having determined the working regions for Borel mass parameter M^2 entering the sum rules, now we are ready to calculate of the masses and residues of the corresponding triply heavy baryons. As example, in Figs. (1) and (2) we present the dependence of the mass of $\Omega_{ccc}(\frac{3^+}{2})$ and $\Omega_{ccc}(\frac{3^-}{2})$ baryons on the Borel mass parameter M^2. We deduce from these

	$M^2(GeV^2)$	$\sqrt{s_0}(GeV)$	$m(GeV)$	$\lambda(GeV^3)$
$\Omega_{ccc}(\frac{3}{2}^+)$	$4.5-8.0$	5.6 ± 0.2	4.72 ± 0.12	0.09 ± 0.01
$\Omega_{ccc}(\frac{3}{2}^-)$	$4.5-8.0$	5.8 ± 0.2	4.9 ± 0.1	0.11 ± 0.01
$\Omega_{ccb}(\frac{3}{2}^+)$	$6.0-10.0$	8.8 ± 0.2	8.07 ± 0.10	0.06 ± 0.01
$\Omega_{ccb}(\frac{3}{2}^-)$	$6.0-10.0$	9.0 ± 0.2	8.35 ± 0.10	0.07 ± 0.01
$\Omega_{bbc}(\frac{3}{2}^+)$	$8.0-10.5$	12.0 ± 0.2	11.35 ± 0.15	0.08 ± 0.01
$\Omega_{bbc}(\frac{3}{2}^-)$	$8.0-10.5$	12.2 ± 0.2	11.5 ± 0.2	0.09 ± 0.01
$\Omega_{bbb}(\frac{3}{2}^+)$	$12.0-18.0$	15.3 ± 0.2	14.3 ± 0.2	0.14 ± 0.02
$\Omega_{bbb}(\frac{3}{2}^-)$	$12.0-18.0$	15.5 ± 0.2	14.9 ± 0.2	0.20 ± 0.02

Table 2: Working regions of auxiliary parameters M^2 and s_0 together with the masses m and residues λ of the triply heavy spin–3/2 baryons. In the numerical analysis we use pole mass of the heavy quarks.

figures that the masses of the $\Omega_{ccc}(\frac{3^+}{2})$ and $\Omega_{ccc}(\frac{3^-}{2})$ baryons are equal to $(4.72\pm0.12)\ GeV$ and $(4.9\pm0.1)\ GeV$, respectively. We have performed similar analysis for the other triply heavy baryons. The results for the masses and residues of all members of the triply heavy. spin-3/2 baryons of both parities are presented in Table 2. From this Table we see that, for the Ω baryons the masses of the negative parity baryons are slightly greater than those of the positive parity baryons. In the case of residues, also, the negative parity baryons have residues slightly higher than those of positive parity baryons.

Now, we compare our results on the masses and residues obtained from using the pole masses of the quarks with the existing predictions of other theoretical approaches. First, in Table 3, we compare our results on the masses with existing predictions of approaches like lattice calculation, QCD bag model, variational method, modified bag model, relativistic quark model, non-relativistic quark mode, QCD sum rules and lattice calculations [8–15, 20, 21]. In all cases the predictions for the masses of the ccc, ccb and bbc baryons are, roughly, in good agreement within the error limits, except than the results of [20], which are considerably low compared to the other predictions. For the bbb baryons, our results are approximately consistent with the existing results of [10, 12, 13], however, our predictions on these baryons are lower compared to the existing predictions of [11, 14, 15, 21]. In the meanwhile, our result on the positive parity bbb baryon is considerably high compared to the result of [20]. The differences between our results for the masses and those presented in [20] can be due to the following reasons. Firstly, in [20] the contributions of the negative parity baryons are not taken into account. Secondly, our spectral densities are different than those presented in [20] for positive parity baryons. Note that predictions for the residues of triply heavy baryons are absent at all in [20].

The comparison of our results on the residues of the triply heavy baryons with those of the [21] as the only existing results in the literature is made in Table 4. From this Table we observe that the predictions of [21] on the residues are approximately (2-5) times grater than our results depending on the quark contents of the baryons. These differences can be attributed to the different interpolating currents that have been used in these two

	Our work	[21]	[10]	[11]	[12]	[13]	[14]	[20]	[15]	[9]	[8]
$m_{\Omega_{ccc}(\frac{3}{2}^+)}$	4.72±0.12	4.99±0.14	4.79	4.925	4.76	4.777	4.803	4.67 ± 0.15	4.965	4.7	
$m_{\Omega_{ccc}(\frac{3}{2}^-)}$	4.9±0.1	5.11±0.15								5.1	
$m_{\Omega_{ccb}(\frac{3}{2}^+)}$	8.07±0.10	8.23±0.13	8.03	8.200	7.98	8.005	8.025	7.45 ± 0.16	8.265	8.05	
$m_{\Omega_{ccb}(\frac{3}{2}^-)}$	8.35±0.10	8.36±0.13									
$m_{\Omega_{bbc}(\frac{3}{2}^+)}$	11.35±0.15	11.49±0.11	11.20	11.480	11.19	11.163	11.287	10.54 ± 0.11	11.554		
$m_{\Omega_{bbc}(\frac{3}{2}^-)}$	11.5±0.2	11.62±0.11									
$m_{\Omega_{bbb}(\frac{3}{2}^+)}$	14.3±0.2	14.83±0.10	14.30	14.760	14.37	14.276	14.569	13.28 ± 0.10	14.834		14.37
$m_{\Omega_{bbb}(\frac{3}{2}^-)}$	14.9±0.2	14.95±0.11									14.72

Table 3: The masses of the triply heavy spin–3/2 baryons in units of GeV, calculated using the pole masses of the heavy quarks, compared with other theoretical predictions.

works. Moreover, in determination of the masses and residues of the negative and positive parity baryons we have coupled four equations but in [21], only two equations exist. In the case of masses, the ratios of two corresponding sum rules are considered and therefore the errors cancel each other. For this reason our predictions for the masses are comparable with those of [21]. But in determination of the residues, there are no ratios of sum rules and one of the obtained equations are used, which leads to the above-mentioned considerable differences. Although triply heavy baryons have not yet been discovered in experiments, their production at LHCb has theoretically been studied in [25], and it is found that 10^4-10^5 events of triply heavy baryons can be produced at 10 fb^{-1} integrated luminosity.

	Present study	[21]
$\lambda_{\Omega_{ccc}(\frac{3}{2}^+)}$	0.09±0.01	0.20±0.04
$\lambda_{\Omega_{ccc}(\frac{3}{2}^-)}$	0.11±0.01	0.24±0.04
$\lambda_{\Omega_{ccb}(\frac{3}{2}^+)}$	0.06±0.01	0.26±0.05
$\lambda_{\Omega_{ccb}(\frac{3}{2}^-)}$	0.07±0.01	0.32±0.06
$\lambda_{\Omega_{bbc}(\frac{3}{2}^+)}$	0.08±0.01	0.39±0.09
$\lambda_{\Omega_{bbc}(\frac{3}{2}^-)}$	0.09±0.01	0.49±0.10
$\lambda_{\Omega_{bbb}(\frac{3}{2}^+)}$	0.14±0.02	0.68±0.16
$\lambda_{\Omega_{bbb}(\frac{3}{2}^-)}$	0.20±0.02	0.86±0.17

Table 4: The residues of the triply heavy spin–3/2 baryons in units of GeV, calculated using the pole masses of the heavy quarks, compared with the predictions of [21].

In summary, we evaluated the masses and residues of the triply heavy spin–3/2 baryons with both positive and negative parities in the framework of the QCD sum rules as one of the most powerful non-perturbative method. The results obtained in this work are compared with the predictions of other theoretical approaches.

We hope it would be possible to measure the masses and decays of the triply heavy baryons in the near future at LHCb.

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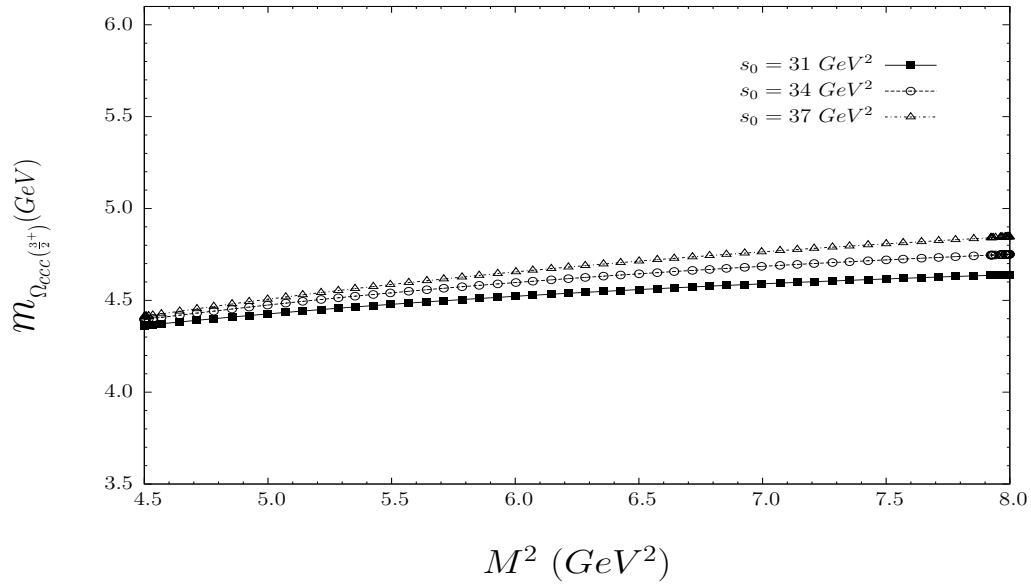[25] Yu-Qi Chen, Su-Zhi Wu, JHEP 1108 (2011) 144, Erratum-ibid. 1109 (2011) 089.

Figure 1: Dependence of the mass of the triply heavy, positive parity $\Omega_{ccc}(\frac{3}{2}^+)$ baryon on the auxiliary Borel mass parameter M^2, at several fixed values of the continuum threshold s_0.
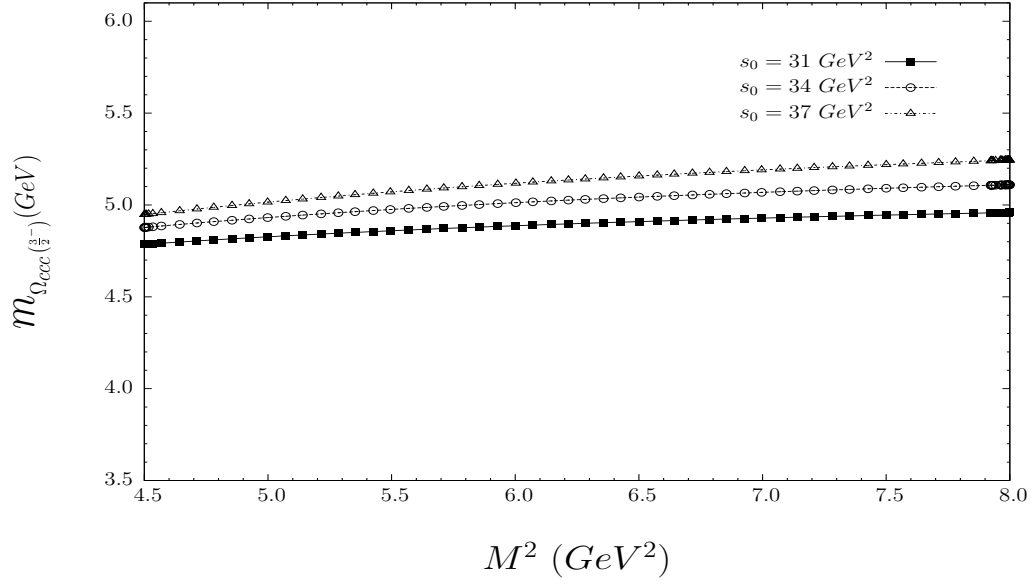
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Figure 2: The same as Fig. 1, but for the negative parity $\Omega_{ccc}(\frac{3}{2}^-)$ baryon.