GENERALIZED MODELING PRINCIPLES OF A NONLINEAR SYSTEM WITH A DYNAMIC FUZZY NETWORK

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ABSTRACT

Generalized modeling principles of a nonlinear system with a dynamic fuzzy network (DFN)- a network with unconstrained connectivity and with dynamic fuzzy processing units called 'feurons', have been given. DFN model has been trained both in open loop and closed loop forms to satisfy these principles. Several system trajectories with a PRBS input have been used for open loop training. DFN model obtained from open loop training was used in a closed loop training with an extended Kalman filter (EKF) in an observer design. For gradient computations adjoint sensitivity method has been used.

I. INTRODUCTION

The application of fuzzy logic technology to modeling and control of dynamical systems has been constrained by the non-dynamical nature of popular fuzzy system architectures. Many of the difficulties that ensue- large rule bases (i.e., curse of dimensionality), long training times, the need to determine buffer lengths- can be overcome with dynamic fuzzy network (DFN)- network realizations that contain unconstrained connectivity and with dynamic fuzzy processing units called "feurons" and may also allow for the incorporation of both heuristics and deep knowledge to exploit the best characteristics of each [12, 21].

The two major complications introduced during the modeling of a nonlinear dynamical system with a DFN are: which principles should be considered to obtain the accurate "model equivalence" of a known model of a nonlinear dynamical system? Moreover, how can we train a DFN for the best convergence? For the first one, the generalized modeling principles have been defined in this study. In algebraic fuzzy systems parameter tuning is relatively simple because the gradients of model error with respect to the parameters are easy to compute [20]. In dynamic fuzzy networks, gradient computation is more involved. Approaches for gradient computation for dynamic systems have been developed in systems and control theories [2, 21], and have been successfully used in identification and control applications [9, 10].

In section 2, we define the generalized modeling principles. In section 3, we introduce the standard fuzzy system. In section 4, we present a model for dynamic fuzzy network (DFN). In sections, we have discussed the employment of approximate second order gradient-based approaches to determine appropriate values for the dynamic fuzzy network parameters for the desired trajectory tracking and modeling.

Simulation examples of phase portrait modeling of a tunnel diode circuit by a DFN and open loop and closed loop modeling of a CSTR system by a DFN are illustrated in section 6.

2. GENERALIZED MODELING PRINCIPLES

To make an effective control system design, there should be a model that accurately represents the real nonlinear dynamical plant. Theoretical models are obtained in general, by four steps [3, 15]. These are 1) problem definition, 2) model formulation by using the first principles, 3) parameter estimation and 4) model validation.

It is asserted that there exists a DFN which can represent all the behaviors of a given dynamical plant. This assertion is based on the experiments done on a mathematical models that accurately represent the physical plant (i.e., reference model). We can develop an equivalent DFN which captures all the qualitative behaviors of this physical plant. In this study these experiments classified into two categories: i) open loop/closed loop validations, ii) local and global validations.

There are some works in the literature for open loop and closed loop verification of a model. Gevers (1999), developed the model error estimation method of Ljung (1987) for the closed loop validation. Some examples of open loop validation can be found in Aguirre's studies [1].

3. STANDARD FUZZY SYSTEM

An important contribution of fuzzy systems theory is that it provides a systematic procedure for transforming a knowledge base into a nonlinear mapping [20]. Figure 3.1 shows the basic configuration of a fuzzy system.
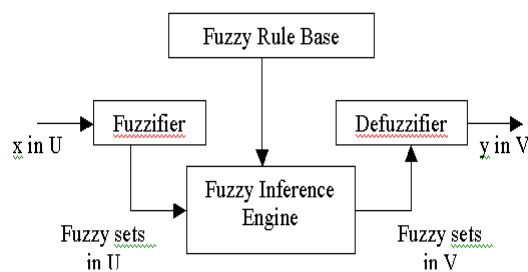

Figure 3.1 Basic configuration of fuzzy systems

We call the fuzzy systems with product inference engine, singleton fuzzifier, center average defuzzifier, and Gaussian membership functions as "standard fuzzy systems" [20]. Nonlinear mapping of a standard fuzzy system is in the following form:

$$f(x) = \frac{\sum_{i=1}^{r} y_i \prod_{j=1}^{n} \exp(-\frac{1}{2}(\frac{x_j - c_{ij}}{\sigma_{ij}})^2)}{\sum_{i=1}^{r} \prod_{j=1}^{n} \exp(-\frac{1}{2}(\frac{x_j - c_{ij}}{\sigma_{ij}})^2)} \qquad (3.1)$$

where r is the number of rules, c_{ij} and σ_{ij} are center and spread parameters of the ith rule's j^{th} input variable's of the Gaussian membership function respectively. y_i, is the output center parameter of the ith rule. Figure 3.2 shows the inference mechanism of a standard fuzzy system with two input variables that uses expert knowledge base.
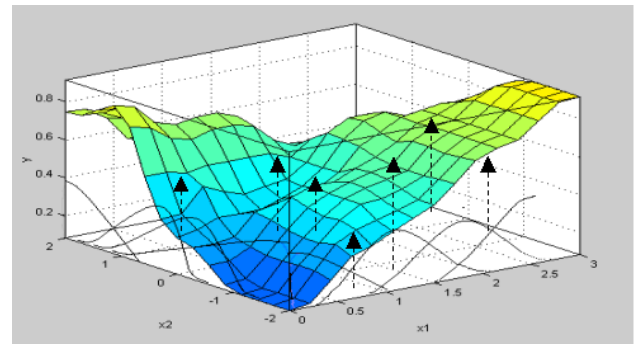

Figure 3.2 The inference mechanism of a standard fuzzy system with two input variables. (Peaks show some expert knowledge)

4. THE DYNAMIC FUZZY NETWORK (DFN) MODEL

By a dynamic fuzzy network, we mean a network with unconstrained connectivity and with dynamic elements in its fuzzy processing units [13, 17]. The processing unit is called the feuron, representing a single dynamic fuzzy neuron. Figure 4.1 shows a DFN with two feurons.

Basically the feuron model represents the biological neuron that fires when its inputs are significantly excited. This firing is through a lag dynamics (i.e., Hopfield dynamics). The manner in which the feuron fires is defined by the fuzzy

activation function ϕ. The feuron's activation model also represents a standard fuzzy system that described before.
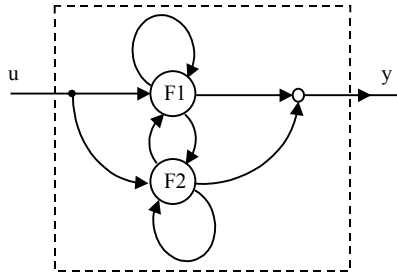

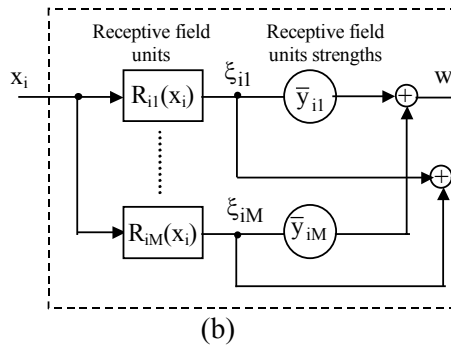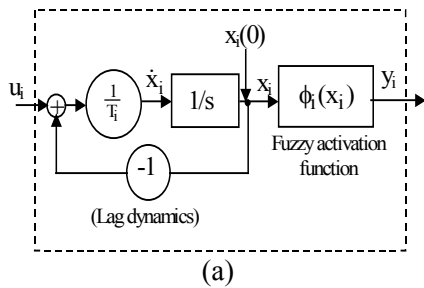**Figure 4.1** Dynamic Fuzzy Network (DFN) with two feurons

Figure 4.2 Block diagram of the feuron
(a) A single feuron model,
(b) Feuron's activation model, $y_i = \phi(x_i)$.

Since the feuron's activation model has a single input, (i.e., the scalar state of the feuron) the product inference output becomes the membership functions itself. The activation function of the ith feuron can be expressed as:

$$\phi_i(x_i) = \frac{\sum_{k=1}^{R_i} b_{ik}\mu_k(x_i)}{\sum_{k=1}^{R_i} \mu_k(x_i)} = \frac{\sum_{k=1}^{R_i} b_{ik}\exp\left(-\frac{1}{2}\left(\frac{x_i - c_{ik}}{\sigma_{ik}}\right)^2\right)}{\sum_{k=1}^{R_i} \exp\left(-\frac{1}{2}\left(\frac{x_i - c_{ik}}{\sigma_{ik}}\right)^2\right)}$$

(4.1)

The boundary membership functions of the feuron may be represented by hard constraints in the form of:

$$\mu_l(x_i) = 1, \quad \text{if } x_i \leq x_{iL} \quad \text{and } \mu_{R_i}(x_i) = 1, \quad \text{if } x_i \geq x_{iU}$$

(4.2)

where x_{iL} and x_{iU} are the lower and upper limits of the state values of the ith feuron respectively. The general computational model that we have used for DFN is summarized in the following equations:

$$z_m = \sum_{j=1}^{n} q_{mj}y_j$$

(4.3)

$$y_i = \phi_i(x_i, \pi_i)$$

(4.4)

$$T_i\dot{x}_i = -x_i + \sum_{j=1}^{n} w_{ij}y_j + \sum_{j=1}^{l} p_{ij}u_j + r_i \quad , \quad x_i(0) = x_{i0}$$

(4.5)

where w, p, q, r are the interconnection parameters of the DFN, T is the time constants and π is the parameter sets (centers **c**, spreads **σ**, output centers **b**) corresponding to fuzzy SISO activation functions of the feurons. This model has some similarities with models encountered in the literature [6, 16].

5. PERFORMANCE INDEX MINIMIZATION IN DFN

In order to obtain DFN model of a process we associate a performance index given by

$$J = \frac{1}{2}\int_0^{t_f} [z(t) - z^d(t)]^T[z(t) - z^d(t)]dt \quad (5.1)$$

where $z^d(t)$ and $z(t)$ are actual and modeled process responses respectively. Tuning dynamic fuzzy networks to mimic a given set of trajectories is equivalent to determining (i.e., learning) DFN parameters so that the performance index is minimized. This is a dynamic nonlinear optimization problem. We require the computation of gradients or sensitivities of the performance index with respect to the various interconnection parameters of the DFN and internal parameters of the feurons. They are respectively:

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$$[\frac{\partial J}{\partial w},\frac{\partial J}{\partial p},\frac{\partial J}{\partial q},\frac{\partial J}{\partial r}] \text{ and } [\frac{\partial J}{\partial T},\frac{\partial J}{\partial c},\frac{\partial J}{\partial \sigma},\frac{\partial J}{\partial b}] \quad (5.2)$$

We have used "Adjoint" method for sensitivity computation [10]. The computation formulas have been found in [15]. We used first and approximate second order gradient-based methods including Broyden-Fletcher-Golfarb-Shanno (BFGS) algorithm to update the parameters of the dynamic fuzzy networks yielding faster rate of convergence [4, 19].

6. GENERALIZED MODELING SIMULATION EXAMPLES

For obtaining the equivalent DFN models due to generalized modeling principles, first of all it is useful to show the efficiency of a DFN in global modeling. Our first set of experiments have used the data of a tunnel diode circuit with two state variables for phase portrait modeling by a DFN with two feurons.

6.1 Phase Portrait Modeling of a Tunnel Diode Circuit

The state-space model of a tunnel diode circuit [7] is given by

$$\dot{x}_1 = \frac{1}{C}\left[-h(x_1)+x_2\right] \quad (6.1)$$

$$\dot{x}_2 = \frac{1}{L}\left[-x_1-Rx_2+u\right] \quad (6.2)$$

where h(x) is a fifth degree polynomial representing the tunnel diode charecteristics and given by

$$h(x)=17.76x-103.79x^2+229.62x^3-226.31x^4+83.72x^5 \quad (6.3)$$

and the other circuit parameters are given as
u=1.2 V, R=1.5 kΩ, C=2 pF, L=5 μH   (6.4)

The tunnel diode circuit system was simulated for several initial conditions to the three equilibrium points A=(0.285,0.61), B=(0.063,0.758), and C=(0.884,0.21) with an integration time step of 0.01 second. These trajectories were taken as the desired trajectories for a DFN to capture. At the beginning, each trajectory segment was used for training. Subsequently, combinations of the segments

were used for fine-tuning based on randomly generated initial conditions. Figure 6.1 shows the resulting phase portraits of the tunnel diode circuit system and the DFN model. The final phase portrait model response was visually indistinguishable in some of the trajectory segments.

Dynamic fuzzy network parameters obtained were:

$$w=\begin{bmatrix} -57.7369 & -39.2152 \\ 28.8822 & 20.2944 \end{bmatrix}, p=\begin{bmatrix} 0 \\ 0 \end{bmatrix}, q=\begin{bmatrix} 1 \\ 1 \end{bmatrix}, r=\begin{bmatrix} 0 \\ 1.2 \end{bmatrix}$$

$$T=\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}, c=\begin{bmatrix} 6.4851 & -5.6517 & -0.7567 & -10.6096 & 4.9055 \\ -23.0508 & -2543668 & 1745070 & -2256273 & 1083446 \end{bmatrix},$$

$$\Sigma=\begin{bmatrix} 31.3802 & 0.4998 & 0.2121 & 64.5575 & 1.0666 \\ 57.8831 & 201.7712 & 40.4817 & 1514181 & 80.4307 \end{bmatrix},$$

$$b=\begin{bmatrix} -15.3997 & -1.1549 & -23.9189 & 16.1915 & 65.1823 \\ -3.1482 & 15.4820 & -24.0564 & -7.4384 & -7.3875 \end{bmatrix}$$
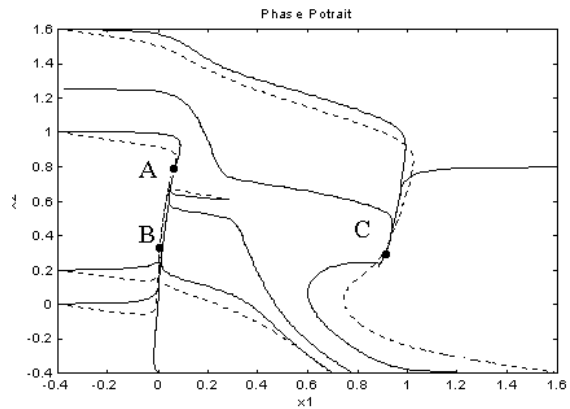

Figure 6.1 The phase portrait of the DFN (dashed curve). The solid curve is the Tunnel Diode circuit phase portrait.

6.2 Equivalent DFN Modeling of a Nonlinear Continuously Stirred Tank Reactor

Generalized modeling principles given in section 2 have been examined for obtaining a DFN model equivalent of a nonlinear continuously stirred tank reactor (CSTR) system with relevant equations given below [18]:

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$$\frac{dx_1}{dt} = -x_1 + Da\,(1 - x_1)e^{\frac{x_2}{1+x_2/\gamma}} \qquad (6.5)$$

$$\frac{dx_2}{dt} = -x_2 + BDa(1 - x_1)e^{\frac{x_2}{1+x_2/\gamma}}$$

$$-\beta(x_2 - x_{2c}) + d + \beta u \qquad (6.6)$$

where x_1, x_2 represent dimensionless reactant conversion and temperature, Da is the Damköhler number and the variables d, u are dimensionless deviation variables for the feed temperature disturbance and control (jacket temperature) respectively. In the simulations the parameters were taken as Da= 0.135, B=11, β=1.5 and γ =20.

For the equivalent DFN modeling of a CSTR, these experiments have been done:
1) Open loop:
   - Zero-input response of the equivalent models,
   - Zero-state response of the equivalent models,
   - Various input and various state responses of the equivalent models,
2) Closed loop:
   - The design of Extended Kalman Filter with equivalent models,

6.2.1 Open Loop Experiments

To obtain an equivalent DFN model, we have used data generated from a simplified nonlinear CSTR model as the target model. Performance index has been chosen as:

$$J = \int_0^{t_f} \frac{1}{2}(xd_1 - x_1)^2 + \frac{1}{2}(xd_2 - x_2)^2\,dt \qquad (6.7)$$

where x_1 are x_2 normalized CSTR concentration and temperatures respectively. xd_1 and xd_2 are state variables (feuron variables) of DFN.

CSTR initial states have been taken as zero. A PRBS has been applied to this system. Approximately in 83 iterations the equivalent DFN model responses converges to the CSTR responses. Figure 6.2 illustrates the change of DFN state responses for some iterations.

To examine the validity of obtained equivalent DFN model, we have also examined the steady-state responses and the eigenvalues of the equivalent systems. Figure 6.3 shows that both system responds exactly the same.

6.2.2 Closed Loop Experiments

For validation of the equivalent DFN model obtained from the open loop experiments, it is necessary to examine some of the closed loop responses of the equivalent models. For this aim equivalent DFN model obtained from open loop training was used in an extended Kalman filter (EKF) in an observer design.

EKF is based on the linearization of a nonlinear system around an operating point or a trajectory and application of linear optimal state estimation to the obtained system [2, 14]. Figure 6.4 shows the block diagram of linearized EKF. The k indices is a discrete time indices. k/k-1 indices represents the k indices variable that uses $(k-1)^{th}$ indices and previous indices values.

Process model can be written as:

$$x_{k+1} = f(x_k, u_k) + w_k,$$

$$z_k = h(x_k) + v_k \qquad (6.8)$$

where w_k and v_k are random variables with Q and R covariance and zero mean respectively.

EKF equations can be summarized as:

1) One-step ahead prediction equations:

$$\hat{x}_{k/k-1} = \hat{f}(\hat{x}_{k-1/k-1}, u_{k-1}), \hat{z}_{k/k-1} = h(\hat{x}_{k/k-1})$$

2) Correction equations:

$$\hat{x}_{k/k} = \hat{x}_{k/k-1} + \hat{K}_k(z_k - \hat{z}_{k/k-1})$$

3) Covariance propagation

$$P_{k/k-1} = F_{k-1}P_{k-1/k-1}F_{k-1}^T + Q_{k-1}$$

4) Covariance correction:

$$P_{k/k} = (I - \hat{K}_k H_k)P_{k/k-1}$$

5) Filter gain:

$$\hat{K}_k = P_{k/k-1}H_k^T(H_k P_{k/k-1}H_k^T + R_k)^{-1}$$

The gradients with respect to system state variables are:

$$F_{k-1} = \left.\frac{\partial \hat{f}}{\partial x}\right|_{\hat{x}_{k-1/k-1}} \quad , \quad H_k = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k-1/k-1}}$$

$$(6.9)$$

For one step ahead prediction equations $\hat{x}_{0/-1}$ and for covariance propagation $P_{0/-1}$ values should be selected first.

To test the generalized modeling principles, the equivalent DFN model obtained from the open loop training has been used in linearized EKF to estimate the CSTR model states. Figure 6.5 shows CSTR response with covariance process noise and zero initial conditions to zero input. When EKF has been applied for state estimation with the initial parameters chosen as:

$$\hat{x}_{0/-1} = x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad P_{0/-1} = \begin{bmatrix} 10^{-2} & 0 \\ 0 & 10^{-2} \end{bmatrix},$$

$$Q_k = \begin{bmatrix} 10^{-4} & 0 \\ 0 & 10^{-4} \end{bmatrix}, \quad R_k = \begin{bmatrix} 10^{-2} & 0 \\ 0 & 10^{-2} \end{bmatrix}$$

it can be seen that estimation was successful (Figure 6.5).

Closed loop experiments can be expanded for various closed loop control appications from PID controller to fuzzy controllers, etc. In [17] the results of the closed loop PID control systems for a CSTR and its equivalent DFN models have been shown. It has been seen that equivalent models exactly gave the same responses to the PID controller under various disturbance effects.
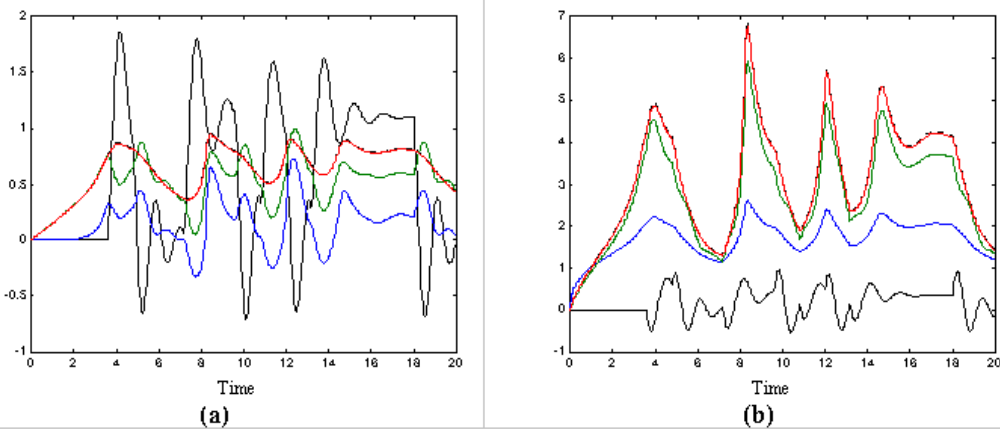

Figure 6.2 DFN (model) response (solid curves at start) (a) concentration (b) temperature. The dashed and near solid curves are Process and DFN (converged model) responses, respectively.
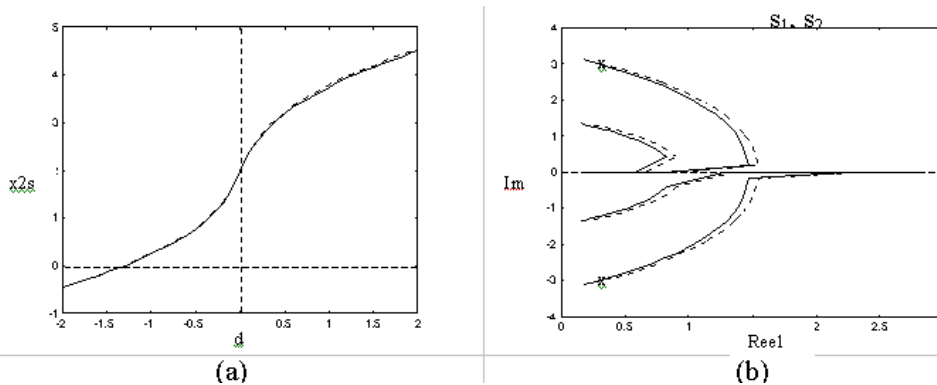

Figure 6.3 The solid and near dashed curves are Process and DFN responses to change in feed temperature respectively (a) steady-state responses, (b) the change of eigenvalues.
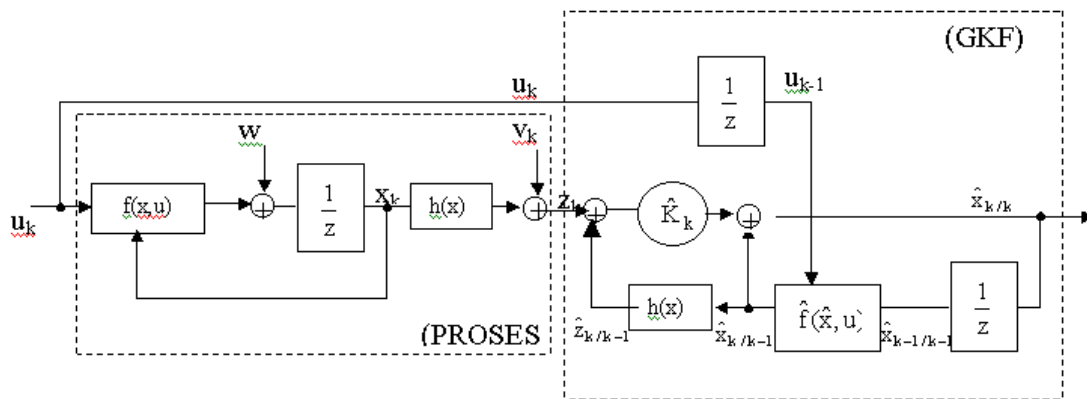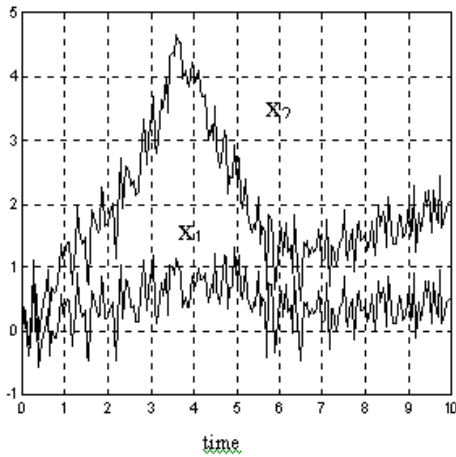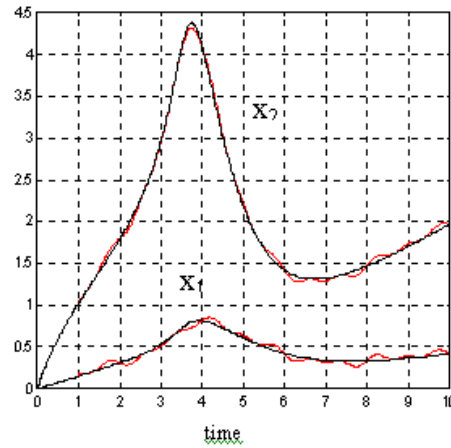
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Figure 6.4 Block diagram of linearized Extended Kalman Filter (EKF)

(a)

(b)

Figure 6.5 (a) CSTR responses with white process and measurement noise, (b) CSTR responses without noise (solid curve) and EKF with DFN estimations (dashed curves)

7. CONCLUSIONS

This work is the result of our ongoing research on Intelligent Systems. It is a companion paper to earlier works given in [8, 10, 12] on Dynamic Neural Networks. The work reported here has concentrated on laying the theoretical and analytic foundations for training of Dynamic Fuzzy Network DFN. They are to be used in real-time applications with process modeling and advanced control.

For modeling with DFN, "extended equivalence conditions" have been defined. DFN model has been trained both in open loop and closed loop forms to satisfy these equivalence conditions.

Several system trajectories with a PRBS input have been used for open loop training. DFN model obtained from open loop training was used in an extended Kalman filter (EKF) in an observer design for a nonlinear system. It has been shown by simulations that DFN system models satisfy all extended equivalence conditions.

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