Springer Link

Triangular mesh parameterization with trimmed surfaces

International Journal on Interactive Design and Manufacturing (IJIDeM)
November 2015, Volume 9, Issue 4, pp 303–316

Authors Authors and affiliations

Oscar E. Ruiz , Daniel Mejia, Carlos A. Cadavid

Original Paper

First Online: 28 April 2015

DOI (Digital Object Identifier): 10.1007/s12008-015-0276-1

Cite this article as:

Ruiz, O.E., Mejia, D. & Cadavid, C.A. Int J Interact Des Manuf (2015) 9: 303. doi:10.1007/s12008-015-0276-1

Abstract

Given a 2-manifold triangular mesh $M \subset {\mathbb {R}}^3$, with border, a parameterization of M is a FACE or trimmed surface $F=\{S,L_0,\ldots ,L_m\}$. F is a connected subset or region of a parametric surface S, bounded by a set of LOOPs $L_0,\ldots ,L_m$ such that each $L_i \subset S$ is a closed 1-manifold having no intersection with the other L_j LOOPs. The parametric surface S is a statistical fit of the mesh M. L_0 is the outermost LOOP bounding F and L_i is the LOOP of the i-th hole in F (if any). The problem of parameterizing triangular meshes is relevant for reverse engineering, tool path planning, feature detection, re-design, etc. State-of-art mesh procedures parameterize a rectangular mesh M. To improve such procedures, we report here the implementation of an algorithm which parameterizes meshes M presenting holes and concavities. We synthesize a parametric surface $S \subset {\mathbb {R}}^3$ which approximates a superset of the mesh M. Then, we compute a set of LOOPs trimming S, and therefore completing the FACE $F=\{S,L_0,\ldots ,L_m\}$. Our algorithm gives satisfactory results for M having low Gaussian curvature (i.e., M being quasi-developable or developable). This assumption is a reasonable one, since M is the product of manifold segmentation pre-processing. Our algorithm computes: (1) a manifold learning mapping $\phi : M \rightarrow U \subset {\mathbb {R}}^2$, (2) an inverse mapping $S: W \subset {\mathbb {R}}^2 \rightarrow {\mathbb {R}}^3$, with W being a rectangular grid containing and surpassing U. To compute ϕ we test IsoMap, Laplacian Eigenmaps and Hessian local linear embedding (best results with HLLE). For the back mapping (NURBS) S the crucial step is to find a control polyhedron P, which is an extrapolation of M. We calculate P by extrapolating radial basis functions that interpolate points inside $\phi (M)$. We successfully test our implementation with several datasets presenting concavities, holes, and are extremely non-developable. Ongoing work is being devoted to manifold segmentation which facilitates mesh parameterization.

Keywords

Triangular mesh parameterization Trimmed surface Manifold learning NURBS RBFs

Abbreviations

LOOP
Closed (piecewise linear or smooth) curve lying on a surface, and bounding a connected region on the surface. In this manuscript, LOOPs are denoted with Γ or γ

B-REP
Boundary representation

HLLE
Hessian locally linear embedding

NURBS
Non-uniform rational B-spline

RBF
Radial basis function

M
Triangular mesh (with boundary), composed by the set of triangles $T=\{t_1,t_2,\ldots ,t_q\}$ with vertex set $X=\{x_1,x_2,\ldots ,x_n\}$ ($X \subset {\mathbb {R}}^3$)

∂M
Boundary of M, whose connected components are LOOPs ($\partial M = \{\Gamma _0, \Gamma _1,\ldots ,\Gamma _k \}$)

ϕ
An homeomorphic map $\phi :M\rightarrow {\mathbb {R}}^2$, implemented here for dimensional reduction or manifold learning. $\phi _{Isom}()$, $\phi _{Lapl}()$, $\phi _{\textit{HLLE}}()$, are the Isomap, Laplacian Eigenmap and Hessian locally linear embedding implementations, respectively. $\phi ()$ is called *forward* map in this manuscript

U

$U=\{u_1,u_2,\ldots ,u_n\}$ is the parametric image of vertices of M ($U=\phi (X)$, $U \subset {\mathbb {R}}^2$)

$\partial (\phi (M))$
Boundary of the parametric image of M. For the sake of simplicity, we assume that $\partial (\phi (M)) = \phi (\partial (M))$

$\gamma _i$
i-th LOOPs of $\partial (\phi (M))$

$\lambda _i$
Re-sampling of a LOOP $\gamma _i$

W
Rectangular grid in R^2 such that U lies in the convex hull of W

$H(W)$
Rectangular point set in R^2 being the convex hull of W

P
Rectangular grid in ${\mathbb {R}}^3$ being the control polyhedron for the parametric surface f

f
Function $f: W \rightarrow {\mathbb {R}}^3$ produces P the control polyhedron of S ($P=f(W)$) by calculating an extrapolation of M in ${\mathbb {R}}^3$

S
$S:{\mathbb {R}}^2 \rightarrow {\mathbb {R}}^3$ is a parametric surface which approximates and extends M in ${\mathbb {R}}^3$. $S()$ is called *backward* map in this manuscript. To simplify notation, S refers here to both: (1) the parametric mapping (i.e., $S()$) and (2) the set of points product of the mapping $S( )$ (i.e., $S(H(W))= \{ S(w_1,w_2) \mid (w_1,w_2) \in H(W)\}$)

L_i
Trimming curve in $M \subset {\mathbb {R}}^3$ defined as $L_i = S(\lambda _i)$

F
Trimmed surface (FACE) such that $F \!=\! (S,\{L_0,L_1,\ldots \})$

∂F
Boundary of F approximated by the union of all L_i

References

1. Zheng, J., Chan, K., Gibson, I.: Constrained deformation of freeform surfaces using surface features for interactive design. Int. J. Adv. Manuf. Technol. (2003). doi:10.1007/s00170-002-1442-8 (http://dx.doi.org/10.1007/s00170-002-1442-8)

2. Yoshizawa, S., Belyaev, A., Seidel, H.P.: A fast and simple stretch-minimizing mesh parameterization. In: Proc. Shape Model. Appl. (2004). doi:10.1109/SMI.2004.1314507 (http://dx.doi.org/10.1109/SMI.2004.1314507)

3. Specht, M., Lebrun, R., Zollikofer, C.P.: Visualizing shape transformation between chimpanzee and human braincases. Vis. Comput. (2007). doi:10.1007/s00371-007-0156-1 (http://dx.doi.org/10.1007/s00371-007-0156-1)

4. Krishnamurthy, A., Khardekar, R., McMains, S.: Optimized GPU evaluation of arbitrary degree NURBS curves and surfaces. Comput.-Aided Des. (2009). doi:10.1016/j.cad.2009.06.015 (http://dx.doi.org/10.1016/j.cad.2009.06.015)

5. Pietroni, N., Massimiliano, C., Cignoni, P., Scopigno, R.: An interactive local flattening operator to support digital investigations on artwork surfaces. IEEE Trans. Vis. Comput. Graph. (2011). doi:10.1109/TVCG.2011.165 (http://dx.doi.org/10.1109/TVCG.2011.165)

6. Liu, X.M., Wang, S.M., Hao, A.M., Liu, H.: Realistic rendering of organ for surgery simulator. Comput. Math. Appl. (2012). doi:10.1016/j.camwa.2011.11.030 (http://dx.doi.org/10.1016/j.camwa.2011.11.030)

7. Tierny, J., Daniels II, J., Nonato, L.G., Pascucci, V., Silva, C.T.: Interactive quadrangulation with reeb atlases and connectivity textures. IEEE Trans. Vis. Comput. Graph. (2012). doi:10.1109/TVCG.2011.270 (http://dx.doi.org/10.1109/TVCG.2011.270)

8. Zhu, X.F., Hu, P., Ma, Z.D., Zhang, X., Li, W., Bao, J., Liu, M.: A new surface parameterization method based on one-step inverse forming for isogeometric analysis-suited geometry. Int. J. Adv. Manuf. Technol. (2013). doi:10.1007/s00170-012-4251-8 (http://dx.doi.org/10.1007/s00170-012-4251-8)

9. Zuo, B.Q., Huang, Z.D., Wang, Y.W., Wu, Z.J.: Isogeometric analysis for CSG models. Comput. Methods Appl. Mech. Eng. (2015). doi:10.1016/j.cma.2014.10.046 (http://dx.doi.org/10.1016/j.cma.2014.10.046)

10. Li, G., Ren, C., Zhang, J., Ma, W.: Approximation of Loop Subdivision Surfaces for Fast Rendering. IEEE Trans. Vis. Comput. Graph. (2011). doi:10.1109/TVCG.2010.83 (http://dx.doi.org/10.1109/TVCG.2010.83)

11. Yang, L., He, D., Zhang, Z.: Construct G1 Smooth surface by using triangular gregory patches. In: Fifth Int. Conf. Image Graph—ICIG '09. (2009). doi:10.1109/ICIG.2009.55 (http://dx.doi.org/10.1109/ICIG.2009.55)

12. Dyken, C., Reimers, M., Seland, J.: Real-time GPU silhouette refinement using adaptively blended Bézier patches. Comput. Graph. Forum **27**(1), 1–12 (2008) CrossRef (http://dx.doi.org/10.1111/j.1467-8659.2007.01030.x)

13. Zhang, Z., Wang, Z., He, D.: A new bi-cubic triangular gregory patch. In: Int. Conf. Comput. Sci. Softw. Eng. (2008). doi:10.1109/CSSE.2008.298 (http://dx.doi.org/10.1109/CSSE.2008.298)

14. Boubekeur, T., Reuter, P., Schlick, C.: Scalar tagged PN triangles. In: Eurographics Short Papers, Eurographics Association and Blackwell, Dublin, Ireland (2005)

15. Mao, Z., Ma, L., Tan, W.: A modified nielsons side-vertex triangular mesh interpolation scheme. In: Gervasi, O., Gavrilova, M., Kumar, V., Laganà, A., Lee, H., Mun, Y., Taniar, D., Tan, C. (eds.) Computational science and its applications ICCSA 2005, pp. 776–785. Springer, Berlin (2005) CrossRef (http://dx.doi.org/10.1007/11424758_80)

16. Vlachos, A., Peters, J., Boyd, C., Mitchell, J.L.: Curved PN triangles. In: Proc. 2001 Symp. Interact. 3D Graph (2001). doi:10.1145/364338.364387

(http://dx.doi.org/10.1145/364338.364387)

17. Acosta, D.A., Ruiz, O.E., Arroyave, S., Ebratt, R., Cadavid, C., Londono, J.J.: Geodesic-based manifold learning for parameterization of triangular meshes. Int. J. Interact. Des. Manuf. (IJIDeM) (2014). doi:10.1007/s12008-014-0249-9 (http://dx.doi.org/10.1007/s12008-014-0249-9)

18. Yu, H., Lee, T.Y., Yeh, I.C., Yang, X., Li, W., Zhang, J.J.: An RBF-based reparameterization method for constrained texture mapping. IEEE Trans. Vis. Comput. Graph. (2012). doi:10.1109/TVCG.2011.117 (http://dx.doi.org/10.1109/TVCG.2011.117)

19. Guo, Y., Wang, J., Sun, H., Cui, X., Peng, Q.: A novel constrained texture mapping method based on harmonic map. Comput. Graph. (2005). doi:10.1016/j.cag.2005.09.013 (http://dx.doi.org/10.1016/j.cag.2005.09.013)

20. Belkin, M., Niyogi, P.: Laplacian Eigenmaps and spectral techniques for embedding and clustering. In: Neural Inf. Process. Syst.: Nat. and Synth.—NIPS, MIT Press, Vancouver, Canada (2001)

21. Sheffer, A., de Sturler, E.: Parameterization of faceted surfaces for meshing using angle-based flattening. Eng. Comput. (2001). doi:10.1007/PL00013391 (http://dx.doi.org/10.1007/PL00013391)

22. Zhao, X., Su, Z., Gu, X.D., Kaufman, A., Sun, J., Gao, J., Luo, F.: Area-preservation mapping using optimal mass transport. IEEE Trans. Vis. Comput. Graph. (2013). doi:10.1109/TVCG.2013.135 (http://dx.doi.org/10.1109/TVCG.2013.135)

23. Zou, G., Hu, J., Gu, X., Hua, J.: Authalic parameterization of general surfaces using lie advection. IEEE Trans. Vis. Comput. Graph. (2011). doi:10.1109/TVCG.2011.171 (http://dx.doi.org/10.1109/TVCG.2011.171)

24. Pietroni, N., Tarini, M., Cignoni, P.: Almost isometric mesh parameterization through abstract domains. IEEE Trans. Vis. Comput. Graph. (2010). doi:10.1109/TVCG.2009.96 (http://dx.doi.org/10.1109/TVCG.2009.96)

25. Liu, L., Zhang, L., Xu, Y., Gotsman, C., Gortler, S.J.: A local/global approach to mesh parameterization. Comput. Graph. Forum (2008). doi:10.1111/j.1467-8659.2008.01290.x (http://dx.doi.org/10.1111/j.1467-8659.2008.01290.x)

26. Sun, X., Hancock, E.R.: Quasi-isometric parameterization for texture mapping. Pattern Recogn. (2008). doi:10.1016/j.patcog.2007.10.027 (http://dx.doi.org/10.1016/j.patcog.2007.10.027)

27. Desbrun, M., Meyer, M., Alliez, P.: Intrinsic parameterizations of surface meshes. Comput. Graph. Forum (2002). doi:10.1111/1467-8659.00580 (http://dx.doi.org/10.1111/1467-8659.00580)

28. Tenenbaum, J.B., de Silva, V., Langford, J.C.: A global geometric framework for nonlinear dimensionality reduction. Sci. (NY) (2000). doi:10.1126/science.290.5500.2319 (http://dx.doi.org/10.1126/science.290.5500.2319)

29. Donoho, D.L., Grimes, C.: Hessian eigenmaps: locally linear embedding techniques for high-dimensional data. Proc. Natl. Acad. Sci. (2003). doi:10.1073/pnas.1031596100 (http://dx.doi.org/10.1073/pnas.1031596100)

30. Bookstein, F.L.: Principal warps: thin-plate splines and the decomposition of deformations. IEEE Trans. Pattern Anal. Mach. Intell. (1989). doi:10.1109/34.24792 (http://dx.doi.org/10.1109/34.24792)

Copyright information

About this article

CrossMark

Print ISSN	Online ISSN	Publisher Name
1955-2513	1955-2505	Springer Paris

About this journal

Reprints and Permissions