Finite difference calculations of permeability in large domains in a wide porosity range

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Abstract

Determining effective hydraulic, thermal, mechanical and electrical properties of porous materials by means of classical physical experiments is often time-consuming and expensive. Thus, accurate numerical calculations of material properties are of increasing interest in geophysical, manufacturing, bio-mechanical and environmental applications, among other fields. Characteristic material properties (e.g. intrinsic permeability, thermal conductivity and elastic moduli) depend on morphological details on the porescale such as shape and size of pores and pore throats or cracks. To obtain reliable predictions of these properties it is necessary to perform numerical analyses of sufficiently large unit cells. Such representative volume elements require optimized numerical simulation techniques. Current state-of-the-art simulation tools to calculate effective permeabilities of porous materials are based on various methods, e.g. lattice Boltzmann, finite volumes or explicit jump Stokes methods. All approaches still have limitations in the maximum size of the simulation domain. In response to these deficits of the well-established methods we propose an efficient and reliable numerical method which allows to calculate intrinsic permeabilities directly from voxel-based data obtained from 3D imaging techniques like X-ray microtomography. We present a modelling framework based on a parallel finite differences solver, allowing the calculation of large domains with relative low computing requirements (i.e. desktop computers). The presented method is validated in a diverse selection of materials, obtaining accurate results for a large range of porosities, wider than the ranges previously reported. Ongoing work includes the estimation of other effective properties of porous media.

Keywords

Effective permeability Porous materials Digital rock physics

List of symbols

$\mathbf{u}$
Fluid velocity on porescale (m/s)

d_s
Sphere diameter (m)

$k^\mathfrak{s}$
Intrinsic permeability ($\hbox{m}^2$)

p
Pressure (Pa)

r_t
Radius of capillary tube (m)

Re
RVE-scale Reynolds number (–)

L
Characteristic size of the investigated RVE domain (–)

u_m
Volume-averaged velocity (m/s)

Δp
Pressure drop in the medium (Pa/m)

η
Effective dynamic viscosity of the fluid (Pa s)

ρ
Density of the fluid ($\hbox{kg}/\hbox{m}^3$)

ϕ
Porosity of the material (–)

${\varOmega}$
Domain of investigated material in $\mathbb{R}^3$

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