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A Fixed Time Convergent Dynamical System to Solve Linear Programming

Juan Diego Sánchez-Torres, Martin J. Loza-Lopez, Riemann Ruiz-Cruz, Edgar N. Sanchez
and Alexander G. Loukianov

Abstract—The aim of this paper is to present a new dynamical system which solves linear programming. Its design is considered as a sliding mode control problem, where its structure is based on the Karush-Kuhn-Tucker optimality conditions, and its multipliers are the control inputs to be implemented by using fixed time stabilizing terms with vectorial structure, based on the unit control, instead of common terms used in other approaches. Thus, the main features of the proposed system are the fixed convergence time to the programming solution and the fixed parameters number despite of the optimization problem dimension. That is, there is a time independent to the initial conditions in which the system converges to the solution and, the proposed structure can be easily scaled from a small to a higher dimension problem. The applicability of the proposed scheme is tested on real-time optimization of an electrical Microgrid prototype.

I. INTRODUCTION

Optimization methods have been widely applied in science and engineering. The optimization goal is to determine the decision variables values, which maximize or minimize an objective function, sometimes, subject to constraints. Some of this problems are large-scale real-time linear programming procedures. For such applications, sequential algorithms as the classical simplex or the interior point methods are often proposed. However, those traditional approaches may not be efficient since the computing time required for a solution is greatly dependent on the problem dimension and structure.

The use of dynamical systems which can solve real-time optimization was introduced in [1] and arises as a promising alternative. A major contribution to this class of solutions is the use of systems with motion on a sliding manifold, as proposed in [2], that is an integral manifold with finite reaching time [3], presented by some non-smooth systems, providing finite time convergence to the problem solution. Extensions of the mentioned schemes were presented for linear programming [4]–[6] and for nonlinear programming [7]. Some of them are finite time convergence approaches [8]–[10] and fixed time convergence [11]. For most of the cases, these systems are presented as the solution to a controller design problem [12] (including the case of sliding

mode control [13]), in the form of circuits [14], [15] or under the computational paradigm of the so-called artificial neural networks where are known as recurrent neural networks [16]. Due to its inherent massive parallelism, those systems are able to solve optimization problems in running time at the orders of magnitude much faster than those of the most popular optimization algorithms executed on general-purpose digital computers [17], with unusual flexibility because the system constantly seeks new solutions as the parameters of the problem are varied [1].

Although the mentioned works exhibit high performance, it is necessary to tune the network parameters such that the optimizer trajectories converge to the optimization solution. For most of the cases, the number of network parameters increases linearly with the optimization problem dimension, since for every decision variable there is an individual selection of each activation function. In addition, the fixed time characteristic is not presented in most of the mentioned references. This last desirable property allows the design of systems with a known and predefined convergence time.

In this paper, a dynamical system for the solution of linear programming is proposed. Its design is considered as a sliding mode control problem, where the network structure is based on the Karush-Kuhn-Tucker (KKT) optimality conditions [18], [19] and the KKT multipliers are regarded as control inputs. At this point, a controller with vectorial structure and fixed time stability is proposed. It allows the problem to be solved without the individual selection of each stabilizing input, instead a multivariable function, based on the unit control [20], [21], is used. On the other hand, the fixed time stability [22], [23] ensures the existence of a time independent to the initial conditions in which the system converges. This controller is used to the KKT multiplier design, enforcing a sliding mode in which the optimization problem is solved.

Thus, the proposed approach have very attractive features as: fixed time convergence to the optimization problem solution and a fixed parameters number (four for this case), regardless of the optimization problem dimension. Therefore, it offers the scalability characteristic, that allows the on-line solution of problems with low and higher dimension without major changes of the system.

On the other hand, the Microgrids are a challenging benchmark for control, optimization and instrumentation. Several studies have been performed to Microgrids, some interesting examples are [24], [25]. Therefore, as case study, this proposal is applied to determine the optimal amounts

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of power supplied by each energy source in a Microgrid prototype. These grids present problems as the time varying load demand and the non-conventional/renewable sources availability, requiring to solve large-scale real-time optimization procedures, most of them in the form of linear programming. As mentioned above, in contrast to the publications which use recurrent neural networks for Microgrid optimization [26], [27], the proposed approach provides fixed convergence time to the solution and the tuning of only four network parameters.

In the following, Section II presents the mathematical preliminaries and some useful definitions. Section III describes the proposed system for the solution of linear programming, including the stability analysis and an academic example which illustrates the fixed time convergence feature of the system. An application as the real-time microgrid optimization results are presented in Section IV. Finally, in Section V the conclusions are presented.

II. MATHEMATICAL PRELIMINARIES

Consider the system

$$\dot{\xi} = f(t, \xi) \quad (1)$$

where $\xi \in \mathbb{R}^n$ and $f : \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n$. If f is a discontinuous (or non-smooth) function, (1) is understood in Filippov sense [28].

Definition 1 (Globally fixed-time attraction [23]): Let a non-empty set $M \subset \mathbb{R}^n$. It is said to be globally fixed-time attractive for the system (1) if any solution $\xi(t, \xi_0)$ of (1) reaches M in some finite time moment $t = T(\xi_0)$ and the settling-time function $T(\xi_0) : \mathbb{R}^n \rightarrow \mathbb{R}_+ \cup \{0\}$ is bounded by some positive number T_{\max} , i.e. $T(\xi_0) \leq T_{\max}$ for $\xi_0 \in \mathbb{R}^n$.

With the definition of a globally fixed-time attractive set, the following lemma provides a Lyapunov characterization of these sets on the state space

Lemma 1 (Lyapunov function [23]): If there exists a continuous radially unbounded function

$$V : \mathbb{R}^n \rightarrow \mathbb{R}_+ \cup \{0\}$$

such that $V(\xi) = 0$ for $\xi \in M$ and any solution $\xi(t)$ satisfies

$$\dot{V} \leq -(\alpha V^p(\xi(t)) + \beta V^q(\xi(t)))^k$$

for $\alpha, \beta, p, q, k > 0$ that $pk < 1$ and $qk > 1$, then the set M is globally fixed-time attractive for the system (1) and $T_{\max} = \frac{1}{\alpha^k(1-pk)} + \frac{1}{\beta^k(qk-1)}$.

III. OPTIMIZER DESIGN

A. Preliminary Result

Before to present the fixed time optimizer, it will be exposed a new class of fixed time stabilizer to be used in the optimizing system design. For this, consider the equation

$$\dot{\xi} = \phi(\xi, t) + u \quad (2)$$

with $\xi, u \in \mathbb{R}^n$ and $\phi : \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n$. The main objective is to drive the system (2) to the point $\xi = 0$ in a predefined fixed time in spite of the unknown non-vanishing disturbance

$\phi(\xi, t)$. A solution to this problem which does not requires an individual selection of each of the n control variables based on the *unit control* is presented in the following theorem:

Theorem 1 (Fixed time multivariable control): Let the function $\phi(\xi, t)$ to be bounded as $\|\phi(\xi, t)\| \leq a + b \|\xi\|^c$, with $a > 0$, $b \geq 0$, $c \geq 1$ known constants. Then, by selecting the control input

$$u = -a \frac{\xi}{\|\xi\|} - b \xi \|\xi\|^{c-1} - \frac{k_1}{2} \frac{\xi}{\|\xi\|^{2(1-p)}} - \frac{k_2}{2} \xi \|\xi\|^{2(q-1)}$$

with $k_1 > 0$, $k_2 > 0$, $0 < p < 1$ and $q > 1$ being scalars, the system (2) is globally fixed-time stable with settling-time $T_{\max} = \frac{1}{k_1(1-p)} + \frac{1}{k_2(q-1)}$.

Proof: Let the Lyapunov function $V = \|\xi\|^2$, its derivative is given by $\dot{V} = 2\xi^T \dot{\xi}$. Therefore

$$\begin{aligned} \dot{V} &= 2\xi^T \phi - 2a \|\xi\| - 2b \|\xi\|^{c+1} \\ &\quad - k_1 \|\xi\|^{2p} - k_2 \|\xi\|^{2q} \\ &\leq 2 \|\xi\| \|\phi\| - 2a \|\xi\| - 2b \|\xi\|^{c+1} \\ &\quad - k_1 \|\xi\|^{2p} - k_2 \|\xi\|^{2q}, \end{aligned} \quad (3)$$

that, by replacing the bound for ϕ , reduces to $\dot{V} \leq -k_1 \|\xi\|^{2p} - k_2 \|\xi\|^{2q}$ which is equivalent to

$$\dot{V} \leq -k_1 V^p - k_2 V^q.$$

Finally, by direct application of *Lemma 1* with $k = 1$, the proof is finished. ■

B. Fixed Time Solution of Linear Programming

Consider the linear programming problem

$$\begin{cases} \min_x & \mathbf{c}^T x \\ \text{s.t} & \mathbf{A}x = \mathbf{b} \\ & l \leq x \leq h \end{cases} \quad (4)$$

where $x = [x_1 \dots x_n]^T \in \mathbb{R}^n$ are the decision variables, $\mathbf{c} \in \mathbb{R}^n$ is a cost vector, \mathbf{A} is an $m \times n$ matrix such that $\text{rank}(\mathbf{A}) = m$ and $m \leq n$; \mathbf{b} is a vector in \mathbb{R}^m and, $l = [l_1 \dots l_n]$, $h = [h_1 \dots h_n] \in \mathbb{R}^n$.

Let $y = [y_1 \dots y_m]^T \in \mathbb{R}^m$ and $z = [z_1 \dots z_n]^T \in \mathbb{R}^n$.

The Lagrangian of (4) is formed as

$$L(x, y, z) = \mathbf{c}^T x + z^T x + y^T (\mathbf{A}x - \mathbf{b}). \quad (5)$$

The KKT conditions establishes that x^* is a solution for (4) if and only if x^* , y and z in (4)-(5) are such that

$$\nabla_x L(x^*, y, z) = \mathbf{c} + z + \mathbf{A}^T y = 0 \quad (6)$$

$$\mathbf{A}x^* - \mathbf{b} = 0 \quad (7)$$

$$z_i x_i^* = 0 \text{ if } l_i < x_i^* < h_i, \forall i = 1, \dots, n. \quad (8)$$

Following the KKT approach, the solution for (4) is such that $x^* \in \Omega$ where $\Omega = \text{int}(\Omega_d \cap \Omega_e)$ with

$$\begin{aligned} \Omega_e &= \{x \in \mathbb{R}^n : \mathbf{A}x - \mathbf{b} = 0\} \\ \Omega_d &= \{x \in \mathbb{R}^n : l \leq x \leq h\}. \end{aligned} \quad (9)$$

Then, y and z must be designed such that Ω is a fixed time attractive set, fulfilling conditions (6)-(8).

In addition to condition (8), z is considered as

$$\begin{cases} z_i \geq 0 & \text{if } x_i \geq h_i \\ z_i \leq 0 & \text{if } x_i \leq l_i \end{cases} \quad (10)$$

and the variable $\sigma \in \mathbb{R}^m$ is defined as $\sigma = \mathbf{A}x - \mathbf{b}$. Hence, with basis on *Theorem 1* and considering the conditions (6)-(10) a continuous fixed time solver for the problem (4) is proposed in the following Lemma:

Lemma 2 (Fixed Time Solver for Linear Programming):

For the dynamical system

$$\dot{x} = -\mathbf{c} + \mathbf{A}^T y + z \quad (11)$$

with the variables y and z proposed as the multivariable control inputs

$$\begin{aligned} y &= \phi(\sigma) \\ z &= \varphi(x, l, h) \end{aligned} \quad (12)$$

defined by

$$\phi(\sigma) = \begin{cases} -\frac{\|\mathbf{c}\| \|\sigma\|}{\|\sigma\|} - \frac{k_1}{2} \frac{\|(\mathbf{A}\mathbf{A}^T)^{-1}\|^{p_1} \sigma}{\|\sigma\|^{2(1-p_1)}} \\ -\frac{k_2}{2} \|(\mathbf{A}\mathbf{A}^T)^{-1}\|^{q_1} \|\sigma\|^{2(q_1-1)} \sigma \\ \text{if } l \leq x \leq h \\ 0 \text{ if } x < l \text{ or } x > h \end{cases} \quad (13)$$

and $\varphi(x, l, h) = [\varphi_1(x, l_1, h_1) \quad \dots \quad \varphi_n(x, l_n, h_n)]^T$, with $\varphi_i(x, l_i, h_i)$ of the form

$$\varphi_i(x, l_i, h_i) = \begin{cases} -\frac{\|\mathbf{c}\| (x_i - l_i)}{\|x - l\|} - \frac{k_3}{2} \frac{(x_i - l_i)}{\|x - l\|^{2(1-p_2)}} \\ -\frac{k_4}{2} (x_i - l_i) \|x - l\|^{2(q_2-1)} \\ \text{if } x_i \leq l_i \\ 0 \text{ if } l_i < x_i < h_i \\ -\frac{\|\mathbf{c}\| (x_i - h_i)}{\|x - h\|} - \frac{k_3}{2} \frac{(x_i - h_i)}{\|x - h\|^{2(1-p_2)}} \\ -\frac{k_4}{2} (x_i - h_i) \|x - h\|^{2(q_2-1)} \\ \text{if } x_i \geq h_i \end{cases}, \quad (14)$$

and $k_1 > 0$, $k_2 > 0$, $k_3 > 0$, $k_4 > 0$, $0 < p_1 < 1$, $q_1 > 1$, $0 < p_2 < 1$ and $q_2 > 1$ are scalars, the point x^* is globally fixed-time stable with the settling-time $T_{\max} = \frac{1}{k_1(1-p_1)} + \frac{1}{k_2(q_1-1)} + \frac{1}{k_3(1-p_2)} + \frac{1}{k_4(q_2-1)}$.

Proof: In order to analyze the stability of the system (11) closed by (13)-(14) to the set Ω defined in (9), the following Lyapunov function is proposed:

$$V = \sigma^T (\mathbf{A}\mathbf{A}^T)^{-1} \sigma + x^T x \quad (15)$$

where it is highlighted the existence of $(\mathbf{A}\mathbf{A}^T)^{-1}$ due to \mathbf{A} is a full rank matrix.

The derivative of the Lyapunov function (15) along the trajectories of system (11) is given by

$$\dot{V} = 2(-\sigma^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{A}\mathbf{c} + \sigma^T \phi(\sigma) + \sigma^T (\mathbf{A}\mathbf{A}^T)^{-1} \times \mathbf{A}\varphi(x, l, h) - x^T \mathbf{c} + x^T \mathbf{A}^T \phi(\sigma) + x^T \varphi(x, l, h)).$$

From (13) and (14), \dot{V} can be written as

$$\dot{V} = \begin{cases} -\sigma^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{A}\mathbf{c} + \sigma^T \phi(\sigma) & \text{if } l \leq x \leq h \\ -x^T \mathbf{c} + x^T \varphi(x, l, h) & \text{if } x < l \text{ or } x > h. \end{cases} \quad (16)$$

Thus, similarly to (3), it follows that

$$\dot{V} \leq \begin{cases} -k_1 \|(\mathbf{A}\mathbf{A}^T)^{-1}\|^{p_1} \|\sigma\|^{2p_1} - k_2 \|(\mathbf{A}\mathbf{A}^T)^{-1}\|^{q_1} \|\sigma\|^{2q_1} \\ \text{if } l \leq x \leq h \\ -k_3 \|x\|^{2p_2} - k_4 \|x\|^{2q_2} & \text{if } x < l \text{ or } x > h \end{cases}$$

which leads to

$$\dot{V} \leq \begin{cases} -k_1 V^{p_1} - k_2 V^{q_1} & \text{if } l \leq x \leq h \\ -k_3 V^{p_2} - k_4 V^{q_2} & \text{if } x < l \text{ or } x > h. \end{cases}$$

By applying *Lemma 1* with $k = 1$, the conditions (7) and (8) are satisfied, guaranteeing fixed time convergence to the set Ω . Now, by using the *equivalent control method* [21], the solution of $\dot{x} = 0$ and $\dot{\sigma} = 0$ in (11) for $t > T_{\max}$ has the form $\mathbf{c} + \mathbf{A}^T \{\phi(\sigma)\}_{\text{eq}} + \{\varphi(x, l, h)\}_{\text{eq}} = 0$. Therefore, the condition (6) is fulfilled, implying the point $x^* \in \Omega$ is globally fixed-time stable. ■

Note that, in contrast to the common approaches presented in the literature, this scheme only needs the tuning of four gains in spite of the problem dimensions.

C. An Academic Example

Consider the linear programming problem [10]

$$\begin{cases} \min_x & 4x_1 + x_2 + 2x_3 \\ \text{s.t.} & x_1 - 2x_2 + x_3 = 2 \\ & -x_1 + 2x_2 + x_3 = 1 \\ & -5 \leq x_1, x_2, x_3 \leq 5. \end{cases} \quad (17)$$

In order to expose the performance of the proposed algorithm, the settling-time is selected as $T_{\max} = \frac{2}{3}$ seconds; as usual, the p and q parameters are selected as $p_1 = p_2 = \frac{1}{2}$ and $q_1 = q_2 = \frac{3}{2}$, respectively. Then, the gains for the system (11) are calculated as: $k_1 = 15$, $k_2 = 15$, $k_3 = 10$ and $k_4 = 10$, fulfilling the T_{\max} design condition. The results are shown in Fig. 1, displaying the obtained results of 35 simulations where the initial conditions are randomly selected within a range from -30 to 30 .

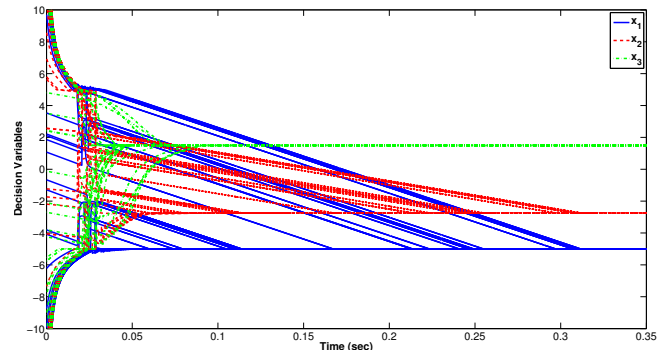


Fig. 1. Transient behavior of the x variables.

Here, it can be observed that the network converges to the optimal solution $x^* = [-5, -2.75, 1.5]$ before the prescribed settling-time.

IV. APPLICATION EXAMPLE: REAL TIME OPTIMIZATION OF A MICROGRID LABORATORY PROTOTYPE

The algorithm previously presented is applied to a Microgrid laboratory energy optimization problem solution.

A. Microgrid Prototype Description

The Microgrid prototype contains a *wind power system*, directly connected to the utility grid, and a DC voltage bus, which interconnects a *solar power system*, a *battery bank system* and a *load bank system*. The Microgrid prototype connection scheme is shown in Fig.2, and a properly picture is displayed in Fig. 3. All these devices are developed by Lab-Volt¹.

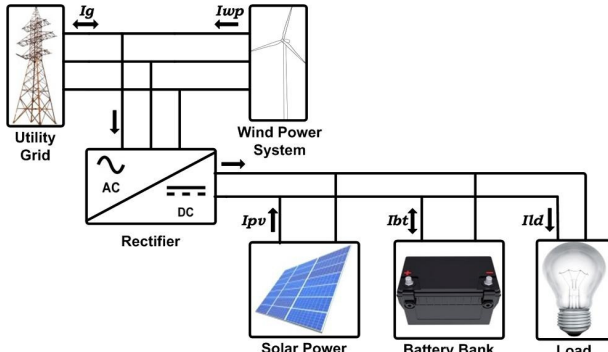


Fig. 2. Microgrid prototype connection scheme.



Fig. 3. Microgrid prototype.

B. Optimization Statement

In order to optimize the Microgrid, the condition for every device is presented as follows:

¹Lab-Volt,675, rue du Carbon G2N 2K7 Quebec, Quebec Canada.

1) *Wind Power System (WPS)*: WPS includes a doubly fed induction generator (DFIG), and a dynamometer which emulates the wind power. The generated power by this system (P_W) is fixed to:

$$P_W = \begin{cases} P_{W_{min}} & S_t \leq S_{min} \\ P_M & S_{min} \leq S_t \leq S_{max} \\ P_{W_{max}} & S_t \geq S_{max} \end{cases} \quad (18)$$

where S_t is the generator speed at time t , S_{min} is the generator minimum allowed speed, S_{max} is the generator maximum allowed speed and P_M is the calculated WPS power. For this test, S_{min} is set to 1840 rpm and S_{max} is set to 2000 rpm for a safe dynamometer functionality. Using these speed values, the minimum WPS power ($P_{W_{min}}$) is equal to 0 watts and the maximum ($P_{W_{max}}$) is 240 watts.

2) *Solar Power System* : The solar power system (SPS) is implemented by means of a two photovoltaic cells workbench. SPS power contribution (P_S) is bounded to:

$$P_{S_{min}} \leq P_{S_t} \leq P_{S_{max}} \quad (19)$$

where P_{S_t} is the SPS power at time t , $P_{S_{min}}$ is the minimum power obtained from this device, in this case 0 watts, and $P_{S_{max}}$ is the SPS maximum power, which for this module is 1.2 watts.

3) *Battery Bank System*: The Microgrid surplus power is stored in a battery bank system (BBS), which includes two lead-acid batteries. BBS power (P_B) must satisfy the next constraints:

$$P_{B_{min}} \leq P_{B_t} \leq P_{B_{max}} \quad (20)$$

where P_{B_t} is the BBS power at time t , $P_{B_{min}}$ is the BBS minimum allowed power and $P_{B_{max}}$ is the BBS maximum allowed power. The BBS maximum and minimum power are fixed in order to increase the batteries lifespan as long as possible. For this purpose $P_{B_{min}}$ is established as a 10% of its full charge value and $P_{B_{max}}$ as a 60%. Therefore, if the batteries have a power rate of 2.5 watts, $P_{B_{min}}$ and $P_{B_{max}}$ are set to 0.25 watts and 1.5 watts respectively.

4) *Utility Grid System*: The Microgrid laboratory has a junction point with the utility grid system all the time, as shown in Fig.2.

The power consumption for this system (P_G) is limited to:

$$P_{G_{min}} \leq P_{G_t} \leq P_{G_{max}} \quad (21)$$

where P_{G_t} is the utility power at time t , $P_{G_{min}}$ is the minimum allowed power consumption from the utility grid, for this test is set to 0 watts, and $P_{G_{max}}$ is the utility maximum allowed power. The utility grid can be considered as an infinite power source; however, for this test $P_{G_{max}}$ bound is set to 250 watts.

C. Proposed Optimizer

The main goal for this test is to optimize the power of the Microgrid based on the previously bounded energy sources, and the required output power of the load (P_L).

The optimization problem, using the equations (18) to (21), can be expressed as follows:

$$\left\{ \begin{array}{l} \text{Minimize } P_G - P_W - P_S - P_B \\ \text{s.t. } P_G + P_W + P_S + P_B = P_L \\ P_{G_{min}} \leq P_{G_t} \leq P_{G_{max}} \\ P_{W_{min}} \leq P_{W_t} \leq P_{W_{max}} \\ P_{S_{min}} \leq P_{S_t} \leq P_{S_{max}} \\ P_{B_{min}} \leq P_{B_t} \leq P_{B_{max}} \end{array} \right. \quad (22)$$

In order to match the form of the equation (4), the needed matrices are established as: $\mathbf{c}^T = [2 \ -1 \ -1 \ -1]^T$, $\mathbf{x} = [P_G \ P_W \ P_S \ P_B]^T$, $\mathbf{A} = [1 \ 1 \ 1 \ 1]$, $\mathbf{b} = [P_L]$, $\mathbf{l} = [P_{G_{min}} \ P_{W_{min}} \ P_{S_{min}} \ P_{B_{min}}]$ and $\mathbf{h} = [P_{G_{max}} \ P_{W_{max}} \ P_{S_{max}} \ P_{B_{max}}]$. The gains for the algorithm are set to: $k_1 = 1.5$, $k_2 = 1.5$, $k_3 = 20$ and $k_4 = 20$, with the parameters $p_1 = p_2 = \frac{1}{2}$ and $q_1 = q_2 = \frac{3}{2}$.

D. Real-Time Results

The presented optimization method uses the measured load power as the vector \mathbf{b} and the matrices defined in section IV-B, to set the references for the interconnected systems in real-time.

The given references for the SPS and BBS systems are continuous values, however, these modules can only be turned on or off to the Microgrid. For this reason a power high limit for activation and a power low limit for deactivation are established; i.e. if BBS power high limit is overcome, this module is connected to the Microgrid or if the power reference is lower than the power low limit, the module is disconnected. WPS has an internal PI controller to change the dynamometer speed and accomplish the power reference set for this module.

In order to test the optimization method on the Microgrid laboratory, at the beginning the prototype is left to stabilize the power without and output load connected. At 30s a 145Ω resistive load is connected to the DC voltage bus; then, at 60s and 90s, same value resistors in parallel configuration are plugged-in. A fourth 19Ω load is connected at 120s; this represents a high disturbance to the system. The loads are disconnected in the same order to show the transient behavior of the Microgrid, as Fig. 4 displays.

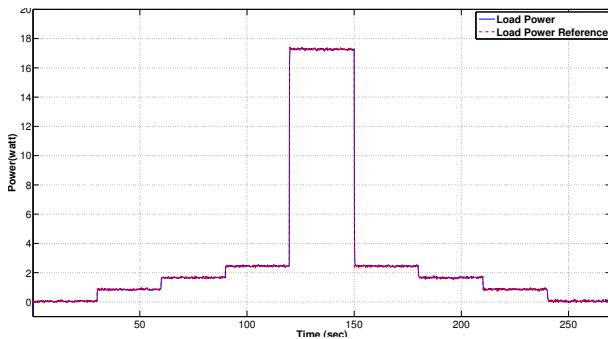


Fig. 4. Load power and references sum.

In Fig. 5, the utility grid behavior is shown. It can be seen that the real power is close to the $P_{G_{min}}$ which is set as 0 watts.

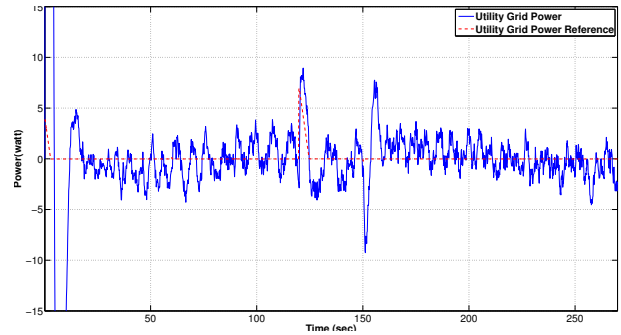


Fig. 5. Utility grid power and utility grid power reference.

In Fig. 6, the WPS power and its reference is displayed. It can be noted that this module approaches to its reference all the time. Fig. 7 and Fig. 8 show how SPS and BBS modules attend to reach their power references even though a related controller for these modules has not been developed yet. BBS tracking error is higher than SPS, because the power contribution of this module depends on the state of charge of the batteries.

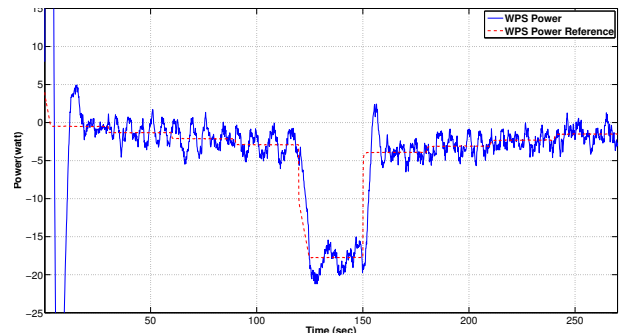


Fig. 6. Wind power and wind power reference.

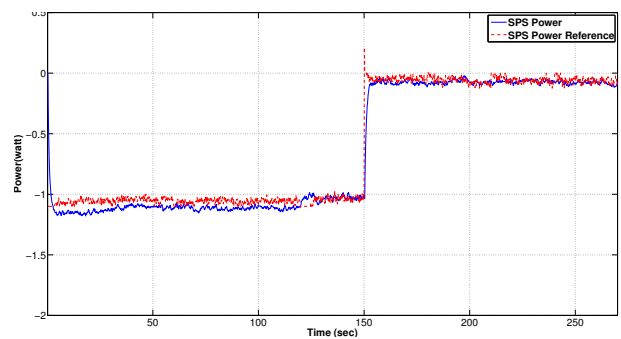


Fig. 7. Solar power and solar power reference.

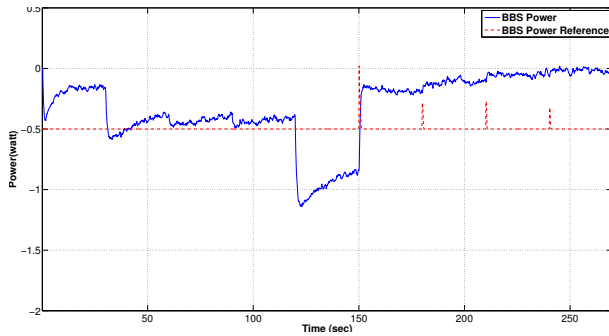


Fig. 8. Battery power and battery power reference.

V. CONCLUSION

In this paper, a new dynamical system which solves linear programming is presented. Its main features are fixed convergence time and fixed parameter number despite the problem dimension. In the simulation results it is shown the independence from the initial conditions and the convergence time. Furthermore, the method structure can be extended from a small to a high dimension problem statement, without a convergence time alteration. The optimization algorithm is used to obtain the optimal solution for a Microgrid laboratory energy distribution problem. The algorithm gives the references to track for all the energy sources connected to the prototype, minimizing the consumed power from the utility grid. The presented results validate the optimization method for a real-time application. As future work, the proposed algorithm will be extend to solve other convex optimization problems.

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