2011-10-01

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J. D. Sánchez-Torres, A. Loukianov, J. Patiño-Muriño, B. Giraldo and H. Botero-Castro, "An equivalent control based second order sliding mode observer using robust differentiators," Robotics Symposium, 2011 IEEE IX Latin American and IEEE Colombian Conference on Automatic Control and Industry Applications (LARC), Bogota, 2011, pp. 1-6.

An equivalent control based second order sliding mode observer using robust differentiators

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***Abstract*—In this paper a sliding-mode observer for a class of non-linear systems is proposed. The observer is based on the equivalent control method. The mathematical tools required to design such an observer are also presented. The proposed scheme can ensure finite time convergence of the observer and the reduction of chattering effect due to relay-type correction terms. Several examples are presented to illustrate the proposed method.**

I. Introduction

Sliding mode approaches have been widely used for the problems of dynamic systems control and observation due to their characteristics of finite time convergence, robustness to uncertainties and insensitivity to external bounded disturbances. In addition, the state observers have another important properties like the possibility of obtaining a step by step design and working with reduced observation dynamics [1, Ch. 4], [2] . Often, sliding mode motion is obtained by means of a discontinuous term depending on the output error, into the controlling or observing system. Additionally, by using the sign of the error to drive the sliding mode observer, the observer trajectories become insensitive to many forms of noise. Hence, some sliding mode observers have attractive properties similar to those of the Kalman filter (i.e. noise resilience) but with simpler implementation [3].

Several researchers have dealt with the issue of designing sliding-mode observers for different applications [4], [5], including the classical problem of non-linear state estimation [6]. Some applications of the sliding mode techniques to control and robust differentiation are presented in [7]. Note that the classical sliding modes techniques are a particular case of the high order sliding mode concept and can be considered as a first order sliding mode [8]. The high order sliding modes allow also to take into account the sampling measurement delays [8]. Some practical examples of the use high order sliding mode observers can be found in [9], [10]. All of these imply that the high order sliding mode observer is very convenient for real implantations.

In this work, our purpose is to discuss an observer design based over the equivalent control method for a class of observable SISO systems with some matching conditions on the input. This method has been treated previously in [11] and [12]. Subsequently, for the calculation of equivalent control, we propose the use of robust sliding mode differentiators (see [13], [14]) instead of the common used classic low-pass filters. In addition, the correction term of the observer is designed using the super-twisting algorithm, in order to ensure finite time convergence of the observer and the reduction of chattering effect due to relay-type correction terms. The other goal of this paper is to show that the proposed observer can also be applied to a chemical process system like the *Continuous Stirred Tank Reactor* (CSTR).

In the following, in section II some mathematical preliminaries are given. In section III an observer design based over the equivalent control method for a class of observable SISO systems with some matching conditions on the input is proposed. Two examples of the proposed observer, including an observer design for a CSTR and the use of an observer for synchronization of a chaotic system are presented in IV. Finally, in section V some conclusions are given.

II. Mathematical Preliminaries

In this section, we present the mathematical concepts required to formulate an observer design based over the equivalent control method. Basic topics of sliding mode observers based on equivalent control method are given and also the arbitrary-order differentiator and the super-twisting algorithm are introduced. These concepts are required for the observer design (and, by extension, to maintain its properties) to be proposed in section III. For the calculation of equivalent control, we employ robust sliding mode differentiators (see [13], [14]) instead of the commonly used classic low-pass filters. The super-twisting algorithm is employed in order to ensure finite time convergence of the observer and the reduction of chattering effect.

A. Sliding Mode Observers Based on Equivalent Control Method

Let us consider the following SISO system

$$\begin{aligned} \dot{x} &= f(x) + g(x)\,u \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the input, $y \in \mathbb{R}$ is the output and f, g, h are sufficiently differentiable function vectors.

For the system (1), let us define the vector of output derivatives, $H(x)$, as follows:

$$H(x) = \begin{pmatrix} h_1(x) \\ h_2(x) \\ \vdots \\ h_n(x) \end{pmatrix} = \begin{pmatrix} h(x) \\ L_f h(x) \\ \vdots \\ L_f^{n-1} h(x) \end{pmatrix}, \tag{2}$$

and the *Observability matrix*, $\mathcal{O}(x)$, as:

$$\mathcal{O}(x) = \frac{\partial H(x)}{\partial x} = \begin{pmatrix} dh(x) \\ dL_f h(x) \\ \vdots \\ dL_f^{n-1} h(x) \end{pmatrix} \tag{3}$$

where $L_f^i h(x)$ represents the i-th Lie derivative of $h(x)$ in the f vector field direction.

In addition, let us suppose the following condition:

Condition 1. *In other words, the system (1) is observable, in the sense that the observability rank condition*

$$\text{rank}\left(\mathcal{O}(x)\right) = n. \tag{4}$$

is fulfilled [15].

For the case of systems without input, $g \equiv 0$, to estimate the state variables of the system (1) using the measurements y, an observer of the form

$$\dot{\hat{x}} = \mathcal{O}^{-1}(\hat{x})\, M \text{sign}\left(V(t) - H(\hat{x})\right) \tag{5}$$

is proposed in [11], [4]. In (5), the function $\text{sign}(\bullet)$ is extended to the form $\text{sign}(\xi) = \text{col}\left(\text{sign}(\xi_1), \ldots, \text{sign}(\xi_n)\right)$, $M(\hat{x}) = \text{diag}\left(m_1(\hat{x}), \ldots, m_n(\hat{x})\right)$ is a $n \times n$ diagonal matrix with positive entries which are the gains of the observer, $V(t) = \text{col}\left(v_1(t), \ldots, v_n(t)\right)$ with $v_1(t) = y(t)$ and

$$v_{i+i}(t) = \left\{m_i(\hat{x})\,\text{sign}\left[v_i(t) - h_i(\hat{x})\right]\right\}_{eq} \tag{6}$$

with $i = 1, \ldots, n-1$, and with $\{\}_{eq}$ denoting an *equivalent value operator* of a discontinuous function in sliding mode [16]. In the case of the sliding mode observer for the system with input, additional matching conditions are needed for the observation error to be independent of the input. So, addressing the relationship between system and inputs, the following condition is needed.

Condition 2. *For any $x \in \mathbb{R}$, the vector*

$$\frac{\partial H(x)}{\partial x} g(x)$$

does not depend on x, it means

$$\frac{\partial}{\partial x}\left[\frac{\partial H(x)}{\partial x} g(x)\right] = 0$$

for all $x \in \mathbb{R}$.

Under **Condition 2**, to estimate the state variables of the system (1) using the measurements y, an observer of the form

$$\dot{\hat{x}} = \mathcal{O}^{-1}(\hat{x})\, M \text{sign}\left(V(t) - H(\hat{x})\right) + g(x)\, u \tag{7}$$

is used.

Drakunov [11] showed that using a suitable choice of $M(\hat{x})$, $m_i(\hat{x})$ as a upper bound of $h_{i+1}(x)$ with $i = 1, \ldots, n-1$ and $m_n(\hat{x})$ of $L_f^n h(x)$, the observers in equations (5) and (7) converge in finite time.

B. Arbitrary-order exact robust differentiator

Real-time differentiation is a well-known problem, and several approaches have been proposed to obtain time derivatives for a given signal. Between these all solutions, sliding mode based methods have demonstrated high accuracy and robustness. For the calculation of higher order exact derivatives, successive implementation of a first order differentiator with finite time convergence is used in [13]. For the same objective, an arbitrary-order exact robust differentiator based in a recursive scheme and which provides the best possible asymptotic accuracy in presence of input noises and discrete sampling is proposed in [14]. Let $f(t) \in \mathcal{C}^{\bar{k}}[0, \infty)$ be a function to be differentiated and let $k \leq \bar{k}$, then the k-th order differentiator is defined as follows:

$$\begin{aligned}
\dot{z}_0 &= \zeta_0, \\
\zeta_0 &= -\lambda_k L^{\frac{1}{k+1}} \left|z_0 - f(t)\right|^{\frac{k}{k+1}} \text{sign}\left(z_0 - f(t)\right) + z_1 \\
\dot{z}_1 &= \zeta_1, \\
\zeta_1 &= -\lambda_{k-1} L^{\frac{1}{k}} \left|z_1 - \zeta_0\right|^{\frac{k-1}{k}} \text{sign}\left(z_1 - \zeta_0\right) + z_2 \\
&\vdots \\
\dot{z}_{k-1} &= \zeta_{k-1}, \\
\zeta_{k-1} &= -\lambda_1 L^{\frac{1}{2}} \left|z_{k-1} - \zeta_{k-2}\right|^{\frac{1}{2}} \text{sign}\left(z_{k-1} - \zeta_{k-2}\right) + z_k \\
\dot{z}_k &= -\lambda_0 L \text{sign}\left(z_k - \zeta_{k-1}\right)
\end{aligned} \tag{8}$$

where z_i is the estimation of the true signal $f^{(i)}(t)$. The differentiator provides finite time exact estimation under ideal condition when neither noise nor sampling are present. The parameters $\lambda_0 = 1.1$, $\lambda_1 = 1.5$, $\lambda_2 = 2$, $\lambda_3 = 3$, $\lambda_4 = 5$, $\lambda_5 = 8$ are suggested for the construction of differentiators up to the 5-th order. For the gain L case, the following condition is provided:

Condition 3. *The parameter L is selected such that be a upper bound for $|f^{(k+1)}|$.*

See [14] and [7] for further details on the estimation of time of convergence, the error bounds for the signal $f(t)$ and their derivatives in presence of noise or discrete sampling and other properties and constraints of the differentiator.

C. The Super-Twisting control algorithm

An important second order sliding mode algorithm for control and observation is the so-called *Super-Twisting*

Algorithm [1, Ch. 3]. To introduce it, consider the following controlled system

$$\dot{x} = u(t) + \varphi(t) \tag{9}$$

where $x \in \mathbb{R}$, $\varphi(t)$ represents external noise or perturbation and u has the form

$$u(t) = u_1(t) + u_2(t) \tag{10}$$

with $u_1(t) = -\alpha_1 |x|^{1/2}\text{sign}(x)$ and $\dot{u}_2(t) = -\alpha_2 \text{sign}(x)$, where $\alpha_1 > 0$ and $\alpha_2 > 0$ are the control parameters. In addition, it is supposed $\varphi(t)$ is differentiable and $|\dot{\varphi}(t)| < N$, $\forall t > 0$ with N a previously known constant.

The stability condition for the system (9) in closed-loop by (10) can be obtained through the transformation

$$y(t) = \varphi(t) - \alpha_2 \int_0^t \text{sign}(x(\tau))\, d\tau$$

reducing the system to

$$\begin{aligned} \dot{x} &= y - \alpha_1 |x|^{1/2}\text{sign}(x) \\ \dot{y} &= \dot{\varphi} - \alpha_2 \text{sign}(x). \end{aligned} \tag{11}$$

In this form, with the Lyapunov function proposed by Polyakov and Poznyak in [17], if the parameters α_1 and α_2 are chosen as $\alpha_2 > 5N$ and $32N < \alpha_1^2 < 8(\alpha_2 - N)$, the finite-time convergence of the system (11) to the origin $(0, 0)$ is provided.

III. Second Order Sliding Mode Observer Using Robust Differentiators

This section presents an sliding mode observer based on a robust differentiator and the super twisting algorithm. Also, the stability analysis for the observer is presented.

A. Observer Design

For the system (1), with reference to the equations (5) and (7), we propose an observer based on an arbitrary-order exact robust differentiator (8) for the calculation of the vector $V(t)$ as follows:

$$V(t) = \text{col}(y, z_1, \ldots, z_{n-1})$$

with

$$\begin{aligned} \dot{z}_0 &= \zeta_0, \\ \zeta_0 &= -\lambda_n L^{\frac{1}{n}} |z_0 - y|^{\frac{n-1}{n}} \text{sign}(z_0 - y) + z_1 \\ \dot{z}_1 &= \zeta_1, \\ \zeta_1 &= -\lambda_{n-1} L^{\frac{1}{n-1}} |z_1 - \zeta_0|^{\frac{n-2}{n-1}} \text{sign}(z_1 - \zeta_0) + z_2 \\ &\vdots \\ \dot{z}_{n-1} &= \zeta_{n-1}, \\ \zeta_{n-1} &= -\lambda_1 L^{\frac{1}{2}} |z_{n-1} - \zeta_{n-2}|^{\frac{1}{2}} \text{sign}(z_{n-1} - \zeta_{n-2}) + z_n \\ \dot{z}_n &= -\lambda_0 L \text{sign}(z_n - \zeta_{n-1}) \end{aligned}$$

instead of the recursive form (6) which generally use classic low-pass filters for the definition of the $\{\}_{eq}$ operator.

Also, the vector $W(t)$ is defined as follows:

$$W(t) = \text{col}(z_1, \ldots, z_n).$$

In addition, the correction term of the observer is designed using the super-twisting algorithm (10), in order to ensure finite time convergence of the observer and the reduction of chattering effect due to relay-type correction terms.

The resultant observer for the system (1) without input, $g \equiv 0$, is presented in the following equation:

$$\dot{\hat{x}} = \mathcal{O}^{-1}(\hat{x})[W(t) + \psi_1 + \psi_2] \tag{12}$$

where $\psi_1 = M_1(\hat{x})|V(t) - H(\hat{x})|^{\frac{1}{2}}\text{sign}(V(t) - H(\hat{x}))$, $\dot{\psi}_2 = M_2(\hat{x})\,\text{sign}(V(t) - H(\hat{x}))$ and the vector $H(\hat{x})$ is defined as in (2), with $M_1(\hat{x}) = \text{diag}(m_{1,1}(\hat{x}), \ldots, m_{1,n}(\hat{x}))$ and $M_2(\hat{x}) = \text{diag}(m_{2,1}(\hat{x}), \ldots, m_{2,n}(\hat{x}))$ are two $n \times n$ diagonal matrices with positive entries which are the gains of the observer. The function $(\bullet)^{\frac{1}{2}}$ is extended to the form $\xi^{\frac{1}{2}} = \text{col}\left(\xi_1^{\frac{1}{2}}, \ldots, \xi_n^{\frac{1}{2}}\right)$ for the expression $|V(t) - H(x)|^{\frac{1}{2}}$.

To estimate the state variables of the system (1) with the input fulfilling the **Condition 2** and using the measurements y, the proposed observer has the form:

$$\dot{\hat{x}} = \mathcal{O}^{-1}(\hat{x})[W(t) + \psi_1 + \psi_2] + g(x)u. \tag{13}$$

B. Observer Convergence

For the observer (12), under the diffeomorphism defined by the observability matrix $\mathcal{O}$, the modified observation error, e, can be written in the transformed states $e = H(x) - H(\hat{x})$, in particular

$$\dot{e} = \dot{H}(x) - \dot{H}(\hat{x}) \tag{14}$$

leading to

$$\dot{e} = \begin{bmatrix} \dot{h}_1(x) \\ \dot{h}_2(x) \\ \vdots \\ \dot{h}_i(x) \\ \vdots \\ \dot{h}_{n-1}(x) \\ \dot{h}_n(x) \end{bmatrix} - [W(t) + \psi_1 + \psi_2] \tag{15}$$

that is

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \vdots \\ \dot{e}_i \\ \vdots \\ \dot{e}_{n-1} \\ \dot{e}_n \end{bmatrix} = \begin{bmatrix} \dot{h}_1(x) \\ \dot{h}_2(x) \\ \vdots \\ \dot{h}_i(x) \\ \vdots \\ \dot{h}_{n-1}(x) \\ \dot{h}_n(x) \end{bmatrix} - \begin{bmatrix} z_1 - m_{1,1}(\hat{x})\,\psi_{1,1} - m_{2,1}(\hat{x})\,\psi_{2,1} \\ z_2 - m_{1,2}(\hat{x})\,\psi_{1,2} - m_{2,2}(\hat{x})\,\psi_{2,2} \\ \vdots \\ z_i - m_{1,i}(\hat{x})\,\psi_{1,i} - m_{2,i}(\hat{x})\,\psi_{2,i} \\ \vdots \\ z_{n-1} - m_{1,n-1}(\hat{x})\,\psi_{1,n-1} - m_{2,n-1}(\hat{x})\,\psi_{2,n-1} \\ z_n - m_{1,n}(\hat{x})\,\psi_{1,n} - m_{2,n}(\hat{x})\,\psi_{2,n} \end{bmatrix} \tag{16}$$

where $\psi_{1,i} = |v_i(t) - h_i(\hat{x})|^{\frac{1}{2}} \operatorname{sign}(v_i(t) - h_i(\hat{x}))$ and $\psi_{2,i} = \operatorname{sign}(v_i(t) - h_i(\hat{x}))$.

The convergence of $V(t)$ does not depend on the observer, but in **Condition 3**, which for this case establishes that the $n+1$-th Lie derivative of $h(x)$ in the f vector field direction, $L_f^{n+1}h(x)$, must be bounded by L. Therefore, under the fulfilment of this condition, there is a time $t_d > 0$ such as if $t > t_d$, then $V(t) \equiv H(x)$ and $W(t) \equiv \dot{H}(x)$.

Then, for $t > t_d$, Eq. (16) becomes

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \vdots \\ \dot{e}_i \\ \vdots \\ \dot{e}_{n-1} \\ \dot{e}_n \end{bmatrix} = \begin{bmatrix} -m_{1,1}(\hat{x})\,\psi_{1,1} - m_{2,1}(\hat{x})\,\psi_{2,1} \\ -m_{1,2}(\hat{x})\,\psi_{1,2} - m_{2,2}(\hat{x})\,\psi_{2,2} \\ \vdots \\ -m_{1,i}(\hat{x})\,\psi_{1,i} - m_{2,i}(\hat{x})\,\psi_{2,i} \\ \vdots \\ -m_{1,n-1}(\hat{x})\,\psi_{1,n-1} - m_{2,n-1}(\hat{x})\,\psi_{2,n-1} \\ -m_{1,n}(\hat{x})\,\psi_{1,n} - m_{2,n}(\hat{x})\,\psi_{2,n} \end{bmatrix}$$

where $\psi_{1,i} = |h_i(x) - h_i(\hat{x})|^{\frac{1}{2}} \operatorname{sign}(h_i(x) - h_i(\hat{x})) = |e_i|^{\frac{1}{2}} \operatorname{sign}(e_i)$ and $\dot{\psi}_{2,i} = \operatorname{sign}(h_i(x) - h_i(\hat{x})) = \operatorname{sign}(e_i)$.

Therefore, with a suitable choice of the gain L and the matrices M_1, M_2 according on (8) and (11) respectively, the convergence of the observation error to zero is achieved in finite time.

IV. Application Case

We highlight in this section the utility and the advantages of the observer design based over the equivalent control method of the previous recalls in the resolution of the observation problem.

At first, we use a system an observer for synchronization of a chaotic system previously treated in the literature [18], [19]. At last, we apply the same procedure to a CSTR system [20].

A. *Rössler chaotic system synchronization*

The problem of synchronization of chaotic systems can be seen as an observer design problem [18], [19]. Full order observer needs to estimate the unmeasurable states and the measurable states at the same time, increasing the complexity of the task. The *Rössler system* [21] is a system conformed by three non-linear ordinary differential equations. These differential equations define a continuous-time dynamical system that exhibits chaotic behavior. The set of equations (17) shows a state representation of the Rössler system.

$$\begin{aligned} \dot{x}_1 &= a x_1 + x_2 \\ \dot{x}_2 &= -x_1 - x_3 \\ \dot{x}_3 &= b + x_3(x_2 - c) \end{aligned} \tag{17}$$

Assuming the state x_1 as the output, the SISO system representation of the Rössler system in the way of (1) is

$$f(x) = \begin{pmatrix} a x_1 + x_2 \\ -x_1 - x_3 \\ b + x_3(x_2 - c) \end{pmatrix} \tag{18}$$

and

$$h(x) = x_1. \tag{19}$$

From (18) and (19) the Lie derivatives of the output are:

$$\begin{aligned} h &= x_1 \\ L_f h &= a x_1 + x_2 \\ L_f^2 h &= a(a x_1 + x_2) - x_1 - x_3 \end{aligned} \tag{20}$$

and the corresponding observability matrix is

$$\mathcal{O} = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a^2 - 1 & a & -1 \end{pmatrix}. \tag{21}$$

Using the equations (20) and (21), we designed and observer with the form shown in (13). The values of the system constants are $a = 0.2$, $b = 0.2$ and $c = 5.7$. The parameter values for the differentiator are $L = 455$, $\lambda_1 = 1.1$, $\lambda_2 = 1.5$, and $\lambda_4 = 3$.

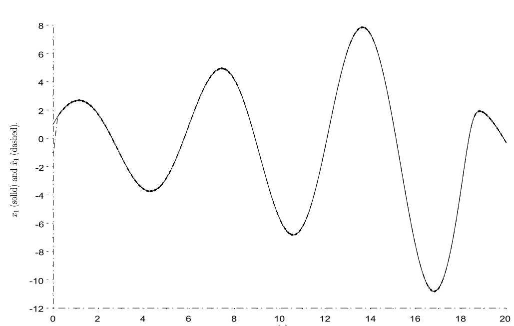

Fig. 1. Real x_1 and observed $\hat{x}_1$(- -) states for system (17)

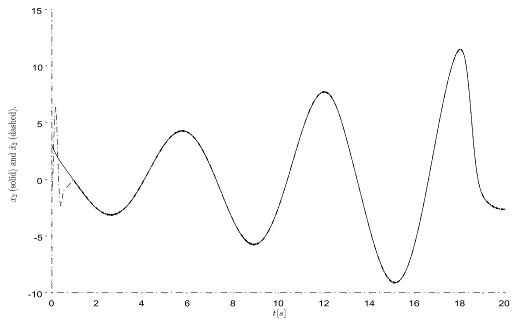

Fig. 2. Real x_2 and observed $\hat{x}_2$(- -) states for system (17)

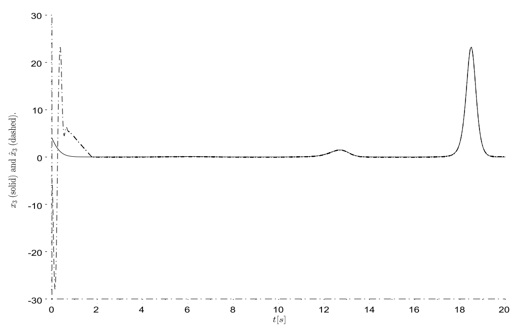

Fig. 3. Real x_3 and observed $\hat{x}_3$(- -) states for system (17)

Figures 1, 2 and 3 show the simulation results for the system (18). The initial conditions for this system were $x(0) = [1\,3\,4]^T$ and $\hat{x}(0) = [-1\ -1\ 5]^T$. The state $\hat{x}_1$ converges to x_1 in finite time of 0.1 seconds. Then $\hat{x}_2$ reaches to x_2 in finite time of about 1.5 seconds. Note that $\hat{x}_2$ reaches x_2 only after $\hat{x}_1$ converges to its state. Finally, at a time of 2.5 second, $\hat{x}_3$ converges to x_3.

B. CSTR Application

The CSTR process is a recognized benchmark frequently used for controller proofs [20] and represents many processes typically employed in industry. The model of the CSTR is described by the set of equations (22). See [20] for a more in-depth description and modeling of the process and its parameters. The simulation values for the system were also extracted from the same reference, and they are summarized in Table I.

TABLE I

NOMINAL PARAMETERS OF CSTR

Parameter	Value	Unit
F	0.1605	$m^3 \cdot min^{-1}$
V	2.4069	m^3
C_{in}	2114.5	$gmol \cdot m^{-3}$
k_0	2.8267·10^{11}	min^{-1}
E	75361.14	$J \cdot gmol^{-1}$
R	8.3174	$J \cdot gmol^{-1} K^{-1}$
T_{in}	295.22	K
ΔH	-9.0712·10^4	$J \cdot gmol^{-1}$
ρ	1000	$kg \cdot m^{-3}$
C_p	3571.3	$J \cdot kg^{-1}$
U	2.5552·10^4	$J \cdot (s \cdot m^2 \cdot K)^{-1}$
A	8,1755	m^{-2}
T_j	279	K

$$\frac{dT}{dt} = \frac{F}{V}\left(T_{in} - T\right) - \frac{\Delta H}{\rho C_p} k_0 C_A e^{-\frac{E}{RT}} + \frac{UA}{\rho C_p V}\left(T_j - T\right)$$
$$\frac{dC_A}{dt} = \frac{F}{V}\left(C_{in} - C_A\right) - k_0 C_A e^{-\frac{E}{RT}}. \tag{22}$$

Assuming as the state variables the temperature T of the reactive mass and the concentration C_A of the reactant respectively, the model in space-state representation is shown in the set of equations (23). We suppose that T can be measured and acts as the model output y. The goal is the estimation of C_A (denoted by x_2) from T (denoted by x_1).

$$\dot{x}_1 = \frac{F}{V}\left(T_{in} - x_1\right) - \frac{\Delta H}{\rho C_p} k_0 x_2 e^{-\frac{E}{Rx1}} + \frac{UA}{\rho C_p V}\left(T_j - x_1\right)$$
$$\dot{x}_2 = \frac{F}{V}\left(C_{in} - x_2\right) - k_0 x_2 e^{-\frac{E}{Rx_1}} \tag{23}$$
$$y = h(x) = x_1$$

let

$$f(x) = \tag{24}$$
$$\begin{pmatrix} \frac{F}{V}\left(T_{in} - x_1\right) - \frac{\Delta H}{\rho C_p} k_0 x_2 e^{-\frac{E}{Rx1}} + \frac{UA}{\rho C_p V}\left(T_j - x_1\right) \\ \frac{F}{V}\left(C_{in} - x_2\right) - k_0 x_2 e^{-\frac{E}{Rx_1}} \end{pmatrix}$$

and

$$h(x) = x_1 \tag{25}$$

from (24) and (25) the Lie derivatives of the output are:

$$h = x_1 \tag{26}$$
$$L_f h = \frac{F}{V}\left(T_{in} - x_1\right) - \frac{\Delta H}{\rho C_p} k_0 x_2 e^{-\frac{E}{Rx1}} + \frac{UA}{\rho C_p V}\left(T_j - x_1\right)$$

and the corresponding observability matrix is

$$\mathcal{O}(x) = \tag{27}$$
$$\begin{pmatrix} 1 & 0 \\ -\frac{F}{V} - \frac{UA}{V\rho C_p} - \frac{E\Delta H}{R\rho C_p}\frac{x_2}{x_1^2} k_0 e^{-\frac{E}{Rx_1}} & -\frac{\Delta H}{\rho C_p} k_0 e^{-\frac{E}{Rx_1}}. \end{pmatrix}$$

The proposed observer has been tested by simulation with the initial conditions $x(0) = [283.225\,2005.838]^T$ and $\hat{x}(0) = [300\,2100]^T$. In Figure 4, we can see $\hat{x}_1$ reaching the real temperature value in about 0.5 minutes.

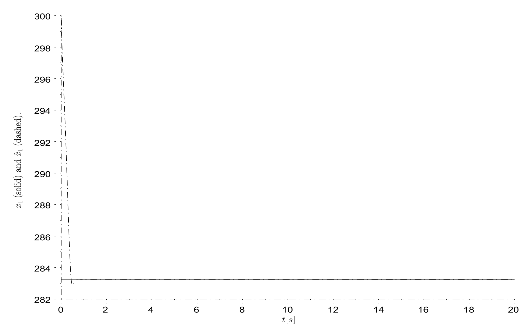

Fig. 4. Temperature x_1

In Figure 5, we see that $\hat{x}_2$ also converges to the real value of the concentration in finite time after 2 minutes.

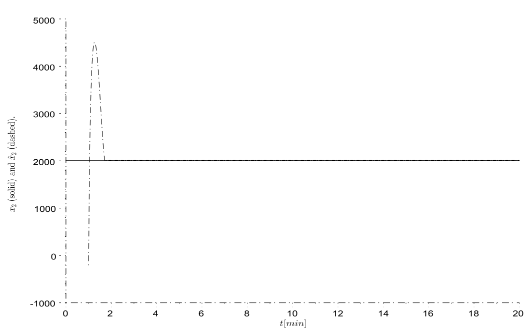

Fig. 5. Concentration x_2

V. Conclusion

We show in this paper the design of a high order sliding mode observer for nonlinear systems using the equivalent control method. The super-twisting algorithm was employed in order to ensure finite time convergence of the observer and the reduction of chattering effect. Moreover, we also showed the application of the proposed schemes to a real process model like the CSTR. This model is a well documented benchmark that includes many dynamical processes; in this regard, the results of this work can be expanded towards a plethora of different application cases.

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