# Colombian Energy Market: Optimal Portfolio

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***Abstract*—This paper focuses on the study of an optimal portfolio in the Colombian Energy Market using the Artificial Intelligence techniques specifically, Fuzzy Modeling and Neural Networks. The methodology at first, is implemented using the Matlab Fuzzy Logic Toolbox and with the help of a script the process is automatized. Secondly, a Neural Network is implemented in Matlab and its results are compared with the ones obtained in the Matlab Neural Network Toolbox. The results of the Fuzzy model and the Neural Network are presented and conclusions of both techniques are discussed. Finally possible future work are proposed.**

***Index Terms*—Energy Markets, Artificial Intelligence, Fuzzy Modeling, Neural Networks.**

## I. INTRODUCTION

In the world of economics and finance, the term market means the aggregate or set of possible buyers and sellers of a certain good or service and the transactions between them. The term market is sometimes used for what are more strictly exchanges, organizations that facilitate the trade in financial securities, for example, a stock exchange or commodity exchange. This may be a physical location or an electronic system. Most trading of stocks takes place on an exchange; nevertheless, corporate actions are outside an exchange, while any two companies or people, for whatever reason, may agree to sell stock from the one to the other without using an exchange [1].

Based on the concept of market, comes a new concept: the financial market. A financial market is a market in which people and entities can trade financial securities, commodities, and other fungible items of value at low transaction costs and at prices that reflect supply and demand. Securities include stocks and bonds, and commodities include precious metals, agricultural goods and energy. There are both general markets (where many commodities are traded) and specialized markets (where only one commodity is traded). Markets work by placing many interested buyers and sellers; including households, firms, and government agencies, in one same place, thus making it easier for them to find each other [2]. An economy which relies primarily on interactions between buyers and sellers to allocate resources is known as a market economy in contrast either to a command economy or to a non-market economy. In finance, depending on the financial markets, it facilitates:

- The raising of capital.
- The transfer of risk.
- Price discovery
- Global transactions with integration of financial markets.
- The transfer of liquidity.
- International trade.

As we have mentioned it, there are several types of financial markets depending on the financial asset that is traded on it. Also, the type of the financial market is given depending on the level of the three most important variables that an investor should quantify: yield, risk and liquidity. The most important financial markets, among other, are:

- Capital markets which consist of: Stock markets, which provide financing through the issuance of shares or common stock, and enable the subsequent trading thereof. Bond markets, which provide financing through the issuance of bonds, and enable the subsequent trading thereof.
- Commodity markets, which facilitate the trading of commodities.
- Money markets, which provide short term debt financing and investment.
- Derivatives markets, which provide instruments for the management of financial risk.
- Futures markets, which provide standardized forward contracts for trading products at some future date; see also forward market.
- Insurance markets, which facilitate the redistribution of various risks.
- Foreign exchange markets, which facilitate the trading of foreign exchange.

Promptly, energy markets are commodity markets that deal specifically with the trade and supply of energy and that trades in the Energy Sector. The energy sector is a category of stocks that are related to producing or supplying energy. This sector includes companies involved in the exploration and development of oil or gas reserves, oil and gas drilling, or integrated power firms [3].

Energy market refers to an electricity market where electricity (both power and energy) is a commodity capable of being bought, sold and traded. An electricity market is a system for effecting purchases, through bids to buy and offers to sell. Bids and offers use supply and demand principles to set the price [3].

In Colombia, in order to get the final rate of the energy, the asset must go through four processes, which are: generation, transmission, distribution, and commercialization. The process of generating electric energy in Colombia, has very specific technical as well as economical characteristics which make the market behave as an oligopoly. Some of these characteristics are: high costs associated to the installation of new plants, long construction periods, restrictions when transporting the energy, impossibility to store the energy in efficient quantities, among others[4]. As it can be seen in table I, the 86% of the generation of electric energy in the country, was focused on just 6 agents, among the 44 agents that trade on stock.

| **Agent** | **Share** |
|---|---|
| EPM | 25.8% |
| EMGESA | 22% |
| ISAGEN | 16% |
| GECELCA | 9% |
| EPSA - CELSIA | 6% |
| AES Chivor | 7.7% |

Table I
SHARE OF THE TOTAL GENERATED ENERGY OF COLOMBIA FOR EACH OF THE BIGGEST AGENTS IN 2012[4]

The technology used to generate electricity in Colombia is as well crucial when determining the prices, because it focuses mainly in hydraulic technology (64% against a 30% from thermoelectric plants). The principal types of plants that exist in Colombia for generating energy are:

1) Hydraulic plants.
2) Thermal power plants.
3) Smaller plants (less than 20 MW).

All the energy produced in the country is traded in the *Mercado Mayorista de Electricidad en Colombia* (MEM), all the generator companies (agents) are linked to the *Sistema Interconectado Nacional* (SIN), who is in charge of satisfying the demand of all the final users who are connected as well to this system. These transactions are made daily through an auction made by the *Administrador del Sistema de Intercambios Comerciales* (ASIC), who acts on behalf of the final consumers[5].

Concerning the plants that are bigger than 20 MW, each agent must present daily its available capacity for each hour of the following day and a selling price for the same day, to the *Centro Nacional de Despacho* (CND). This has to be done for each one of its resources. The CND is in charge of making the "economical dispatch", which consists of sorting from lowest to highest the generation plants according to the price, until they reach the demand[5].

The smaller plants provide smaller selling price, but they do not present it to the CND, so they are all included in the "real dispatch" which is a variation of the "economical dispatch" after adding the smaller plants to the list. The operation and dispatch is done by XM S.A E.S.P.

The electrical sector in Colombia is controlled by the *Comisión de Regulación de Energía y Gas* (CREG), *Unidad de Planeación Minero Energética* (UPME), and the *Superintendencia de Servicios Públicos Domiciliarios* (SSPD)[4].

Now, reviewing the background of general techniques of artificial intelligence (such as fuzzy logic, neural networks or a hybrid between the two named ANFIS) applications to energy markets, especially optimal investment portfolios, we find that there is no specific application in this field. However, there are different applications of artificial intelligence techniques focused on solving problems of financial markets.

It is possible to observe that fuzzy and neural networks approaches have been under high level of interest among all the researchers in this field, for example, in 1991 [6] the effectiveness of applying fuzzy logic and neural networks to securities trading decision support systems (STDSS) is demonstrated through some examples. First, the characteristics of STDSS are established, for then being able to propose examples such as buy/sell timing detection or stock portfolio selection using fuzzy logic and neural networks by showing their algorithms and simulation results. Moving on a few years, we find out that recently in 2014 [7] the authors focus on the forecasting of financial time series data by giving to it a fuzzy artificial neural network approach. Empirical results of financial markets, especially exchange rate market, forecasting indicate that the proposed model by the authors performs significantly better than its components used separately which indicates a fully functional behavior of the financial information when it is being modeled by artificial intelligence tools.

Going further into the field, one of the main advantages that fuzzy models and fuzzy logic as such, is the ability to give values to linguistic variables and through membership functions determine intermediate values between true and false for each one of it. This capability was exploited in 2012 [8] where the authors establish that the main problem in portfolio selection is the problem of how to diversify investments in the most efficient and profitable way possible. Portfolio selection is a field of study that has been broached from several perspectives, including, among others, recommender systems. On this paper the authors propose a tool based on semantic technologies and fuzzy logic techniques named SINVLIO. This tool recommends investments grounded in both psychological aspects of the investor and traditional financial parameters of the investments. The results are really good and show that SINVLIO makes good recommendations depending on the investor inputs. Last but not least, assuming that portfolio management decision is usually made on the basis of product value, project risk and business strategies. Due to both the nature and timing of new product development, portfolio selection is associated with uncertainty and complexity, and conventional evaluation methods not can handle such decisions suitably and effectively. This is why in 2007 [9] the authors define fuzzy logic as a well suited tool for decision making with uncertainty and they propose a method for portfolio selection decision using fuzzy logic. The procedure is evaluated on a new product portfolio and the results show that fuzzy logic is a good tool to use when a decision of portfolio selection must be done.

Now, changing of technique, we found that Neural Network is quite a strong tool to use when analyzing an optimal portfolio is required. For example, in [10], the authors propose a novel Neural Network named "neural network-based mean-variance-skewness model" for optimal portfolio selection. This Neural Network or model integrates different forecasts and trading strategies, as well as the investor risk preference. The model seeks to provide solutions satisfying the trade-off conditions of mean-variance-skewness. The authors verify the feasibility of the model with a simulation experiment. The conclusions are remarkable since the proposed model is a fast and efficient way of solving the trade-off in the mean-variance-skewness portfolio problem.

In regard to portfolio optimization, we found that the authors in [11] develop a portfolio optimization for index investing based on self-organizing neural network. Index investing is an important issue for researchers and practitioners so first, the authors construct a self-organizing neural network clustering model to complete the stock clustering based on stock trend which regards stock price as input. In result, the index portfolio optimization model is proposed to determine the optimal investment proportion of each cluster sampling and achieve the minimum tracking error. Also, they improve the Back-propagation algorithm to benefit the optimization calculation of stock weights. Finally the empirical results show that the approach achieves smaller tracking error and better index tracking effect than the random sampling, which in addition, shows the excellent development of Neural Networks in this field. Last but not least, we found that in [12] the authors focus on Optimizing portfolio construction using artificial intelligence. Basically, the paper aims to enhance the practicability of Artificial Intelligence using Neural Network in the actual market. The authors generalizes the standard Markowitz Theory's Efficient Frontier to mimic and optimize the portfolio construction, and develops a neural network heuristic to better understand the mechanism of how Artificial Intelligence can construct optimal portfolio and provide advantages to all levels of investors. Concluding, we can establish that Neural Network is quite an excellent tool to aboard and develop investigations related to optimal portfolios.

Therefore, the objective of this paper is to apply the theory and techniques of Artificial Intelligence, to build an optimal portfolio for the Colombian Energy Market, representing a game of strategies. In particular, the techniques of Artificial Intelligence that will be applied are: Fuzzy Modeling, Neural Networks and the Adaptive Neuro Fuzzy Inference System (ANFIS), which is the hybrid of the first two techniques. The study will focus on the first part of the process defined for Colombia, it means, generation; and taking into consideration the four most important agents that influence the market, namely EPM, EMGESA, CELSIA-EPSA, and Chivor.

The toolboxes that will be used throughout the work are: the Fuzzy Logic Toolbox for fuzzy modeling and the development of the ANFIS; and Neural Network Toolbox. All of them implemented in Matlab.

### A. Problem Definition

The methodology is applied on the basis of an optimal portfolio for the Colombian Energy Market. An *optimal portfolio* is the one that minimizes the risk of the agent, while striving for the highest return possible, or said in other words, maximizing its utility. The theory states that investors will act rationally, always making decisions aimed at maximizing their return for their acceptable level of risk [**?**].

The paper is organized as follows: in section 2 the methodology to follow is presented, giving a brief introduction to the various techniques of artificial intelligence and the elements necessary for its implementation along with brief explanations of the concepts used in the study. All results obtained by implementing and developing the methodology presented are exposed in Section 3. Discussion and analysis of the results obtained in the previous section are presented in Section 4 and finally, in section 5 the conclusions are given and some future work are proposed. Finally the references consulted are presented.

## II. METHODOLOGY

### A. Fuzzy Theory

Fuzzy theory holds that all things are matters of degree[13]. It reduces "black-white" logic and mathematics to special cases of "gray" relationships. It doesn't follow the traditional laws of logic, and solves some paradoxes that the classical logic theory generates. Fuzziness also provides a fresh and deterministic interpretation of probability and randomness. Fuzziness means, mathematically, multivaluedness or multivalence. It is, degrees of indeterminacy or ambiguity, partial occurrence of events or relations.

*1) History:* Fuzzy logic started being developed between the 1920s and 1930s, motivated by the logical paradoxes and the Heisenberg uncertainty principle. Quantum theorists included a third truth value, between TRUE and FALSE, in the logical framework. Afterwards, they allowed degrees of indeterminacy, being TRUE and FALSE the two limiting cases of the spectrum of indeterminacy. Jan Lukasiewicz[14] extended twice the range of truth values, to arrive finally to al numbers in the range $[0, 1]$. Logics that use the general truth function $t : \{Statements\} \rightarrow [0, 1]$ define continuous or fuzzy logics. In 1965, Zadeh formally developed multivalued set theory and introduced the term *fuzzy* into the technical literature.

*2) Definition of a Fuzzy Set:* Let $X$ be a set of elements (space of points) from the fuzzy universe. A fuzzy set $A$ is characterized by a membership function (or characteristic function) $\mu_A(x)$ which associates with each point in $X$, a real number in the interval $[0, 1]$. The value of $\mu_A(x)$ at $x$ represents the grade of membership of $x$ in $A$.

*3) Fuzzy Modeling[15]:* Systems can be modeled in a great variety of ways. Particularly fuzzy set theory and fuzzy logic can be employed for this purpose. Rule based fuzzy systems, are systems where the relationships between variables are represented by a means of fuzzy if-then rules of the form:

**If** $antecedent\, proposition$ $\mathbf{then}$ $consequent\, proposition.$

Depending on the particular structure of the consequent proposition, there are three types of models:

1) *Linguistic fuzzy model.*
2) *Fuzzy relational model.*
3) *Takagi-Sugeno (TS) fuzzy model.*

In this study, the first model will be used. In this case, both the antecedent and consequent are fuzzy propositions. A general form of this model is:

$$R_i : \; If\, x\, is\, A_i\, then\, y\, is\, B_i, \quad i = 1,\, 2, ..., K, \tag{1}$$

Where $x$ is the antecedent variable, which represents the input to the fuzzy system; and $y$ is the consequent variable, representing the output of the fuzzy system.

Linguistic terms can be seen as qualitative values used to describe a particular relationship by linguistic rules. Typically, a set of $N$ linguistic terms $\mathcal{A} = \{A_1, A_2, ..., A_N\}$ is defined in the domain of a given scalar variable $x$. A linguistic variable $\mathcal{L}$ is defined as a quintuple:

$$\mathcal{L} = (x,\; ,\mathcal{A},\; X,\; g,\; m) \tag{2}$$

where:
$x$: base variable.
$\mathcal{A}$: set of linguistic terms of $x$.
$X$: domain (universe of discourse).
$g$: syntactic rule for generating linguistic terms.
$m$: semantic rule that assigns to each linguistic term its meaning.

*4) Inputs and Outputs:* In order to determine the optimal portfolio according to the fuzzy theory, we used the *fuzzy* toolbox that offers Matlab. Before the implementation, we needed to define the inputs and outputs of the system:

The inputs of the system in this case are:
$u_1 =$ Marginal price of the system in $t - 1$.
$u_2 =$ Average price quoted by the agent in $t - 1$.
$u_3 =$ Price of the resource 1 (hydraulic) in $t - 1$.
$u_4 =$ Price of the resource 2 (thermal power) in $t - 1$.

And its outputs are:
$y_1 =$ Change in the price of the resource 1 (hydraulic).
$y_2 =$ Change in the price of the resource 2 (thermal power).

### *B. Neural Networks*

The inspiration for the neural networks came from examination of central nervous systems, artificial nodes are called "neurons" and are connected together to form a network which mimics a biological neural network. Commonly, a class of statistical models may be called neural if they consist of sets of adaptive weights, numerical parameters that are tuned by a learning algorithm, and are capable of approximating non-linear functions of their inputs.The adaptive weights are conceptually connection strengths between neurons, which are activated during training and prediction.

Neural Networks started being developed between the 1940s and 1950s, specifically; Warren McCulloch and Walter Pitts created in 1943 a computational model for neural networks based on mathematics and algorithms known as “threshold logic”. This model made the way for neural network research to split into two distinct approaches: One approach focused on biological processes in the brain and the other focused on the application of neural networks to artificial intelligence. From then up until now, Neural Networks had become an important center of research and focus from the different investigators and many advances, both theoretical as practical, had been done. We now count with diverse algorithms and computer programs that let the Neural Network tool be one of the most important ones now a day.

*1) Theory:* For more information on this topic please check the following book [16]. In a Neural Network, the network is the inter–connections between the neurons in the different layers of each system. For example, consider a system that has three layers. The first layer has input neurons which send data to the second layer of neurons, and then to the third layer of output neurons. More complex systems will have more layers of neurons with some having increased layers of input neurons and output neurons. The synapses store parameters called "weights" that are the ones in charge of manipulate the data in the calculations. The layers between de input layer and the output layer as referred to as hidden layers and the number of neurons in each layer is totally random and depends on the researcher. A Neural Network is typically defined by three types of parameters:

1) The interconnection pattern between the different layers of neurons.
2) The learning process for updating the weights of the interconnections.
3) The activation function that converts a neuron’s weighted input to its output activation.

Nevertheless, what really is the center point of interest in the study of how efficient and effective a Neural Network is its capability of learning. In this case, given a specific task to solve, and a set of functions, to learn means to use a set of observations to find the ones that solves the task in some optimal sense. There exist three types or paradigms of learning: supervised learning, unsupervised learning and reinforcement learning. Each of them consider a different learning task. That is why it is so important to choose correctly the activation function of each neuron and the general cost function to be optimized so that the optimal weights be assigned.

The learning algorithm is in charge of training a neural network model by selecting one model that minimizes the cost criterion. There are numerous algorithms available for training neural network models most of them are an application of optimization theory and statistical estimation and employ some form of gradient descent, in other words, taking the derivative of the cost function with respect to the network parameters and then changing those parameters in a gradient-related direction. Evolutionary methods, gene expression programming, simulated annealing, expectation-maximization, non-parametric methods and particle swarm optimization are some commonly used methods for training neural networks.

Particularly we have decided to work and implement the back-propagation algorithm proposed in [16]. The back-propagation algorithm is a supervised learning method and it requires a dataset of the desired output for many inputs, making up the training set or in other words from a desired output, the network learns from many inputs. It is most useful for feed-forward networks and requires that the activation function used by the neurons be differentiable.

*2) Inputs and Outputs:* In order to evaluate de network, we defined the following inputs:

$u_1 =$ Marginal price of the system in $t$.
$u_2 =$ Price of the resource 1 (hydraulic) in $t$.
$u_3 =$ Price of the resource 2 (thermal power) in $t$.
$u_4 =$ Availability of resource 1 in $t$.
$u_5 =$ Availability of resource 2 in $t$.

Whereas the output is an objective function, which we want to be maximized:

$J = (u_1 - u_2) \times u_4 + (u_1 - u_3) \times u_5$

## III. Implementation and Results

We have data from 2250 days, of different Colombian agents and resources. Initially, the implementation is done for the case of EPM (*Empresas Públicas de Medellín*).

### A. Fuzzy Sets

### B. Neural Networks

*1) Our Approach:* Different proofs were held in order to determine the best neural network for this approach. Specifically, the number of layers, number of neurons per layer, the parameter $\eta$ and the activation functions were changed, in order to compare and select the best combination of parameters, which is the one that returns the smallest global error. The following tables show the results. The graphical results are shown only for the best adjustment of each parameter.

| Network | Number of Layers | Neurons per Layer | Error |
|---|---|---|---|
| **1** | **1** | **20** | **0.277** |
| 2 | 2 | 15 | 0.345 |
| 3 | 3 | 10 | 7.322 |
| 4 | 4 | 7 | 6.720 |
| 5 | 5 | 5 | 6.745 |

Table II
Results of Experiment 1

| Network | Neurons per Layer | Error |
|---|---|---|
| **6** | **5** | **0.238** |
| 7 | 10 | 0.249 |
| 8 | 20 | 0.261 |
| 9 | 50 | 0.420 |
| 10 | 100 | 182.301 |

Table III
Results of Experiment 2

In total we held four different experiments which are:

- Experiment 1: changing the number of layers.
- Experiment 2: changing the number of neurons per layer.
- Experiment 3: changing the parameter $\eta$.
- Experiment 4: changing the types and parameters of the activation functions.

In table II, the results of the experiment 1 are shown. The parameter $\eta = 0.4$ is always constant, and all the activation functions are sigmoidal with parameter $a = 0.7$, because after making different runs, this function provided the best results. The number of epochs is fixed as well in 300. We decided to change the number of neurons per layer, however the results were very similar if we didn't change them. They depended more on the number of layers, than the number of neurons per layer.

As it can be seen, the best results came from the network with just one layer and 20 hidden neurons. This is the reason why in the ongoing analyses, the number of hidden layers will be 1. Figure 1 shows the graphical performance of the best network, which is network 1.

Since in the previous experiment, the best result was gotten with just one layer, this parameter will be fixed for all the subsequent experiments in this value. In table III, the number of layers and all the other parameters are fixed (in the same values as for experiment 1), and in this case, just the number of neurons per layer change.

As for experiment 1, in experiment 2, the error increased when the number of neurons increased. We decided to check if with less than 5 neurons per layer, the result was better, but it appears to be that with 5 neurons in the hidden layer, we get the best results.

For experiment 3, the best parameters obtained in the previous experiments are used, as well as the fixed ones since the beginning. So now we have a fixed value of 5 neurons in the only hidden layer of the network.

As it can be seen, as the value of $\eta$ increases, the error decreases, finally network 15 provides the best results, with $\eta = 0.9$.

| Network | $\eta$ | Error |
|---|---|---|
| 11 | 0.2 | 0.479 |
| 12 | 0.4 | 0.245 |
| 13 | 0.6 | 0.164 |
| 14 | 0.8 | 0.155 |
| **15** | **0.9** | **0.125** |

Table IV
RESULTS OF EXPERIMENT 3

| Network | Activation Functions | Parameters | Error |
|---|---|---|---|
| 16 | Sigmoidal - Sigmoidal | $a_1 = 1$, $a_2 = 1$ | 0.103 |
| 17 | Tanh - Tanh | $a_1 = 1$, $b_1 = 1$ $a_2 = 1$, $b_2 = 1$ | 0.133 |
| 18 | Sigmoidal - Tanh | $a_1 = 1$ $a_2 = 1$, $b_2 = 1$ | 0.1068 |
| **19** | **Tanh - Sigmoidal** | $a_1 = 1$, $b_1 = 1$ $a_2 = 1$ | **0.083** |
| 20 | Sigmoidal - Tanh | $a_1 = 0.2$ $a_2 = 0.2$,$b_2 = 0.3$ | 62.543 |

Table V
RESULTS OF EXPERIMENT 4

In order to determine the best activation function, and its optimal parameters, we developed different proofs with just sigmoidal functions, just hyperbolic tangent functions, and then mixing them. Each of these three proofs were done with different parameters. In table V, the best result for each type of proof is shown. Since the best result is gotten with just one layer, we only need two activation functions for the back propagation algorithm. This is the reason why we only have 4 different combinations of the functions and therefore, one result for each of them. Additionally, network 20 is the worst result we got, and we show it to highlight which parameters not to use for this data.

As it can be seen, network 19 provides the best result, by mixing a Tanh and a sigmoidal function, with all its parameters equal to 1. Since this network has the smallest error among all the 20 networks, we propose these parameters as the best we tried to use with this data. The mentioned parameters are summarized in table VI. We propose more epochs because the error may slightly decrease, however, the results provided with 300 epochs are good enough and the computation time is better.

| Parameter | Value |
|---|---|
| Number of hidden layers | 1 |
| Neurons per layer | 5 |
| $\eta$ | 0.9 |
| Activation Function 1 | Tanh |
| Parameters of Activation Function 1 | $a_1 = 1$, $b_1 = 1$ |
| Activation Function 2 | Sigmoidal |
| Parameters of Activation Function 2 | $a_2 = 1$ |
| Number of Epochs | 600 |

Table VI
BEST PARAMETERS FOR OUR APPROACH

*2) Matlab's nntool:* Using Matlab's *nntool*, we also held different experiments changing the number of hidden layers, neurons per layer, and the types of activation functions. Most of the proofs were made with TANSIG functions, because they always provided better results. The results are shown in table VII. The best result is given by network 5. For this network, some graphic results are shown in figure 5.

## IV. CONCLUSIONS

Generally, determining an optimal portfolio is a really interesting subject in many fields and since there are many researchers trying to do it, the artificial intelligence techniques worked on this paper are a really solid and helpful tool to do so. Our main objective was to help into the procedure of applying correctly these techniques to find an optimal portfolio in Colombian energy market and to obtain interesting results during the process.

We found in the literature that no others authors tried to apply artificial intelligence techniques (specifically Fuzzy Logic, Neural Networks and ANFIS) to solve the problem of finding an optimal portfolio in an energy market, making our job get a huge added value in this topic and making it totally original.

When neural networks were implemented really good results were obtained. The objective this time was to maximize the objective function J by means of the five different inputs proposed earlier. We worked with two tools: our approach (an implemented neural network in Matlab) and Matlab Neural Network Toolbox. For testing both tools several experiments were proposed that consisted on changing the number of layers, number of neurons per layer, the parameter N, and the types and parameters of the activation functions. With both techniques the results were reliable and proved the consistence, efficiency and effectiveness of the Neural Networks as a tool for solving this problem.

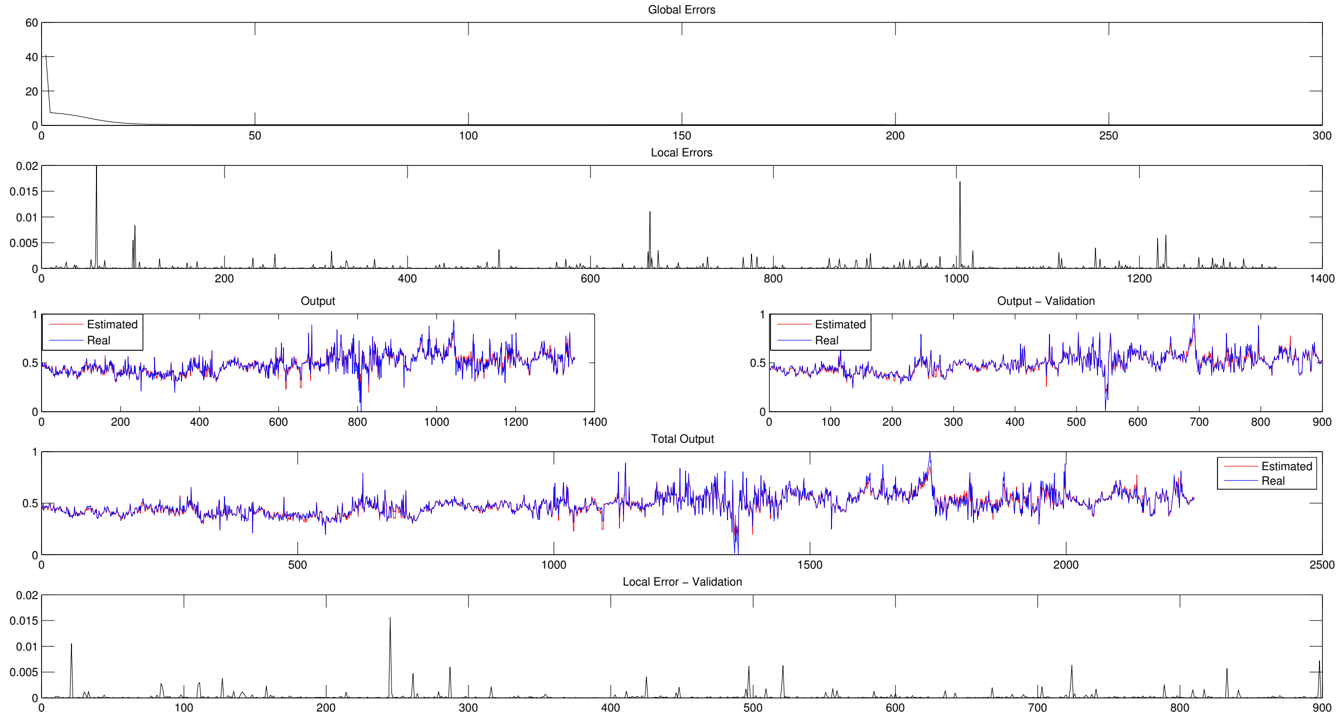



Figure 1. Graphic Result of Experiment 1

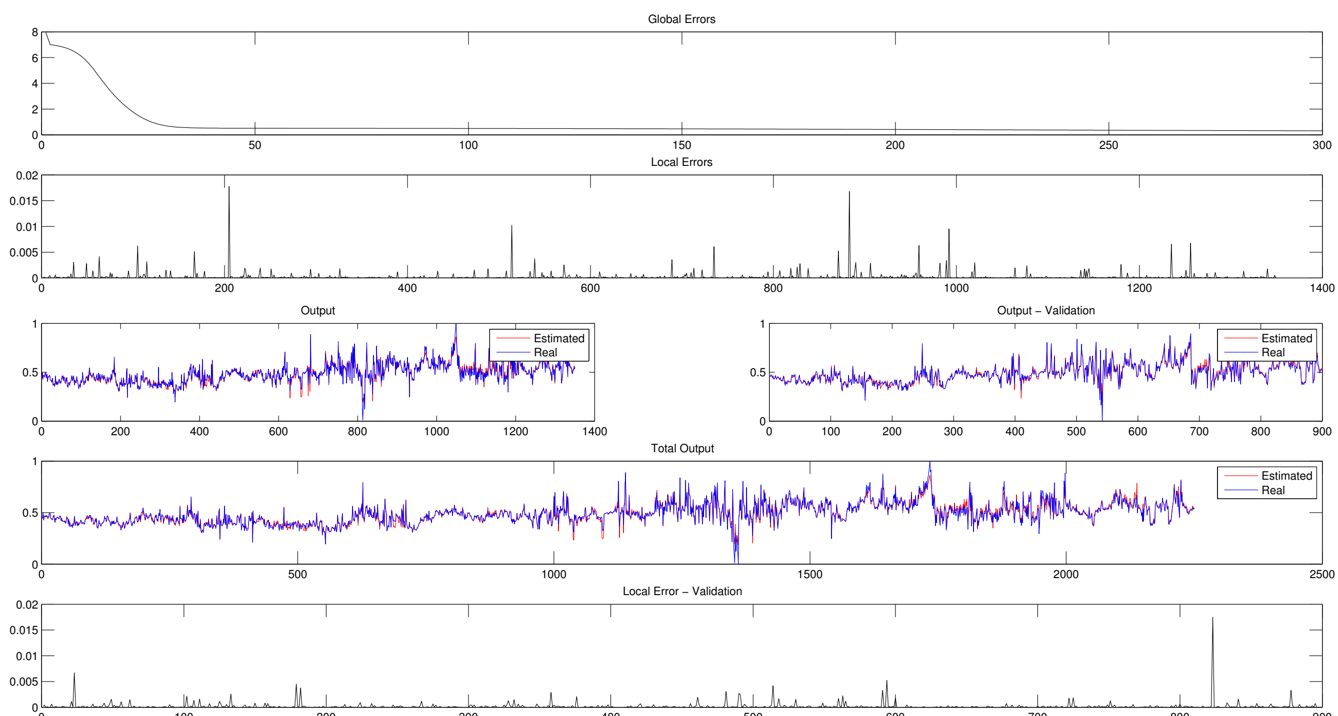



Figure 2. Graphic Result of Experiment 2

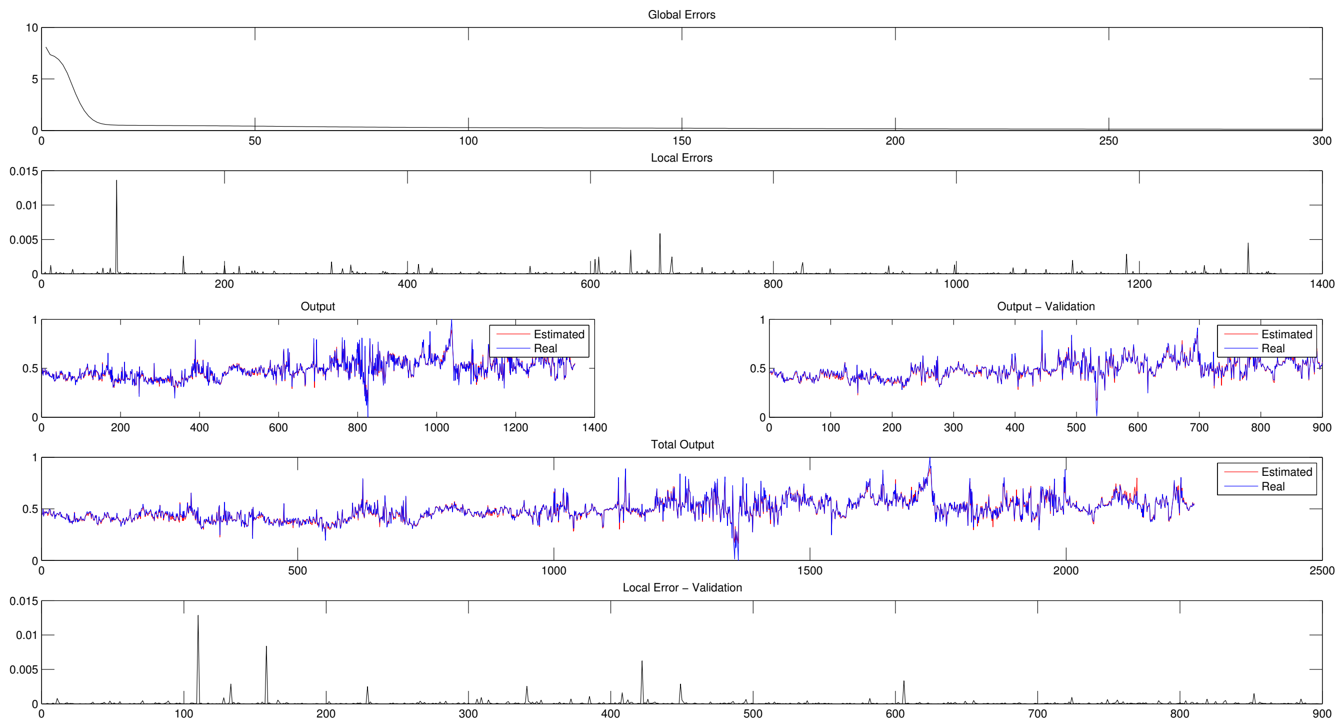



Figure 3. Graphic Result of Experiment 3

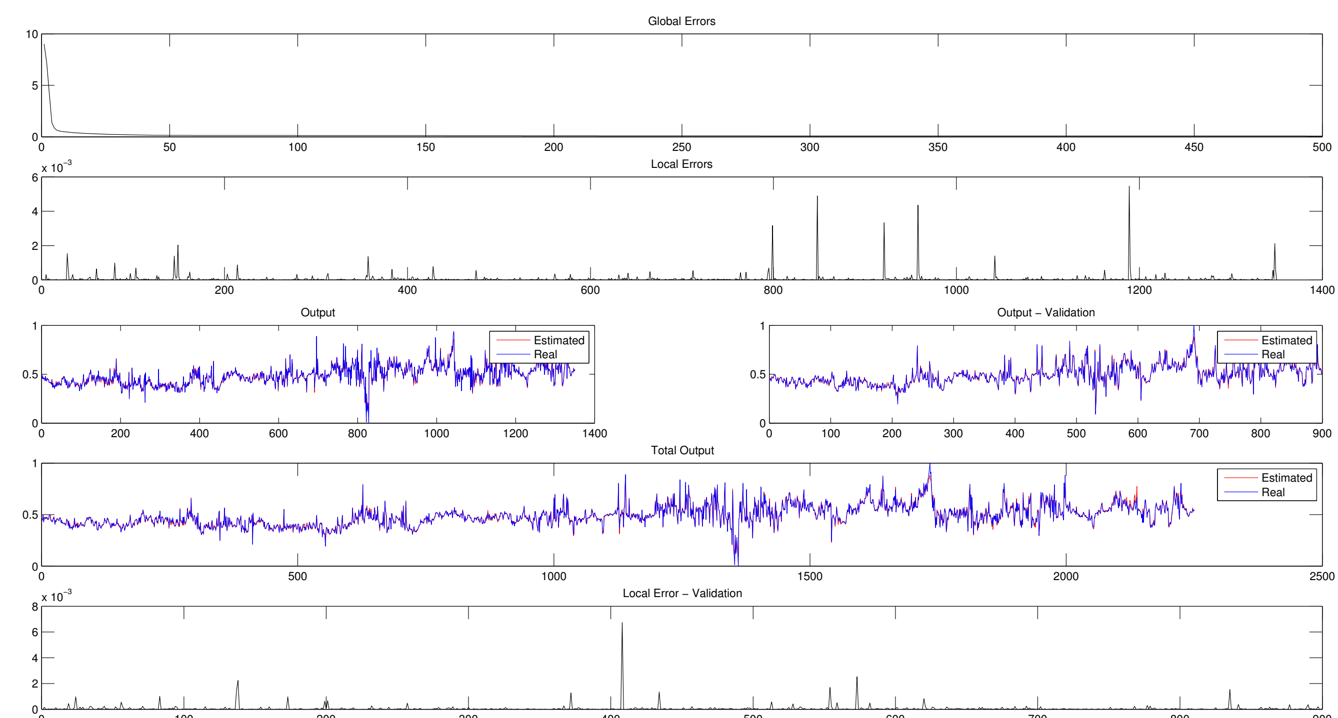
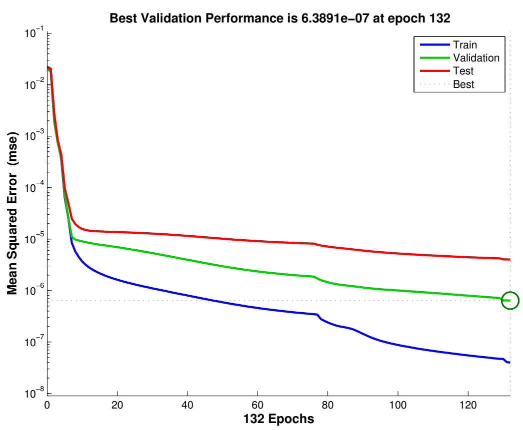


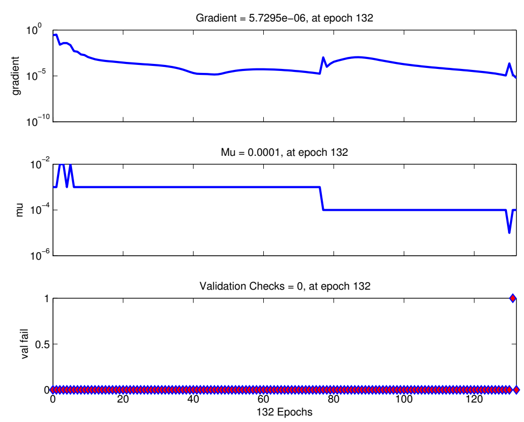

Figure 4. Graphic Result of Experiment 4

| Network | Hidden Layers | Neurons per Layer | Activation Functions | Error |
|---|---|---|---|---|
| 1 | 1 | - | LOGSIG | 0.0058 |
| 2 | 2 | 10 | LOGSIG | 0.00976 |
| 3 | 2 | 10 | TANSIG | $2.32 * 10^{-6}$ |
| 4 | 3 | 10 | TANSIG | $2.69 * 10^{-7}$ |
| **5** | **4** | **10** | **TANSIG** | $\mathbf{7.61 * 10^{-8}}$ |
| 6 | 4 | 20 | TANSIG | $1.47 * 10^{-6}$ |
| 7 | 5 | 10 | TANSIG | $3.11 * 10^{-5}$ |
| 8 | 5 | 20 | TANSIG | $6.63 * 10^{-7}$ |

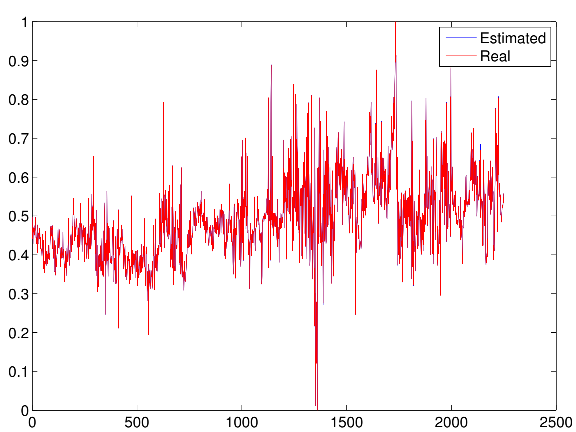
Table VII
RESULTS (*nntool* MATLAB)



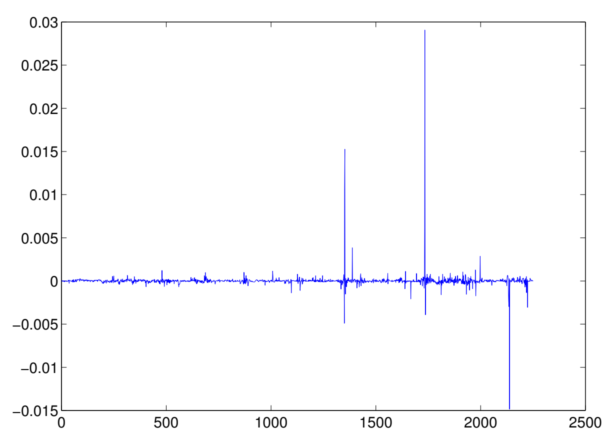

(a) MSE




(b) Training State




(c) Real Output Vs. Estimated Output



(d) Error

Figure 5. Graphic results of Network 5

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