EN ROUTE SPEED OPTIMIZATION FOR CONTINUOUS DESCENT ARRIVAL

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EN ROUTE SPEED OPTIMIZATION FOR CONTINUOUS DESCENT ARRIVAL

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This thesis work is dedicated to my parents, Kevin and Dianne Lowther, who have worked incessantly to provide the opportunity for me to be where I am today.

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LIST OF SYMBOLS

δ_i	Binary variable indicating whether this aircraft has a change in Mach
ΔM_i	Change in Mach number
Δt_i	Change in time from initial ETA
ΔT_l	Δt_i grid point
η	Thrust specific fuel consumption
η_l	λ_{kl} summing term
λ_{kl}	Grid point selector
μ_k	λ_{kl} summing term
σ	Ambient pressure ratio
a_i	Speed of sound
$a_{i,m}$	Slope of the mth linear segment of a fuel curve
$b_{i,m}$	y-intercept of the mth linear segment of a fuel curve
cc_i	Integer variable for discrete Mach number change
D	Distance traveled by aircraft
E	Number of engines
$\dot{f}_i$	Fuel burn rate for an aircraft at a given Mach number
$\dot{f}_{\min}$	Minimum fuel burn rate
F_k	$\dot{f}\Big\vert_{M_{d_i}}$ grid point
j	Selected number of aircraft able to make a speed change
m	Number of lines in the linear interpolation
M	Large number in the big M method
$M_{bound,i}$	Maximum or minimum bound on the Mach number change

M_{d_i}	Final Mach number
M_i	Initial Mach number
N	Total number of aircraft performing CDA
P	Arbitrary large number in separation constraints
P_{f_i}	Percentage difference in fuel burn rate change
r_i	Variable approximating nonlinear $\dot{f}\big\|_{M_{d_i}} \Delta t_i$ term
$t_{0,i}$	Start time of first Mach change
t_D	Calculated ETA
t_i	Initial ETA
T_i	Total time of Mach speed change
$T_{i,r}$	Time interval during which the aircraft returns to its original Mach
t_r	Time at which aircraft returns to its original Mach
V	Ground speed of aircraft
V_r	Resultant ground speed
$\boldsymbol{W}$	Wind vector
W_C	Cross wind component of wind vector
W_H	Head wind component of wind vector
z_1, z_2, z_3	Binary variables for separation constraints
z_k	Integer variable
z_{k0}	Integer variable prior to convergence on an integer value

LIST OF ABBREVIATIONS

ABESS	Airline-Based en Route Sequencing and Spacing
ACARS	Aircraft Communication Addressing and Reporting System
ADS-B	Automatic Dependent Surveillance Broadcast
AOC	Airline Operations Center
ATA	Actual Time of Arrival
CDA	Continuous Descent Arrival (and Approach)
ESCORT	En route Speed Change Optimization Relay Tool
ETA	Estimated Time of Arrival
FAA	Federal Aviation Administration
FAR	Federal Aviation Regulation
FMS	Flight Management System
ILS	Instrument Landing System
MILP	Mixed Integer Linear Program
MIP	Mixed Integer Program
MITRE	Non-Profit Research Company MITRE
NOAA	National Oceanic and Atmospheric Administration
OCC	Operations Control Center
RTA	Required Time of Arrival
RUC	Rapid Update Cycle
SMART	Self-Managed Arrival Resequencing Tool
TASAT	Tool for Analysis of Separation and Throughput
TRACON	Terminal Radar Approach Control
UPS	United Parcel Service

SUMMARY

Continuous Descent Arrival (CDA) procedures have been shown to minimize the thrust required during landing, thereby reducing noise, emissions, and fuel usage for commercial aircraft. Thus, implementation of CDA at Atlanta's Hartsfield-Jackson International Airport, the world's busiest airport, would result in significant reductions in environmental impact and airline operating costs. The Air Transportation Laboratory at Georgia Tech, Delta Air Lines, and the local FAA facilities (Atlanta Center and Atlanta TRACON) collaborated to design CDA procedures for early morning arrivals from the west coast. Using the Tool for Analysis of Separation and Throughput (TASAT), we analyzed the performance of various aircraft types over a wide range of weights and wind conditions to determine the optimum descent profile parameters and to find the required spacing between aircraft types at a fixed metering point to implement the procedure. However, to see the full benefits of CDA, these spacing targets must be adhered, lest there will be a loss in capacity or negation of the noise, emissions, and fuel savings benefits. Thus a method was developed to determine adjustments to cruise speeds while aircraft are still en route, to achieve these spacing targets and to optimize fleet wide fuel burn increase. The tool in development, En route Speed Change Optimization Relay Tool (ESCORT), has been shown to solve the speed change problem quickly, incorporating aircraft fuel burn information and dividing the speed changes fairly across multiple airlines. The details of this tool will be explained in this thesis defense. Flight tests were conducted in April-May of 2007, where it was observed that the spacing targets developed by TASAT were accurate but that delivery of these aircraft to the metering point with the desired spacing targets was very challenging without automation. Thus,

further flight tests will be conducted in 2008 using the en route spacing tool described above to validate the improvement it provides in terms of accurately delivering aircraft to the metering point.

CHAPTER 1: INTRODUCTION

The number of domestic airport operations in 2006 in the United States was recorded as 15.4 million; and by 2025 this number is expected to increase to 23.8 million, an increase of over 150% [18]. However, with only three airports to be built to alleviate these busy hubs in the next 10 years, increased airport operations will most likely occur at the same busy airports [16]. With this increased number of operations, there are a number of issues to consider, particularly the issue of noise from increased operations on the same runways, and additional noise and emissions from the operations on newly created runways [6].

Studies show increased noise pollution (aircraft noise levels above 50 dB) deteriorates general health, increases the risk of cardiovascular disease, is associated with the intake of non-prescribed sleep medication [19], and lowers property values of the surrounding communities [14]. In addition to contributing to noise pollution, expanding airports also increase emissions close to the airport. Extraneous CO_2, N_2, H_2O, SO_X, and ozone-depleting NO_X emissions close to populated areas affect respiratory health and increase greenhouse effects [3]. These noise and emissions pollution concerns predicate a need to reduce the noise and emissions from aircraft close to the airport in the terminal descent area.

Yet another concern for air transportation expansion is the rising cost of fuel and its contribution to an airline's costs. Although future fuel prices cannot be estimated with much accuracy, fuel currently comprises 27% of airline operating costs [23]. As the second largest operating cost (and sometimes the largest depending on the airline), there is significant incentive for fuel efficiency.

Currently, there exists a procedure, among many other research efforts, that meets all of these goals—reducing noise and emissions in the terminal area, and reducing the fuel consumed during the descent of each flight. This procedure is continuous descent arrival (CDA). Continuous descent arrival is a procedure for both approach and arrival where an aircraft flies a higher altitude and lower thrust, eliminating the need for level flight segments for air traffic control spacing purposes prior to the aircraft aligning with the glide slope. The idea of CDA has been common pilot knowledge for a long time, and the first formal introductions for a widespread procedure in busy areas began in the 1960's and 1970's at NASA, Lufthansa, and British Airways [12, 31, 13]. These initial studies created the formal definition of CDA as a descent from 6000 ft to the interception of the glide slope that contains no or at most one level segment not longer than 2 NM [26]. However, it was not until the advent of modern avionics technologies such as the Flight Management System (FMS) and Instrument Landing System (ILS) that CDA could be considered seriously. These improvements in navigation technologies have extended CDA to begin even further from the runway. The modern CDA starts at cruise altitude tens of thousands of feet above the original 6000 ft definition. The phrase CDA in this report will refer to arrival (outside of the terminal area) and approach (inside the terminal area) procedures beginning from the cruise altitude.

CDA solves a number of problems in the ever-expanding air transportation system. Clarke et al. demonstrated a reduction of fuel consumed by 400-500 lb per flight,

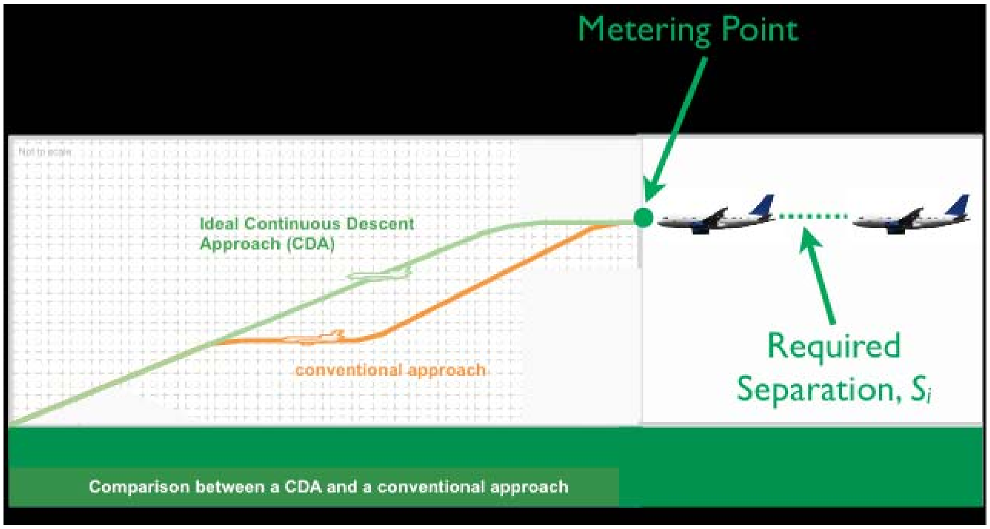


Figure 1.1: Comparison between a CDA and a conventional approach [14]

and 3.9 to 6.5 dBA reductions in the peak noise level along some portions of the flight path in a 2004 CDA flight test [10]. Both are promising results; in addition, CDA allowed select aircraft to reduce their standard arrival procedure flight time by up to 147 seconds. However, to ensure that aircraft meet FAR separation constraints throughout the CDA, their separation prior to, and thus time passing, the descent fix must be controlled. This task lies outside the realm of normal air traffic control duties, as it requires detailed knowledge about an aircraft's trajectory, i.e. both an aircraft's three dimensional path and accurate time trajectory, as well as relative timing between aircraft.

Clarke et al. reports, "One of the key issues preventing widespread implementation of [CDA] procedures is the inability of air traffic controllers to predict the future trajectory of aircraft with enough accuracy and confidence that they would use these procedures during periods of high-density traffic... This suggests that an appropriate solution for the system would be to provide a tool that will translate the predicted trajectory of each aircraft into a form that controllers can easily monitor and use to predict future separation" [10]. The issue in expanding CDA does not lie in an aircraft's ability to fly the procedure; rather, the limiting factor of CDA expansion is the spacing of the aircraft prior to descent and how to implement that information. The excerpt from Clarke et al.'s paper states that the future trajectories of the aircraft participating in the CDA is not known with enough precision to implement CDA during periods of high-density traffic.

This limitation in CDA implementation can be divided into three main areas—knowing how far apart aircraft must be spaced prior to the top of descent in order to maintain separation requirements close to the runway, knowing the precise time at which an aircraft will arrive at a given point, and how to arrange aircraft to achieve the necessary spacing during the en route portion of flight while being fair to the airlines involved. The combination of these three limitations has not been addressed with a unified effort until now. Although this thesis details the solution to the third problem, that of arranging en route aircraft with the proper spacing, for this project to be completely successful in expanding the implementation of CDA's, the other two pieces must be solved as well.

The first piece, knowing how far apart aircraft must be spaced prior to beginning the CDA, has previously been addressed. The Tool for the Analysis of Separation and Throughput (TASAT) is a fast-time aircraft simulator which runs a variety of aircraft types, weights, and wind conditions for a given area and determines what the minimum time spacing for two aircraft flying a CDA should be. The output to TASAT gives this time separation based on the desired percentage controller involvement in spacing the aircraft is desired to occur. Although no controller involvement for 100% of the flights is desirable, a reduction in this non-involvement number is more practical to decrease the separation time required [C]. TASAT provides a concrete time separation number necessary for flights to fly a CDA.

The second limitation of CDA, knowing the precise time at which an aircraft will arrive at a metering point, is a separate project from this thesis. However, a developing solution for an improved ETA tool (IET) and trajectory predictor is explained in Section 3.3. The exact ETA is an important piece of CDA implementation because if the separation required by TASAT is known, but the time at which the aircraft arrive at a metering point is unknown ahead of time, there can be no prior planning to achieve this separation. As the intended expansion of CDA depends on all three pieces working concurrently, this thesis work has been developed with the functioning of the IET in mind.

The last piece of the puzzle, the ability to calculate en route trajectory changes so that the TASAT-calculated separation can be achieved, is the subject of this thesis. Figure 1.2 is an image taken from Delta's Operational Control Center (OCC) during a CDA flight test [11]. The difficulty of arranging flights flying a CDA at ATL is apparent from this diagram. For the fifteen flights involved, all must arrive at the same point with the spacing calculated by TASAT. However, there are several factors to consider to achieve this spacing: the minimum speed change necessary for each aircraft, the minimum fuel burn increase to achieve the spacing so that the benefits of these aircraft flying the CDA are not negated, how to be fair to the different airlines operating the aircraft if speed changes are necessary, minimizing the delay incurred by the necessary speed changes, and determining the landing sequence of the aircraft which will achieve the lowest net fuel burn increase while achieving the TASAT-calculated separation.

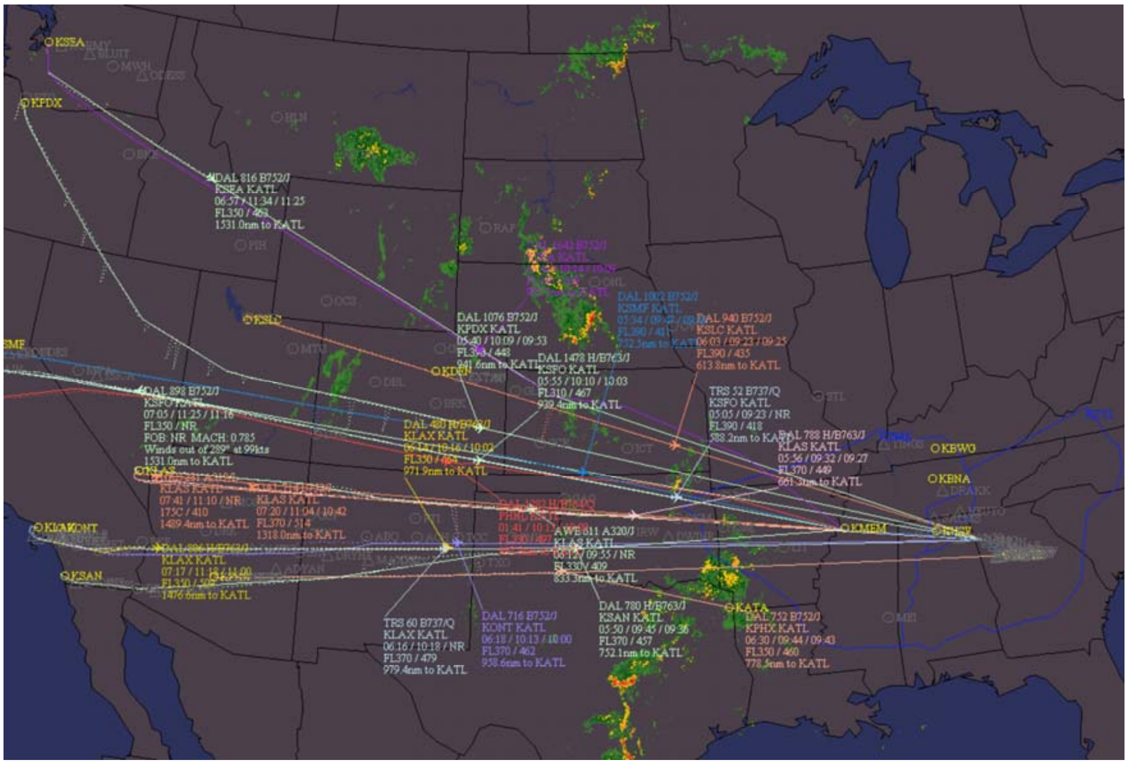


Figure 1.2: Aircraft trajectories to a metering point prior to descent during a 2007 CDA flight test [11]

These five concerns are the guiding principles for this thesis work and the creation of The En route Speed Change Optimization Relay Tool (ESCORT). Combined with TASAT and the IET, ESCORT aims to eliminate the difficulties previously experienced in implementing CDA because of the lack of information for necessary initial separations of CDA aircraft, and how to achieve these separations once they are known. ESCORT has been designed for either air traffic controllers or airline dispatchers to use while communicating with aircraft in the en route environment.

CHAPTER 2: CURRENT STATE OF THE ART FOR CDA & PROJECT GOALS

2.1 CURRENT STATE OF THE ART FOR CDA

Researchers in the past few years have studied the problem of modifying the trajectories of aircraft along a route to have a more exact spacing for increased arrival efficiency, or for CDA implementation, but there are a number of key differences between those studies and the work to be presented here.

In several studies [33,6], 4D trajectories (spatial trajectory plus temporal) have been used to issue speed changes along the aircraft flight route in order to effect better spacing without vectoring or holding patterns at a fix, or prior to the runway. Some of these studies have assumed that the aircraft in question will be ADS-B in and ADS-B out equipped. ADS-B, or Automatic Dependent Surveillance Broadcast allows pilots and controllers to see radar-like displays with traffic data, without depending on radar. These displays also will give pilots access to weather services, terrain maps and flight information services. This improved situational awareness will allow pilots to fly at safe distances from one another with less assistance from air traffic controllers [17].

For United Parcel Service (UPS) aircraft (used for many CDA flight tests), which are all ADS-B equipped, such an assumption is not an issue. However, with plans to equip commercial aircraft with ADS-B-out technology up until 2020, ADS-B is many years away from practical implementation in conjunction with CDA [22]. Thus, to implement CDA on a wide level using aircraft with varying levels of equipage, assuming ADS-B capabilities sets back the implementation possibilities several years. The plan for this research is to develop a spacing tool that can be tested at Delta's OCC within the next six months and flight-tested within the next nine months without the necessity of ADS-B equipped aircraft.

A similar study for “trajectory-oriented time-based arrival operations” used an additional assumption that is not met by this thesis’ near-term implementation goal [28]. This study assumed the spacing tool will be managed by air traffic controllers. Thus, workload becomes a greater consideration, as they have a number of other tasks to perform. In addition, controllers speak with the pilot only through radio communication; ESCORT aims to uplink speed changes to the aircraft directly via ACARS, saving time and possible confusion with misunderstood oral commands. Yet, there was some relevant information gained from the study for this thesis. Prevot et al. asked the controllers to rank the tools that were most useful to them in modifying the aircraft’s trajectories. The final list was stated as: “timeline, speed info block, speed advisories, trajectory preview, route modification tool, color coding, and a conflict list.” The controllers themselves, put in a similar situation with what this thesis aims to do, said that having an accurate time line, accurate speed information, and then the ability to issue speed advisories are the most important components of an ETA trajectory modification tool [28]. ESCORT will incorporate this core functionality and will automatically provide the conflict list. However, ESCORT will not allow for route modification, as only minimal speed adjustments will be made. With ESCORT, air traffic controllers will perform their normal duties, with speed changes so small that ATC does not need to be asked for approval. By limiting the speed change to a 0.02 Mach increase or decrease from the flight plan cruise speed, airline operations control centers (AOC) can bypass air traffic control and issue the speed changes directly to the planes. By focusing on an

AOC, the requirements of the regulatory environment of ATC can be met within the operation so that testing and implementing the speed change solution can occur on a

Table 2.1: Summary of literature review

Authors	Year	Key Findings	Limitations
Wichman et al.	2001	-Used a Required Time of Arrival (RTA) capable aircraft to prove that RTA capability could place an aircraft at a way-point with an accuracy of 7 seconds	-Limited to RTA-equipped aircraft -Single B737-600 tested
Prevot et al.	2003	-Potential benefits in throughput, efficiency, and workload shown for a trajectory-oriented approach for ATC -Can be used in conjunction with relative operations	-Full benefits of study require ADS-B equipped aircraft
Weitz et al.	2005	-Simulation results show Airborne Precision Spacing tool capable of reducing inter-aircraft spacing errors for CDA	-Limited to one aircraft type -Speed changes made during descent
Baxley et al.	2006	-Use a ground ATC tool to separate aircraft prior to a metering fix -Incorporates an on-aircraft system to make frequent minor speed adjustments to increase timing precision	-Frequent minor speed changes issued -Optimization component for fuel and airline schedule not included -Assumes ADS-B aircraft

much faster time scale.

2.2 SIMILAR TOOLS

Currently, there are a number of similar tools in development. The differences in assumptions for previous research have been explained in the previous section. However, it is worth naming and describing similar tools that have been developed, or are under development. As part of the Airline-Based en Route Sequencing and Spacing (ABESS) program, the NASA Ames-developed Cruise Speed Calculator (CRZ) uses flight closure rates to a common fix for speed advisory calculation. This tool allows for both speed-ups and slow-downs. However, limited flight plan and wind information is taken into account with this basic tool.

Another tool that is a part of ABESS is the Self-Managed Arrival Resequencing Tool (SMART), developed by MITRE. This is a more sophisticated tool, utilizing both flight plan and wind data to not only calculate necessary speed changes, but also to sequence and merge the aircraft while ensuring an appropriate time interval between each aircraft. A display from SMART is shown in Figure 2.1. One can see a sequence of flights organized by arrival time, with space for a displayed speed advisory. The capabilities of SMART are exactly what ESCORT aims to emulate. Although the details of SMART—exactly how the calculations are performed and calculated—are not known at this point, ESCORT will have the same functionality in addition to a key added component, that of minimizing the overall fuel consumption for en route aircraft flying a CDA. This minimization will be discussed in following sections. In addition, the spacing to be achieved with ESCORT will be determined by the Tool for the Analysis of Separation and Throughput (TASAT) developed in support of previous CDA flight tests. This tool ensures that, for a given leading and following aircraft pair, their spacing will allow them to fly a CDA without encroaching on each other.

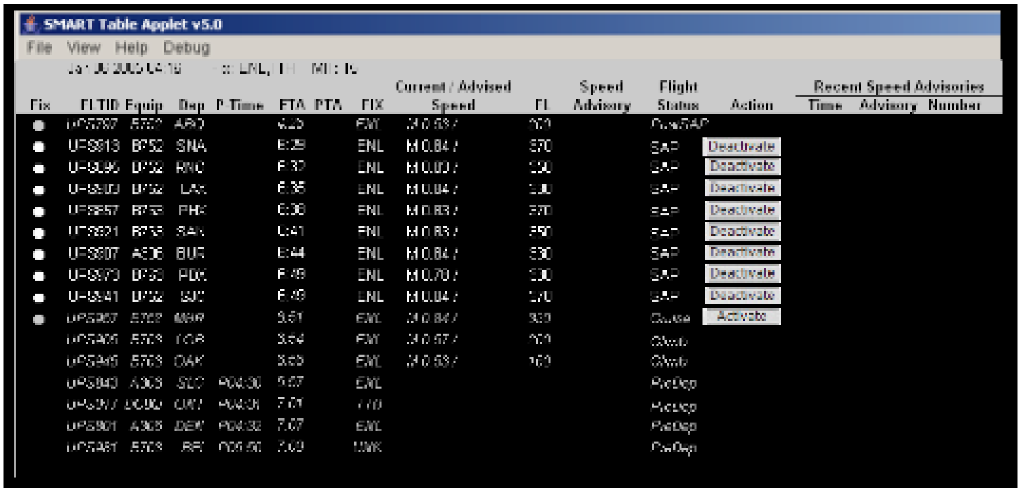


Figure 2.1: SMART display

2.3 ESCORT CORE CAPABILITIES

Each of these tools, SMART and CRZ, calculates the minimum speed change to reach a desired speed change at a metering fix. In contrast, ESCORT will calculate a limited number of speed changes. Both one speed change and two speed change formulations have been examined, each occurring no earlier than three hours prior to the aircraft's initial ETA at the metering fix. This discreet number of speed changes will be easier to implement by the pilots, although more speed changes may be necessary during the course of the flight. If such is the case, ESCORT could be run multiple times with updated information. With ADS-B spacing, previous studies encountered issues of issuing speed changes so frequently that it required constant adjustment by the pilot [6].

The most important difference between ESCORT and the previous en route spacing prior to a fix tools is the inclusion of an optimization component in ESCORT. ESCORT is designed to not only calculate the minimum speed change necessary to achieve the desired spacing; it will also include fuel burn as a consideration for aircraft participating in the CDA program. While all aircraft flying a CDA are capable of speeding up and slowing down, the flight plan cruise Mach number is often designed to

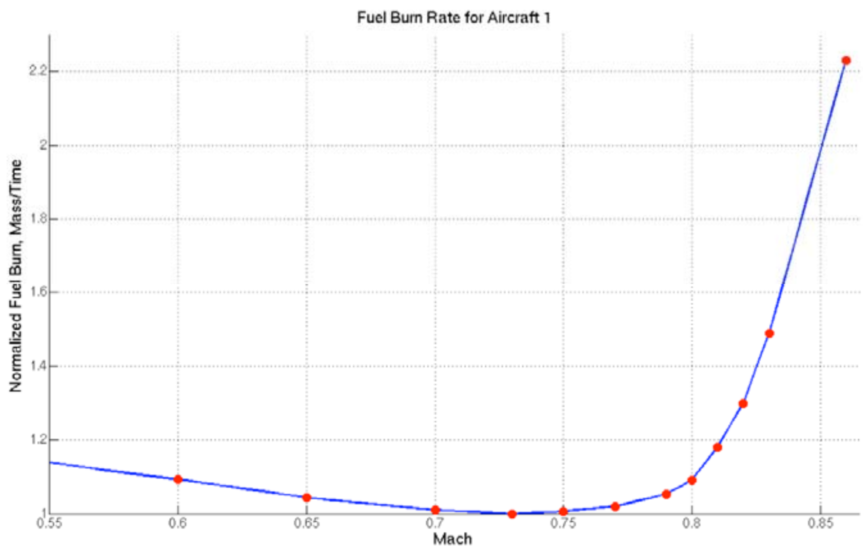


Figure 2.2: Fuel burn rate vs. Mach, for a commercial wide body jet

be close to the minimum fuel burn rate for that particular aircraft.

Figure 2.2 shows the minimum fuel burn as a function of Mach for a commercial wide body aircraft at 37,000 ft. A speed change increasing the cruise speed by 0.02 Mach, corresponds to increased fuel consumption, depending on the initial cruise speed of the aircraft. Since one of the goals of the CDA program is to reduce an aircraft's fuel burn, it would not make sense to increase the fuel burn (if it can be avoided) during the cruise portion only to save fuel during the descent. By considering fuel burn over both the cruise phase and CDA, an additional fuel savings may be attained.

CHAPTER 3: RELEVANT SUPPORTING CONCEPTS

3.1 REVIEW OF LINEAR OPTIMIZATION TECHNIQUES

3.1.1 CPLEX Branch and Cut Overview

CPLEX, the linear optimization solver used in the implementation of ESCORT was created by ILOG to solve many types of linear programming problems. The problem type of interest in the current setting is a mixed integer program (MIP), or more specifically, a mixed integer linear program (MILP), a problem class where there are no quadratic, or higher order, terms in the objective function. As MILP's are a subset of the more general MIP, a brief description of how CPLEX solves MIP problems will be given here.

As stated in the CPLEX user's manual, CPLEX solves a series of continuous sub problems as a part of the branch and cut (also referred to as branch and bound) algorithm [24]. To manage those sub problems efficiently, CPLEX builds a tree in which each sub problem is a node. The root of the tree is the continuous relaxation of the original MIP problem and is first solved with relaxed integer constraints using the simplex method. If the solution satisfies the integer conditions, the process stops. However, if the solution to the relaxation has one or more fractional variables, CPLEX will select some integer variable z_k whose value z_{k0} in the continuous solution is not integer. To bring z_k to an integer value, there are two necessary constraints, $z_k \leq [z_{k0}]$ and he second, $z_k \geq [z_{k0}] + 1$, with the brackets indicating, "the integer part of." These constraints are then attached to two sides of the previous continuous problem, creating the branches. These problems are then solved by the simplex method, and if either problem is infeasible, that branch is excluded from further consideration. For the feasible solutions, in a minimization problem, if the objective function value is less than the value of previously known integer solutions, then that branch is added to an active consideration list. If the objective function is larger, then that branch is bounded (cut), and eliminated from continued consideration [20].

For the problems remaining in the active problem list, the one with the lowest objective function is selected. If its solution satisfies the integer constraints, then it is optimal. However, if this is not the case, an integer variable that has a fractional value is selected (usually the one with the largest fraction) and the branching process described above is repeated. The steps are repeated, with feasible problems being added to the active list, and objective values compared to the best-known integer solution. This process exhausts all possibilities, and eventually selects the optimal integer value.

For the problem relevant to ESCORT, the integer constraints are the binary variables described in Section 4.4.2, as well as the binary variables described in Section 4.3 necessary to modify the objective function. In addition, the possibility of using integer variables to limit the Mach changes to discrete, rather than continuous, decimal values were considered in Section 4.4.3. CPLEX was used in all instances of ESCORT to solve the optimization using its MIP solvers.

3.2 REVIEW OF AIRCRAFT PERFORMANCE CHARACTERISTICS

For steady, level flight, a force balance on an aircraft in motion yields information necessary to determine the fuel use rate for the aircraft. In Figure 3.2.1, the force balance is displayed [1].

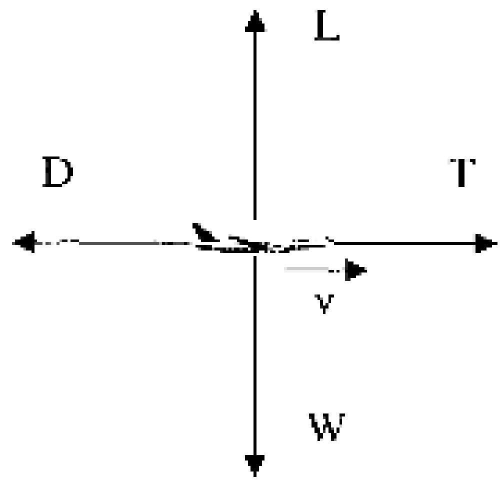


Figure 3.1: Aircraft in steady, level flight

In ESCORT, the data relating to aircraft's performance comes from TASAT's aircraft dynamics model [29]. As such, the lift and drag forces in Figure 4.1 are written as [2]

$$L = \frac{1}{2}\rho V_r^2 S C_l, \tag{3.2.1}$$

$$D = \frac{1}{2}\rho V_r^2 S C_d, \text{ and} \tag{3.2.2}$$

$$W = mg, \text{ with} \tag{3.2.3}$$

$$\dot{m} = -\dot{f}. \tag{3.2.4}$$

However, the derivative of aircraft mass was only used to compute the instantaneous aircraft mass, meaning the aircraft mass was assumed static. Examining Figure 4.1, we see that to find the thrust, *T*, we need an equation for the drag, *D*:

$$\sum F_x = T - D = 0. \tag{3.2.5}$$

In order to solve for *D*, it was assumed that in cruise

$$C_d = k_0 + k_1 C_l + k_2 C_l^2, \qquad (3.2.6)$$

with k_0, k_1, and k_2, being drag polar parameters. Using this equation in conjunction with the basic lift and drag force equations yields an equation for the thrust,

$$T = \frac{1}{2}\rho V_r^2 S k_0 + mgk_1 + \frac{2(mg)^2}{\rho V_r^2 S} k_2. \qquad (3.2.7)$$

Knowing the thrust specific fuel consumption η and a fuel correction factor C_f would then yield the fuel burn rate for a given weight and altitude

$$\dot{f} = \eta T C_f. \qquad (3.2.8)$$

However, TASAT uses specific collected aircraft performance data to supplement the above calculation. The fuel burn rate curves used in ESCORT and described in the remaining document were derived using TASAT simulations of each of the aircraft flying at 37,000 ft with a specified weight [29]. In TASAT, corrected engine fuel flow was represented as a lookup table given fuel flow as a function of corrected thrust and Mach number at different altitudes. In addition, TASAT calculates thrust as corrected net thrust per engine, defined as net thrust per engine F_n divided by the ambient pressure ratio σ, with E being the number of engines,

$$T = E\sigma\left(\frac{F_n}{\sigma}\right). \qquad (3.2.9)$$

An operational version of ESCORT will use fuel burn values calculated by TASAT for the given flight levels and aircraft weights using the aircraft performance data in the lookup tables. In the remainder of this report, fuel burn curves are used assuming 37,000 ft flight level and an average aircraft weight for the given aircraft type.

3.3 REVIEW OF ETA TECHNIQUES

To meet separation constraints during all portions of a CDA, aircraft must be accurately and sufficiently spaced at the metering point at or before the top of the descent portion of the flight. By ensuring that flights are a certain distance, or time apart, both wake vortex and trailing rule restrictions can be satisfied. Traditionally, air traffic control has used relative operations, meaning spacing aircraft with respect to one another (for example, a miles-in-trail requirement), as opposed to absolute operations, where aircraft are directed to be in a certain place at a certain time on the same time scale. Yet, with relative operations, air traffic controllers are not accustomed to sequencing aircraft coming to a point from different directions. It is much easier to direct aircraft that are traveling on the same trajectory (same straight line) using miles in trail requirements, than to visualize and detect the spacing of aircraft traveling to the same point from multiple directions.

However, time-based techniques utilizing ETA and flight path information are becoming more commonplace, as traffic flow tools aim to maximize airport and TRACON capacity without compromising safety. One such tool is Traffic Management Advisory, a system using ETA, flight path, and capacity information for airports, sectors, fixes, and runways to schedule and sequence flights for a given sector. Time-based techniques like this one are developing, but their usefulness and accuracy remains to be tested [34].

In addition, in the future, if aircraft are to self-merge and self-space, the aircraft are limited by range relevant technologies such as traffic collision avoidance systems, automatic dependent surveillance broadcast systems, or traffic information services broadcast systems. This equipment also provides a limited size of the displays in the cockpit. In order to merge behind or follow another aircraft, flight crews need to be able to identify traffic and they need to be in a position close to the time or distance interval target [28].

Using an absolute time scale to examine spacing as opposed to using relative spacing will help to ensure the flyability of CDA during busy operations. ESCORT functionality will depend on accurate estimated time of arrivals (ETA) and flight plan information to have an initial sequencing of the aircraft. An accurate ETA is crucial to the success of ESCORT, as a flight test at Delta's AOC in May 2007 demonstrated [11]. While currently airlines have an ETA for all of their flights, these estimates change significantly during the course of the flight. An example of this variation is shown in Figure 3.2 for a sample of Delta aircraft reporting FMS information during the May flight test. The graph demonstrates that, for each flight, the ETA deviates 5-15 minutes from the final reported ETA. This inaccuracy means that, if someone were trying to sequence the flights, he or she would have to account for a possible fifteen minute ETA deviation. When small speed changes are desired, accounting for a possible fifteen-minute leeway means making much more drastic speed change decisions and changing the sequencing significantly as the flights near their destination.

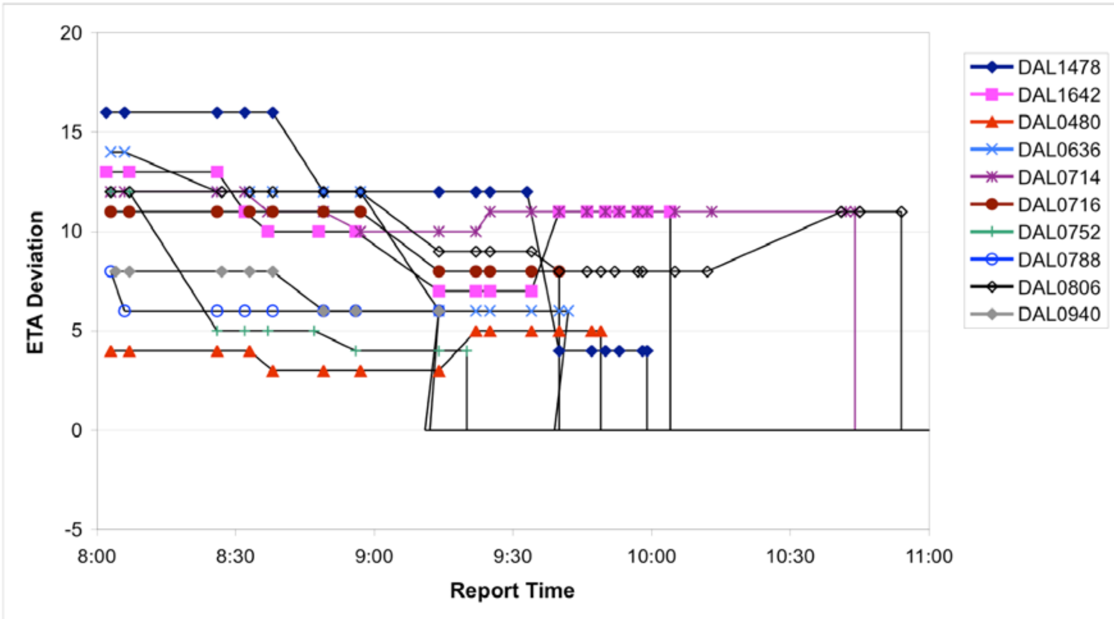


Figure 3.2: ETA variation vs. report time for a May 2007 flight test

ESCORT works in conjunction with an improved ETA tool (IET) that was developed separately, which will take into account real-time wind information, flight plan information, and probabilities of trajectory disturbance. Researchers in the Air Transportation Laboratory have developed a prototype IET which has a basic trajectory predictor, incorporating forecast information from the National Oceanic and Atmospheric Administration's (NOAA) Rapid Update Cycle (RUC) model and flight plan information to give an ETA. The development of this tool has shown that the RUC data provides unbiased forecast information for forecasts up to five hours. Since current FMS computers onboard aircraft only use weather information that is a 6 hour forecast, using the RUC data will allow greater accuracy for the trajectory predictor. In addition, the trajectory predictor incorporates meteorological data provided by the Aircraft Communications Addressing and Reporting System (ACARS). Aircraft in flight record the actual wind velocities along their flight path and report this information to other aircraft and to NOAA archives in a telex-based system. By incorporating both of these data sources, the trajectory predictor is expected to make use of better information to provide a more accurate ETA.

In addition, it is recognized that an ETA is not a deterministic calculation. In every estimate, there is likely be some expected variation. Yet, this variation can be quantified, and it is the goal of the IET to incorporate this variance by including a confidence interval on the final ETA calculations. By determining how forecast data is correlated between adjacent ACARS data points and historical data, the exact correlation of the data can be determined. Once this correlation is known, the IET will use this information to provide confidence intervals on the ETA estimate [32]. The simplified trajectory predictor architecture is shown in Figure 3.3.

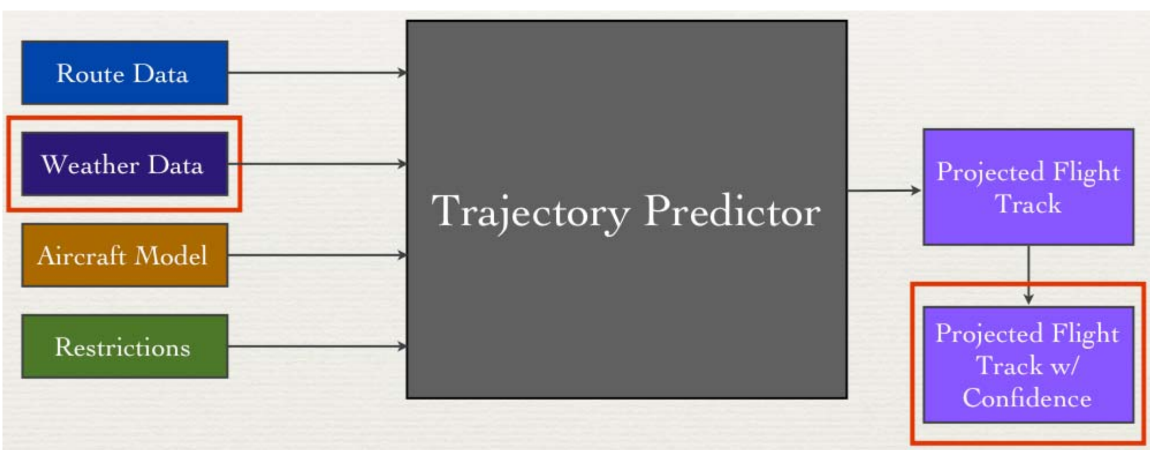


Figure 3.3: Simplified IET/Trajectory Predictor Architecture

The IET is being developed simultaneously with ESCORT so that both tools function together. Starting from the ETA estimation provided by IET, ESCORT will optimally pick up to two speed changes for each flight, making allowances for the necessary spacing, and sequence the aircraft, while minimizing the net fuel burn for the aircraft involved.

3.4 FAIRNESS IN THE CONTEXT OF EN ROUTE SPEED ADJUSTMENT

Fairness is an important aspect of CDA, because for CDA to have the most effect, as many pilots and airlines must agree to perform the procedure as possible. If each party knows that fuel burn and schedules of participating flights are all being altered as fairly as possible, it is assumed that more parties will be willing to participate.

In determining a quantifiable measure for fairness, there were five traits identified, as explained in [7] and [8]. The measurement should be population size independent, the unit of measure for the resource should not matter, the measure should be on a set scale varying from 0 to 1 (or 0 to 100 if a percentage), the measure should exhibit continuity—reflecting any change in the resource allocation (i.e. not a minimum or a maximum), and the fairness measure should be able to be tracked instantaneously, as well as on an extended time scale. In addition, a fairness measurement should satisfy five criteria for a concrete definition of fairness. These five criteria are described below.

3.4.1 Fairness Criteria

3.4.1.1 Proportionality

According to Aristotle, goods should be divided in proportion to each claimant's contribution. This characteristic means that if there are three parties, but party 1 contributes 20% of the resources for the construction of a good, party 2 contributes 30% of the resources, and party 3 contributes 50% of the resources, then a proportional division would allocate 20% of the divisible good to party one, 30% to party 2, and 50%

to party 3. This portion of the fairness definition is seen to be inherent as a part of envy-free division.

3.4.1.2 Envy-freeness

No party is willing to give up the portion it receives in exchange for the portion someone else receives, meaning that no party envies any other party. An envy-free division is always proportional, because each party does not want anything from the other parties involved, meaning that the resource is divided proportionally (at least in the eyes of the resource division participants). Strategies that the other players select cannot prevent you from obtaining a portion that you think is the largest or most valuable.

3.4.1.3 Equitability

All parties think they receive the same fraction of the total, as each of them values the different resources. A difficult issue of how to measure whether both parties are equally happy is solved using a point-allocation system. In such a system, each party assigns points to the things being divided, and if when divided, the parties receive items with equal point values for the respective parties' scale, then the division is equitable. In the case of dividing en route airspace, dividing fuel burn may be a starting point for a measure of equitability.

3.4.1.4 Efficiency

A division is efficient when there is no other allocation of the system that would raise the allocation for the participant with the minimum allocation without decreasing the other participants' allocations. In other words, all users would benefit from a more efficient distribution. When such a possibility no longer exists, then the division is efficient. This efficiency will come from the general optimization algorithm being developed.

3.4.1.5 Truthfulness

For the conditions such as envy-freeness and equitability to be met, the individuals involved in the 'bargaining' must have their true intentions and desires known to all other parties. For a fair division, one party cannot try and "play the game," saying that one resource allocation scheme would be more desirable to his own party than another in hopes that another party will change their resource allocation preference, eventually helping the first airline. This condition, sometimes not included in fairness measurement and evaluation schemes, may be the most important, as an incentive to tell the truth must be inherent in any resource allocation in order for the allocation algorithm to be effective.

3.4.2 Suggested Fairness Measurement

The suggestion for a fairness measurement to allocate en route speed adjustment is to divide equally possible percent increase in fuel burn among different aircraft (or aircraft groups). Each airline seeks to operate their aircraft at, or close to the minimum fuel burn, reducing costs and maximizing aircraft range. By identifying the type of aircraft involved in the optimization calculation, the fuel burn characteristics are known for each aircraft. Then, a constraint is needed so that the percent fuel burn increase is equalized for all the aircraft involved. Such constraints are:

$$P_{f_i} = \frac{\dot{f}_i\big|_{M_d}}{\dot{f}_{\min}} \cdot 100 \qquad (3.4.1)$$

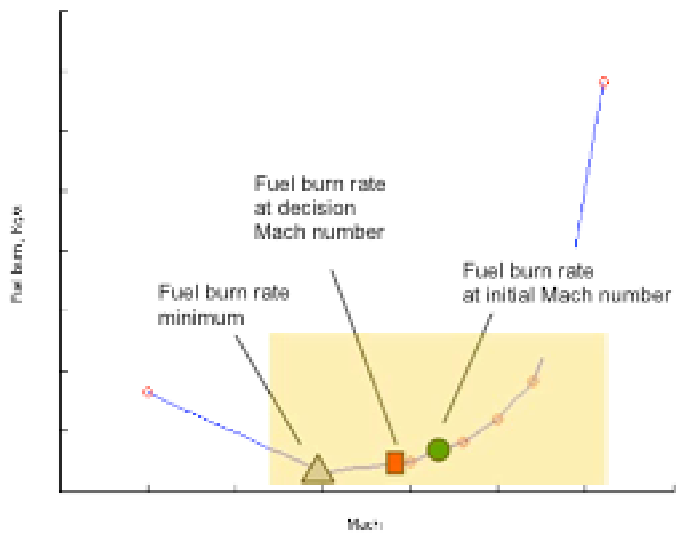


Figure 3.4: Chart showing the minimum fuel burn rate with the percentage difference between the initial fuel burn rate and the fuel burn rate at the decision Mach number compared to the fuel burn rate wanting to be minimized in order to account for fairness (values not shown because it is proprietary information)

$$P_{f_i} - P_{f_{i+1}} \leq |tolerance| . \tag{3.4.2}$$

Equation 3.4.1 is an expression for the percentage fuel burn, and Equation 3.4.2 ensures that consecutive airplanes in the CDA sequence have equivalent percentage fuel burn increases to within some tolerance (i.e. 0.01%).

An example fuel burn curve is shown in Fig. 3.4. The highlighted area shows an example of a range of possible values, centering on the initial cruise Mach of the aircraft (indicated by the green circle). The percentage deviation from the gray triangle, the fuel burn minimum, to the red square will be equalized for each flight in the CDA-performing fleet. These constraints can be included as a part of the optimization model so that each aircraft, or groups of aircraft belonging to separate airlines, has the same percentage increase in fuel burn for their re-routings.

An examination of the fairness constraint shows that it meets five of the five fairness criteria described in section 3.4.1, as well as the five necessary measurement traits. For a more complete literature review and explanation of fairness in the context of traffic flow management, please refer to Appendix A.

Proportionality- The contribution of differing aircraft performance is taken into account with the fact that different aircraft types are assigned different fuel burns, so that using a percentage, fuel inefficient aircraft would still have the same percentage fuel burn increase compared to efficient aircraft. The fuel burn increase would be proportional, since it is a percent, increasing or decreasing for the number of aircraft an airline operates.

Envy-freeness- Envy-freeness may be the toughest characteristic to meet. However, it can be implied that if all airlines lose the same amount of increased fuel use, there is no more enviable package among the different participants. The only envy that would be created would come from airlines using different aircraft with different minimum cruise fuel burns, but the fuel burn is already taken into account in the purchase of an aircraft.

Equitability- The equitability constraint requires different airlines creating different scales to measure the usefulness of the resource divided. It is assumed that airlines all want to reduce fuel use, and thereby operating costs, making equitable divisions a purely economic concern. This fairness constraint takes this equitability criterion into account, although a bidding process could be more effective in the future.

Efficiency- Since this fairness measure would be an additional constraint of the overall optimization algorithm; the efficient division of the airspace will indeed be possible with the suggested fairness measure.

Truthfulness- Truthfulness will also be met with the suggested fuel burn equalization fairness measure since the measurement is unbiased as it is simply a function of aircraft performance. Airlines will not be able to lie about their desired minimum fuel burn rate, because this data can be checked with other airlines and with the aircraft manufacturer.

CHAPTER 4: FORMULATION DESCRIPTION

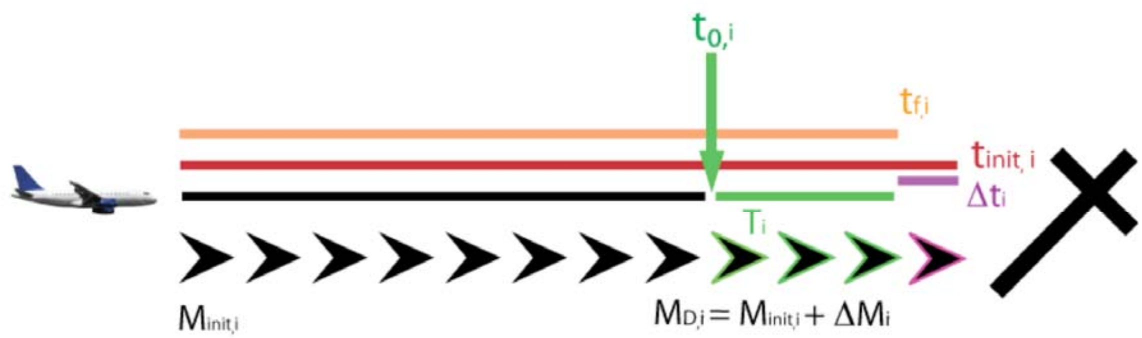


Figure 4.1: Diagram of formulation variables

4.1 MACH-TIME DERIVATION

The most important constraint in the problem formulation is the constraint relating the change in velocity to the change in time. This section describes the assumptions and derivation leading to the relatively simple equation 4.1.13. The derivation begins with the common equation distance equals rate times time, and develops from there.

We assume the speed adjustment is implemented at initial point at t_0, the time to travel from initial point to the virtual metering point is T, the original ETA at the virtual metering point is t_i, and the absolute value of the ETA adjustment is Δt. The positive sense of the adjustment Δt is an advance. The relationship between these parameters is shown in Figure 4.1 for an advance with a value of Δt. For n aircraft, we can assume that, although the initial ETA provides a sequence for the aircraft, this sequence is capable of being altered. The necessary separation for all possible aircraft following combinations is determined from TASAT analysis, and is the difference between the initial ETA t_{init} and the desired time, t_f, to meet separation requirements.

Assuming the ground speed of the aircraft is *V*, the distance *D* traversed by the aircraft during time *T* can be obtained as

$$D = \int_{t_0}^{t_i} V dt \tag{4.1.1}$$

Assuming the ground speed is increased by a constant value Δ*V* to achieve a time advance of Δ*t*, the distance traversed by the aircraft during time *t* - Δ*t* remains the same

$$D = \int_{t_0}^{t_i - \Delta t} (V + \Delta V) dt = \int_{t_0}^{t_i} (V + \Delta V) dt - \int_{t_i - dt}^{t_i} (V + \Delta V) dt \tag{4.1.2}$$

$$D = \int_{t_0}^{t_i} V dt + \int_{t_0}^{t_i} \Delta V dt - \int_{t_i - \Delta t}^{t_i} V dt - \int_{t_i - \Delta t}^{t_i} \Delta V dt = D + \Delta V \cdot T - \int_{t_i - \Delta t}^{t_i} V dt - \Delta V \cdot \Delta t \tag{4.1.3}$$

Assuming further that the ground speed *V* during time period [t_i - δ*t*, t_i] remains the same, i.e. $V = V_1$ (corresponding to the original mach number) for this time period. Thus, we have

$$\Delta V \cdot T - V_1 \cdot \Delta t - \Delta V \cdot \Delta t = 0 \tag{4.1.4}$$

This gives

$$T = \frac{(V_1 + \Delta V) \cdot \Delta t}{\Delta V} \tag{4.1.5}$$

This is to say, given ground speed at the virtual metering point, the time duration *T* needed to achieve a time advance of Δ*t* for a selected speed increase Δ*V* can be obtained or vice versa.

To determine the final $M_{d,i}$, it is necessary to examine the vector relationship between ground speed and true airspeed and winds. This relationship is shown in Figure 4.2.

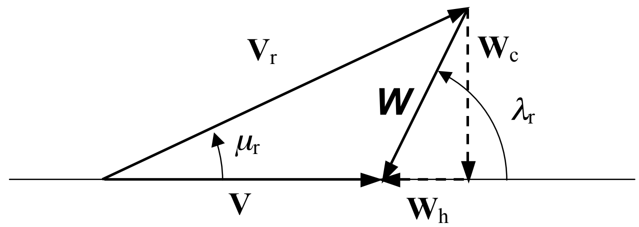


Figure 4.2: Diagram of wind vectors

W_h and W_c denote head wind and cross wind components respectively. In vector form, we have:

$$\mathbf{V} = \mathbf{V}_r + \mathbf{W} \, . \tag{4.1.6}$$

This gives

$$V = \sqrt{V_r^2 - W_c^2} - W_h \, . \tag{4.1.7}$$

The increase in ground speed is then

$$\Delta V = \left[\sqrt{(V_r + \Delta V_r)^2 - W_c^2} - W_h\right] - V = \sqrt{(V_r + \Delta V_r)^2 - W_c^2} - \sqrt{V_r^2 - W_c^2} \tag{4.1.8}$$

Using Taylor expansion and ignoring higher order small terms, we have

$$\Delta V \approx \frac{V_r}{\sqrt{V_r^2 - W_c^2}} \Delta V_r \tag{4.1.9}$$

If the cross wind component is relatively small comparing to the true airspeed, then

$$\Delta V \approx \Delta V_r \tag{4.1.10}$$

With the true air speed change known, the Mach number change, dependent on true airspeed and altitude can be found:

$$\Delta M_i = \frac{\Delta V_i}{a_i}. \qquad (4.1.11)$$

Where a^i is the speed of sound at the cruise altitude of aircraft *i*. Similarly,

$$M_i = \frac{V_i}{a_i}. \qquad (4.1.12)$$

Since it is assumed that the initial cruise speed M_{init} and final cruise speed M_d occur at the same altitude for each flight, the speed of sound for each speed change calculation is constant, meaning that Equation 4.1.5 above can be written as

$$T = \frac{(M_{imit} + \Delta M) \cdot \Delta t}{\Delta M}. \qquad (4.1.13)$$

However, when expanded, this equation gives a nonlinear problem, with bilinear unknowns, $\Delta M * \Delta t$. Yet, further simplification is possible, assuming that the change in Mach is much smaller than the initial Mach number. If the initial Mach is assumed to be close to 0.8, and the maximum change in Mach is 0.02, this assumption introduces a 2.5% error. With desired accuracy on the order of seconds for separation, a 2.5% error added to Δt's that are at most 300 seconds, would introduce an error of +/- 8 seconds. This introduced error may prove to be significant during flight trials that require drastic repositioning of flights, but for the cases observed during the April-May 2007 flight test [11], Δt's on the order of 120-180 seconds were common. In these cases, an error of 3-4.5 seconds is less significant. If, during the next flight test testing of this algorithm, this introduced error is an issue, this simplification must be addressed again. For now, it is assumed that the simplified equation, assuming ΔM in the numerator of Equation 4.1.13 goes to zero ($\Delta M << M$), gives the final Mach-time relationship for each flight *i*:

$$\Delta t_i = T_i \frac{\Delta M_i}{M_i}. \tag{4.1.14}$$

And finally, we have the final Mach number:

$$M_{d,i} = M_{init,i} + \Delta M_i. \tag{4.1.15}$$

4.2 FUEL BURN LINEARIZATION

As explained in Section 2.3, it is possible to derive the fuel burn rate as a function of Mach number. Once this curve is known, it is then possible to approximate the fuel curve into a series of linear segments. However, in this case, the fuel burn rate vs. Mach number for each aircraft was given as a series of distinct points. With such information, it was then possible to treat each data point as the end points of a series of lines that spanned the range of fuel burn rates for the given aircraft. Figure 4.3 demonstrates this

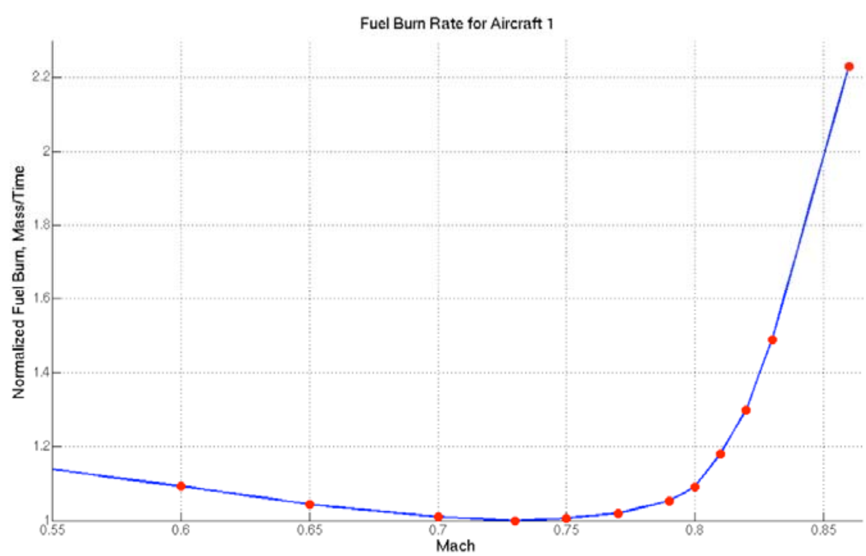


Figure 4.3: Fuel burn rate vs. Mach for a commercial wide body jet

process.

Each blue line in the figure denotes a constraint in the form

$$\dot{f} = a M + b ,\qquad (4.2.1)$$

where $\dot{f}$ is the fuel burn rate, a the slope of the line, and b is the y-intercept for that line. As will be explained later, the problem will be solved in order to minimize the fuel burn experienced during the speed change. Because this problem is a minimization problem, the above graph can be approximated with the following series of constraints:

$$\begin{aligned} \dot{f}_{M_{d,i}} &\geq a_{1,i} M_{d,i} + b_{1,i} \\ \dot{f}_{M_{d,i}} &\geq a_{2,i} M_{d,i} + b_{2,i} \\ &\vdots \\ \dot{f}_{M_{d,i}} &\geq a_{m,i} M_{d,i} + b_{m,i} \end{aligned} \qquad (4.2.2\text{-}4.2.4)$$

For each aircraft i, there are m lines connecting the fuel burn rate data points. By writing each line in slope-intercept form, the Mach number selected, $M_{d,i}$, will be forced to the correct line, because the overall goal is to minimize the fuel burn. In this manner, the fuel burn characteristics of the aircraft intending to fly the CDA are taken into account in the optimization problem. It is important to note that this fuel burn rate calculation is altitude-specific, and for any flight scenario, the appropriate fuel burn information must be acquired.

4.3 THE OBJECTIVE FUNCTION

4.3.1 Minimizing Fuel Burn Only

As explained in the introduction, the overall goal of this thesis is to provide a tool that enables closely spaced flights to be arranged so that minimum spacing necessary to fly a CDA is achieved. As one of CDA's main benefits is fuel reduction for each landing flight, setting flights up to enable CDA should likewise avoid increased fuel burn. With this goal in mind, an obvious objective function for the linear program was one that minimized the fuel burn for the time period during which the speed change occurred. Since we are assuming a speed change (for the one speed change solution) to take place at known time duration prior to the aircraft's initial ETA, this minimization objective translates to

$$\min Z = \sum_{i=1}^{N} \left(\dot{f}_i \Big|_{M_{d_i}} T_i \right). \tag{4.3.1}$$

In other words, for each aircraft *i*, the fuel burn rate for the decision (final) Mach number is multiplied by the fixed time during which the speed change takes place, and this quantity is summed for the *N* aircraft involved.

4.3.2 Minimizing Fuel Burn in Conjunction with Delay

However, this objective function does not take into account the fact that, if there is a speed change for each aircraft, the time duration during which the speed change occurs is no longer fixed. The previous objective function amounts to finding the speed change for each aircraft which brings it as close as possible to the aircraft's maximum endurance cruise speed. While this objective function would seem to reduce the overall fuel burn, as explained by Abad, "Further consider that the coupling between fuel burn and flight time is complex due to the occasional trade-off in optimizing for one at the expense of the other. This coupling is obvious when considering the causality of flight time upon fuel burn: a longer flight time necessitates a greater fuel burn" [1]. This scenario is exactly the case in the present problem.

If the flights in question all had initial cruise speeds below the speed of minimum fuel burn rate, the above objective function would be adequate. The flights would all increase speed by some amount so that they reach the required separation, the flights would arrive earlier, and fuel would be saved during the en route segment of the flight, as well as during the CDA. However, this is not the reality. Most flights during the 2007 flight test [11] were cruising above the maximum endurance speed, at the cruise speed for maximum range. Since this higher cruise speed was the observed scenario, the above objective function decreased the speed of each flight, actually resulting in increased fuel burn for each flight. These results are presented in Chapter Eight. With this result, the objective function was reexamined, and it was necessary to include the time change in the objective function as well. A simple addition to the above function is:

$$\min Z = \sum_{i=1}^{N}\left(\dot{f}_i\Big|_{M_{d_i}} \left(T_i - \Delta t_i\right)\right). \tag{4.3.2}$$

While this addition of Δt is straightforward in terms of logic, the resulting math is not as simple. Including two unknowns in the objective function when multiplied create a nonconvex problem. Fortunately, the problem has been encountered previously, and Babayev [4] has provided a way to deal with such a situation by adding additional constraints as follows.

The approximation method is equivalent to creating a new variable that replaces the nonlinear terms,

$$r_i = \dot{f}\Big|_{M_{d_i}} \Delta t_i. \tag{4.3.3}$$

The following approximations create a series of planes outlining the values of the function. It is then necessary to create a grid for the possible values of r_i. These grid points are denoted by λ_{kl_i}, with each λ being a grid point for a specific r_i value. Specifically,

$$\lambda_{kl_i} \approx F_k \Delta T_l, \tag{4.3.4}$$

with k and l being the number of grid points in the fuel burn and time ranges respectively. In this case, the range of possible fuel burn rate values are needed, as well as the range of Δt's. To keep the calculation capable of being solved in a reasonable amount of time, a small range of Δt's are used. It is assumed that the maximum range for Δt is within ten minutes of the original ETA, meaning the range of Δt is -300 seconds < Δt < 300 seconds.

By summing these grid values, and enforcing the condition that at most four λ's are nonzero (by SOS2 variables), the appropriate portion of the function can be approximated. In mathematical terms, these constraints are as follows.

$$\dot{f}\Big|_{M_{d_i}} = \sum_k \sum_l F_k \lambda_{kl_i} \tag{4.3.5}$$

$$\Delta t_i = \sum_k \sum_l \Delta T_l \lambda_{kl_i} \tag{4.3.6}$$

$$r_i = \sum_k \sum_l \left(F_k \Delta T_l\right) \lambda_{kl_i} \tag{4.3.7}$$

$$\sum_k \sum_l \lambda_{kl_i} = 1 \tag{4.3.8}$$

$$\mu_{k_i} = \sum_l \lambda_{kl_i} \tag{4.3.9}$$

$$\eta_{l_i} = \sum_k \lambda_{kl_i} \tag{4.3.10}$$

$$\lambda_{kl_i} \geq 0 \tag{4.3.11}$$

$$\mu_{k_i} \in SOS2 \tag{4.3.12}$$

$$\eta_{l_i} \in SOS2 \tag{4.3.13}$$

$$r_i \rightarrow free \tag{4.3.14}$$

4.4 ADDITIONAL CONSTRAINTS

4.4.1 Variable Sequence Constraints

It was initially thought that keeping a fixed sequence of aircraft, based on the initial ETA's of the aircraft group, would help to simplify the optimization problem; the results derived from a keeping a fixed aircraft sequence are presented in Section 6.1. By keeping a fixed sequence, it is only necessary to know the required separation between pairs of leading and following aircraft. These *n*-1 constraints are easy to work with, and minimal coordination with TASAT is necessary, since only a few combinations of leading and following aircraft are needed.

However, by freezing the sequence of the aircraft, improved aircraft ordering are ignored, and the solution giving the lowest possible increase in fuel burn may be missed entirely. In addition, for terminal area landing scenarios where there is limited time to set up aircraft with a necessary separation, enabling a variable sequence in the formulation is even more important.

The easiest way to see how a series of variable sequence constraints can be implemented can be seen by examining just three aircraft with a separation distance for each leading and following scenario. The conditions to be satisfied are:

$$\left|T_{SC_1} - T_{SC_2}\right| \geq \alpha_{1,2} \tag{4.4.1}$$

$$\left|T_{SC_1} - T_{SC_3}\right| \geq \alpha_{1,3} \tag{4.4.2}$$

$$\left|T_{SC_2} - T_{SC_3}\right| \geq \alpha_{2,3}, \tag{4.4.3}$$

with alpha being the required separation distance based on aircraft type. Here, the separation time is written as being the same regardless of the aircraft order, but there will be a different $\alpha_{i,j}$ value depending on the order of the aircraft. For example, the separation time for a B767-300 following a B737-800 would be significant lower than for the case where the B737-800 follows the B767-300, due to the size difference of the aircraft.

However, absolute values create a non-convex problem and these constraints must be rewritten as

$$T_2 - T_1 + \alpha_{1,2} \le P z_1 \tag{4.4.4}$$

$$2\alpha_{2,1} - (T_2 - T_1 + \alpha_{2,1}) \le P(1 - z_1) \tag{4.4.5}$$

$$T_3 - T_1 + \alpha_{1,3} \le P z_2 \tag{4.4.6}$$

$$2\alpha_{3,1} - (T_3 - T_1 + \alpha_{3,1}) \le P(1 - z_2) \tag{4.4.7}$$

$$T_3 - T_2 + \alpha_{2,3} \le P z_3 \tag{4.4.8}$$

$$2\alpha_{3,2} - (T_3 - T_2 + \alpha_{3,2}) \le P(1 - z_3), \tag{4.4.9}$$

according to Hillier and Lieberman [21], with z_1, z_2, and z_3 binary variables, and P sufficiently large so that one constraint is made invalid (~10000). In addition, the different separation time for the reciprocal aircraft orders are indicated by $\alpha_{i,j}$ and $\alpha_{j,i}$ for each pair of constraints. Only one separation constraint will be active because of the binary variables and large value of P.

For multiple aircraft, determining all possible combinations of orders and the necessary separation adds many constraints, but does not change the solution time of the problem significantly. For n aircraft, it is necessary to have $\sum_{1}^{N} i$ constraints (3 aircraft necessitates 6 constraints, 5 aircraft necessitates 15 constraints, 16 aircraft necessitates 120 constraints, etc.).

4.4.2 Speed Change Limitation Constraint

A constraint is needed to limit speed changes to at most one speed change per aircraft:

$$\delta_i \geq \frac{|\Delta t_i|}{M_i} \tag{4.4.10}$$

$$\delta_i \leq M|\Delta t_i| \tag{4.4.11}$$

$$\delta_1, \delta_2 \ldots \delta_n \text{ binary} \tag{4.4.12}$$

In Equation 4.4.10, δ_i is a binary variable. By setting $\delta_i = 0$, an aircraft that is for some reason is unable to fly a CDA would be eliminated from the speed change calculation but still considered in the spacing requirements for the remaining aircraft.

In order to implement these constraints in CPLEX, absolute values had to be accounted for differently in order to avoid a nonconvex problem. By creating a new variable, z_i, this nonconvexity can be avoided. Equations 4.4.13-4.4.16 replace equations 4.4.10 and 4.4.11 above:

$$z_i \geq \Delta t_i \tag{4.4.13}$$

$$z_i \geq -\Delta t_i \tag{4.4.14}$$

$$\delta_i \geq \frac{z_i}{\mathrm{M}} \tag{4.4.15}$$

$$\delta_i \leq \mathrm{M} z_i \tag{4.4.16}$$

In these equations, M is a very large number, from the "big-M method" [21] so that each constraint can be active at all times.

In addition, the maximum number of aircraft to which speed changes can be issued is specified with the following constraint:

$$\sum_{j=1}^{J} \delta_i \leq j, \tag{4.4.17}$$

with j most likely being the total number of aircraft flying the CDA. However, an airline may only want a maximum number of flights per day making a speed change. This maximum number could be adjusted here.

4.4.3 Integer Constraints for a Discrete Mach Change

Since only one or two speed changes are possible in versions of ESCORT, an important assumption is how well aircraft are able to hold a chosen Mach, and the accuracy with which a cruise Mach number can be chosen. Based on observations for the 2007 flight test [11], it was seen that aircraft could hold Mach numbers up to the thousandth place. In order to include a discrete Mach change in the formulation, an additional constraint was needed:

$$cc_i = 1000 \Delta M_i \tag{4.4.18}$$

$$cc_i \text{ integer, free} \tag{4.4.19}$$

While this constraint allowed for the calculation of a discrete Mach change, as will be seen in Chapter 6, the inclusion of this constraint in various ESCORT algorithms created solutions with very high solution times. In these cases, this constraint was removed and the solution gave Mach changes up to 16 decimal places. The results in these cases would be rounded to the nearest thousandth to achieve the same discrete calculation as if 4.4.18 and 4.4.19 were included. However, rounding may increase the optimality gap of the solution.

4.5 CODING PROCEDURES IN MATLAB & CPLEX

Figure 4.4 shows the program's flow as currently implemented in Matlab and CPLEX. The four shades of green correspond to different sections of the code—initialization, problem creation, problem solution, and post-processing.

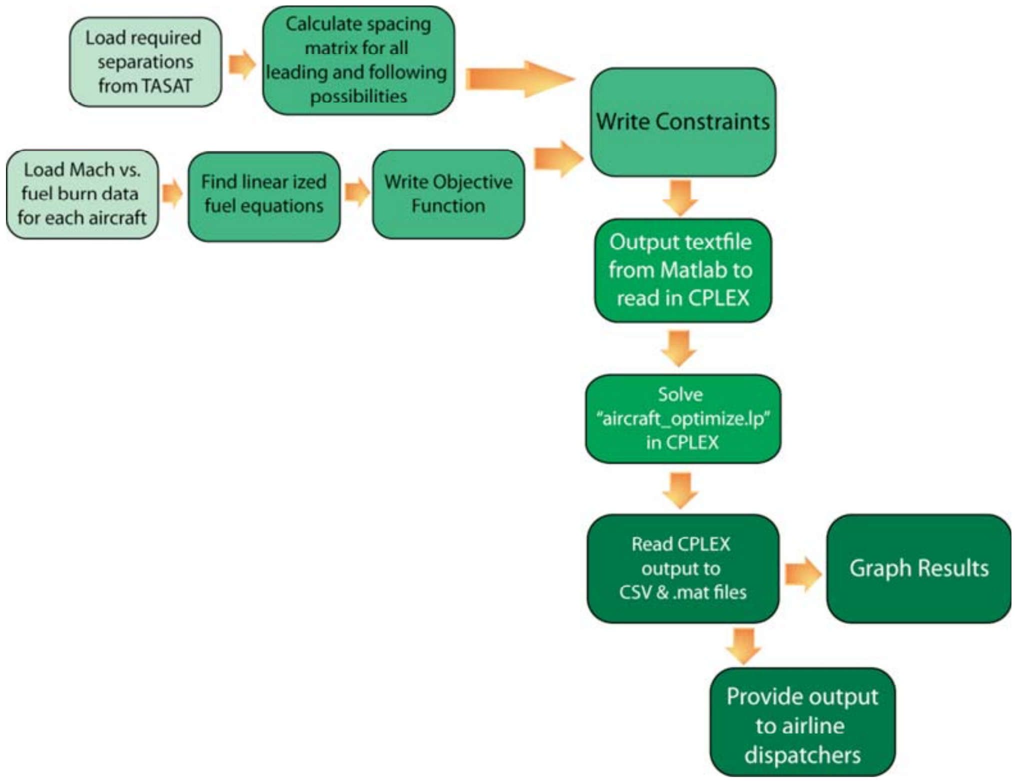


Figure 4.4- ESCORT Program Flow

4.5.1 Initialization

The first section, initialization, is where the relevant data files are loaded. The program requires a table for each aircraft that provides a schedule of fuel burn rate for the range of cruise speeds possible. In addition, the separation requirements for that day's flights must be loaded from TASAT. All possible leading and following combinations for the day's aircraft must be extracted at this point.

4.5.2 Problem Creation

The second group of functions is the problem creation steps. These three blocks—calculate spacing matrix for all leading and following possibilities, find linearized fuel equations, and write objective function—are the sub-steps to this portion of the algorithm. While the data from TASAT was loaded in the previous program section, the possible leading and following combinations for each flight combination are enumerated here. This process creates a matrix of all the flight numbers as seen in Table 4.1:

Table 4.1: Sample separation constraints

Flight Number	**940**	**788**	**780**	**1002**	**752**
940	-	131.1	131.1	135	135
788	115	-	107.2	115	115
780	115	107.2	-	115	115
1002	135	131.1	131.1	-	135
752	135	131.1	131.1	135	-

Writing out the possible separation constraints for each possible leading and following combinations gives $\sum_{i=1}^{N} i$ constraints, as described in Section 4.4.1. Care is taken not to write repeat constraints.

The next block, that of finding the linearized fuel equations is explained in detail in Section 4.2. By taking two consecutive points, and calculating the line that connects them in slope-intercept form ($y = mx+b$), these linearized fuel equations are derived. In addition, at this point if the value of Δt is included in the objective function for the given algorithm, the grid of possible values for r_i is created. As described in Section 6.1.6, high grid resolutions for the problem's sample space are possible but drastically affect the solution time of the problem. In practice, a grid with 3 – 5 points will be the most likely for running ESCORT. The objective function is then written, using the initial fuel burn rate values found from these linear equations and the initial Mach number and including each r_i variable.

Following the writing of the objective function, the constraints described in Sections 4.1 -4.4 are written in full. This requires using the separation table created in the previous step as well as using the grid values in Eq. 4.3.3 to 4.3.14.

4.5.3 Problem Solution

Once the objective function and constraints are written, the full linear program formulation is saved in an .lp file format. In addition, files are created in which the solutions and run time will be recorded. Using the command

```
run_command=['!ILOG_LICENSE_FILE=/u1/ilog/meta/access.ilm
/u1/ilog/cplex101/bin/x86-64_RHEL3.0_3.2/cplex < ' cplex_commands];
```

CPLEX is called directly from Matlab, so long as Matlab is running from the Air Transportation Laboratory's computational cluster called Ironman. Using this command, CPLEX runs and reads the solution progress directly to Matlab's command window. If the problem is infeasible or unreadable, this information will be displayed at this time. However, if the problem is feasible, solutions are written to a file entitled sols_out.

4.5.4 Post Processing

In this series of steps, the CPLEX sols_out file is transformed to a long text file detailing each constraint and variable's solution to results that are more useful to ESCORT users. A Matlab function was created to read the sols_out file and save only the important variable solution values into a .mat matrix entitled all_data.mat. This matrix contains all of the initial flight information provided to ESCORT, as well as the fuel burn rate at the initial Mach, the decision Mach number, fuel burn rate at this Mach, final arrival time, net fuel burn difference for each flight, the time at which the Mach change is made, and, if two speed changes are possible, the time at which the aircraft returns to its original cruise Mach. This data is also saved to a CSV file if later analysis is desired.

A series of graphs is produced from this data. In future versions of ESCORT, there will be more of a graphical user interface to input the initial information and display the final information. It is assumed that this GUI will be designed in Matlab for initial use and later transferred for development in another language once the layout is finalized. The types of graphs generated are shown in Chapter 6. The graphs include a comparison of the initial and final separation times for each aircraft, a bar chart displaying the Mach change for each aircraft, a graph displaying the time at which the Mach change is made (a trajectory chart), and a final graph showing the net fuel burn difference compared to the fuel used had the aircraft remained at its initial Mach for each aircraft. Although the input and output features of ESCORT are rudimentary at this time, the purpose of this thesis is to show that the equations behind the program serve their intended purpose, and user-friendly GUI’s can be developed as the program progresses.

CHAPTER 5: FLIGHT TEST SCENARIO & SAMPLE DATA

During the April-May 2007 flight test, CDA operations were observed for a period of 4 weeks [11]. During this time, 31 days of flights flying early morning operations from the west coast were observed to fly a CDA path shown Figure 5.1. While the fuel savings and noise data are still currently unavailable for these flights, this flight scenario provides a range of sample scenarios for late night CDA operations. A

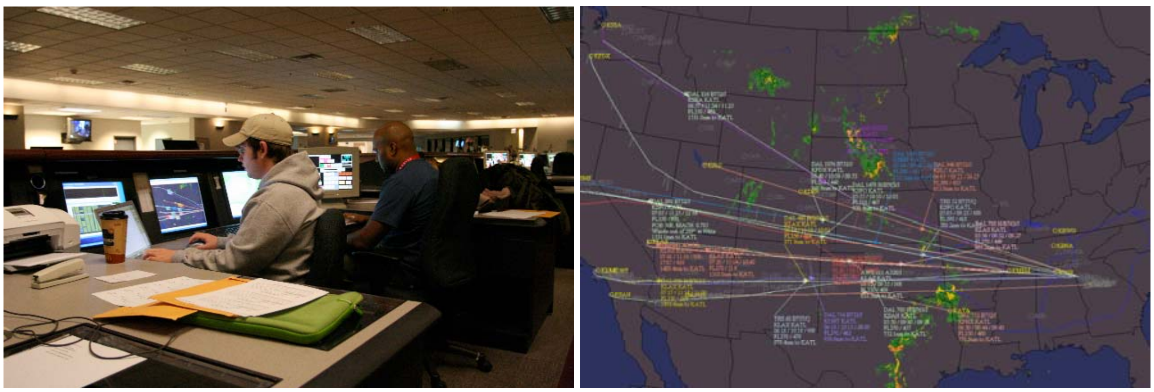


Figure 5.1: Delta's Operational Control Center and Graphical Flight Following display [11]

range of aircraft—B737-800, B757-200, B767-300, and B767-400—were involved in the flight tests, as well as a range of wind conditions for each night. The information collected from each day's flight test was the date, flight number, tail number, origin airport of the flight, whether the plane flew the CDA or not, the runway to which the flight was directed, cruising altitude, cruising Mach, takeoff weight, required time of arrival at the metering point RMG, the actual time of arrival at RMG, the scheduled arrival time for the destination (ATL), the ATA at ATL, and wheels on time at ATL.

This information provided cases to test ESCORT. On one day, due to the tight initial spacing of the flights (among other factors not recorded), 10 of the 16 flights were unable to fly the CDA. A key usefulness test for this thesis is to show the ease and elegance with which ESCORT provides a solution to previous problems encountered, and also to show that more complex can also be handled. On this sample, worst-case scenario day, there were 15 flights, all scheduled to arrive over the metering point RMG between 9:06 AM and 11:40 AM. While on that night care was taken to space these flights as well as possible, increasing and decreasing the speeds of the flights without creating new separation conflicts was beyond the capacity of myself and the other flight test researchers present without the air of ESCORT.

Table 5.1. Sample initial conditions for a CDA scenario, taken from May 22, 2007

Flight Number	Aircraft Type	Initial Mach	Flight Departure Time	Initial ETA	Required Sep. (s)	Initial Sep. (s)
940	752	0.78	3:35 AM	9:05 AM	131.1	240
788	763	0.785	3:39 AM	9:09 AM	107.2	720
780	763	0.785	3:51 AM	9:21 AM	115	240
1002	752	0.78	3:55 AM	9:25 AM	135	**60**
752	752	0.78	3:56 AM	9:26 AM	131.1	1080
1478	763	0.785	4:14 AM	9:44 AM	115	180
716	752	0.78	4:17 AM	9:47 AM	135	**0**
1076	752	0.775	4:17 AM	9:47 AM	107.2	300
1282	764	0.79	4:22 AM	9:52 AM	107.2	**60**
480	763	0.785	4:23 AM	9:53 AM	115	180
1642	752	0.78	4:26 AM	9:56 AM	135	2400
714	752	0.78	6:06 AM	10:36 AM	131.1	780
806	763	0.78	6:19 AM	10:49 AM	115	540
898	752	0.775	6:28 AM	10:58 AM	135	1020
816	752	0.78	6:45 AM	11:15 AM	135	1500
636	752	0.78	7:10 AM	11:40 AM		

The details of this difficult May 22, 2007 scenario are given in Table 5.1.

CHAPTER 6: RESULTS

6.1 DETERMINING THE BEST ESCORT ALGORITHMS

As described in Chapter 4, there were many variations of ESCORT as the program developed. It is the intention of this section to provide data to determine the most meaningful and time-sensitive constraints to include in the final version of ESCORT. The possible variations in the code include the inclusion/exclusion of fairness, having a fixed vs. variable sequence, the inclusion/exclusion of arrival time deviation in the objective function, the time prior to the metering point ETA at which the speed change is made, including an integer constraint for the final Mach number (keeping the final Mach value discrete), a one, or two speed change formulation, and how the time deviation solutions perform with some constraints relaxed.

Equations 6.1.1 – 6.1.27 below present the baseline formulation in its entirety. In order to examine how varying each of the above parameters affects ESCORT's results, each variation will be compared to this baseline formulation. The metrics used to evaluate the choice of algorithm will be the solution time, objection function values, net fuel burn difference, and the optimality gap. The value of these metrics for each algorithm will be based on the sample scenario described in Chapter 5. The baseline formulation is as follows:

$$\min Z = \sum_{i=1}^{N}\left(\dot{f}_i\Big|_{M_{d_i}} \left(T_i - \Delta t_i\right)\right) \tag{6.1.1}$$

Subject to:

$$T_i = t_i - t_{0,i} \forall i \in N \tag{6.1.2}$$

$$\begin{aligned}&\dot{f}_i \geq a_{i,1}M_d + b_{i,1}\\&\dot{f}_i \geq a_{i,2}M_d + b_{i,2}\forall i \in N\\&\vdots\\&\dot{f}_i \geq a_{i,m}M_d + b_{i,m}\end{aligned} \tag{6.1.3}$$

$$z_i \geq \Delta t_i \forall i \in N \tag{6.1.4}$$

$$z_i \geq -\Delta t_i \forall i \in N \tag{6.1.5}$$

$$\delta_i \geq \frac{z_i}{\mathrm{M}} \forall i \in N \tag{6.1.6}$$

$$\delta_i \leq \mathrm{M} z_i \forall i \in N \tag{6.1.7}$$

$$\sum_{j=1}^{J} \delta_i \leq j \forall i \in N \tag{6.1.8}$$

$$\Delta t_i = T_i \frac{\Delta M_i}{M_i} \forall i \in N. \tag{6.1.9}$$

$$M_{d_i} = M_i + \Delta M_i \forall i \in N \tag{6.1.10}$$

$$t_{f_i} = t_i - \Delta t_i \forall i \in N \tag{6.1.11}$$

$$\dot{f}\Big|_{M_{d_i}} = \sum_k \sum_l F_k \lambda_{kl_i} \forall i \in N \tag{6.1.12}$$

$$\Delta t_i = \sum_k \sum_l \Delta T_l \lambda_{kl_i} \forall i \in N \tag{6.1.13}$$

$$r_i = \sum_k \sum_l \left(F_k \Delta T_l\right) \lambda_{kl_i} \forall i \in N \tag{6.1.14}$$

$$\sum_k \sum_l \lambda_{kl_i} = 1 \forall i \in N \tag{6.1.15}$$

$$\mu_{k_i} = \sum_l \lambda_{kl_i} \forall i \in N \tag{6.1.16}$$

$$\eta_{l_i} = \sum_k \lambda_{kl_i} \forall i \in N \tag{6.1.17}$$

$$\lambda_{kl_i} \geq 0 \forall i \in N \tag{6.1.18}$$

$$T_j - T_i + \alpha_{i,j} \le Pz_i \forall i \in N, j \in N \mid i \ne j \tag{6.1.19}$$

$$2\alpha_{j,i} - (T_j - T_i + \alpha_{j,i}) \le P(1 - z_i) \forall i \in N, j \in N \mid i \ne j \tag{6.1.20}$$

$$\mu_{k_i} \in SOS2 \forall i \in N \tag{6.1.21}$$

$$\eta_{l_i} \in SOS2 \forall i \in N \tag{6.1.22}$$

$$r_i \rightarrow free \forall i \in N \tag{6.1.23}$$

Bounds

$$-\infty \le \Delta t_i \le \infty \forall i \in N \tag{6.1.24}$$

$$-0.02 \le dM_i \le 0.02 \forall i \in N \tag{6.1.25}$$

$$z_i --> free \forall i \in N \tag{6.1.26}$$

Binary

$$\delta_1, \delta_2, \ldots \delta_n \tag{6.1.27}$$

In order to summarize these results, a series of sample cases are presented for many combinations of the above variables. While it would be arduous to examine the details of each solution, by examining the solution time, objection function values, and net fuel burn difference for the sample cases, the most promising solutions will be identified with these parameters and these solutions will then be examined in greater detail in Section 6.2.

6.1.1 Baseline Algorithm

The Baseline Algorithm has the following characteristics:

- No fairness constraints
- Variable Sequence
- Δt inclusion in the objective function (Eq. 4.3.2 as opposed to Eq. 4.3.1)
- Full SOS2 Constraints (referring to Eq. 6.1.21 and 6.1.22)
- 4 grid points
- Speed change made 2 hours prior to arriving at the metering point
- No integer constraints to ensure a discrete decision Mach number calculation

The following sections will describe how varying each of these characteristics affects the performance metrics for ESCORT—the objective function value, solution run time, and overall net fuel burn. Table 6.1 gives the baseline performance metric values

Table 6.1: Baseline algorithm performance metrics

Algorithm Description	Objective Function value (kg)	CPLEX runtime (s)	Net Fuel Burn Difference (kg)	Optimality Gap (%)
BASELINE	**108951**	**0.4**	**-117**	**0.01**

for the Baseline solution.

6.1.2 Fairness Inclusion

Including the fairness constraints (Eq. 3.4.1 and 3.4.2) in the formulation with the baseline algorithm is the algorithm that will be referred to as Baseline with Fairness from here on. Table 6.2 shows the difference in performance metrics for the Baseline and Baseline with Fairness Algorithms.

Table 6.2: Comparison of Baseline and Baseline with Fairness Algorithms

	Objective Function value (kg)	CPLEX runtime (s)	Net Fuel Burn Difference (kg)	Optimality Gap (%)
BASELINE	**108951**	**0.4**	**-117**	**0.01**
BASELINE w/ Fairness	109636	1.1	476	0.01

For the inclusion of fairness, in this case, it is assumed that each of the sixteen flights in the sample case belongs to a different airline. In such a case, the percentage increase in fuel burn would be equalized for each flight. This case is the worst-case scenario for sixteen flights, because it is more likely that groups of aircraft would belong to one airline, and that entire aircraft group could have the same percentage fuel burn increase as other participating groups. From these results, it is apparent that although more airlines may be more likely to participate in CDA knowing that the penalty of increased fuel burn will be shared, including fairness creates a net increase in fuel burn that would not be present otherwise. Only if there are multiple airlines involved should fairness be included.

6.1.3 Variable Sequence

Section 4.4.1 detailed the variable sequence constraints. Had the sequence remained fixed, equations 6.1.28 would have been used in place of Eq. 6.1.19 and 6.1.20.

$$(t_{i+1} - \Delta t_{i+1}) - (t_i - \Delta t_i) \geq S_{i,i+1} \tag{6.1.28}$$

By assuming a fixed sequence, the number of constraints in the problem is reduced from $\sum_{i=1}^{N} i$ constraints to *n-1* constraints. The effect of this reduced number of constraints shows in the CPLEX runtime column of Table 6.3.

Table 6.3: Comparison of Baseline and fixed sequence algorithms

	Objective Function value (kg)	CPLEX runtime (s)	Net Fuel Burn Difference (kg)	Optimality Gap (%)
BASELINE	**108951**	**0.4**	**-117**	**0.01**
FIXED SEQUENCE	109057	0.3	-3	0.01

While there is a faster runtime compared to the Baseline Algorithm, the net fuel burn difference is significantly higher (by 114 kg), and the time difference achieved by the reduced number of constraints is not seen to be sufficient to have a fixed, instead of a variable, aircraft sequence.

6.1.4 Δt Inclusion in the Objective Function

As described in Section 4.3, there were two possible objective functions considered in ESCORT. Here, it will be shown that while not including a time difference between initial ETA and calculated ETA term (Δt) in the objective function reduces the objective function value, the net fuel burn increase is the real metric of usefulness. Table 6.4 gives the comparison between the Baseline algorithm and one that does not include Δt in the

Table 6.4: Comparison of Baseline and no Δt inclusion algorithms

	Objective Function value	CPLEX runtime (s)	Net Fuel Burn Difference (kg)	Optimality Gap (%)
BASELINE	**108951**	**0.4**	**-117**	**0.01**
NO DT Inclusion	107622	0.1	658	0.05

objective function.

While the objective function value is lower (the sum of fuel burn for the duration of the speed change), in the case of no Δt inclusion, including Δt would actually be an increase to the time during which the speed change takes place, so that the objective function is deceptive in showing a lower total fuel burn. However, by comparing the fuel burn of flights making a speed change to the fuel burn had the speed change not taken place and summing this difference, the net fuel burn difference gives a good measure of the usefulness of the different algorithms. Although the no Δt inclusion algorithm has a faster run time, the net fuel burn difference is significantly higher (775 kg) for this algorithm, and the reduced solution time does not compensate for the increase in fuel burn that would be observed.

6.1.5 Relaxed SOS2 Constraints

While including Δt in the objective function is beneficial in reducing the net fuel burn difference, it is possible to relax some of the constraints in the formulation, particularly the SOS2 constraints, Eq.6.1.21 and 6.1.22. Relaxing these constraints still gives feasible solutions, although all of the vertices of the linear program may not be fully examined. Yet, this relaxation reduces the solution run time and achieves only a marginally higher net fuel burn difference, as shown in Table 6.5. However, the decreased run time is still not enough to replace the utility of the Baseline Algorithm.

Table 6.5: Comparison of Baseline and relaxed SOS2 algorithms

	Objective Function value (kg)	CPLEX runtime (s)	Net Fuel Burn Difference (kg)	Optimality Gap (%)
BASELINE	**108951**	**0.4**	**-117**	**0.01**
SOS2 Relaxed	107812	0.1	-93	0.01

6.1.6 Number of Grid Points Variation

Another parameter that can be varied if Eq. 4.3.2 is used as the objective function is the number of grid points. This number has the most drastic effect on the solution time out of any algorithm parameter. In addition, the net fuel burn difference also depends heavily on the number of grid points. The selection for the Baseline algorithm to use four grid points was a tradeoff between a low solution run time and a low net fuel burn

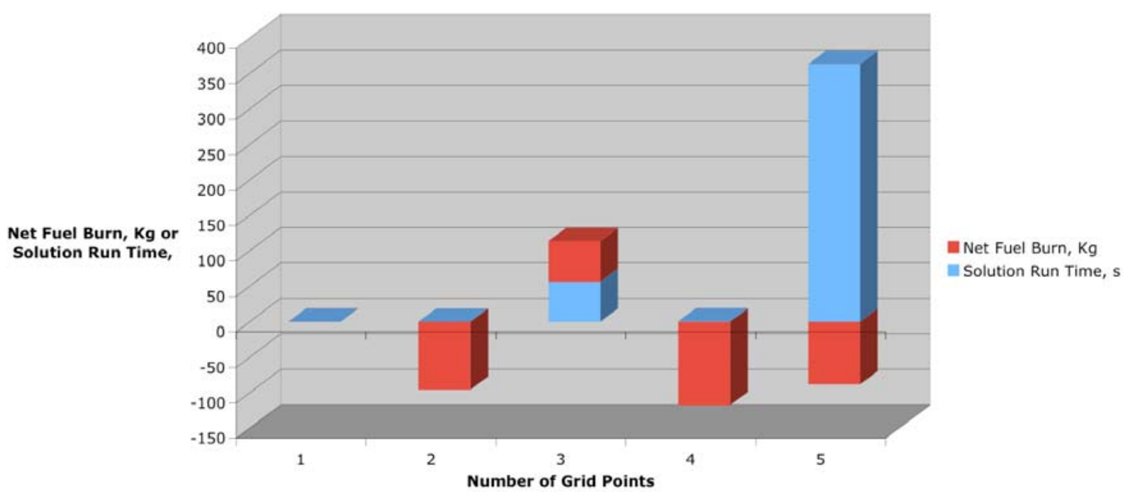


Figure 6.1: Net fuel burn and solution run time vs. number of grid points

difference.

The Baseline Algorithm was chosen to be the lowest sum of solution time and net fuel burn, which occurs with four grid points. In Figure 6.1, the bar corresponding to four grid points clear has the lowest (most negative) bar. It should be noted that 1 grid point gives an infeasible solution, and 6 grid points gives solution run times upwards of two hours (not displayed in Figure 6.1). The data in table 6.5 gives a summary of the

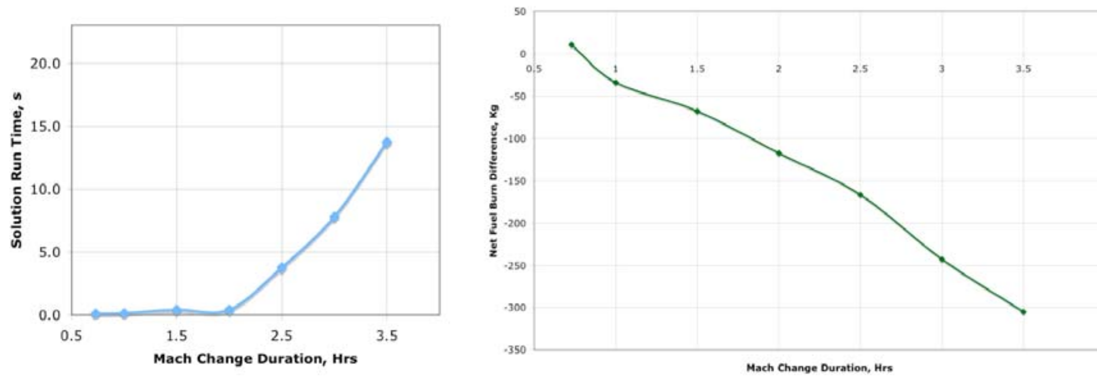
Table 6.6: Baseline algorithm compared to varying numbers of grid points

	Objective Function value (kg)	**CPLEX runtime (s)**	**Net Fuel Burn Difference (kg)**	**Optimality Gap (%)**
BASELINE	**108951**	**0.4**	**-117**	**0.01**
1 Grid Point	Infeasible	Infeasible	Infeasible	Infeasible
2 Grid Points	107799	0.0	-96	0.04
3 Grid Points	108941	56.5	57	0.01
4 Grid Points (Baseline)	108951	0.4	-117	0.01
5 Grid Points	109123	363.2	-88	0.01
6 Grid Points	109196	6962.2	-121	0.01

algorithm performance metrics for varying numbers of grid points.

6.1.6 Altered Speed Change Duration with and without Fairness

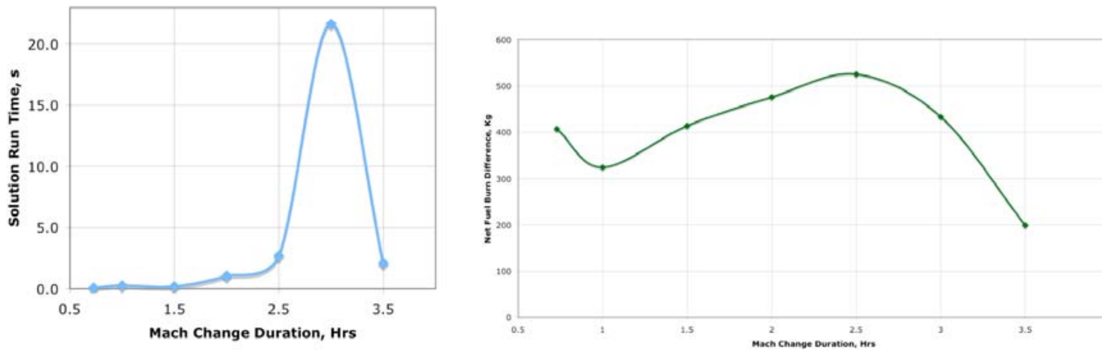
The way the formulation has been written, the time at which a speed change is made must be fixed. For the Baseline Algorithm, it is assumed that the speed change for each aircraft is made two hours prior to the aircraft's initial ETA to the metering point. While further fuel savings may be gained by making the speed change earlier in the aircraft's flight plan, the speed change must also me made as late in the flight as possible so that the most accurate weather information can be used in the IET trajectory predictor. Figure 6.2 shows the additional benefits making a speed change further in advance would have for algorithms that do not include fairness. In addition, for the given scenario, making a speed change later than 44 minutes (0.73 hours) creates an infeasible problem, and the solution run time increases in an almost linear relationship with the Mach change



Figures 6.2.A and 6.2.B: Solution run time and net fuel burn difference vs. Mach change duration, no fairness

duration greater than 2 hours.

For algorithms including fairness, the benefits of choosing a greater Mach change duration are not apparent. In fact, a Mach change duration of 1 hour gives a lower net fuel burn difference than the 2 hour mach change duration. Because this low net fuel difference with a short Mach change duration is such an interesting result, this algorithm will be examined further in Section 6.2. This algorithm will be referred to as the Quick Mach Duration Algorithm.



Figures 6.3.A and 6.3.B: Solution run time and net fuel burn difference vs. Mach change duration, no fairness

6.1.7 Discrete Mach Number Integer Constraints

The last parameter varied in the algorithms was the inclusion of integer constraints to make the decision Mach a discrete value. While the inclusion of these constraints makes the solution output easier to relay to the pilots, it was found that compared to the Baseline Algorithm, incorporating these integer constraints vastly increased the solution run time. In addition, the optimality gap was the largest of any of the other solutions if integer constraints were included. While certain combinations of the algorithms, such as including integer constraints but relaxing the SOS2 constraints, may have a reduced run time, the benefit of having discrete Mach changes did not outweigh the impracticality of

Table 6.7: Discrete mach number integer constraints compared to Baseline Algorithm

	Objective Function value (kg)	CPLEX runtime (s)	Net Fuel Burn Difference (kg)	Optimality Gap (%)
BASELINE	**108951**	**0.4**	**-117**	**0.01**
Integer Constraint	109140	7250.8	-71	1.19

a solution that that takes two hours to compute. Table 6.6 details these results.

6.2 EXAMINING ESCORT'S SOLUTIONS

The results of the three algorithms presented in this section are those corresponding to the Baseline, Baseline with Fairness and Quick Mach Duration Algorithms. The Baseline Algorithm assumes one speed change, with full SOS2 constraints for the inclusion of the altered arrival time in the objective function, solving for four grid points, making the speed change 2 hours prior to the aircraft's initial ETA at the fix, and not including integer constraints on the final Mach number. The Baseline Algorithm with Fairness has all of the same characteristics, except it includes fairness constraints. Lastly, the differences for the Quick Mach Duration Algorithm are that it is solved assuming a speed change made one hour prior to the initial metering fix ETA. As stated previously, these Algorithms were selected for further examination, because the Baseline Algorithm and the Baseline Algorithm with Fairness corresponded to the pair with the lowest net fuel burn values for an assumed speed change two hours from the fix for basic constraints and fairness constraints, respectively. The Quick Mach Change Algorithm gave the lowest net fuel burn for any algorithm including fairness.

The first results to be examined are those showing the initial and final ETA separation calculated by ESCORT in Figures 6.4 – 6.6.

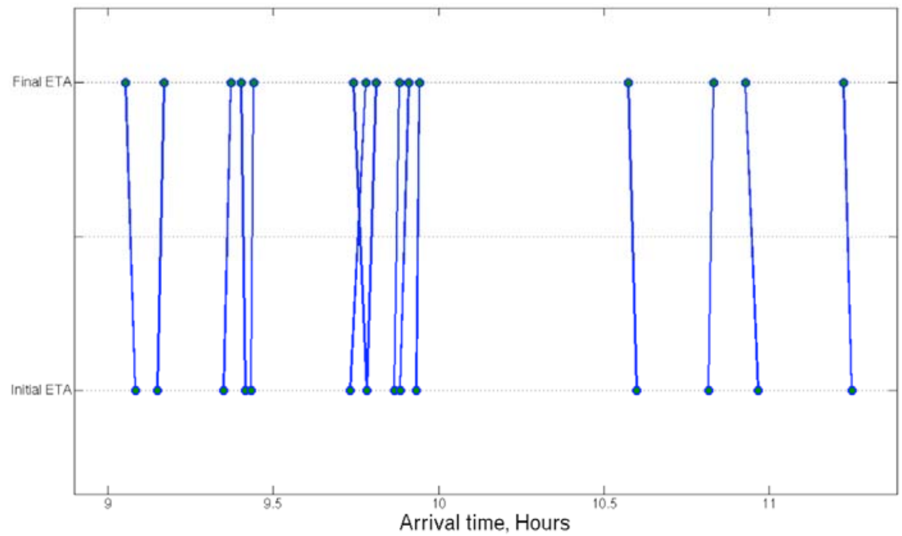


Figure 6.4: Initial and final ETA separation of Baseline Algorithm

Figure 6.5: Initial and final ETA separation of Baseline Algorithm with Fairness

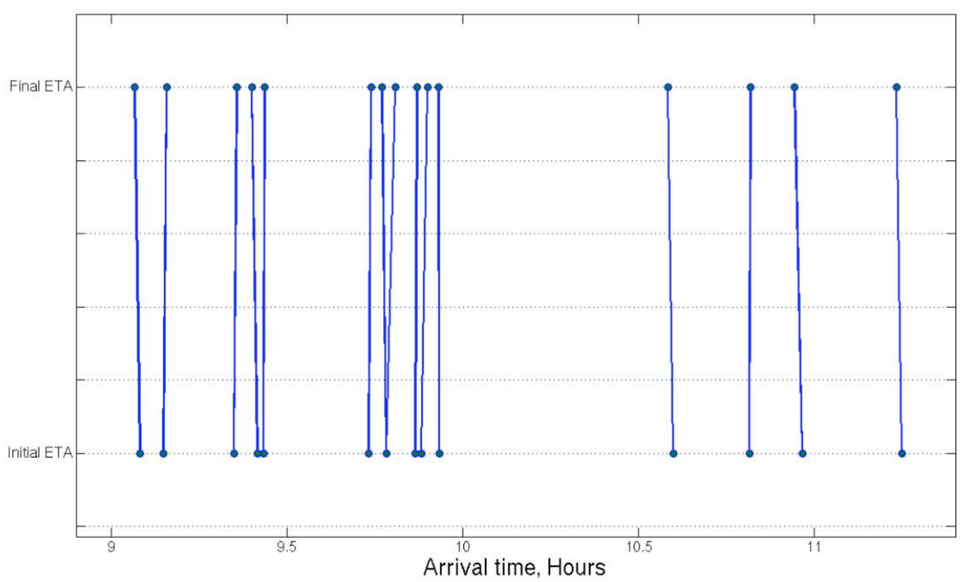


Figure 6.6: Initial and final ETA separation of Quick Mach Duration Algorithm

Although these figures appear similar, there are important features to note. While it is necessary for aircraft in each Algorithm to make a speed change to meet the separation constraints provided by TASAT, the Baseline Algorithm provides a solution offering little deviation from the aircraft's initial cruise speed. However, because the Baseline Algorithm with Fairness endeavors to fairly divide the increased fuel usage among the sixteen aircraft, treating each aircraft as if it were operated by a separate airline, more movement is necessary to still meet the separation requirements. Finally, in the Quick Mach Duration Algorithm, because the speed change is assumed to be made so much closer to the arrival time, more drastic speed changes are necessary indicated by the slopes of the lines from initial to final ETA. While more speed changes are needed for a solution one hour prior to the aircraft's arrival at the metering fix, the problem still remains a feasible one.

Figures 6.7, 6.8, and 6.9 further demonstrate the difference in speed change calculations, showing a trajectory of the Mach change for each aircraft plotted against flight time, in hours. With these graphs, it is straightforward to observe the relative complexities of each solution. In the Baseline Algorithm, it is worth noting that five flights do not need to alter their speeds at all, demonstrating that care is taken by the airline to select a cruise speed that is optimal in terms of fuel burn and schedule. The other flights are made to change speeds because of the CDA separation constraints in place. In future versions of ESCORT, it may be practical to have a constraint indicating that if a flight is not in conflict, it should not be made to change speeds. However, as seen in the Baseline Algorithm, such a constraint may not be necessary, as a third of the flights already remain at the same cruise speed. Yet for small speed changes, it may be better for practical reasons to avoid requesting a very small (0.001 Mach or less) speed

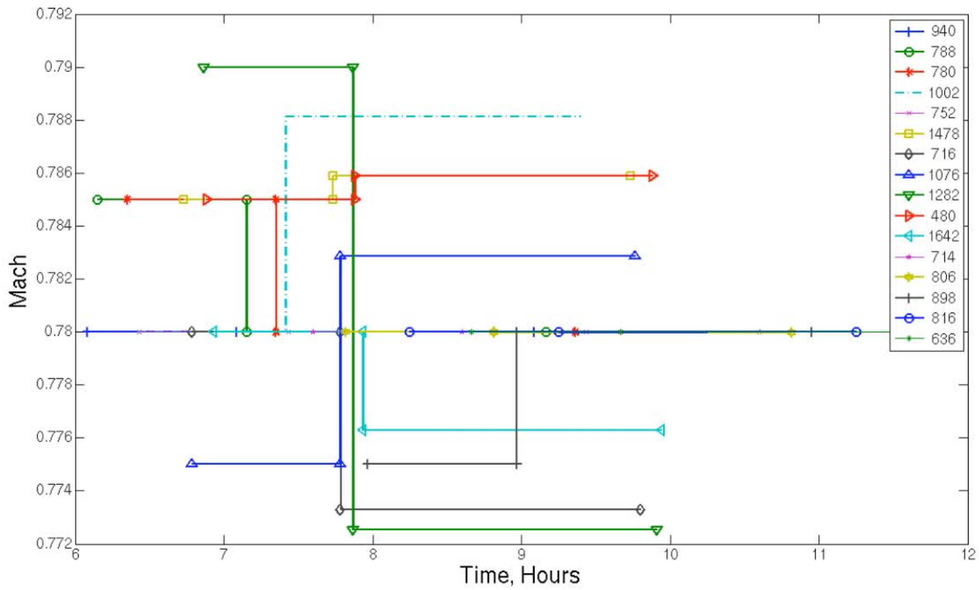


Figure 6.7: Mach trajectory for Baseline Algorithm

change.

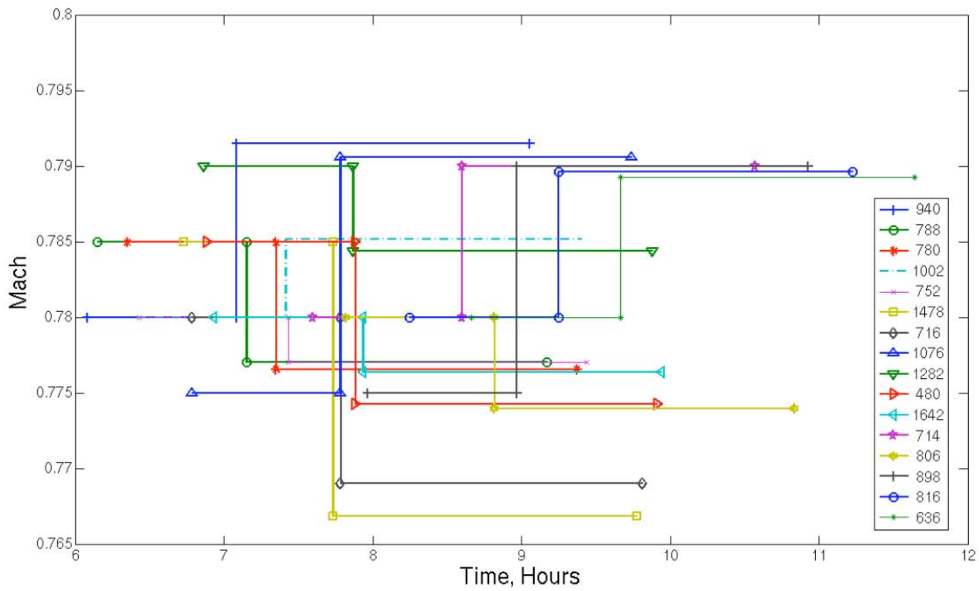


Figure 6.8: Mach trajectory for Baseline Algorithm with Fairness

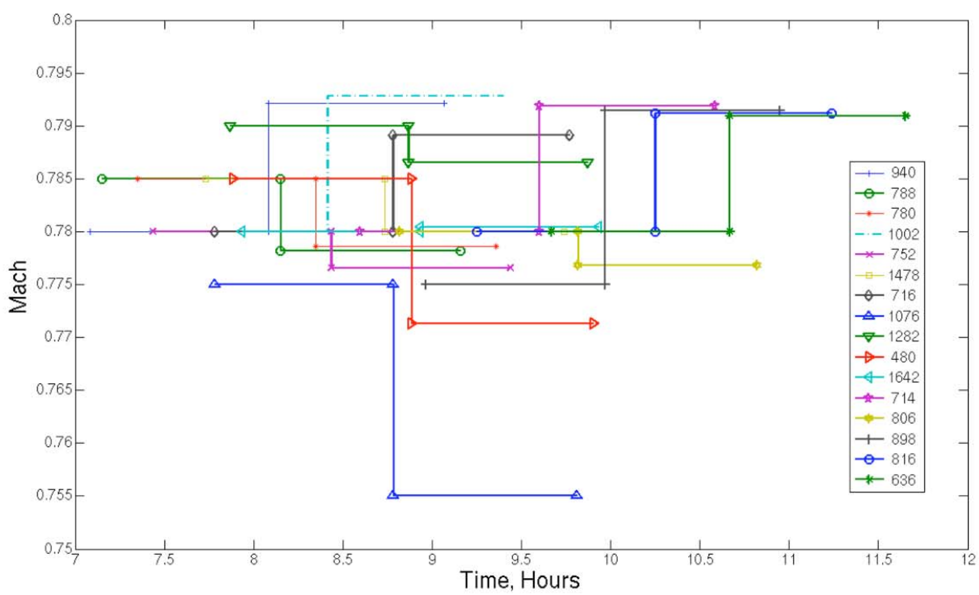


Figure 6.9: Mach trajectory for Quick Mach Duration Algorithm

The remaining graphical results (Figures 6.10-6.12) show the difference in fuel burn for each algorithm. As with the summing of the net fuel burn metric described in Section 6.1, ESCORT calculates net fuel burn for each aircraft. This calculation compares the fuel used throughout the duration of the speed change to the fuel which would have been used had the aircraft remained at its initial cruise speed.

The key result from this series of figures is the Baseline Algorithm calculates a potential fuel savings for a group of aircraft assigned to fly a CDA, so long as they all belong to the same airline looking for an overall net benefit. The Baseline Algorithm calculates a potential fuel savings of 117 kg for the sixteen combined aircraft. While this a best-case scenario, the net sum fuel burn for the Baseline Algorithm with Fairness is a fuel penalty of 476 kg and is the worst-case scenario, with each flight belonging to a separate airline. It will most likely be the case that a series of CDA flights will belong to groups of different airlines, and the fuel penalty (or savings) will lie somewhere within this range (-117 kg to 467 kg). Much of the savings will depend on the particular combination of flight paths and fleet mix for the given day's CDA grouping.

The large value of fuel burn difference for flights 716 and 1076 in the Baseline Algorithm with Fairness and the Quick Mach Duration Algorithm, respectively, is due to these aircraft being scheduled to arrive at the same time. Even if these aircraft were not flying the CDA, a rerouting by air traffic control would be likely with the ETA's of each aircraft being so close. In this case, the fuel burn difference between the calculated route for CDA spacing and the conventional route may be less than what is indicated in these figures.

In addition, the Quick Mach Duration Algorithm actually gives a net fuel burn increase lower than that found with the Baseline Algorithm with Fairness. This result is surprising, because the Quick Mach Duration Algorithm assumes a speed change being made an hour prior to the aircraft's arrival at a metering fix, whereas the Baseline Algorithm with Fairness assumes a change two hours prior to arrival. Yet, while this lower net fuel burn for a speed change made closer to the aircraft's arrival time does not make intuitive sense, fixing the point at which the speed change is made keeps the problem solvable with fast solution times. Future ESCORT versions may want to run multiple cases to determine what is the optimal time at which the speed change should be

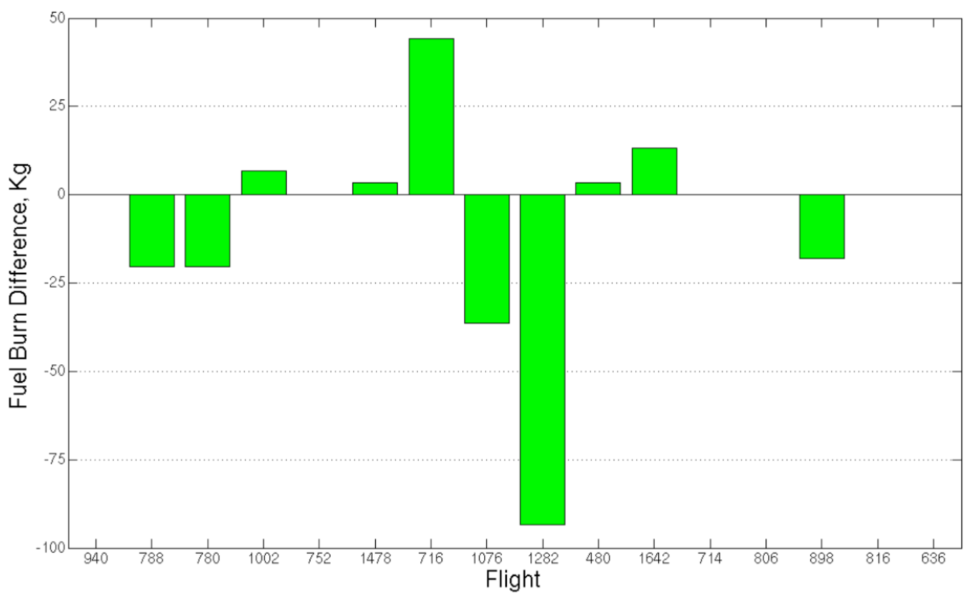


Figure 6.10: Fuel burn difference for en route flight segment for the Baseline Algorithm

made.

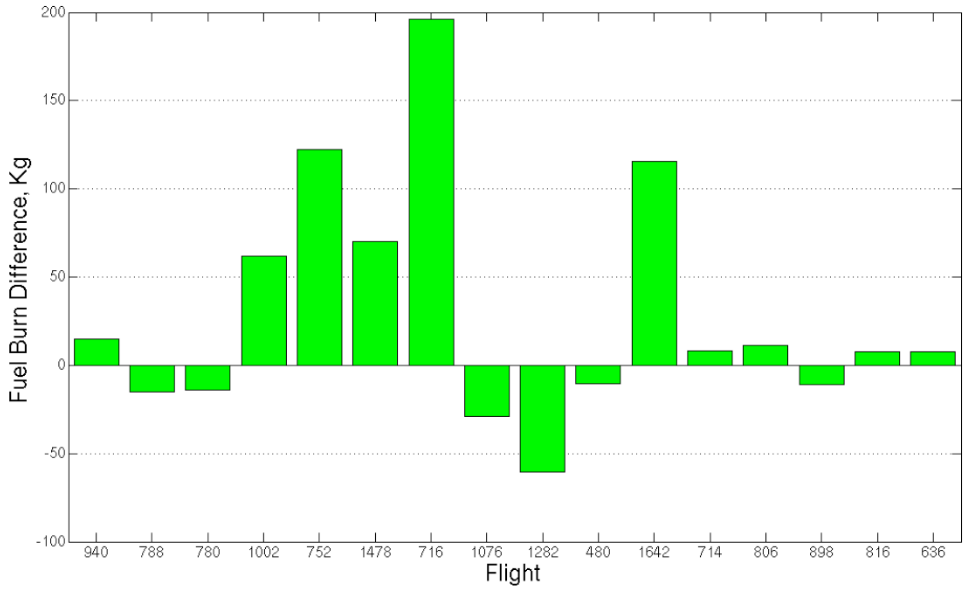


Figure 6.11: Fuel burn difference for en route flight segment for the Baseline Algorithm with Fairness

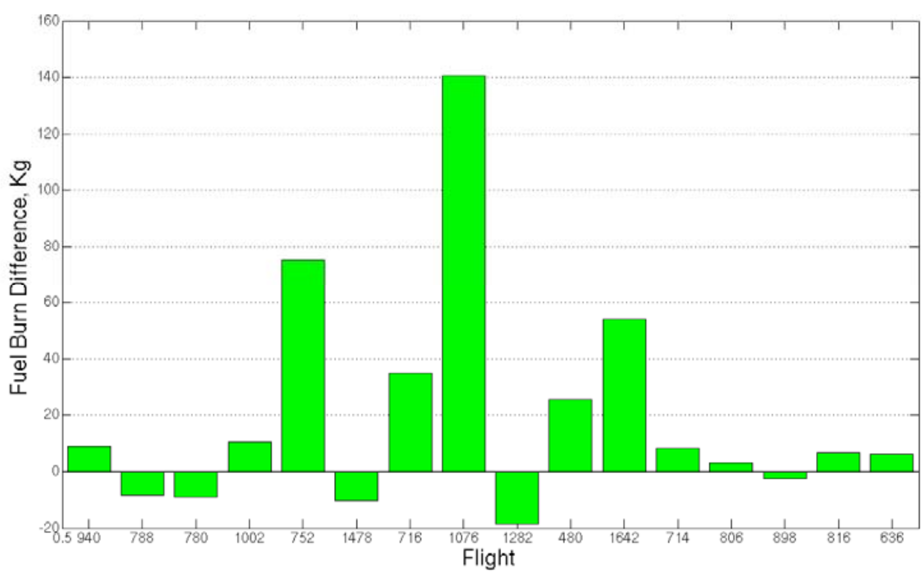


Figure 6.12: Fuel burn difference for en route flight segment for the Quick Mach Duration Algorithm

CHAPTER 7: CONCLUSIONS AND FUTURE WORK

This thesis aimed to provide a tool for near-term CDA expansion so that further fuel and emissions savings could be achieved by the nation's airports. The limitations to CDA expansion were described as threefold—knowing how far apart aircraft should be spaced prior to beginning the CDA, knowing exactly when aircraft will arrive at a metering point prior to their arrival, and knowing the optimal way to set up the aircraft's necessary spacing at a metering point prior to CDA so that the fuel savings inherent in the CDA are maintained. ESCORT functions with two other tools—TASAT, to calculate the necessary separations, and the IET, to provide an accurate ETA—to advance CDA's implementation. By incorporating fuel burn information based on aircraft type and fairness constraints in a mixed integer linear programming problem, ESCORT goes beyond the current state of the art in CDA trajectory tools to provide a tool for speed calculation for en route flights flying a CDA.

7.1 FUTURE WORK

While ESCORT, which is an en route speed change program to implement CDA, has addressed many concerns, such as fairness, maintaining linearity, and giving logical results, there are areas in which more work needs to be completed. For example, it was assumed that the ETA was an absolute discrete time. However, observations during the 2007 flight test showed that a constant ETA is not nearly the case, and these estimations could vary widely during the course of a flight, as shown in Figure 2.4.1. ESCORT is being developed concurrently with an improved ETA estimator at Georgia Tech so that the ETA variation can be minimized. Future versions of the en route speed change tool must handle ETA's as a mean arrival time with a probability distribution as opposed to the current discrete ETA assumption.

In addition, the sample case presented here assumed the first speed change being made two hours prior to arrival at the metering point. This was done to ensure feasibility of the solution in all formulations and was a fairly realistic assumption. However, a more realistic time to make the first speed change will depend on the flight scenario for that day. Future versions of this code must work iteratively from a two-hour speed change solution to one that begins prior, with the output being the solution that gives the lowest net fuel burn. This inclusion will need to be an iterative procedure outside of the optimization itself.

Another extension to this project involves testing ESCORT's results in a real-time flight scenario. Before this can happen, user-friendly graphical user interfaces must be developed so that the flight information for that day is simple and intuitive to enter. The end goal of this relay tool is to interface directly with ACARS once the speed change schedule is calculated. In this final version, the information would then be uploaded to the relevant aircraft for the pilot to observe and execute the speed change advisory.

Yet before this tool can be taken to that level of automation, it will be important to test its accuracy. A testing procedure for ESCORT before actually giving speed changes to the aircraft is to observe a series of flights similar to the previous CDA situations observed during 2007. While observing these flights from an AOC, the IET would be tested concurrently, entering the flight path and wind information for that day and calculating the ETA for each flight. At the same time, a sample speed change schedule would be created using ESCORT. Assuming that the IET provides accurate

ETA's for the flights on that night, the speed change schedule from ESCORT would be entered into the IET to observe the final separation of the aircraft had they actually executed the speed changes. If successful, a flight test using the IET and ESCORT, while actually requesting aircraft to make these speed changes, would follow. In this case, the fuel savings information from the actual CDA portion and the estimated fuel difference for the en route portion with and without speed changes issued could be compared.

There are clearly many directions where this initial research can be applied, and this thesis will be the basis for that future work.

7.2 CONCLUSION

The motivation for this project is to automate the separation of CDA flights so that more parties can experience the fuel, cost, and noise savings of CDA. The formulations presented for the ESCORT (En route Speed Change Optimization Relay Tool) program above are a large first step in meeting this goal, with sample results using initial conditions from a May 2007 CDA flight test validating the solution procedures.

There are two main strengths to this en route speed optimization calculation. The first is a fast solution time. By keeping the optimization problem linear (a mixed integer linear programming problem) for all three of the formulations—the Baseline Algorithm, the Baseline Algorithm with Fairness, and the Quick Mach Duration Algorithm—the combined MATLAB and CPLEX code was able to solve the sample problem in less than two seconds. The fast-solving formulations therefore will expand well to CDA scenarios where more than 16 flights are involved, while still being able to be run in real time. The end goal of the en route speed change program is to calculate a Mach trajectory for each aircraft, relay this information to the aircraft's airline operations center (AOC), and then have the AOC dispatchers transmit the Mach schedule to the pilots while in the air. A fast solution time ensures that this end capability is feasible.

A second strength of the current program is that the results found so far are in line with expectations. For example, although the fairness formulation is more "expensive," in terms of fuel burn increase, the cost is shared among all aircraft, which should be pleasing to the different airlines involved. While this cost is shared, the upper range of the combined fuel penalty was on the order of 450 kg. The CDA fuel benefits would likely offset this fuel penalty, and these numbers do not bring into play the added noise reductions that are one of the main motivations for CDA. Another example of the program meeting expectations is that the spacing constraints are met with all formulations, with the two speed-change solutions being more accurate. As two speed-changes allow for greater flexibility, this result makes sense.

Although there are many future steps to full-scale implementation of this en route speed change program to implement CDA, the groundwork has been laid. Development of the speed change will continue with real time data being tested in an airline's AOC, as well as a future flight test using the program in 2008. This tool will go a long way toward expanding the cost, fuel, and emissions savings of CDA to many more air transportation parties once the current limitations have been addressed.

APPENDIX A

FAIRNESS IN THE CONTEXT OF EN ROUTE SPEED OPTIMIZATION

A.1 Conditions of Fairness

The qualities of fairness can be divided into five distinct categories. The combination of these five distinctions has been seen by those in the field of negotiations to make up the fairest allocation possible. Although en route airspace is not a settlement, its allocation is similar, as there is no Best Alternative to a Negotiated Agreement (BATNA), because the involved parties cannot simply walk away from the 'table.' If all of these five criteria are met, described in Brams' and Taylor's *The Win-Win Solution* [8], and *Fair Division* [7], then the solution found is as close to fair as possible.

A.1.1 Proportionality

According to Aristotle, goods should be divided in proportion to each claimant's contribution. This characteristic means that if there are three parties, but party 1 contributes 20% of the resources for the construction of a good, party 2 contributes 30% of the resources, and party 3 contributes 50% of the resources, then a proportional division would allocate 20% of the divisible good to party one, 30% to party 2, and 50% to party 3. This portion of the fairness definition is seen to be inherent as a part of envy-free division.

A.1.2 Envy-freeness

No party is willing to give up the portion it receives in exchange for the portion someone else receives, meaning that no party envies any other party. An envy-free division is always proportional, because each party does not want anything from the other parties involved, meaning that the resource is divided proportionally (at least in the eyes of the resource division participants). Strategies that the other players select cannot prevent you from obtaining a portion that you think is the largest or most valuable.

A.1.3 Equitability

All parties think they receive the same fraction of the total, as each of them values the different resources. A difficult issue of how to measure whether both parties are equally happy is solved using a point-allocation system. In such a system, each party assigns points to the things being divided, and if when divided, the parties receive items with equal point values for the respective parties' scale, then the division is equitable. In the case of dividing en route airspace, dividing fuel burn may be a starting point for a measure of equitability.

A.1.4 Efficiency

A division is efficient when there is no other allocation of the system that would raise the allocation for the participant with the minimum allocation without decreasing the other participants' allocations. In other words, all users would benefit from a more efficient distribution. When such a possibility no longer exists, then the division is efficient. This efficiency will come from the general optimization algorithm being developed.

A.1.5 Truthfulness

For the conditions such as envy-freeness and equitability to be met, the individuals involved in the 'bargaining' must have their true intentions and desires known to all other parties. For a fair division, one party cannot try and 'play the game,' saying that one resource allocation scheme would be more desirable to his own party than another in hopes that another party will change their resource allocation preference, eventually helping the first airline. This condition, sometimes not included in fairness measurement and evaluation schemes, may be the most important, as an incentive to tell the truth must be inherent in any resource allocation in order for the allocation algorithm to be effective.

Proportionality is inherent in envy-freeness, so there are really four criteria to consider when making a fair division. The next questions to answer, obviously are how can this definition of fairness be allocated, implemented and measured effectively?

A.2 Means of Allocation

A study of fairness would not be complete without including different methods of allocating resources with fairness in mind. For fair allocation, it may be necessary to couple the means of allocating the resource with different ways of measuring fairness and the optimization algorithm. The means of allocation are presented separately from fairness in order to give a clear distinction.

A.2.1 Random/Lotteries

For a random allocation of airspace re-routings, it will be assumed that a history of all re-routings of an airline's aircraft can be maintained. If a set of aircraft with conflicting flight paths is redirected, there will always be an aircraft that has the worst redirection penalty (fuel use increase). If the worst redirection penalty, for example can be assigned to different aircraft at random, then for a large enough time period, *t*, airlines will have a proportional distribution of worst rerouting penalties based on the number aircraft they operate. Keeping track of the random re-routings for a long enough time span will create a proportional, envy-free, and efficient allocation scheme. However, airlines may judge that certain re-routings may be more advantageous at certain times than at others. Airlines may not be satisfied with a static random distribution system when compromise between airlines may be possible for a given situation. In this sense, a random distribution is not equitable.

A.2.2 First-Come, First-Served

The method of first-come first-served, or Grover Jack, has long been in place in the Air Traffic system when dealing with aircraft arrivals. However, this allocation of the landing resource, or in the en route case, the original intended flight path of the airline, will not meet the fairness constraint of envy-freeness. In the en route situation, the point to which the first to arrive (first-come) is measured from is arbitrarily defined. Since aircraft are flying on different trajectories, there can be no set point where they are 'arriving,' so the first-come first-served allocation is an arbitrary decision, which will always create a situation of envy. In addition, Grover Jack had consistent problems of airlines giving false information [5], exacerbating delay problems and making fair allocation impossible by violating the fifth fairness constraint of truthfulness above.

A.2.3 Auctions/Sealed Bidding

Auctions are an efficient mechanism to assigning a nominal value on a scarce resource and are implemented when the seller does not know the bidders' willingness to pay for an item for which there is no well-functioning market (find source). In the En Route case, bundles of flight path re-routings would be auctioned in a real-time situation so that the equitability characteristic lacking in a random allocation could be addressed. Outlining the details of an auction is a fitting forward-thinking solution that will be invaluable in presenting a best-case scenario report for en route airspace optimization. Auctioning airspace and landing slots has been considered as an alternative to dividing NAS resources, but has many auction details to address before its implementation is possible.

Ball and Donahue have presented a possible auctioning scheme for landing and takeoff slots, assuming that a simultaneous multiple round ascending bid combinatorial auction (combinatorial auction is one in which combinations of items are bid for at once) would be the model auction to pursue. The idea is attractive as it shows strong promise for improving the safety, delay performance and economic efficiency of the NAS [5]. However, before such an auction scheme could be implemented, auction concerns involving bidding activity, bidding language, bidding aid tools, pricing of all items, winner determination calculations, bidding increment, and stopping rules would all have to be addressed. In addition, developing an auction system for relatively minor speed adjustments most likely will induce a greater cost than the small fuel savings being divided.

A.2.4 Collaborative Decision Making and Collaborative Routing

Collaborative decision making is the process in which airlines communicate with the FAA in order to provide real time information to Air Traffic Management personnel so that they can ration arrival and departure slots based on the airlines' schedules instead of the traditional Grover Jack first come, first serve method. Flights are then filled into openings created by the Ration by Schedule (RBS) method in order to fill all slots. CDM, in conjunction with RBS and compression have been seen to be effective in allocating scarce airport slots. According to the FAA, the implementation of CDM solutions in GDPs saved more than four million minutes of scheduled ground delay between September 1998 and December 1999 [25].

Collaborative routing uses an idea similar to CDM, where airlines coordinate with the FAA and Traffic Flow Management personnel to assign flight paths [9]. Collaborative Routing is the domain in which the En Route Optimization algorithm will function so an entire airspace sector's flights will be redirected slightly so that the en route airspace can allow more flights at a single time. While collaborative routing is not inherently fair, it meets the restraint of meeting efficiency, and fairness can be implemented from this baseline fairness condition. Any of the allocation methods described here are a form of collaborative routing, because it will be necessary to obtain the intended flight paths from the airlines and then display to them the optimized re-routings. From here, the Collaborative Routing will diverge into random allocation, auctioning methods, or a fair allocation inherent in the optimization algorithm, which will use a fair division measure, discussed in section 5.4.

A.2.5 Ultimatum Bargaining and Divide-and-Choose

The allocation methods of ultimatum bargaining and divide-and-choose are classic game theory methods. In the first, two or more users are involved. The first user divides the pie, and the user either accepts the division, or rejects the division and neither user receives anything. Divide-and-choose is similar in that an individual partitions the desired resource and the remaining user picks the half he wants. Both allocation methods can be extended to more than two users, but in the allocation of airspace in a dynamic situation, there is no real equivalent to cutting airspace and asking for users to either accept or reject it. In the ultimatum case, neither user has the capacity to refuse anything, and for the Divide-and-choose case, the airspace is not readily divisible, and there is no equivalent to the knife to divide the pie in the airspace application.

A.3 Possible traits for a Fairness Measure

The following characteristics would create an ideal metric to measure fairness.

A.3.1 Population Size Independence

The amount of fairness should not depend on the number of individuals present among whom the resource is to be divided.

A.3.2 Scale and Metric Independence

The unit of measure for the resource should not matter

A.3.3 Boundedness (Vary from 0-1)

In order to compare fairness measures, there needs to be a set range of values, and fairness ranging from 0 to 1 provides a readily-understandable scale, with 0 being completely unfair, and 1 being completely fair.

A.3.4 Continuity

Any change in the resource allocation should reflect in the index, meaning that the fairness metric cannot just measure minimum and maximum resource allocations.

A.3.5 Individual Characterization

Since individuals will come to the ‘bargaining table’ with varied backgrounds, these backgrounds must be able to be accounted for, necessary for any envy-free or equitability factor to be taken into account in the fairness measure.

A.3.6 Adequate Timescale

A fairness measure should be able to track fairness both for an instantaneous case and for a given time scale, keeping track of important quantities for the fairness to be measured.

A.4 Proposed Fairness Criterion

The suggestion for a fairness criterion to allocate en route speed adjustment is to divide equally possible percent increase in fuel burn among different aircraft (or aircraft groups). Fairness is an important aspect of CDA because for CDA to have the most effect, as many pilots and airlines must agree to perform the procedure as possible. If each party knows that fuel burn and schedules of participating flights are all being altered as fairly as possible, it is assumed that more parties will be willing to participate. Each airline seeks to operate their aircraft at, or close to the minimum fuel burn, reducing costs and maximizing aircraft range. By identifying the type of aircraft involved in the optimization calculation, the fuel burn characteristics are known for each aircraft. Then, a constraint is needed so that the percent fuel burn increase is equalized for all the aircraft involved. Such constraints are

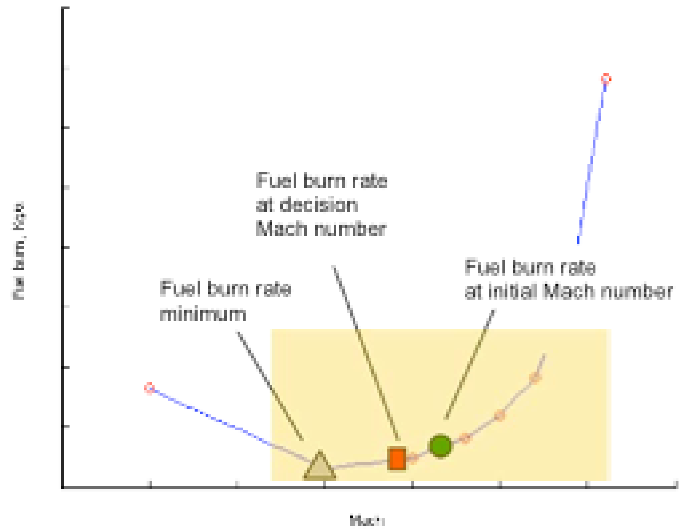


Figure A.1: Chart showing the minimum fuel burn rate with the percentage difference between the initial fuel burn rate and the fuel burn rate at the decision Mach number compared to the fuel burn rate wanting to be minimized in order to account for fairness (values not shown because it is proprietary information).

$$P_{f_i} = \frac{\dot{f}_i\big|_{M_d}}{\dot{f}_{\min}} \cdot 100 \tag{A.1}$$

$$P_{f_i} - P_{f_{i+1}} \le |tolerance|. \tag{A.2}$$

The first constraint is simply an expression for the percentage fuel burn, and the second equation ensures that consecutive airplanes in the CDA sequence have equivalent percentage fuel burn increases to within some tolerance (i.e. 0.01%).

An example fuel burn curve is shown in Fig. A.1. The highlighted area (between the triangle and circle) shows an example of a range of possible values, centering on the initial cruise Mach of the aircraft (indicated by the green circle). The percentage deviation from the gray triangle, the fuel burn minimum to the red square will be equalized for each flight in the CDA-performing fleet. These constraints are a part of the optimization model so that each aircraft has the same percentage increase in fuel burn for their re-routings.

An examination of this fairness criteria shows that it meets four of the five fairness characteristics. In addition, as a fairness measure, all six fairness measure traits are satisfied.

Proportionality- The contribution of differing aircraft performance would be taken into account with the fact that different aircraft types would be assigned different fuel burns, so that using a percentage, gas guzzling aircraft would still have the same percentage fuel burn increase compared to efficient aircraft. The fuel burn increase would be proportional, since it is a percent, increasing or decreasing for the number of aircraft an airline operates.

Envy-freeness- Envy-freeness may be the toughest characteristic to meet. However, it can be implied that if all airlines lose the same amount of increased fuel use, there is no more enviable package among the different participants. The only envy that would be created would come from airlines using different aircraft with different minimum cruise fuel burns, but the fuel burn is already taken into account in the purchase of an aircraft. More work in speaking with actual airline representatives is necessary to see if the envy-free characteristic is met from their perspective.

Equitability- The equitability constraint requires different airlines creating different scales to measure the usefulness of the resource divided. It is assumed that airlines all want to reduce fuel use, and thereby operating costs, making equitable divisions a purely economic concern. This fairness constraint takes this equitability criterion into account, although a bidding process could be more effective in the future.

Efficiency- Since this fairness measure would be an additional constraint of the overall optimization algorithm; the efficient division of the airspace will indeed be possible with the suggested fairness measure.

Truthfulness- Truthfulness will also be met with the suggested fuel burn equalization fairness measure since the measurement is unbiased as it is simply a function of aircraft performance. Airlines will not be able to lie about their desired minimum fuel burn rate, because this data can be checked with other airlines and with the aircraft manufacturer.

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