# THREE-DIMENSIONAL CONTOURING WITH DIGITAL HOLOGRAPHY

Daniel Velásquez Prieto [1,2] Jorge Garcia-Sucerquia, [2,3]
[1] *Basic Sciences Department, Universidad EAFIT*
[2] *Physics School, Universidad Nacional de Colombia Sede Medellín,*
[3] *Department of Physics, Dalhousie University, Halifax, NS, Canada*
(Recibido 05 de Oct.2005; Aceptado 22 de Feb.2006; Publicado 28 de Abr. 2005)

## RESUMEN

Presentamos una técnica para determinar contornos tridimensionales de objetos con dimensiones de al menos cuatro órdenes de magnitud mayor que la longitud de onda de la iluminación. Nuestra propuesta esta basada en la reconstrucción numérica del frente de onda objeto a partir de un holograma registrado digitalmente. El mapa de fase módulo-2π requerido en cualquier proceso de contorneado es obtenido por medio de la substracción directa de dos imágenes de fase (imagen de diferencia de fase) de un objeto bajo diferentes ángulos de iluminación. La obtención de la imagen de diferencia de fase es posible gracias a la capacidad de reconstrucción numérica del campo óptico complejo proporcionado por la holografía digital. Esta característica particular nos proporciona un robusto, confiable y rápido procedimiento que sólo requiere de dos imágenes para su ejecución. La propuesta es soportada con el análisis teórico del sistema de contorneado y verificado por medio de resultados numéricos y experimentales.

**Palabras claves:** holografía digital, 3D-contorno

## ABSTRACT

We report on a technique to determine the 3D contour of objects with dimensions of at least four orders of magnitude larger than the illumination optical wavelength. Our proposal is based on the numerical reconstruction of the object optical wave field of digitally recorded holograms. The required 2π-module phase map in any contouring process is obtained by means of the direct subtraction of two phase-contrast images (phase-difference image) of a still object under different illumination angles. Obtaining phase difference images is only possible by using the capability of numerical reconstruction of the complex optical field provided by the digital holography. This unique characteristic leads us to a robust, reliable, and fast procedure that only requires of two images. We support our proposal with the theoretical analysis of the contouring system verified by means of numerical and experimental results.

**Keywords:** holograms, 3D contour.

## 1. INTRODUCTION

The key problem considered in interferogram analysis is to determine the intensity-coded phase-difference resulting from interferometric experiments. The aims of those interferometric essays range from the study of the micro-deformation experimented by a silicon wafer in a temperature gradient to human body postural studies [1]. The most accurate methods are the several versions of phase-shifting [2]. All of them are based on numerical computation of the phase-difference through the solution of system of simultaneous equations of three unknown variables; hence at least three different interferograms with a phase shift between them are required.

Through a point-to-point subtraction of phase images of “large” object, that we simply call phase-difference image, we determine the 3D-contour of “large” objects. We compute the phase difference image of a still object, illuminated with two different angles. This phase-difference directly provides us a $2\pi$-module phase map from which it is possible to recover the 3D contour of and “large” object via conventional algorithms of phase unwrapping [2]. With this procedure we avoid the need of acquiring more than two interferometric registers in the contouring process, reducing computation complexity and of course the elapsed time. Based on the description of our experimental setup, we theoretically model our proposal. Then numerical modelling and experimental results are shown to prove the feasibility of this technique.

## 2. Phase-difference images

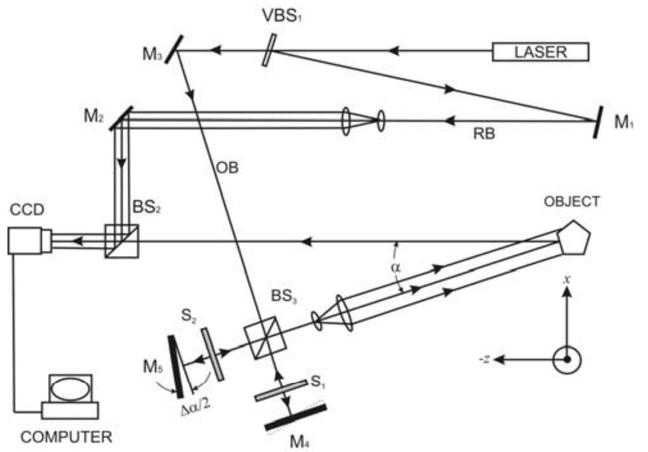


**Figure 1.** Schematic representation of the experimental setup: see text for details.

Let us consider the experimental setup illustrated in Figure 1. By tilting the $M_5$ mirror an angle $\Delta\alpha/2$ in the x’-z’ plane (where the ’ stands for a coordinate system rotated with respect to the illustrated one), the beam reflected on it will be angularly displaced by an amount $\Delta\alpha$ with respect to that coming from $M_4$. Then the object illumination can be chosen, by means of $S_1$ and $S_2$ shutters, from a couple of object beams $OB_1$ and $OB_2$. Any approach to calculate the object optical field, provide it expressed as a complex function of the discrete reconstruction coordinates $(m,n)$ for a particular distance *z*. This fact allows us to numerically evaluate intensity, amplitude and phase for this field. The intensity image of the object optical field can be calculated as:

$$I(m,n,z) = E(m,n,z)\,E^{*}(m,n,z) = \mathrm{Re}^{2}\left[E(m,n,z)\right] + \mathrm{Im}^{2}\left[E(m,n,z)\right] \qquad (1)$$

and the amplitude image is given by the squared root of (1). In the former equation * stand for the complex conjugate and Re and Im denote the real and imaginary part of the complex field $E(m,n,z)$, respectively. In optical holography to recover an accurate version of the object optical field we must make a diffraction experiment of the recorded hologram when it is illuminated by the used reference wave in the recording stage [1]. In digital holography this procedure is carried out numerically by means of the solution of the Kirchhof-Fresnel diffraction formula when the recorded hologram by a CCD camera is multiplied by the reference wave [3]. The result is a complex-valued function $E(m,n,z)$ of the discrete coordinates $(m,n)$ and the reconstruction distance *z*. From that complex field it is also possible to calculate the phase-contrast image as:

$$\phi(m,n) = \arctan 2\left(\frac{\mathrm{Im}[E(n,m)]}{\mathrm{Re}[E(n,m)]}\right). \tag{2}$$

Several procedures can be found in the literature to produce a 3D-contour of "large" objects by means of digital holography [3-5]. The most widely utilized techniques rely on the use of illumination optical sources with multiple wavelengths or on the use of laterally displaced illumination to produce fringe projection systems. The calculation of the phase image that provides the information about the 3D-contour of the object is obtained by means of phase shifting techniques or by the evaluation of the phase of two holograms acquired with different wavelengths.
In our experimental setup, when the object is illuminated by the $OB_1$ we can calculate a phase image $\phi_1(x,y) = 2\pi/\lambda[x\sin\alpha + h(x,y)\cos(\alpha)]$. If the object is now illuminated by $OB_2$ the phase image will be $\phi_2(x,y) = 2\pi/\lambda[x\sin(\alpha+\Delta\alpha) + h(x,y)\cos(\alpha+\Delta\alpha)]$. This phase-contrast images can be calculated by means of equation (2), such that the phase-difference image $\Delta\phi(x,y) = \phi_2(x,y) - \phi_1(x,y)$ will be given by :

$$\Delta\phi(x,y) = \frac{2\pi}{\lambda} 2\sin\frac{\Delta\alpha}{2}\left[x\cos\left(\alpha+\frac{\Delta\alpha}{2}\right) - h(x,y)\sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right]. \tag{3}$$

The first term of this phase-difference image represents the linear phase provided by the carrier of the system; it is totally independent of the object and determines the sensitivity of our procedure. The second term represents the phase introduced by the object. This term must be unscrambled from the phase-difference image in order to get the object topography, i.e. the height map $h(x,y)$. According to equation (3), to recover height map $h(x,y)$, the phase introduced by the linear carrier should be subtracted from the phase-difference to obtain version of the contour's object. The recovered contour by this procedure will be affected by a constant factor that entirely depends on the geometry of the setup.

**3. Results and analysis.**

In this section we show by means of numerical modelling and experimental results the feasibility of our proposal to determine the 3D-contour of a "large" object. We have considered a sphere of 7.8 mm of radius illuminated by a red laser beam (632.8 nm). The illumination angle $\alpha$ was set to $23^o$ with respect to the $-z$ axis and the $M_5$ mirror was titled $\Delta\alpha/2 = 0.033^o$ with respect to $M_4$ mirror.

In Figure 2 we show the numerically calculated phase-difference images resulting of the direct subtraction of the phase-contrast images A) before and B) after the subtraction of the linear phase carrier present in equation 3. C) and D) Same for experimental results. The possibility of setting the tilt angle to any value, makes possible to tune the experiment such that the phase-unwrapping stage can be avoided, as can be seen from Figure 2D.

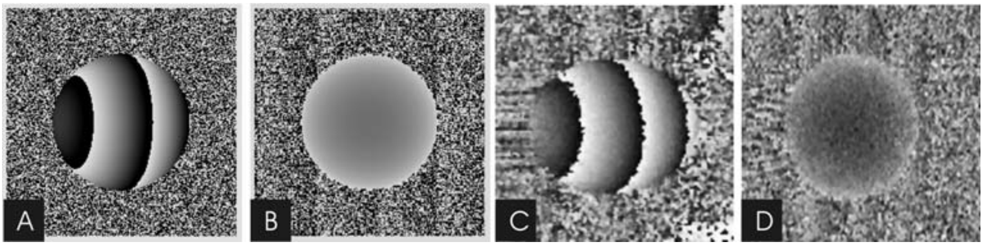


**Figure 2.** Phase-difference Images: Numerical result of subtracting the corresponding two phase-contrast maps A) before and B) after subtraction the linear carrier in equation 3. C) and D) same for experimental results.

**Figure 3** represents the 3D-object reconstructed from the experimental phase-difference image.

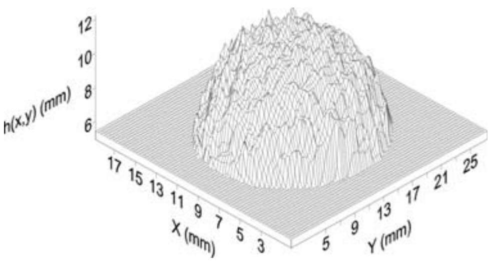


**Figure 3**. Experimental 3D-reconstruction via phase-difference image: The original object, a semi-sphere of 7.8 mm of radius is successfully reconstructed.

The reconstructed semi-sphere has a radius of 6.5 ± 0.5 mm in contrast to the measure of 7.8±0.1 mm obtained with other method.

## CONCLUSION

The feasibility of the phase-difference images to obtain the 3D-contour of "large" objects has been shown. This method is based on the possibility of getting phase-contrast images by means of digital holography. A couple of phase-contrast images of a still object are obtained from two holograms recorded with different angle illumination. The calculation of point-to-point subtraction of those phase images lead us to a $2\pi$-module phase image, which allow us to reconstruct the 3D contour of an object at least four order of magnitude larger than the optical wavelength. Our experimental system allows to set the right conditions such that the phase-unwrapping stage can be skipped, avoiding the many times bottle-neck in optical metrology studies.

## REFERENCES

Williams D.C. "*Optical methods in engineering metrolog***y**". New York: Chapman & Hall, (1993).

D. Malacara, M. Servín, Z. Malacara. "*Interferogram analysis for optical testing*". Marcel Dekker. (1998).

L. P. Yaroslavskii and N. S. Merzlyakov, "*Methods of Digital Holography*", translated from Russian by D. Parsons, Consultants Bureau, New York, (1989)

I Yamaguchi, S. Ohta, J. Kato, "Surface contouring by phase-shifting digital holography", *Opt. Laser Eng*. **36**, 417-428 (2001).

Yunlu Zou, G Pedrini, H.J. Tiziani, "Surface contouring in video frame by changing the wavelength of diode laser", *Opt. Eng*. **35** 1074-1079 (1996).

C Wagner, W Osten and S Seebacher, "Direct shape measurement by digital wavefront reconstruction and multiwavelength contouring", *Opt. Eng*. **39**, 79–85 (2000).