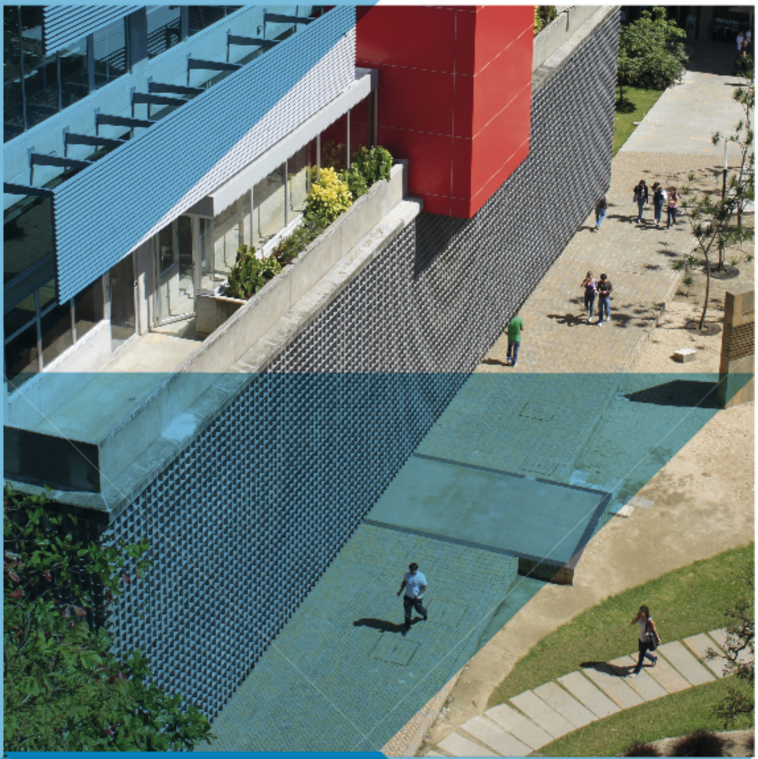

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Which team will win the 2014 FIFA World Cup? A Bayesian approach for dummies

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Which team will win the 2014 FIFA World Cup?

A Bayesian approach for dummies*

Andrés Ramírez Hassan† and Johnatan Cardona Jiménez ‡

February 14, 2014

Abstract

This paper presents several *"ex ante"* simulation exercises of the 2014 FIFA World Cup. Specifically, we estimate the probabilities of each national team advancing to different stages, using a basic Bayesian approach based on conjugate families. In particular, we use the Categorical-Dirichlet model in the first round and the Bernoulli-Beta model in the following stages. The novelty of our framework is given by the use of betting odds to elicit the hyperparameters of prior distributions. Additionally, we obtain the posterior distributions with the Highest Density Intervals of the probability to being champion for each team. We find that Brazil (19.95%), Germany (14.68%), Argentina (12.05%), and Spain (6.2%) have the highest probabilities of being champion. Finally, we identify some betting opportunities with our simulation exercises. In particular, Bosnia & Herzegovina is a promising, whereas Australia shows the lowest betting opportunities return.

JEL Classification: C11, C15, C53
Keywords: Bayesian Approach, Conjugate Families, Simulation, World Cup.

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1 Introduction

The FIFA World Cup is one of the most important sporting competitions in the world. The event, organized by the Fédération Internationale de Football Association, is an international football competition contested by the senior men's national teams of countries that are FIFA members. The tournament takes place every four years since its beginning in 1930, except in 1942 and 1946 when it was not held due to the Second World War. This year, the World Cup Finals are hosted by Brazil whose national team has won the competition five times. Other teams that have won this prominent competition include Italy (four times), Germany (three times) Uruguay and Argentina (twice each), and England, France and Spain (once each). Spain also won the 2010 FIFA World Cup held in South Africa, an event that was broadcasted to 204 countries.

209 FIFA members in six continental confederations participate in qualifying matches over a previous three year period to determine, the 31 countries that qualify to the World Cup Finals. Those countries plus the host nation compete for a total of €25.5 million in prize money for the champion. In total the prize money is €260 million. The 32 teams are divided in 8 groups where the top two teams of each group advance to the round of 16 (see Table 1). After that, the single-elimination stages begin, where teams play each other in one-off matches until the semi finals where losers play for the third place and winners play for the championship. Overall, this sums up to a total of 64 matches (see figure 1).

Although the FIFA World Cup is one of the biggest sporting competitions around the world, to our knowledge, few authors have performed simulation exercises to predict its outcome. A possible explanation for this negligence maybe the availability of information, as few football competitions join forces across different continents. In addition, the time elapsed between tournaments is substantial. Dyte and Clarke (2000) propose a Poisson model, where the dependent variable is the number of goals scored by a team in a football match, and control variables consist of the Fédération Internationale de Football Association rating of each team and the match venue. After estimation, Dyte and Clarke (2000) use their results to simulate the 1998 FIFA World Cup. Specifically, they perform 10,000 runs of each match, and derive probabilities for

Figure 1: Matches beyond second round: 2014 FIFA World Cup

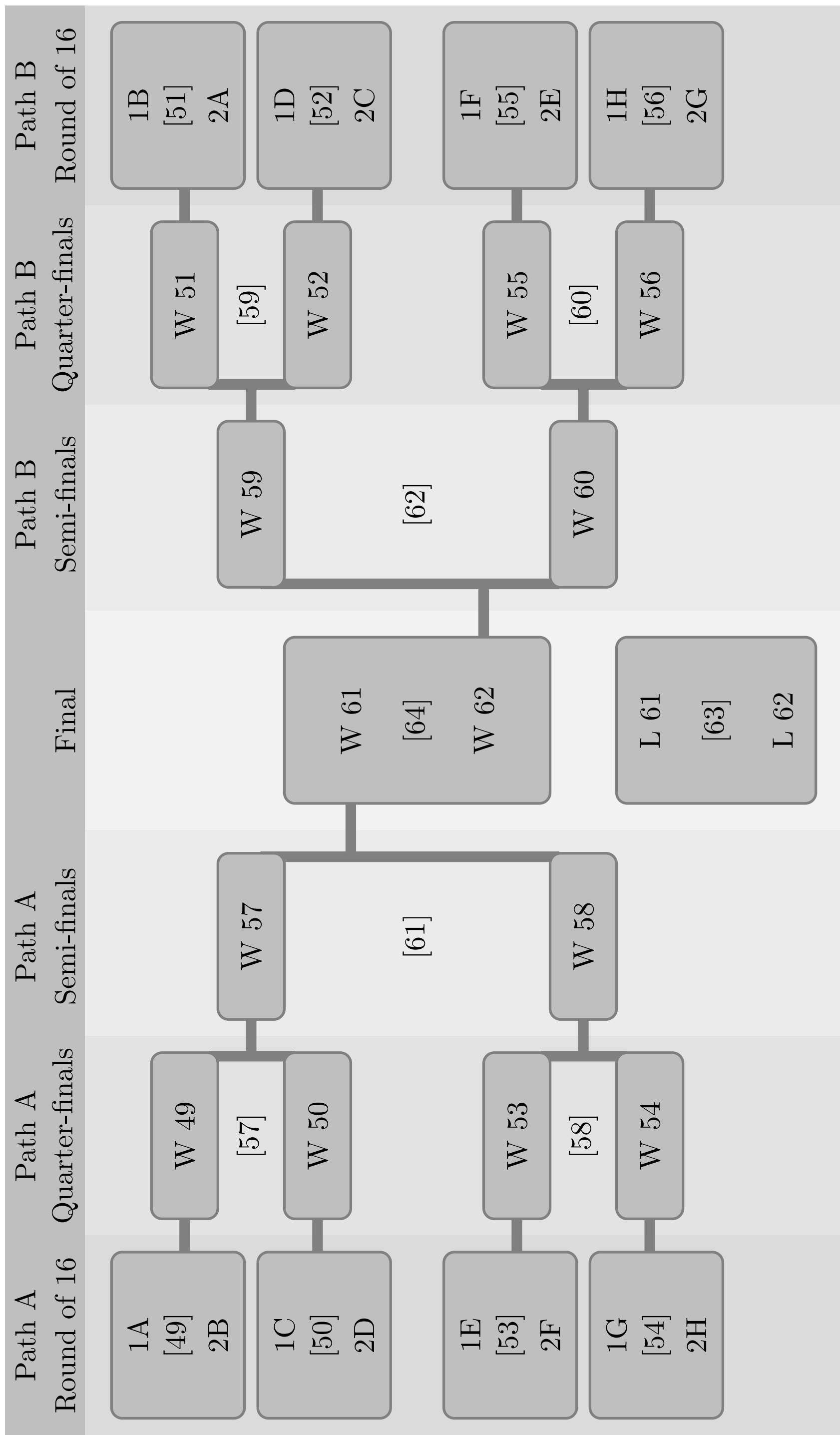

Table 1: Groups: 2014 FIFA World Cup

Group	Team	Group	Team	Group	Team	Group	Team
A	Brasil	C	Colombia	E	Switzerland	G	Germany
A	Croatia	C	Greece	E	Ecuador	G	Portugal
A	Mexico	C	Ivory coast	E	France	G	Ghana
A	Camerun	C	Japan	E	Honduras	G	USA
B	Spain	D	Uruguay	F	Argentina	H	Belgium
B	Netherlands	D	Costa Rica	F	Bosnia & Herzegovina	H	Algeria
B	Chile	D	England	F	Iran	H	Russia
B	Australia	D	Italy	F	Nigeria	H	South korea

Source: the Fédération Internationale de Football Association.

each possible result. Finally, they compare their simulations with observed scores, and obtain a good performance of their model. Volf (2009) modelled a sequence of scored goals in a match as the interaction of two dependent random point processes, where the score intensity is modelled using a semi-parametric regression approach and control variables are red cards, a current positive score, and a current negative score. This model is then simulated using data from the matches that follow the round of 16 teams in the 2006 FIFA World Cup in Germany, and produces satisfying results. Suzuki et al. (2010) follow a Bayesian approach, where the Likelihood associated with the scored goals by a team is given by a Poisson process using the Fédération Internationale de Football Association rating of each team as a control variable. Additionally, Suzuki et al. (2010) use a predictive elicitation procedure based on specialists' opinion to build a Power prior. A posterior Negative Binomial distribution is obtained once the Poisson Likelihood and the Power prior are combined. Suzuki et al. (2010) simulate 10,000 times the 2006 FIFA World Cup to obtain the probability of winning the competition for each national team. Those probabilities are sensitive to prior information and weights given to specialists' opinion in prior distribution.

Beyond that, there exist numerous papers related to predicting outcomes in national leagues, potentially due to an abundance of historical information, which facilities statistical inference. For instance, Rue and Salvesen (2000) suggest a Bayesian dynamic generalized linear model to simultaneously estimate the time-dependent skills of all teams in a league using the Markov Chain

Monte Carlo iterative framework. Their model is then used to predict the football matches of the following weekend. Rue and Salvesen (2000) show various applications of their model based on data from the English Premier League in 1997-1998. A different approach is introduced by Brillinger (2008), who models the probabilities of tie, loss, and win through an ordinal-valued model using the results of each game in the Brazilian Series A. Karlis and Ntzoufras (2009) use a Bayesian framework modelling the difference of scored goals in a match, that is, the margin of victory. They state that this way can incorporate correlation due to the fact that the two opposing teams compete against each other. But there exist another advantage in their methodology because is not necessary to assume that scored goals by each team are marginally Poisson distributed. A Poisson difference distribution known as the Skellman's distribution is used as Likelihood, and a vague Normal distribution as prior. Karlis and Ntzoufras (2009) use Markov Chain Monte Carlo algorithms to generate values from the posterior distribution. Those authors apply their model to simulate the English Premiership for the 2006-2007 season. Finally, Baio and Blangiardo (2010) propose a Bayesian hierarchical model to address the problem of estimating the characteristics that lead a team to lose or win a game, and to predict the score of a particular match. Specifically, they use a Poisson distribution for scored goals where the logarithm of the scoring intensity is a linear function of the attack and defense ability of the two teams involved, in addition to a hosting dummy variable. The first stage parameters have normal prior distributions, as well as the mean hyperparameters, additionally, precision hyperparameters have vague Gamma distributions. They test the performance of their approach using as example the Italian Series A championship 2007-2008 season.

This paper proposes a Bayesian methodology to *"ex ante"* predict the probability that the national teams which participate in the 2014 FIFA World Cup achieve different stages in this competition. The novelty of our approach are twofold: first, we use simple Bayesian rules based on conjugate families, specifically, we use a Categorical-Dirichlet model in the first round, and a Bernoulli-Beta model in the following stages to model the World Cup, so we explicitly incorporate the dependence between opponents in each match in our framework. And second, we use betting odds from bookmakers to build the hyperparameters of prior distributions where betting odds is a measure that summarizes different sources of knowledge related to matches and tournaments, and implicitly incorporates probabilistic forecasts with good performance (Forrest et al., 2005; Strumbelj and Robnik, 2010). Thus, the strength of our Bayesian approach is that our two sets of knowledge to build our posterior distributions, betting odds and historical World Cup information, can easily be updated overcoming the lack of information that characterizes the FIFA World Cup. And finally, our approach can be easily implemented due to being based on simple probabilistic rules.

The following section presents our methodology, followed by some preliminary outcomes in section 3. Section 4 presents the results of our main simulation exercises. Finally, section 5 concludes.

2 Methodology

The point of departure of our Bayesian approach to predict the possible match outcomes of the first stage in the 2014 FIFA World Cup is the Categorical-Dirichlet model. In particular, we assume that the Likelihood is given by a Categorical or Generalized Bernoulli distribution with three possible outcomes $\{Win, Draw, Lose\}$, that is, $p(x = i) = p_i$, $x = \{Win, Draw, Lose\}$, $\sum_i p_i = 1$. On the other hand, prior information is summarized in a Dirichlet distribution such that $\pi(\boldsymbol{p}) \sim \mathcal{D}(\boldsymbol{\alpha})$, that is, $\pi(\boldsymbol{p}) = \frac{\Gamma(\alpha_1+\alpha_2+\alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)} p_1^{\alpha_1-1} p_2^{\alpha_2-1} p_3^{\alpha_3-1}$.

Following the Bayes' theorem, the posterior distribution is $\pi(\boldsymbol{p}|Data) \sim \mathcal{D}(\boldsymbol{\alpha} + \boldsymbol{c})$ where $\boldsymbol{c} = (c_1, c_2, c_3)$ is the vector with the number of occurrences of category i, $c_i = \sum_{j=1}^{N} [x_j = i]$.

$$\pi(\boldsymbol{p}|Data) = \frac{\Gamma(\alpha_1 + \alpha_2 + \alpha_3 + N)}{\Gamma(\alpha_1 + c_1)\Gamma(\alpha_2 + c_2)\Gamma(\alpha_3 + c_3)} p_1^{\alpha_1+c_1-1} p_2^{\alpha_2+c_2-1} p_3^{\alpha_3+c_3-1} \quad (1)$$

We use betting odds to obtain the hyperparameters of our prior distributions using Dirichlet regression models (Thomas and Jacob, 2006; Hizaji and Jernigan, 2009). In particular, we take betting odds associated with the matches of the first stage of the World Cup from different sporting bookmakers, and transform them in outcome probabilities using the procedure developed by Shin (1991, 1993) and Jullien and Salanié (1994). The reason for this decision is the well known argument that betting odds are inherently biased due to bookmakers profit from their service, thus they offer unfair odds. Specifically, the booksum (sum of inverse odds) is greater than one, and as a consequence quoted odds overestimate the probability of every possible outcome (Strumbelj, 2014). A common practice is to use a procedure called Basic Standardization; However, Strumbelj (2014) shows that probabilities deduced from betting odds using the Shin (1993) procedure are more accurate than probabilities produced by Basic Standardization.

Given the odds for a match $O = \{o_{Win}, o_{Draw}, o_{Lose}\}$ the probability estimates are given by

$$p(z)_i^{betting\ odds} = \frac{\sqrt{z^2 + 4(1-z)\frac{(1/o_l)^2}{\sum_l (1/o_l)}} - z}{2(1-z)},\ i = \{Win, Draw, Lose\}\,, \tag{2}$$

such that $\sum_{i=1}^{3} p_i^{betting\ odds} = 1$. We find the proportion of insider trading z as

$$\arg\min_z \sum_{i=1}^{3} \left\{p(z)_i^{betting\ odds} - 1\right\}^2. \tag{3}$$

Once we obtain the probabilities from different bookmarkers for each match in the first stage, we use them as dependent variables in an empirical Bayes approach where hyperparameters of the Dirichlet distribution associated with each match are estimated using Maximum Likelihood.

Additionally, we use historical FIFA World Cup information of the 32 national teams that participate in the 2014 FIFA World Cup to build the Likelihood of each possible match in this competition. Therefore, we count the number of wins, draws, and loses between the different opponents in the first round (48 matches). Finally, we get the posterior Dirichlet distribution of each match in the first stage, and use it to simulate possible outcomes from a Categorical distribution whose outcomes are $\{Win, Draw, Lose\}$ for each match.

Once we sum the points that each team gets in the first round, we determine its group position and which teams qualify to the second stage. However, we must account for the possibility of a draw in points, so we use a second criterium to define which team classifies to the following stage, this is the probability of being champion of the World Cup obtained from betting odds from bookmakers.

We use the Bernoulli-Beta model to simulate following stages, that is, we use as prior a Beta distribution such that $\pi(p) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)+\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1}$ and the Likelihood is given by a Bernoulli process with two possible outcomes $\{Win, Lose\}$, that is, $p(x = Win) = p$ and $p(x = Lose) = 1 - p$. As a consequence the posterior distribution is $\pi(p|Data) \sim \mathcal{B}(\alpha + \sum_{j=1}^{N}[x_j = Win], \beta + N - \sum_{j=1}^{N}[x_j = Win])$. Specifically,

$$\pi(p|Data) = \frac{\Gamma(\alpha+\beta+N)}{\Gamma(\alpha+\sum_{j=1}^{N}[x_j = Win])\Gamma(\beta+N-\sum_{j=1}^{N}[x_j = Win])} p^{\alpha+\sum_{j=1}^{N}[x_j=Win]-1}(1-p)^{\beta+N+\sum_{j=1}^{N}[x_j=Win]-1} \quad (4)$$

Once again, we use historical FIFA World Cup information to build the Likelihood of each match from the round of sixteen to the final match, that is 496 ($\binom{32}{2}$) possible matches. In these stages, we do not allow for draws as long just two outcomes are possible $\{Win, Lose\}$.

Given that betting odds from the second stage to semi finals are not available yet, we assume non informative prior distributions, that is, we use $\mathcal{B}(1,1)$ distributions which imply $U(0,1)$ distributions. However, we use the probability to be champion calculated from betting odds to elicit the hyperparameters of the prior Beta distribution to be used in the final match. In particular, we obtain the probability to be champion of each national team from different bookmakers using the Shin (1993) procedure, and given that just two teams achieve the final match, we standardize the probability between each possible final match combination, so we get 496 combinations for four bookmakers.[1] Finally, we estimate the mean and standard deviation for each combination, and from those estimates, we calculate the hyperparameters, α and β, for each prior Beta distribution associated with the final match.

Finally, we get the posterior Beta distributions for each match from the second stage to

[1]bet-at-home, bet365, Unibet and Winner.

the final match, and use them to get a possible outcome from a Bernoulli distribution, whose outcomes are $\{Win, Lose\}$.

3 Preliminary Results

First, we present basic descriptive statistics in Table 2. We can see that Brazil, Argentina, Germany, Spain, and Belgium are the top five national teams in betting odds whereas Honduras, Costa Rica, Algeria, Iran, and Australia almost have no chance of winning the 2014 FIFA World Cup given this criterium. Specifically, if a person bets €1 on Brazil being the champion and Brazil indeed wins the tournament, this person gains €4.04, while if a person bets €1 on Honduras and Honduras wins, this person gains €2,500.6. Regarding the winning percentage in qualifier rounds, Germany and Netherlands obtain the highest score (90%) while Ecuador has the lowest (38%). Also, Ecuador presents the lowest average scored goals with 1.9 while Germany presents the highest with 3.6. Additionally, we have Spain with just 0.38 goals conceded while the maximum average is 1.56, which pertains to Uruguay. Brazil is the national team whose market value is the highest (€509 million). On the other end of the spectrum, Costa Rica is the team with the lowest value (€21 million). Finally, the FIFA establishes that Spain is in first place in its ranking while South Korea in place 54 is the team with the least favorable position here.

As mentioned in the previous section, we use the Shin (1993) procedure to elicit hyperparameters of prior distributions. In particular, we use betting odds from bookmakers associated with the first stage to calculate hyperparameters of the Dirichlet distribution.[2] Table 3 shows the mean of the proportion of insider trading z for each match of this round.[3] For instance, the match Argentina against Iran has the highest proportion 3.7% whereas the match Netherlands versus Chile has the lowest insider trading proportion with 3.17%. Additionally, Table 3 displays the mean betting odds for each match, and the maximum average is 16.62 which implies that if a person bets against Argentina in the match Argentina versus Iran, and Iran wins, this person

[2]All calculations were performed with the R package (Team, 2013), and specifically, we use the dirichletreg library (Maier, 2012) to estimate these hyperparameters by Maximum Likelihood.

[3]We use the rgenoud library (Mebane and Singh, 2013) to minimize the insider trading factor.

Table 2: Descriptive Statistics: 2014 FIFA World Cup

Team	Championship Odds[1]	Winning Percentage[2]	Scored Goals[2]	Own Goals[2]	Market Value[3]	FIFA Ranking[4]
Brazil	4.04	0.78	3.00	0.78	509	10
	(0.09)	(0.44)	(1.32)	(0.97)		
Croatia	155.60	0.50	1.20	0.80	163	16
	(27.48)	(0.53)	(0.79)	(0.79)		
Mexico	130.60	0.50	1.38	0.69	47	21
	(20.44)	(0.52)	(1.36)	(0.87)		
Cameroon	630.60	0.63	1.50	0.50	123	50
	(244.30)	(0.52)	(1.41)	(0.76)		
Spain	8.00	0.75	1.75	0.38	504	1
	(0.00)	(0.46)	(1.04)	(0.52)		
Netherlands	25.80	0.90	3.40	0.50	162	9
	(3.49)	(0.32)	(1.84)	(0.71)		
Chile	48.80	0.56	1.56	0.56	135	15
	(11.05)	(0.51)	(1.46)	(0.51)		
Australia	650.60	0.57	1.79	0.86	24	46
	(223.94)	(0.51)	(1.31)	(0.77)		
Colombia	23.60	0.56	1.69	0.81	219	4
	(3.91)	(0.51)	(1.49)	(0.91)		
Greece	270.60	0.80	1.20	0.40	65	12
	(27.75)	(0.42)	(0.63)	(0.97)		
Ivory coast	146.60	0.63	2.38	0.88	129	17
	(22.61)	(0.52)	(1.06)	(0.83)		
Japan	155.60	0.57	2.14	0.57	106	47
	(27.71)	(0.51)	(2.35)	(0.65)		
Uruguay	27.00	0.44	1.56	1.56	185	6
	(1.87)	(0.51)	(1.46)	(1.26)		
England	33.40	0.60	3.10	0.40	312	13
	(0.89)	(0.52)	(2.51)	(0.52)		
Italy	27.40	0.60	1.90	0.9	316	7
	(1.82)	(0.52)	(0.88)	(0.88)		
Costa Rica	2150.60	0.56	1.75	0.69	21	31
	(1728.81)	(0.51)	(1.81)	(0.79)		
Switzerland	102.60	0.70	1.70	0.60	124	8
	(23.82)	(0.48)	(1.06)	(1.26)		
France	21.40	0.63	1.88	0.75	373	20
	(2.30)	(0.52)	(1.55)	(0.71)		
Ecuador	151.60	0.38	1.19	1.06	69	23
	(19.14)	(0.50)	(1.11)	(1.06)		
Honduras	2500.60	0.44	1.56	0.94	24	42
	(1500.33)	(0.51)	(2.00)	(0.85)		
Argentina	5.65	0.56	2.19	0.94	475	3
	(0.42)	(0.51)	(1.47)	(0.77)		
Bosnia & Herzegovina	175.60	0.80	3.00	0.60	121	19
	(35.54)	(0.42)	(2.45)	(0.52)		
Iran	1400.60	0.50	1.79	0.50	22	33
	(223.94)	(0.52)	(1.81)	(0.65)		
Nigeria	266.60	0.63	1.38	0.50	89	37
	(23.05)	(0.52)	(0.52)	(0.53)		
Germany	6.40	0.90	3.60	1.00	466	2
	(0.22)	(0.32)	(1.17)	(1.41)		
Portugal	32.40	0.60	2.00	0.90	310	5
	(5.68)	(0.52)	(1.25)	(0.99)		
USA	170.60	0.69	1.63	0.88	51	14
	(27.30)	(0.48)	(0.89)	(0.96)		
Ghana	190.60	0.75	3.13	0.75	84	24
	(54.68)	(0.46)	(2.42)	(0.71)		
Belgium	15.40	0.80	1.80	0.40	320	11
	(0.89)	(0.42)	(0.63)	(0.52)		
Russia	77.60	0.70	2.00	0.50	169	22
	(7.06)	(0.48)	(1.63)	(0.53)		
Algeria	1450.60	0.75	2.00	0.88	63	26
	(671.25)	(0.46)	(1.20)	(1.13)		
South Korea	370.60	0.57	1.93	0.79	46	54
	(120.29)	(0.51)	(1.59)	(0.70)		

[1] Source: ODDSPORTAL.COM (http://www.oddsportal.com/soccer/world/world-cup/outrights/ January 25, 2014 at 13:30.).
[2] Information related to qualifying matches except for Brazil where home matches were used since Luiz Felipe Scolari as coach. Source: the Fédération Internationale de Football Association.
[3] Million Euros. Source: PLURI Sportmetric (www.pluriconsultoria.com.br).
[4] Source: the Fédération Internationale de Football Association (http://es.fifa.com/worldranking/index.html December 27, 2013.).

gets €16.62 for each €1. Finally, we depict the last three columns in this table to show that there are arbitrage opportunities in betting odds. For example, we can assume that a person owns

€95.5 and bets in the match Brazil versus Croatia in the following way: Brazil wins €75.76, Brazil loses €5.88 and draw €13.89. So, this person wins €100 without any risk which implies a profit of €4.5. There are 23 matches with arbitrage opportunities.

Additionally, we have information regarding the betting odds from four bookmakers regarding the 2014 FIFA World Cup champion. We use this information to elicit the hyperparameters of the Beta distributions that are used in the final match. Specifically, we have information from Bet-at-home, bet365, Unibet and Winner, and after the optimization process required to get the insider trading in this stage, we have the following outcomes: 0.94%, 0.71%, 0.47%, and 0.68% for Bet-at-home, bet365, Unibet and Winner. Thus, the average insider trading in this bet (0.70%) is lower than the insider trading of the first round (3.34%).

Table 3: Betting odds first stage: 2014 FIFA World Cup

Team One	Team Two	Insider trading	Average			Maximum		
			Win Team 1	Draw	Lose Team 1	Win Team 1	Draw	Lose team 1
Brazil	Croatia	3.38	1.28	5.40	10.87	1.32	7.20	17.00
		(1.16)	(0.03)	(0.62)	(2.37)			
Mexico	Cameroon	3.26	2.11	3.21	3.61	2.20	3.40	4.08
		(1.13)	(0.22)	(1.54)	(1.97)			
Brazil	Mexico	3.55	1.32	4.96	9.13	1.36	6.00	10.00
		(1.25)	(0.69)	(0.78)	(2.47)			
Cameroon	Croatia	3.40	3.93	3.29	1.97	4.20	3.40	2.10
		(1.28)	(0.19)	(0.06)	(0.08)			
Croatia	Mexico	3.23	2.37	3.39	2.89	2.65	3.50	3.10
		(1.22)	(0.15)	(0.07)	(0.15)			
Camerron	Brazil	3.55	14.47	5.88	1.21	17.00	7.00	1.25
		(1.36)	(2.21)	(0.58)	(0.02)			
Spain	Netherlands	3.23	2.06	3.24	3.71	2.11	3.45	4.00
		(1.15)	(0.04)	(0.10)	(0.16)			
Spain	Chile	3.33	1.69	3.63	5.06	1.80	3.90	5.50
		(1.20)	(0.06)	(0.20)	(0.46)			
Australia	Spain	3.55	13.59	5.95	1.22	17.00	7.00	1.25
		(1.34)	(1.89)	(0.69)	(0.03)			
Australia	Netherlands	3.41	9.09	4.95	1.33	10.50	5.40	1.36
		(1.20)	(0.68)	(0.25)	(0.03)			
Netherlands	Chile	3.17	2.23	3.33	3.18	2.34	3.40	3.40
		(1.16)	(0.08)	(0.06)	(0.14)			
Chile	Australia	3.24	1.50	3.97	7.05	1.53	4.33	7.99
		(1.27)	(0.03)	(0.22)	(0.46)			
Colombia	Greece	3.24	1.63	3.56	5.93	1.70	4.00	6.70
		(1.21)	(0.06)	(0.22)	(0.38)			
Colombia	Ivory coast	3.40	1.91	3.36	4.09	2.15	3.50	4.50
		(1.29)	(0.12)	(0.12)	(0.20)			
Japan	Colombia	3.29	4.86	3.54	1.74	5.80	3.75	1.85
		(1.26)	(0.53)	(0.14)	(0.06)			
Ivory coast	Japan	3.23	2.55	3.12	2.85	2.70	3.30	2.95

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Table 3 – *Continue from previous page*

Team One	Team Two	Insider trading	Average			Maximum		
			Win Team 1	Draw	Lose Team 1	Win Team 1	Draw	Lose team 1
		(1.13)	(0.09)	(0.12)	(0.08)			
Greece	Ivory coast	3.26	3.05	3.30	2.31	3.35	3.35	2.42
		(1.22)	(0.18)	(0.03)	(0.08)			
Japan	Greece	3.28	2.41	3.31	2.89	2.60	3.40	3.00
		(1.20)	(0.11)	(0.06)	(0.12)			
Uruguay	Costa rica	3.46	1.32	4.74	10.35	1.37	5.30	13.00
		(1.17)	(0.03)	(0.23)	(1.32)			
Uruguay	England	3.34	2.44	3.27	2.86	2.60	3.35	3.00
		(1.08)	(0.08)	(0.04)	(0.11)			
Italy	Uruguay	3.28	2.32	3.31	3.01	2.50	3.45	3.30
		(1.21)	(0.11)	(0.06)	(0.16)			
England	Italy	3.40	3.02	3.10	2.42	3.15	3.35	2.55
		(1.18)	(0.09)	(0.11)	(0.08)			
Costa rica	England	3.49	7.17	4.25	1.45	7.80	4.50	1.50
		(1.08)	(0.34)	(0.24)	(0.04)			
Italy	Costa rica	3.31	1.32	4.96	10.04	1.36	6.00	11.75
		(1.21)	(0.04)	(0.50)	(1.28)			
Switzerland	Ecuador	3.25	2.26	3.12	3.34	2.38	3.30	3.75
		(1.20)	(0.08)	(0.10)	(0.24)			
Switzerland	France	3.33	3.82	3.24	2.02	4.10	3.40	2.10
		(1.26)	(0.21)	(0.07)	(0.05)			
Honduras	Switzerland	3.36	6.70	3.79	1.54	8.00	4.10	1.60
		(1.26)	(0.84)	(0.15)	(0.05)			
France	Honduras	3.41	1.26	5.25	12.38	1.30	5.90	15.00
		(1.31)	(0.02)	(0.32)	(1.60)			
Honduras	Ecuador	3.36	5.29	3.57	1.68	6.00	3.90	1.80
		(1.21)	(1.45)	(0.49)	(3.44)			
Ecuador	France	3.34	3.81	3.44	1.96	4.75	3.60	2.05
		(1.24)	(0.39)	(0.08)	(0.05)			
Argentina	Bosnia & Herzegovina	3.30	1.38	4.62	8.16	1.41	5.30	9.36
		(1.19)	(0.02)	(0.26)	(0.64)			
Argentina	Iran	3.70	1.17	6.62	16.62	1.22	7.20	21.00
		(1.42)	(0.03)	(0.43)	(2.57)			
Nigeria	Argentina	3.41	8.35	4.58	1.38	9.80	5.80	1.40
		(1.20)	(0.57)	(0.04)	(5.88)			
Iran	Nigeria	3.22	4.15	3.20	1.96	4.60	3.55	2.04
		(1.25)	(0.24)	(0.16)	(0.04)			
Nigeria	Bosnia & Herzegovina	3.21	2.87	3.28	2.46	3.40	3.30	2.65
		(1.08)	(0.26)	(0.05)	(0.17)			
Bosnia & Herzegovina	Iran	3.30	1.54	3.86	6.42	1.60	4.45	7.20
		(1.13)	(0.03)	(0.29)	(0.48)			
Germany	Portugal	3.28	1.85	3.43	4.32	1.91	3.60	5.00
		(1.18)	(0.05)	(0.10)	(0.33)			
Germany	Ghana	3.48	1.42	4.49	7.42	1.45	4.85	9.20
		(1.21)	(0.04)	(0.23)	(1.00)			
USA	Gernamy	3.35	8.21	4.37	1.41	10.00	4.50	1.45
		(1.21)	(1.40)	(0.14)	(0.06)			
Ghana	USA	3.18	2.50	3.17	2.88	2.63	3.35	3.07
		(1.13)	(0.07)	(0.09)	(0.08)			
USA	Portugal	3.38	5.07	3.73	1.68	5.90	4.33	1.80
		(1.32)	(0.41)	(0.37)	(0.11)			
Portugal	Ghana	3.35	1.87	3.44	4.19	1.95	3.55	5.00
		(1.18)	(0.06)	(0.09)	(0.46)			
Belgium	Algeria	3.43	1.35	4.66	9.15	1.42	5.00	10.60

Continue on next page

Table 3 – *Continue from previous page*

Team One	Team Two	Insider trading	Average			Maximum		
			Win Team 1	Draw	Lose Team 1	Win Team 1	Draw	Lose team 1
		(1.30)	(0.03)	(0.22)	(0.84)			
Belgium	Russia	3.34	2.14	3.26	3.44	2.40	3.35	3.55
		(1.22)	(0.12)	(0.06)	(0.09)			
South korea	Belgium	3.46	4.98	3.59	1.70	5.50	3.75	1.80
		(1.26)	(0.35)	(0.10)	(0.07)			
Russia	South korea	3.30	1.95	3.26	4.06	2.07	3.45	4.20
		(1.21)	(0.07)	(0.12)	(0.19)			
South korea	Algeria	3.38	1.96	3.25	4.03	2.00	3.40	4.80
		(1.19)	(0.04)	(0.07)	(0.37)			
Algeria	Russia	3.34	6.12	3.76	1.58	7.00	4.00	1.67
		(1.18)	(0.66)	(0.17)	(0.07)			

Source: ODDSPORTAL.COM (http://www.oddsportal.com/soccer/world/world-cup/outrights/ January 25, 2014 at 13:45.)

4 Simulation Exercises

This section shows the results of our simulation exercises of the 2014 FIFA World Cup. First, we show the outcomes using the Shin (1993) procedure, and then use these outcomes to spot several betting opportunities.

4.1 Main results: Posterior probabilities

Table 4 displays the probabilities of qualifying to each stage of every national team. For instance, Brazil, Spain, Colombia, Uruguay, France, Argentina, Germany, and Belgium have the highest probabilities to qualify to the Round of 16 in their groups. Regarding Quarter-finals we observe that Brazil (65.87%), Argentina (58.16%), and Germany (61.33%) have the highest probabilities to qualify to this round. Also, we can deduce from Table 4 that the most probable Semi-final matches are Brazil versus Germany and Uruguay versus Argentina. Moreover, the final match with the highest probability is Brazil against Argentina, and although Germany has the second highest probability to achieve this stage, the Germans play against Brazil in the Semi-final match, and most of the times the Brazilians win. Overall, Brazil has the highest probability to be the 2014 FIFA World Cup champion with 19.95%, followed by Germany, Argentina and Spain with probabilities of 14.68%, 12.05%, and 6.20%. Table 6 of the Appendix shows the probabilities to advance to each stage conditional on the path, and additionally, Figure 2 displays the most probable development of the 2014 FIFA World Cup.

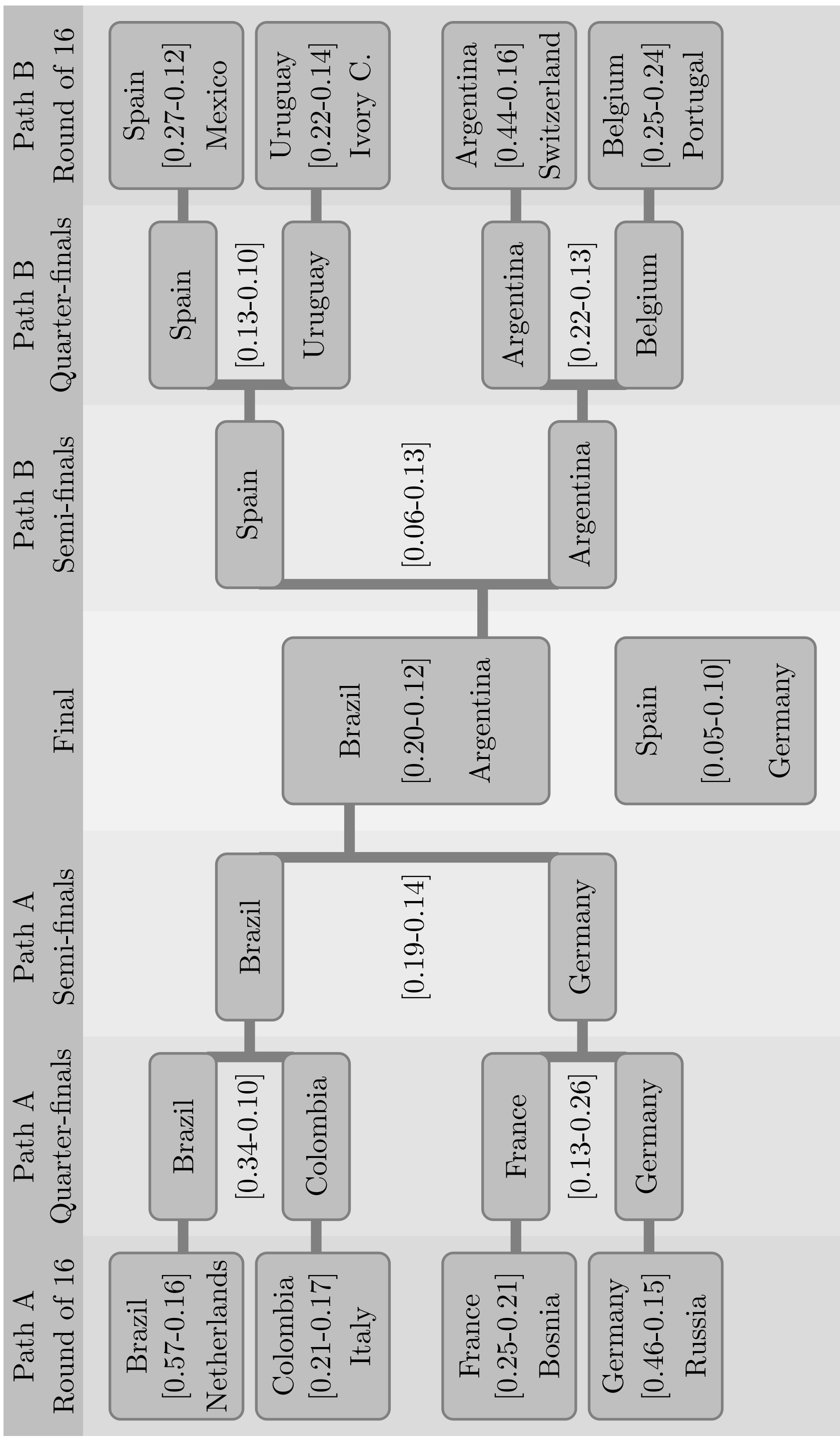
Table 4: Mean probability of qualifying each stage using Shin's procedure: 2014 FIFA World Cup

Team	Group	Round of 16	Quarter-finals	Semi-finals	Final	Champion
Brazil	A	0.9674	0.6587	0.3861	0.2223	0.1995
Croatia	A	0.3678	0.1983	0.1075	0.0276	0.0097
Mexico	A	0.3841	0.1456	0.0603	0.0240	0.0054
Cameroon	A	0.2807	0.1228	0.0606	0.0310	0.0012
Spain	B	0.8164	0.3439	0.1702	0.0779	0.0620
Netherlands	B	0.5648	0.2812	0.1720	0.0757	0.0360
Chile	B	0.5398	0.2172	0.1022	0.0484	0.0170
Australia	B	0.0790	0.0323	0.0165	0.0070	0.0003
Colombia	C	0.8019	0.3190	0.1511	0.0796	0.0396
Greece	C	0.2676	0.1382	0.0683	0.0288	0.0046
Ivory coast	C	0.5493	0.2742	0.1345	0.0638	0.0140
Japan	C	0.3812	0.1864	0.0833	0.0403	0.0071
Uruguay	D	0.7111	0.3889	0.1884	0.0877	0.0431
England	D	0.6009	0.3441	0.1355	0.0742	0.0319
Italy	D	0.6335	0.3244	0.1527	0.0984	0.0502
Costa rica	D	0.0545	0.0248	0.0108	0.0054	0.0003
Switzerland	E	0.6448	0.2767	0.1270	0.0628	0.0188
France	E	0.8448	0.3342	0.1767	0.0757	0.0469
Ecuador	E	0.4267	0.2114	0.1008	0.0457	0.0102
Honduras	E	0.0837	0.0411	0.0229	0.0091	0.0001
Argentina	F	0.8977	0.5816	0.2895	0.1566	0.1205
Bosnia & Herzegovina	F	0.6047	0.3032	0.1505	0.0682	0.0139
Iran	F	0.1142	0.0577	0.0329	0.0170	0.0004
Nigeria	F	0.3834	0.1941	0.0975	0.0505	0.0079
Germany	G	0.8911	0.6133	0.3592	0.1922	0.1468
Portugal	G	0.6807	0.3627	0.1698	0.0861	0.0400
Usa	G	0.1987	0.1196	0.0496	0.0236	0.0047
Ghana	G	0.2295	0.1131	0.0547	0.0238	0.0057
Belgium	H	0.8502	0.3352	0.1654	0.0750	0.0435
Russia	H	0.7105	0.2526	0.1156	0.0557	0.0162
Algeria	H	0.1222	0.0641	0.0333	0.0170	0.0004
South korea	H	0.3171	0.1394	0.0546	0.0269	0.0021

Source: Authors' estimations.

We depict the densities associated with being the champion of the tournament in figures 3

Figure 2: The most probable development of the 2014 FIFA World Cup



and 4. Furthermore, we can find in these figures the Highest Density Intervals at 95% credibility of being champion. As we can see in Figure 3, Brazil has a HDI equal to $(16.7\%, 21.3\%)$, which implies that there is a 95% of credibility that the probability of the Brazilians winning the 2014 FIFA World Cup is between 16.7% and 21.3%. On the other hand, we can see in Figure 4 that Honduras has a 95% credibility to win the tournament equal to 0%. In the Appendix we can find the densities of the probabilities to achieve all stages for each team. These results are displayed in figures 5, 6, 7, 8, 9, 10, 11 and 12.

4.2 Probable betting opportunities

We can observe in Table 5 the betting odds from different bookmakers in the market associated with the winner of the championship. Additionally, we calculate the implicit posterior odds as the inverse posterior probabilities from our simulation exercises. We know that our implicit odds are fair as the booksum (sum of the posterior probabilities) is equal to one, whereas the market's odds are unfair because bookmakers profit from their service. Specifically, bookmakers overestimate the probability of an event and this implies a reduction of their betting odds. As a consequence, if there are posterior odds that are less than market betting odds we can find betting opportunities. For instance, we observe in Table 5 that if a person bets €1 in bet-at-home that Croatia wins the 2014 FIFA World Cup and Croatia wins, this person gains €150. However, we can see that the posterior odds for Croatia are 103.1, which implies that in our fair world this person should gain just €103.1. As a consequence, we identify a possible betting opportunity that implies a 45.5% of return.

We calculate in the last three columns of Table 5 the betting opportunities as a percentage. We can see that Bosnia & Herzegovina has good return opportunities because in three out of four bookmakers this team has the highest returns: 109.9%, 178% and 179.4% for bet365, Unibet and Winner. Regarding bet-at-home we find Nigeria whose possible betting opportunity stands at 97.5%. On the other hand, Australia is not a good team to bet on, as it has the lowest returns in three out of four bookmakers: -85%, -85% and -77.5% for bet365, Unibet, and Winner. Additionally, Honduras has the lowest return in bet-at-home (-90%).

Table 5: Possible betting opportunities: 2014 FIFA World Cup

Team	Betting Odds[1]				Implicit Posterior Odds[2]	Margin[2]			
	Bet-at-home	bet365	Unibet	Winner		Bet-at-home	bet365	Unibet	Winner
Brazil	4.2	4.0	4.0	4.0	5.0	-16.2%	-20.2%	-20.2%	-20.2%
Croatia	150.0	151.0	150.0	126.0	103.1	45.5%	46.5%	45.5%	22.2%
Mexico	150.0	101.0	150.0	126.0	185.2	-19.0%	-45.5%	-19.0%	-32.0%
Cameroon	400.0	501.0	500.0	751.0	833.3	-52.0%	-39.9%	-40.0%	-9.9%
Spain	8.0	8.0	8.0	8.0	16.1	-50.4%	-50.4%	-50.4%	-50.4%
Netherlands	20.0	29.0	28.0	26.0	27.8	-28.0%	4.4%	0.8%	-6.4%
Chile	40.0	51.0	45.0	41.0	58.8	-32.0%	-13.3%	-23.5%	-30.3%
Australia	500.0	501.0	500.0	751.0	3333.3	-85.0%	-85.0%	-85.0%	-77.5%
Colombia	17.0	26.0	26.0	23.0	25.3	-32.7%	3.0%	3.0%	-8.9%
Greece	250.0	301.0	250.0	251.0	217.4	15.0%	38.5%	15.0%	15.5%
Ivory Coast	125.0	126.0	180.0	151.0	71.4	75.0%	76.4%	152.0%	111.4%
Japan	125.0	151.0	150.0	151.0	140.8	-11.3%	7.2%	6.5%	7.2%
Uruguay	25.0	26.0	26.0	29.0	23.2	7.7%	12.1%	12.1%	25.0%
Italy	25.0	29.0	28.0	26.0	31.3	-20.3%	-7.5%	-10.7%	-17.1%
England	33.0	34.0	32.0	34.0	19.9	65.7%	70.7%	60.6%	70.7%
Costa Rica	1000.0	2501.0	750.0	1501.0	3333.3	-70.0%	-25.0%	-77.5%	-55.0%
Switzerland	65.0	101.0	120.0	101.0	53.2	22.2%	89.9%	125.6%	89.9%
France	18.0	21.0	24.0	21.0	21.3	-15.6%	-1.5%	12.6%	-1.5%
Ecuador	150.0	126.0	180.0	151.0	98.0	53.0%	28.5%	83.6%	54.0%
Honduras	1000.0	2001.0	2000.0	2501.0	10000.0	-90.0%	-80.0%	-80.0%	-75.0%
Argentina	6.0	5.0	5.7	5.5	8.3	-27.7%	-39.8%	-30.7%	-33.7%
Bosnia-Herzegovina	125.0	151.0	200.0	201.0	71.9	73.8%	109.9%	178.0%	179.4%
Iran	1000.0	1501.0	1500.0	1501.0	2500.0	-60.0%	-40.0%	-40.0%	-40.0%
Nigeria	250.0	251.0	280.0	251.0	126.6	97.5%	98.3%	121.2%	98.3%
Germany	6.0	6.5	6.5	6.5	6.8	-11.9%	-4.6%	-4.6%	-4.6%
Portugal	25.0	34.0	40.0	34.0	25.0	0.0%	36.0%	60.0%	36.0%
USA	150.0	151.0	200.0	151.0	212.8	-29.5%	-29.0%	-6.0%	-29.0%
Ghana	150.0	251.0	250.0	151.0	175.4	-14.5%	43.1%	42.5%	-13.9%
Belgium	15.0	15.0	17.0	15.0	23.0	-34.8%	-34.8%	-26.0%	-34.8%
Russia	65.0	81.0	80.0	81.0	61.7	5.3%	31.2%	29.6%	31.2%
Algeria	1000.0	1501.0	750.0	1501.0	2500.0	-60.0%	-40.0%	-70.0%	-40.0%
South Korea	500.0	301.0	300.0	251.0	476.2	5.0%	-36.8%	-37.0%	-47.3%

[1] Source: ODDSPORTAL.COM (http://www.oddsportal.com/soccer/world/world-cup/outrights/ January 25, 2014 at 13:30.).
[2] Source: Authors' estimations.

5 Concluding Remarks

Although the FIFA World Cup is one of the most important sporting competitions around the world, there are not many published works that simulate this tournament. Moreover, most of these papers perform *"ex post"* simulation exercises. This is because there are few football competitions that join national teams from different continents, and also there is a large elapsed time between competitions. Those facts imply limited information that complicates statistical inference. Therefore, we propose a basic Bayesian approach based on conjugate families to simulate the 2014 FIFA World Cup. Specifically, we use betting odds from different bookmakers in the market to elicit the hyperparameters of the Categorical-Dirichlet and the Bernoulli-Beta models that are used to simulate the tournament.

We find that the most probable final match is Brazil versus Argentina, and the match for the third place is projected to be Germany against Uruguay. Overall, Brazil (19.95%), Germany

(14.68%), Argentina (12.05%), and Spain (6.2%) have the highest probabilities to be the champion of the 2014 FIFA World Cup.

Finally, we identify some betting opportunities. In particular, Bosnia & Herzegovina is the team with the highest betting opportunities (135.3% in average), whereas Australia has the lowest return (-83.1% in average).

Figure 3: Density of being champion using Shin procedure: 2014 FIFA World Cup

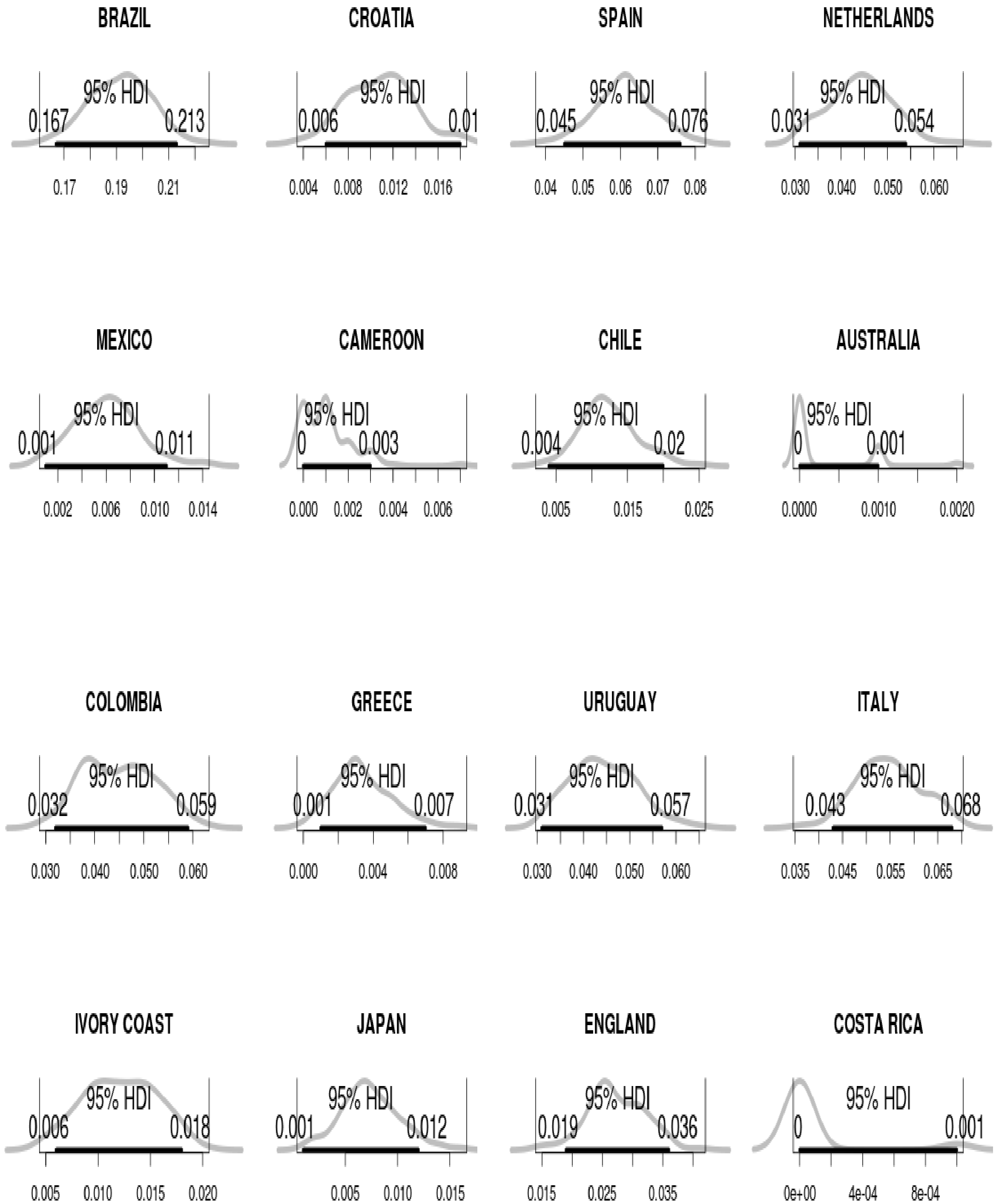
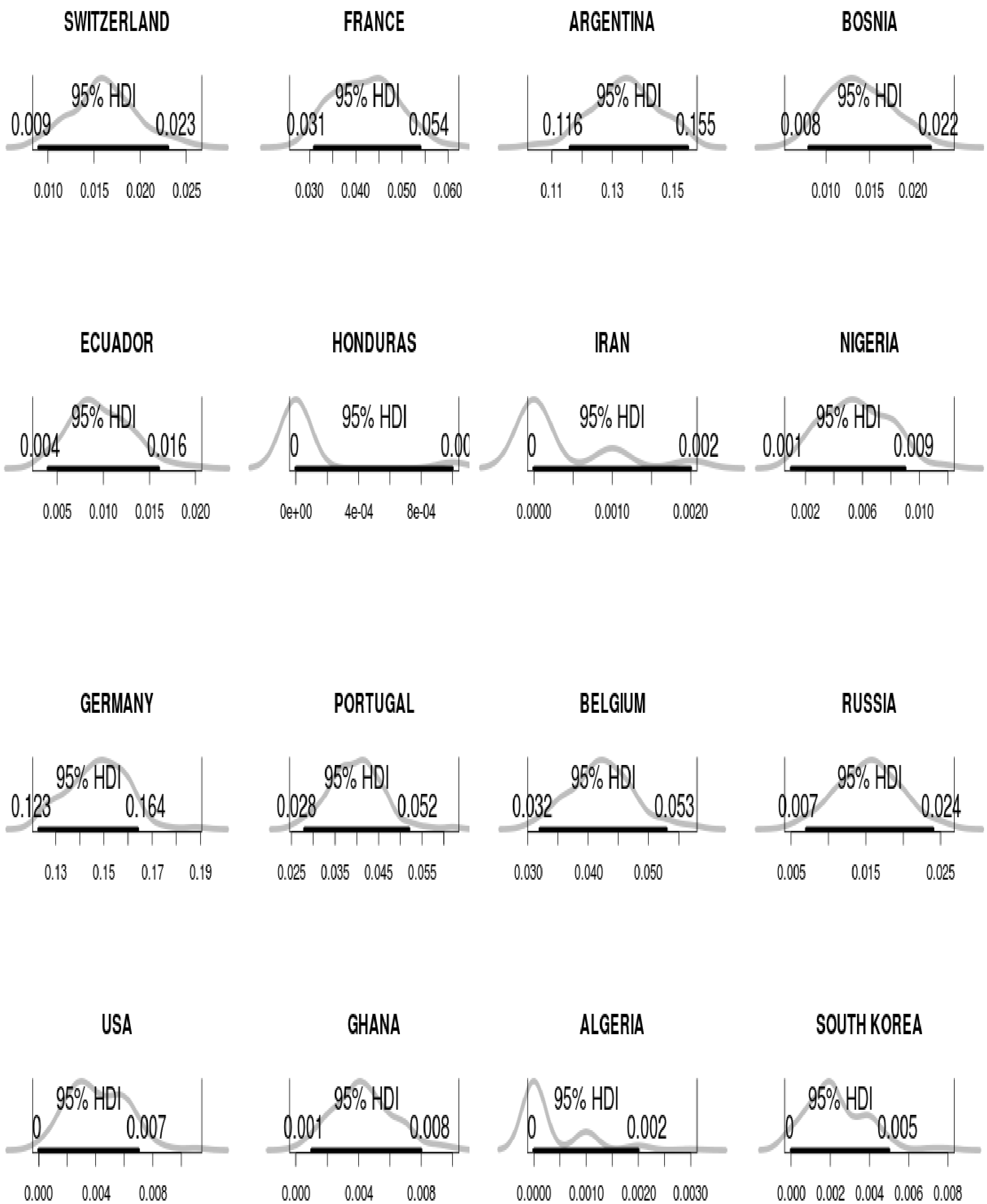


Figure 4: Density of being champion using Shin procedure: 2014 FIFA World Cup



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6 Appendix

Figure 5: Density of qualifying in the first stage using Shin procedure: 2014 FIFA World Cup

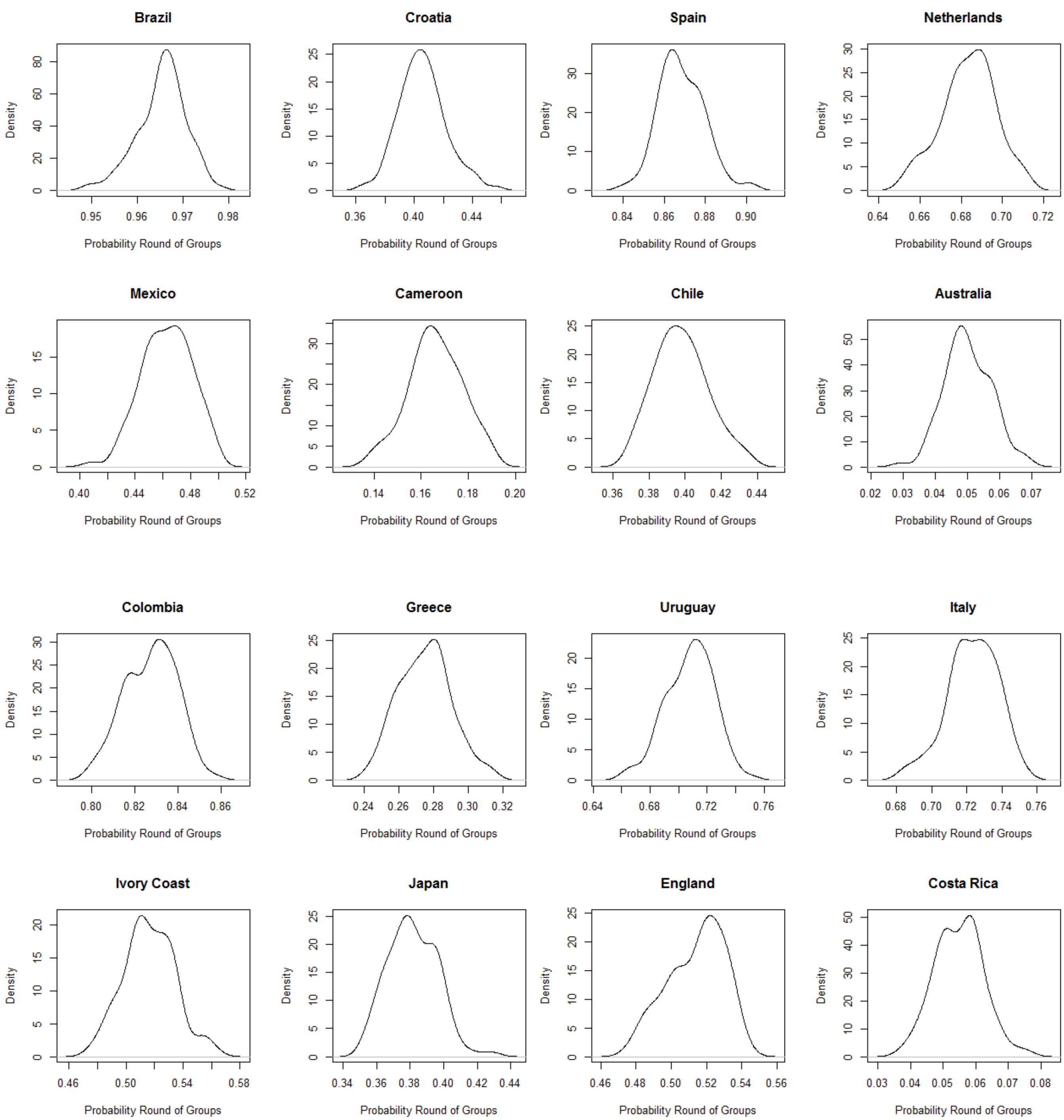

Figure 6: Density of qualifying in the first stage using Shin procedure: 2014 FIFA World Cup

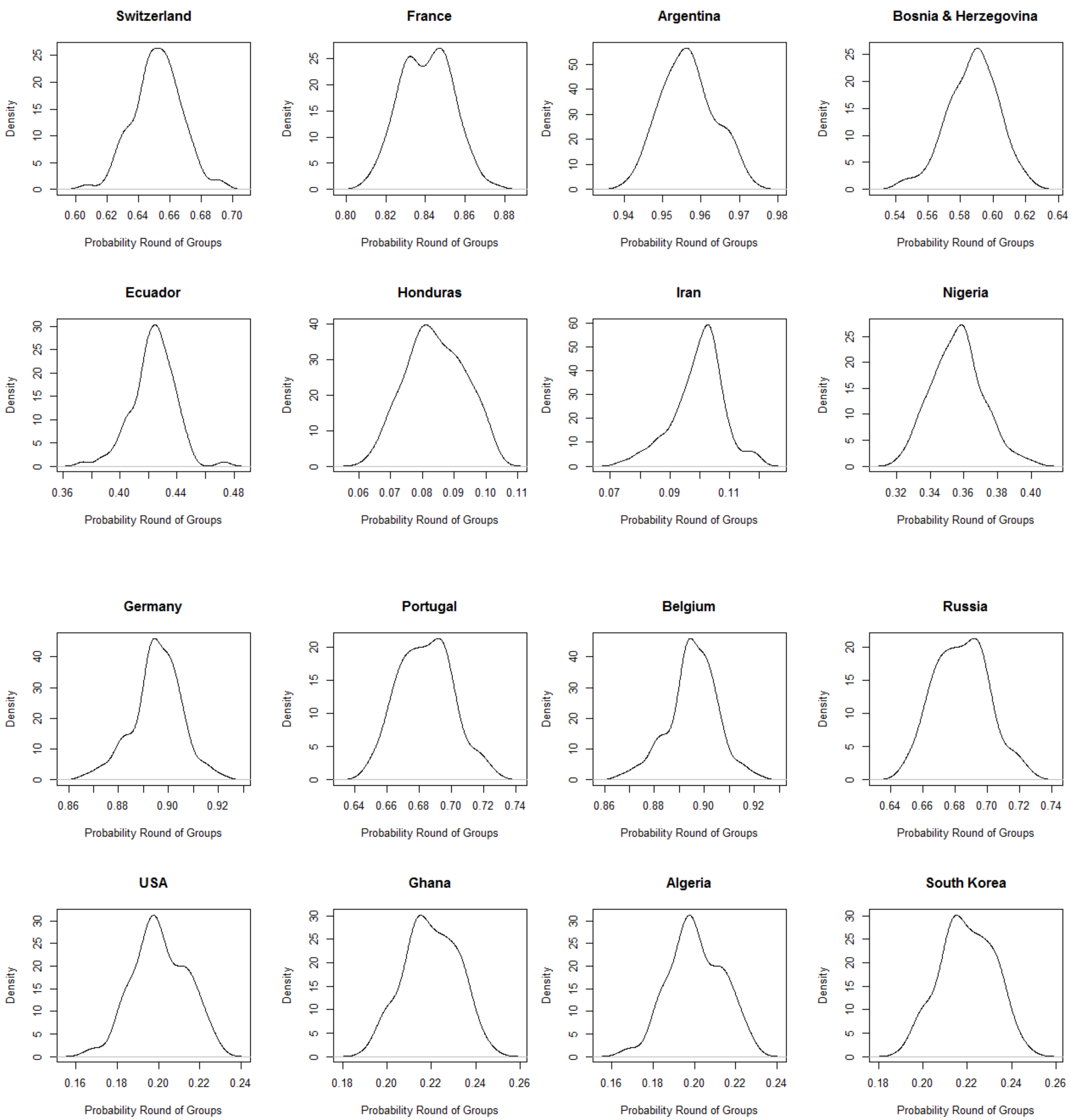

Figure 7: Density of qualifying in the second stage using Shin procedure: 2014 FIFA World Cup

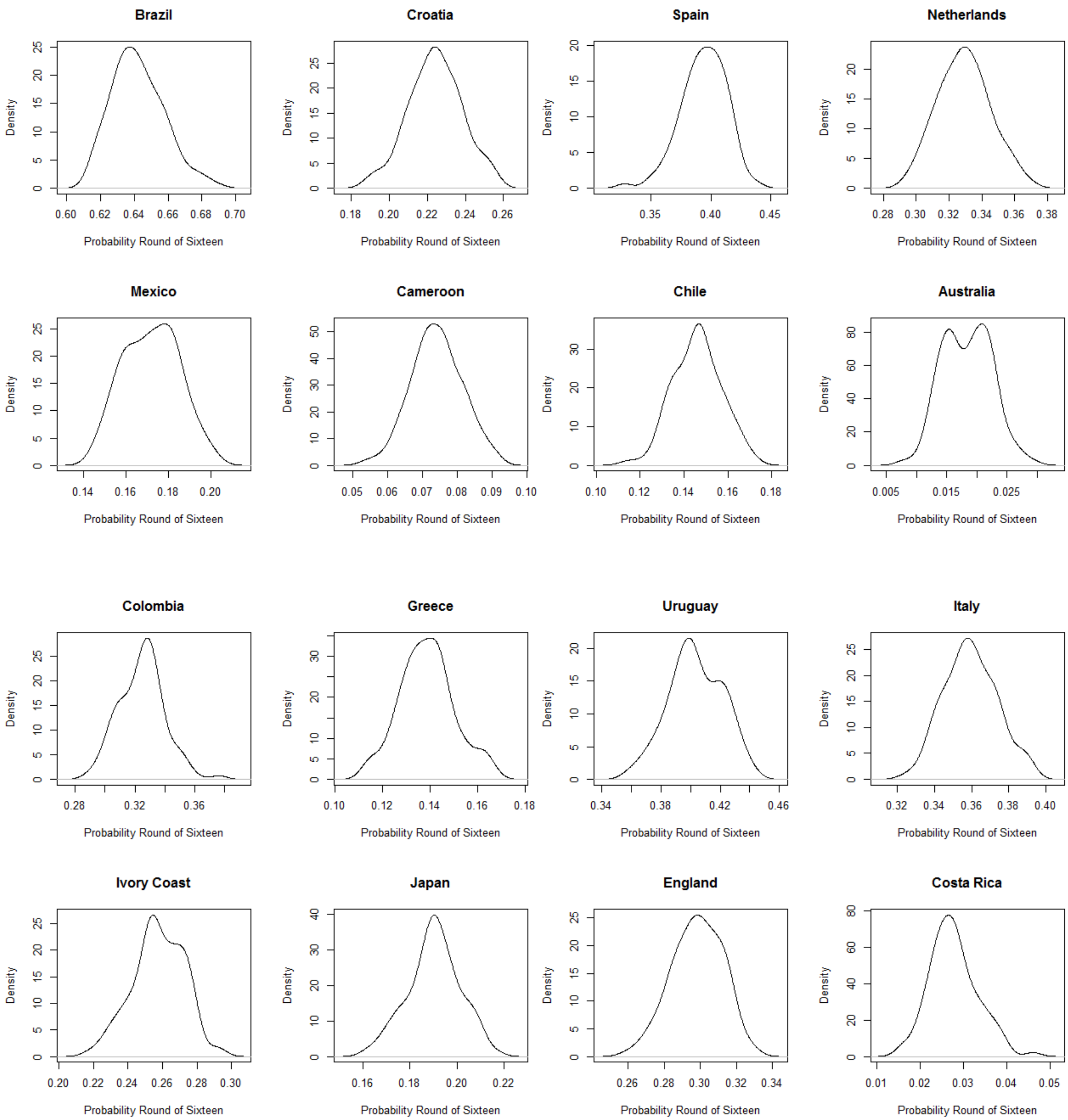

Figure 8: Density of qualifying in the second stage using Shin procedure: 2014 FIFA World Cup

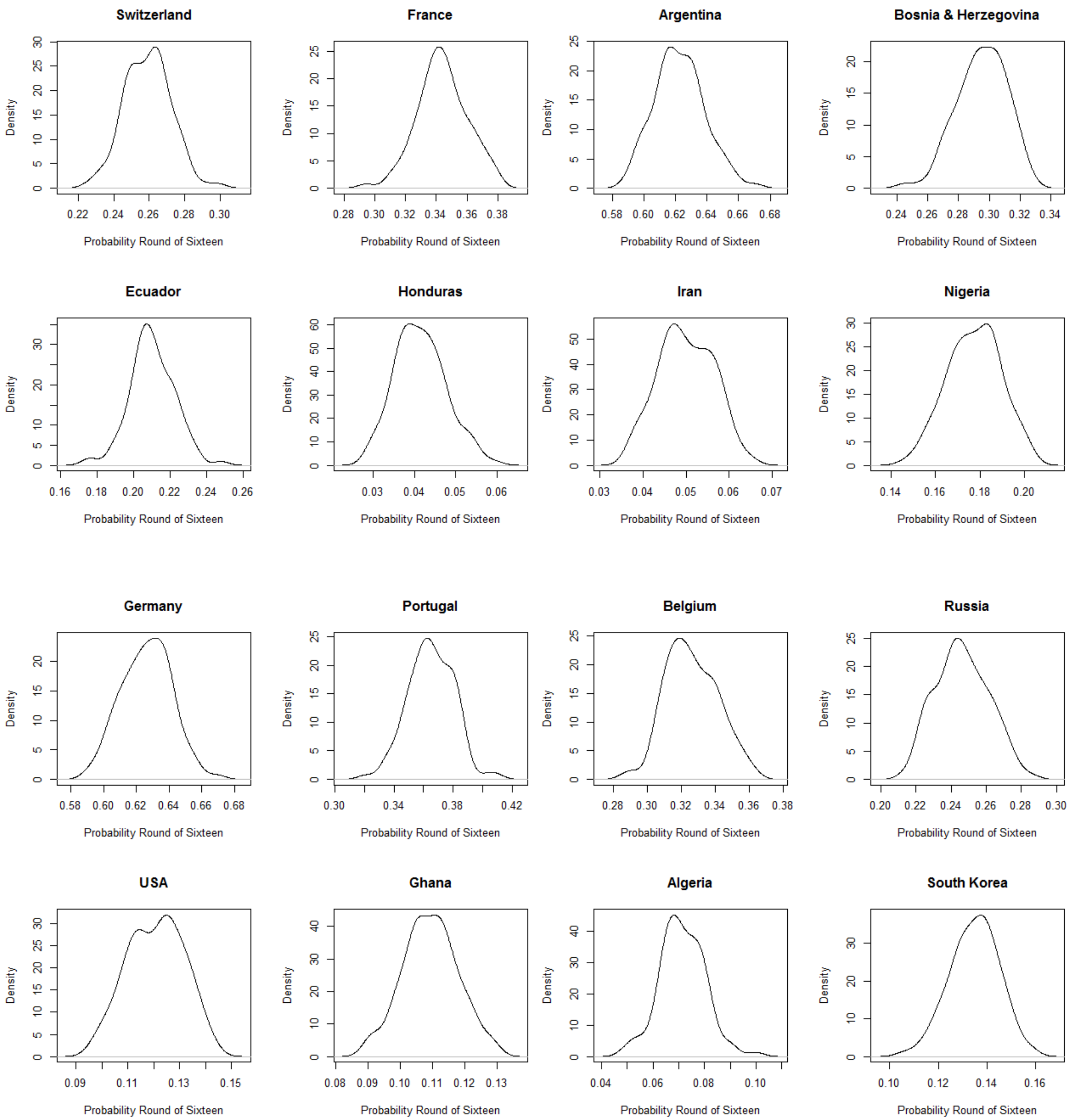

Figure 9: Density of qualifying in the quarter finals stage using Shin procedure: 2014 FIFA World Cup

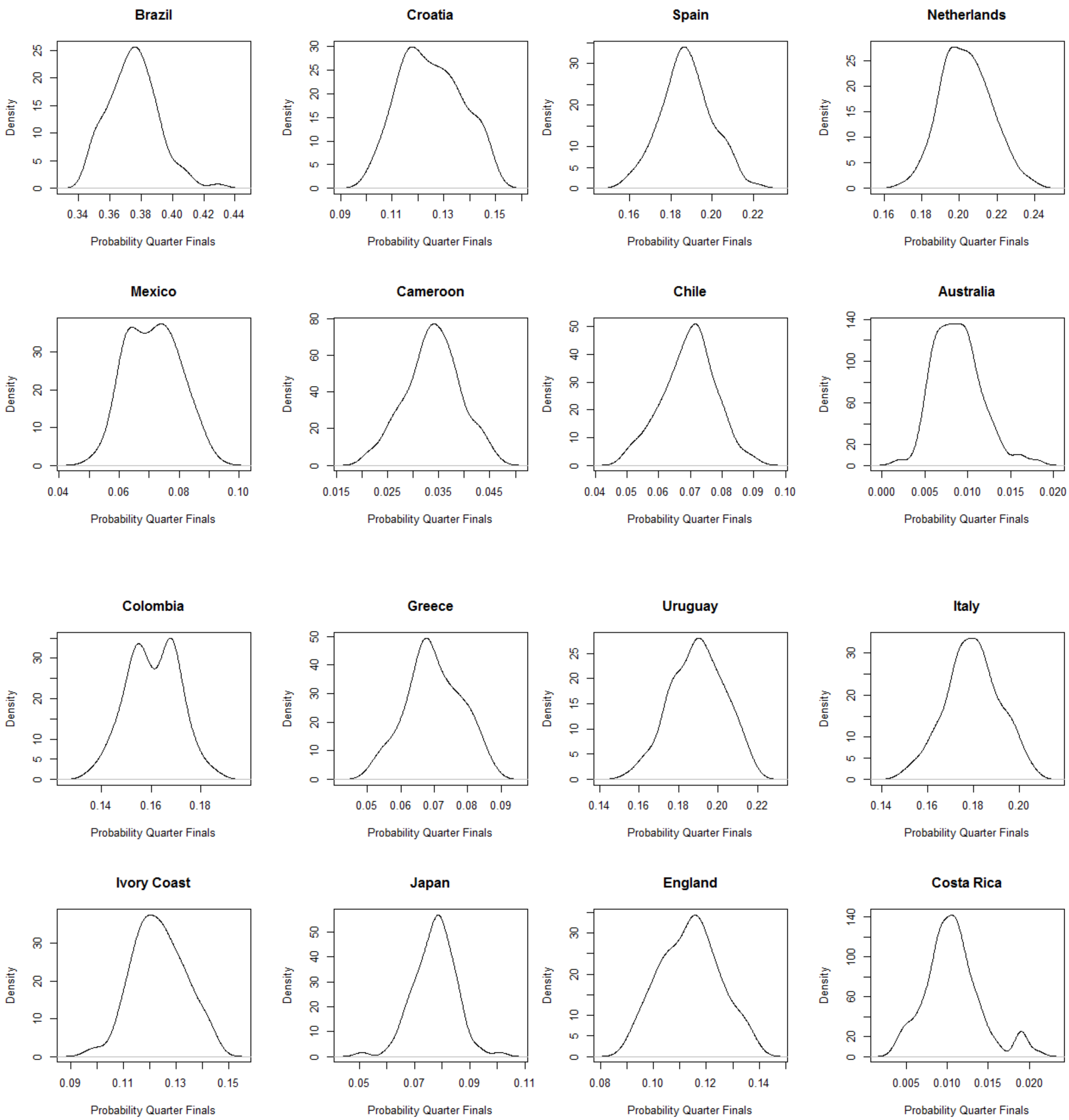

Figure 10: Density of qualifying in the quarter finals stage using Shin procedure: 2014 FIFA World Cup

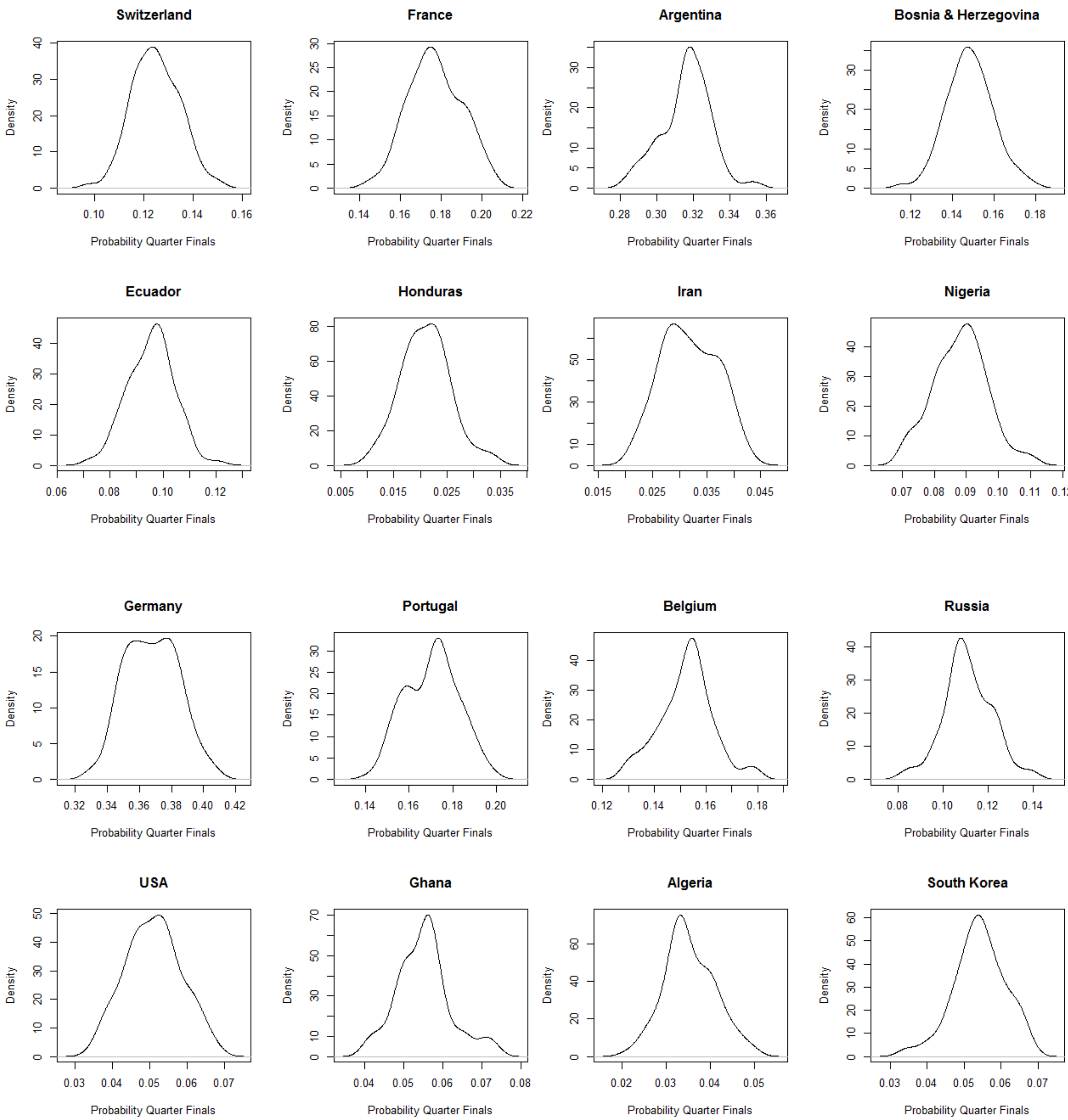

Figure 11: Density of qualifying in the semi finals stage using Shin procedure: 2014 FIFA World Cup

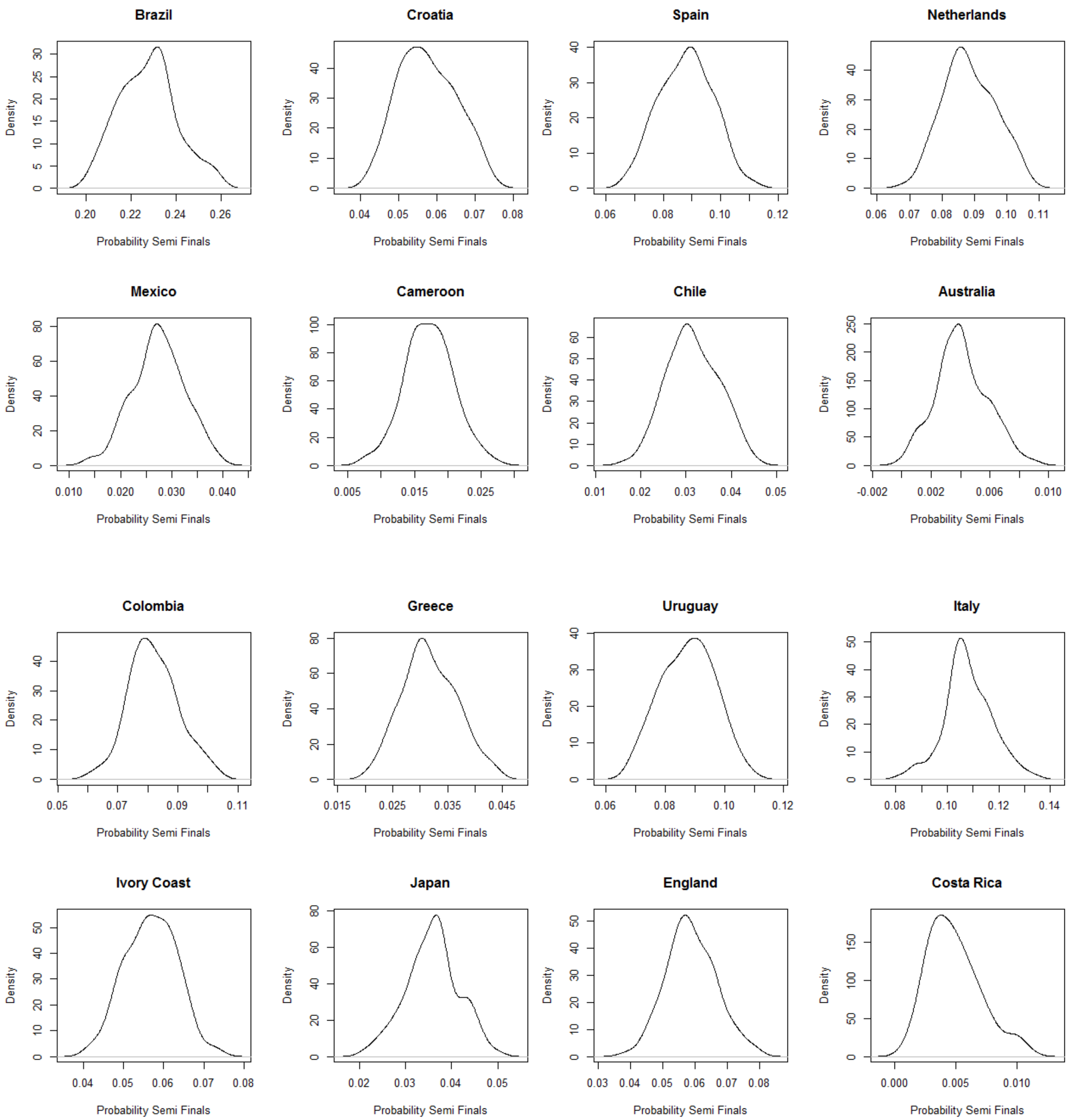

Figure 12: Density of qualifying in the semi finals stage using Shin procedure: 2014 FIFA World Cup

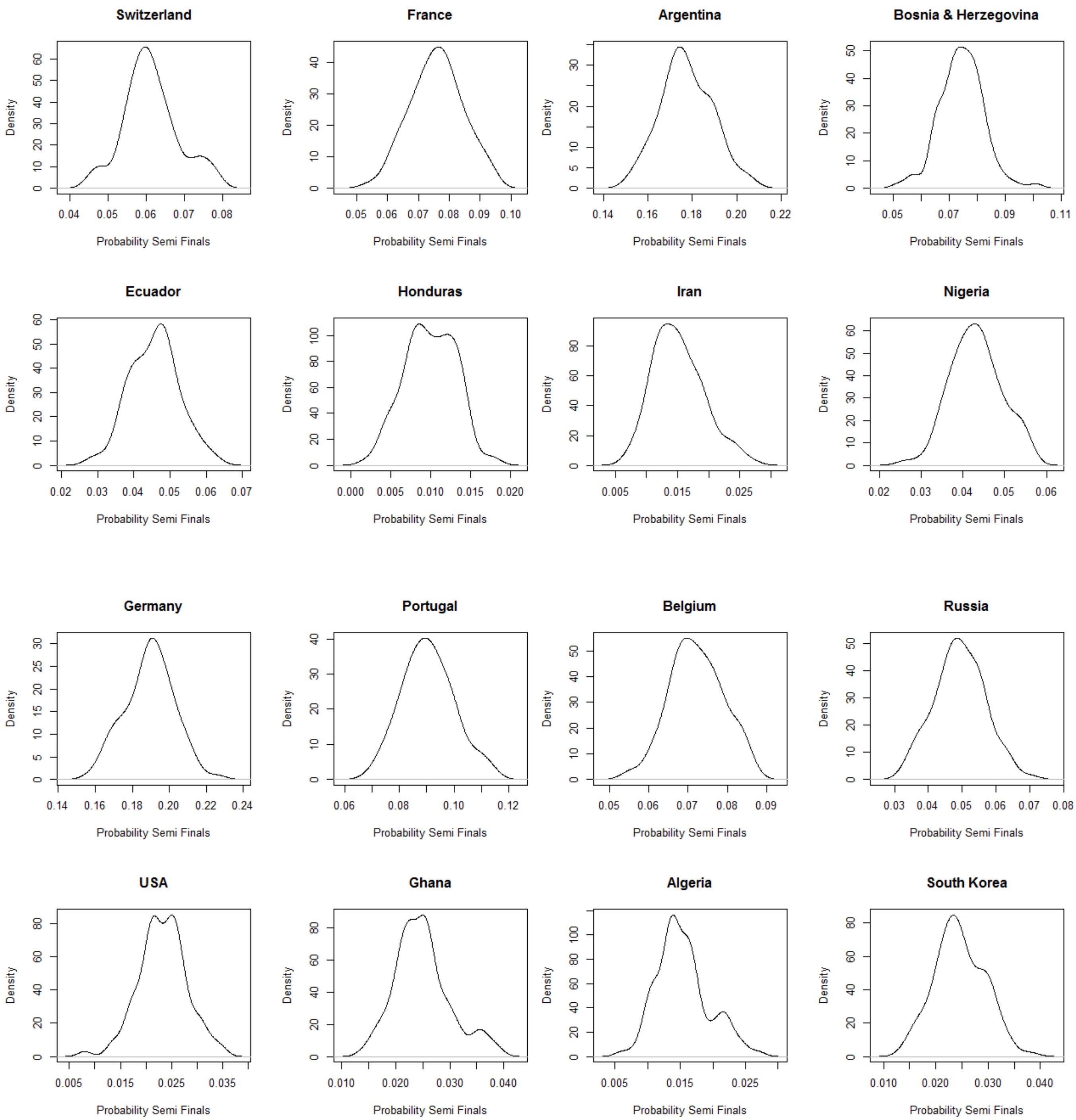

Table 6: Mean probability of qualifying each stage conditional on path using Shin's procedure: 2014 FIFA World Cup

		Group stage		Round of 16		Quarter-finals		Semi-finals	
Team	Group	First place	Second place	Path A	Path B	Path A	Path B	Path A	Path B
Brazil	A	0.8557	0.1117	0.5750	0.0837	0.3363	0.0498	0.1935	0.0288
Croatia	A	0.0602	0.3076	0.0324	0.1659	0.0178	0.0897	0.0097	0.0179
Mexico	A	0.0586	0.3255	0.0257	0.1199	0.0114	0.0489	0.0037	0.0203
Cameroon	A	0.0255	0.2552	0.0125	0.1103	0.0068	0.0538	0.0022	0.0288
Spain	B	0.5291	0.2873	0.0711	0.2728	0.0355	0.1347	0.0157	0.0622
Netherlands	B	0.2236	0.3412	0.1657	0.1155	0.1033	0.0687	0.0432	0.0325
Chile	B	0.2276	0.3122	0.0947	0.1225	0.0431	0.0591	0.0196	0.0288
Australia	B	0.0197	0.0593	0.0229	0.0094	0.0110	0.0055	0.0051	0.0019
Colombia	C	0.5201	0.2818	0.2102	0.1088	0.0998	0.0513	0.0508	0.0288
Greece	C	0.0886	0.1790	0.0446	0.0936	0.0233	0.0450	0.0097	0.0191
Ivory coast	C	0.2566	0.2927	0.1290	0.1452	0.0603	0.0742	0.0311	0.0327
Japan	C	0.1347	0.2465	0.0672	0.1192	0.0257	0.0576	0.0133	0.0270
Uruguay	D	0.4050	0.3061	0.1669	0.2220	0.0843	0.1041	0.0379	0.0498
England	D	0.2665	0.3344	0.1973	0.1468	0.0685	0.0670	0.0336	0.0406
Italy	D	0.3171	0.3164	0.1664	0.1580	0.0650	0.0877	0.0462	0.0522
Costa rica	D	0.0114	0.0431	0.0184	0.0064	0.0079	0.0029	0.0035	0.0019
Switzerland	E	0.2403	0.4045	0.1178	0.1589	0.0493	0.0777	0.0248	0.0380
France	E	0.5887	0.2561	0.2517	0.0825	0.1308	0.0459	0.0540	0.0217
Ecuador	E	0.1514	0.2753	0.0739	0.1375	0.0358	0.0650	0.0176	0.0281
Honduras	E	0.0196	0.0641	0.0096	0.0315	0.0051	0.0178	0.0024	0.0067
Argentina	F	0.6779	0.2198	0.1459	0.4357	0.0663	0.2232	0.0293	0.1273
Bosnia & Herzegovina	F	0.1849	0.4198	0.2139	0.0893	0.1059	0.0446	0.0503	0.0179
Iran	F	0.0256	0.0886	0.0452	0.0125	0.0267	0.0062	0.0141	0.0029
Nigeria	F	0.1116	0.2718	0.1420	0.0521	0.0721	0.0254	0.0368	0.0137
Germany	G	0.6723	0.2188	0.4571	0.1562	0.2623	0.0969	0.1368	0.0554
Portugal	G	0.2166	0.4641	0.1156	0.2471	0.0516	0.1182	0.0291	0.0570
Usa	G	0.0469	0.1518	0.0266	0.0930	0.0116	0.0380	0.0047	0.0189
Ghana	G	0.0642	0.1653	0.0292	0.0839	0.0139	0.0408	0.0043	0.0195
Belgium	H	0.6007	0.2495	0.0851	0.2501	0.0358	0.1296	0.0156	0.0594
Russia	H	0.2724	0.4381	0.1513	0.1013	0.0722	0.0434	0.0307	0.0250
Algeria	H	0.0307	0.0915	0.0501	0.0140	0.0253	0.0080	0.0127	0.0043
South korea	H	0.0962	0.2209	0.0850	0.0544	0.0353	0.0193	0.0180	0.0089

Source: Authors' estimations.