THREE ESSAYS ON RISK MANAGEMENT IN ELECTRIC POWER MARKETS

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This work is especially dedicated to my son Sebastian, for his constant and profound love, which has always been my support.

THREE ESSAYS ON RISK MANAGEMENT IN ELECTRIC POWER MARKETS

Abstract

This dissertation has arisen in the context of the electric power markets, the study of risk management and the relations between physical production and the electricity transactions using financial contracts in particular. Electricity is very difficult to compare with any other commodity, since it has a peculiar characteristic; electricity "must be produced at exactly the same time as it is consumed". The technological inability to store electricity efficiently and the characteristics of marginal production costs create jumps in the spot price. The electricity power market is heavily incomplete. Load-matching problems occur because electricity prices show volatility because of unexpected variations due to climatic conditions and other associated risk factors.

A branch of the literature in risk management has tried to give a definitive answer to the question of how agents in the markets with non-storable underlying asset could hedge their exposure to volatile price and quantity. The first essay tackles the basis of this question, which is the implication of the price of risk when forward risk premia are presented. This essay also shows how the properties and variations of forward risk premia is explained by risk factors variations on expected spot prices, and unexpected changes on the available quantity of water to generate electric power. Forward risk premia are the measure, hour by hour throughout the day, of the price of risk that the agents pay to trade electric power using forward contracts. In this essay forward premia were measured from the unregulated market segment. The results indicate that the average expected forward risk premia could have a positive behavior in seventeen out of twenty-four hours. These results represent the equilibrium compensation for bearing the price risk of the electric power for one year. In the Colombian market, the risk taker is the marketer, specifically in the unregulated market segment, because they are assuming the price risk in the long-term negotiations. The marketer, represented by this demand, tries to ensure their future Profit and Losses P&L and so they sacrifice their premia. It is relevant for further studies to evaluate the efficiency of this market, and the characteristics to determine why the marketer is willing to pay forward risk premia and why the generator has a better position to receive this bonus.

Exploring the optimization problem of portfolios my second essay asks whether the agents in the electric power market could hedge their exposure to uncertainties; price and quantity. We propose a close form solution for the optimization problem of portfolios composed by two claims, price and weather, according to factors influencing electric power markets such as price volatility, price spikes, and climatic conditions that influence volume volatility. Results show a positive correlation among price, quantity, and the weather variable.

In order to apply the optimal static hedging that includes the second claim on weather indexes for seasonal countries such as United States and tropical countries such as Colombia, the third essay shows an application of the static hedging model, using parameters from US market(PJM[1]), and Colombian market

[1] Pennsylvania, New Jersey and Maryland system; PJM

(WPMC[2]). For the PJM, I used weather indexes from Chicago Mercantile Exchange Group, and the hydrological index from WPMC which is based on the hydrological contributions of rivers on dam levels. We verify that El Niño and La Niña phenomena also influence quantity variations, and the agents in those markets are exposed to both price and quantity volatiles.

[2]Wholesale Power Market in Colombia; WPMC

Contents

Three essays on Risk Management in Electric Power Markets

"The revolutionary idea that defines the boundary between modern times and the past is the mastery of risk: the notion that the future is more than A whim of the gods and that, men and women are not passive before nature"
Peter Bernstein

The word "risk" is defined as the danger of not having the expected but adverse situations and the inauspicious consequences shown in loss and damages. Therefore, the concept of risk is strongly linked to uncertainty, and consequently, to the notion of randomness and probability (Kolmogorov (1933)).

In financial terms, risk can be defined as the quantifiable probability of obtaining loss or reducing the expected profits. The risk of market, which is probably the most well-known type of risk in economy and finance, can be explained as the risk of changing value of a financial position due to variation in the value of underlying assets that depends on the bid, stock prices, bonds and commodity prices, among others. The credit risk is understood as the risk of not obtaining the pending reimbursements of investments due to the borrower's default, whereas the impossibility of trading a financial asset quickly enough to avoid or minimize loss is known as liquidity risk. Thus, it is evident that the study of risk in financial markets is the cornerstone of the portfolio theory and the starting point for future decision making in terms of financing and investment that could provide an adequate capital structure Sharpe (2000).

The financial contracts for the trading of electricity represent one of the most complex processes in financial securities traded, in both the appraisal and the mechanics of the transaction itself. The transaction of a commodity that relates physical production to financial transactions through contracts presents peculiar characteristics. It technically presents first the singularity of requiring balance between the offer and the demand in real time and the possession of geographical limitations, and secondly, the impossibility of generating inventory that mitigates the price fluctuation, taking into account that those prices present the most numerous peaks, jumps and fat tails. [3]

Few researches are really focused on studying the uncertainty impact or risk market in the energy power markets, and most of them have arisen in economics. The models of instrument valuation and stochastic

[3] Fat tail is a property of some probability distributions (alternatively referred to as heavy-tailed distributions) Figuring extremely large kurtosis particularly relative to the ubiquitous normal which itself is an example of an exceptionally thin tail distribution. Fat tail distributions have power law decay. More precisely, the distribution of a random variable X is said to have a fat tail if $Pr[X > x] \sim x^{-(1+\alpha)} \;\; as \;\; x \to \infty, \; \alpha > 0$ Some reserve the term "fat tail" for distributions only where $0 < \alpha < 2$ (i.e. only in cases with infinite variance), source:Asmussen (2003).

behavior for asset prices different from energy power propose assumptions that are not applicable in this case, because electricity is difficult to compare with other commodities. For instance, its impossibility of storage, among others, has several implications such as the need for electricity to be produced at the time of consumption. This means that the non-arbitrage principle is not applicable in this case. Moreover, there are equilibrium problems due to the variation of prices generated by volatility and unexpected variations caused by climatic conditions and other connected risk facts. Subsequently, the energy power markets become incomplete.

Fama and French (1987) argue that the presence of risk premium in forward prices is difficult to detect because of the limited number of maturities in the available contracts to study and the observed premia variances are very high. Eydeland and Geman (1999) propose a valuation model that is based on assumptions of development of forward prices of electricity based on an equilibrium focus. Pirrong and Jermakyan (1999) observe that the non-arbitrage focus is not a consideration that could be taken into account in the case of derivatives of energy power, also proposing the existence of a forward risk premium due to an endogenous market price in the energy power demand. Routledge, Seppi, and Spatt (2000)also consider the fixation of equilibrium prices for energy power contracts. They focus on the existent relations between the natural gas and energy power market, even proposing that gas could be stored or transformed into electricity. From their model, Routledge, Seppi, and Spatt (2000), obtain predictions of the mean-reversion of spot prices, as well as correlations between electricity and fuel prices. Their model also proposes that electricity prices present a positive bias.

Bessembinder and Lemmon (2002)) similarly discuss this issue for energy power. They present an equilibrium model that explains how electricity forward prices are biased estimates of the futures spot price, provided that the expected demand is low and the risk perception is moderate. Correspondingly, they conclude that electricity forward contracts cannot be valued applying the typical cost-of-carry focus because electricity is not storable. The authors also show evidence that the market behaves as a Contango [4] if the expected demand or demand volatility is high due to the presence of a positive skeweness, which is induced in the electricity price distribution. Geman and Vasicek (2001) empirically confirm findings from Bessembinder and Lemmon and demonstrate, using a database from The United States, that forward contracts in the short term are a biased estimate of futures spot prices accordingly to the high volatility and risk associated with American energy power markets.

Additionally, the verification of the normality assumption, as an enigma, is proposed. This concept assumes that prices of financial assets and commodities are subject to random walks, verifying the hypothesis of Random Walk 1, 2 and 3. Nevertheless, multiple empirical works have found that such assumption is not met in reality. On the contrary, the price series in both financial assets and especially commodities present characteristics that set them apart from being considered a normality case due to diverse factors. Firstly, returns from different periods are not independent because they present autocorrelation and do not follow a random walk. Secondly, the presence of extreme values suggests strong evidence of fat tails. Thirdly, the evidence of negative skewness is weak. Campbell, Lo and Mackinlay (1998),show that financial assets are limited responsibility; the highest loss an investor can suffer is the total investment. Losses of the investment will show as negative returns, which are a violation of the normality assumption.

Diverse explanations have been given for the fact that these series escape the normal assumption. The de-

[4] Market condition in which forward or futures prices are trading above the current spot prices.

pendence of residuals makes the white noise difficult to obtain and the heteroscedasticity provided in such. Engle (1982) proposes GARCH models (General Autoregresive Conditional Heteroscedasticity) Bollerslev and Tim (1986) shows a generalization of model known as GARCH (p,q). This has had some variations, creating further proposals such as the IGARCH model (Integrated ARCH) of Bollerslev Tim and Engle Robert (1993), EGARCH model (Exponential GARCH) of Nelson (1991), the A-PGARCH model (Asymmetric Power GARCH) of Ding et al (1993), TARCH model (Threshold GARCH) Zakoian (1994), and GARCH - Jump Model Duan (1997). Deng (2000) presents mean-reversion models: jump-diffusion, which is deterministic, stochastic, and switching-regime models, and achieves an analytical solution to energy power futures and options pricing. Other authors also suggest models based on the jump-diffusion model of Merton (1976) that takes into account the discontinuity of stock price trajectory and incorporates a Poisson component to represent big jumps in energy power prices.

Lucia and Schwartz(2002) model the energy power prices following an Ornstein-Uhlenbeck process (Vasicek (1977)) introducing a deterministic component and a stochastic factor. They also extend their model by capturing seasonal patterns in the forward and futures curves that are directly implicated by the seasonal behavior of electricity spot prices, which describe price behaviors in terms of those two component types. Additionally, Lucia and Schwartz(2002) argue the importance of regular patterns in the behavior of electricity prices and the implications for derivative pricing purposes. They were focused on the Nordic energy power market (NordPool), both in the spot and forward and futures market, and it was found that spot prices could be highly dependent on the temporary and local conditions of the offer and demand such as the climatic conditions, population habits, among others, due to the impossibility of storage and the transportation limitation. On the other hand, the non-storability condition of the energy power significantly affects the derivative prices, which influences the forward curve's shape. Thus, Lucia and Schwartz(2002) apply models of one and two factors.

Pilipovic (1997) agrees with Lucia and Schwartz(2002) on the periodical seasonal behavior of electricity prices and the mean-reversion, which is possibly non-seasonal. He presents a complete study of financial series connected to energy, in which the spot series behavior, volatility, derivative pricing and risk policies are analyzed. Also, he presents several energy series in which the Mid Columbia (MC) electricity spot price stands out, which applies mean-reversion models in fuction of prices and price natural logarithms and finds that the best adjustment is the mean-reversion model in prices. Geman and Roncoroni (2006) introduce a calibrated market point process to calibrate the recording trajectory, statistical characteristics of energy power markets. The processes used were jump-reversion, and they allow recording of both the trajectory and the stochastic variability. Geman and Roncoroni (2006) utilize data from the American energy power markets to recreate the adjustment achieved through their model.

Different authors explore the problem of risk hedging, modeling both prices and loads, taking as a start point the stochastic modeling of energy power prices. Brown and Toft (2002) show, as an attempt to answer questions like why and how companies decide to hedge, hedging strategies using vanilla and exotic derivatives to maximize the company value against risks that could be hedged such as price risk and those unhedged as volumetric risk. These authors suggest that in addition to using hedging for exotic derivatives design, it is more proper to utilize derivatives such as vanilla when the correlation between prices and quantities are high and the volumetric risk is substantially higher than the price risk.

The portfolio construction problem follows the model Markowitz (1952), where the investor's goal defines the portfolio construction in order to maximize expected future returns given a certain level of risk. The Markowitz model establishes that the volatility of portfolio returns measure the risk. Campbell et al (2001) introduced a similar portfolio selection problem using Value at Risk (VaR) of the risk of potential portfolio losses due to unexpected changes; prices of the traded assets.

In the electric power literature, several authors follow Markowitz's methodology to address hedging strategy using vanilla derivatives. Particularly, Nasakkala and Keppo (2005), who study the interaction between stochastic consumption volumes and electricity prices, and proposed a mean-variance type model to determine optimal hedging strategies. Vehvilainen and Keppo (2006) optimize hedging strategies taken into account the Value at Risk as risk measure. Huisman (2007) introduced a one period framework to determine optimal positions in peak and off-peak contracts in order to purchase future consumption volume. In this framework, hedging strategy is assumed to minimize expected costs respecting to an ex-ante risk limit defined in terms of Value at Risk.

In their model, Nasakkala and Keppo (2005), take as a starting point the optimal hedging rate, to subsequently used this result to solve the optimal hedging timing problem. Results show that when there is a negative correlation between forward prices and load patterns, the agents with high load volatility get hedged after those who observed low volatility. That is, the uncertainty about the presence of high load could put off the decision of hedging up to the point in which a better load estimate is obtained. Furthermore, the authors suggest that a load estimate positively correlated with the forward prices generate a hedging rate higher than one, and the negative correlation will generates hedging rate much lower than the unit. In addition to this, the authors explain that the positive correlation has an effect of bringing hedging time forward and agents with negative correlation seek to postpone the hedging decisions, taking the negative correlation as an additional tool for hedging.

Oum et al (2006) present a static hedging strategy for LSE and for marketers whose objective is to minimize the mean-variance of their profit function on the gross profit, given the self-financing restrictions. Provided that risk measurement is not a tradeable asset, the hedging is established through a portfolio that is based on the financial instruments of prices of electricity, including bonds, forward contracts and a wide variety of Call and Put options with different strike prices. The optimal hedging is jointly optimized with the contract maturities under the price and quantity dynamic and the assumption that the hedging portfolio that matures right in the moment of the physical delivery of electricity is purchased in only one moment of time. The authors developed a methodology to mitigate volumetric risk, in which they work on the positive correlation between the spot price of the wholesale market and the demand volume through a hedging strategy to obtain a short position on an unknown electricity volume using energy power derivatives. The optimal hedging strategy was developed based on the expected profit maximization, obtained from the derivation of the optimal revenue function that represents the optimal revenue of a cost-zero, as a price function, exotic option. Therefore, the optimal revenue of the exotic option could be replicated using a portfolio composed by forward contracts and European options.

Oum and Oren (2008), as a modification to Oum et al (2006), introduce the concept of maximization of expected hedged profits, which are subject to a VaR restriction under certain assumptions of distribution that allow finding of the optimal portfolio with the VaR restriction on the efficient mean-variance frontier. The results presented in this work were achieved due to the assumption that profits are normally distributed and depend on the result of two correlated variables, quantity and price. The hedging strategy is characterized by the non-lineal function of a random variable, which is purposely done in order to find a numerical solution to the problem.

In terms of the weather derivatives literature, authors are focused on weather derivatives pricing for instance, Cao and Wei (1999) suggest an equilibrium-pricing model based on the Euler equation Stillwell (2002) and the fact that in equilibrium both the financial market and the goods market are clear so that aggregate consumption equals the dividends generated from the risky stock. Thus, they calculated a stochastic discount factor SDF and used it to price weather derivatives. Brody, Syroka and Zervos (2002), proposed the resolution of a partial differential equation in order to price weather derivatives. Alaton et al (1960), use historical data to suggest a stochastic process that describes changes in temperature. Brix, Jewson and Ziehmann (2002), Marteau et al (2004), find the expected outcome under a probability distribution adjusted by the risk measure; they model the temperature using historical data and a Montecarlo simulation.

Platen and West (2004) provide a fairly pricing model, based on the idea that the growth-optimal portfolio is used as a numeraire, and all derivative price processes discounted by the growth-optimal portfolio (benchmarked) are martingales. Richards et al (2004) in a continuous time framework suggested an equilibrium-pricing model based on temperature processes of a mean-reverting Brownian motion with discrete jumps and autoregressive conditional heteroscedastic errors. A standard Euler equation from the Lucas general equilibrium valuation model was applied to pricing CDD weather options. Also see Ankirchner et al (2006) for the indifference pricing approach in continuous time.

Chaumont et al (2005) discuss the market price of risk, which is determined by a backward stochastic differential equation that can be translated into semi-linear partial differential equations. These approaches can be viewed as an actuarial perspective since the price of the weather derivatives are based on the exposure of the weather derivative underwriter to weather risk. Hamisultane (2007) infers the risk-neutral distribution by minimizing the second derivatives of the simulated risk-neutral distribution.

Other authors have investigated the effect on enterprise revenues due to weather. Dutton (2002) estimates that one third of private industry activities, representing some three trillion dollars annually, bears some degree of weather and climate risk. Energy, agriculture, leisure and insurance are good examples of weather-sensitive industries.

This dissertation has been organized as follows: Chapter I is the essay “Modelling risk for electric power markets”, which represents the starting point on risk management for the energy power markets, and studies the risk premium that this market offers to the agents for bearing the risk. In Chapter II, the second essay is “Optimal Static Hedging of Energy Price and Volume Risk: closed-form results”. This chapter proposes the VaR-constrained Static Hedging Model, as a hedging strategy, using a replicating portfolio formed by two claims; price and weather. Chapter III is the third essay “Applications of Optimal Static Hedging of Energy Price and Volume Risk to markets in the US and Colombia”, and Chapter IV presents the final discussion of the thesis.

Chapter I

Modelling Risk for Electric Power Market

Abstract

The inability to store electric power efficiently is an important consideration when analyzing the electric power market, and prevents the use of the cost-of-carry approach, which explains the use of equilibrium models to understand the electric power market behavior. Electric power is strongly difficult to compare with any other commodity, since electric power has a peculiar characteristic; "it must be produced at exactly the same time as it is consumed". The technological inability to store electric power efficiently and the characteristics of marginal production costs create jumps in the spot price. Several authors agree with the fact that the no-arbitrage approach does not apply to electric power and notice that there are differences between electric power forward prices and expected spot prices, which implies the presence of the Forward premia, suggesting that the Forward premia represent compensation for bearing the risk. This chapter first presents a study of the Forward Risk Premia FRP in Wholesale Electric Power Market in Colombia (WPMC) showing how the FRP vary throughout the day and how its properties are explained by risk factors. Secondly, it shows that expected forward risk premia depend on factors such as variations in expected spot prices, due to the climatic conditions generated by the Oceanic Niño Index (ONI) and its impact on the available quantity of water to generate electric power. The case of study is the Colombian market as an example of the emerging markets.

Keywords: Forward Risk Premia Electric Power Markets, Conditional Volatility, Normal Backwardation, Oceanic Niño Index (ONI), GARCH Model.

JEL Classification: G0, G13, C32.

Acknowledgements: Rich discussions with the participants to the XM seminar on November 13, 2009, and also the participants to the 17th Global Finance Conference on June 27-30, 2010 in Poznan (Poland), are gratefully acknowledged.

1 INTRODUCTION

The transaction of a commodity, which relates physical production with financial transactions through contracts, presents peculiar characteristics. It technically first presents the singularity of requiring balance between the offer and the demand in real time and the possession of geographical limitations; and second, the impossibility of generating inventory that mitigates the price fluctuation, taking into account that those prices present the most numerous peaks, jumps and fat tails. In addition, the question that has got to be asked is if electric forward prices reflect economic fundamentals or if the traders can manipulate those prices; moreover, the answers become extremely relevant in the market on study due to regulatory issues and the expectations of rational agents.

In the case of a commodity as particular as electric power, the volatility presents firstly, excess of kurtosis, which in most cases reflects the length of tail distribution - that is, the longer the tail is, the higher the probability of obtaining prices is extremely high or low. Secondly, discontinuous price jumps take place; thirdly, there is evidence of volatility patterns, which are periods of high or low volatility, followed by behaviors that are more moderate. Finally, there is evidence of convergence that is contrary to what happens to the underlying prices that move freely in any direction, implying that an underlying asset tends to present average long-term volatility. Figure 1.1 shows the characteristics of non-constant volatility and prices with jumps on the average daily spot price.

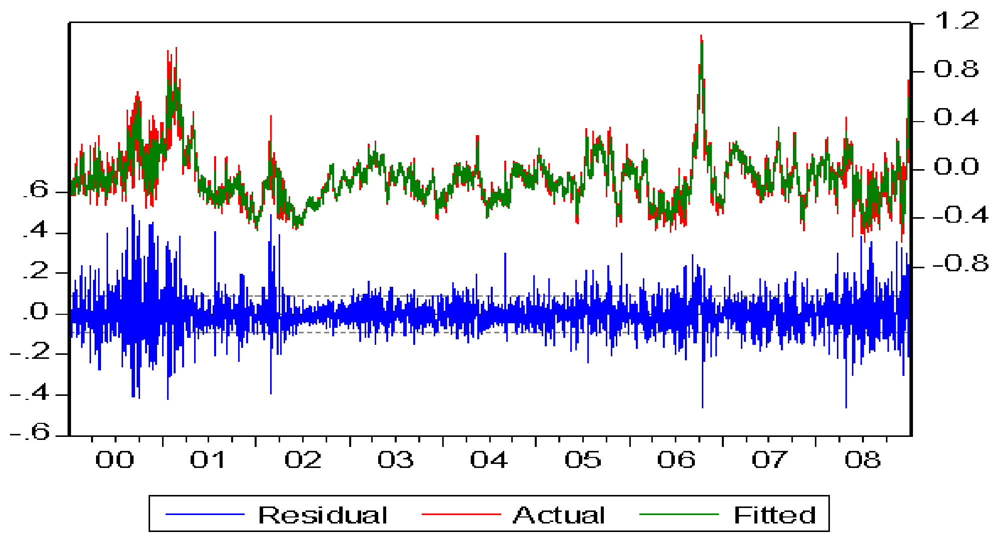

Figure 1.1: Volatility in Electric Power Prices, (XM data).

Diverse explanations have been given to the fact that these series, based upon its behavior, escape the normal assumption. The dependence of residuals that makes difficult to obtain white noise and the Heteroskedasticity provided in such. Figure 1.2 shows movements of traded prices: Sharp peaks can be observed during some periods due to macroclimatic effects as drought periods (El NIÑO phenomenon). The histogram shows the pattern prices and sharp peaks, these patterns cannot be associated with any specific distribution. Often, the volatility of the electric power market takes characteristics such as: i) excess kurtosis, which means high probability of extremely high or extremely low prices, known as "Fat tails"; ii) periods with high or low volatility; iii) when volatility follows a mean-reversion process: the underlying asset tends to have an average volatility or long-term average, known as convergence.

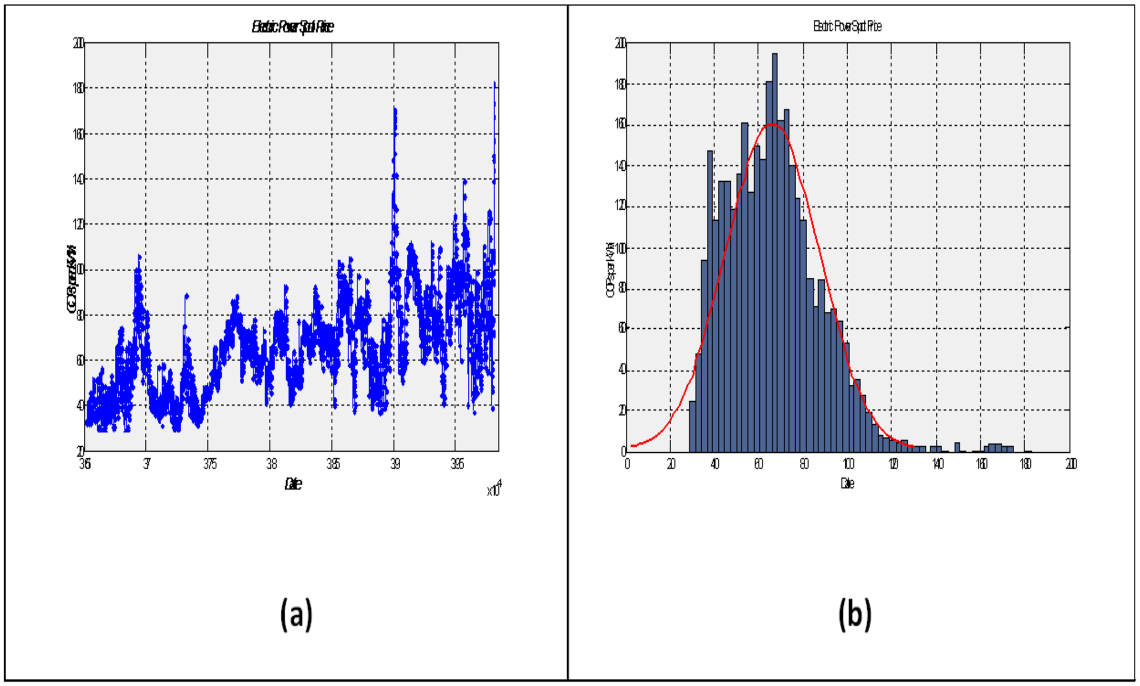

Figure 1.2: Daily average of prices from the Colombian Electric Power market. Fig 2(a) shows daily average of Spot price; fig 2(b), exhibits histogram of spot price distribution.

Empirical studies about electric power markets from several authors show that there are two basic approaches to define Risk Premia; first, the standard no-arbitrage or cost-of-carry (Brennan (1958)), and second, the equilibrium model (Hicks (1939), Cootner (1960), Bessembinder and Lemmon (2002), and Longstaff and Wang (2004)). Firstly, if storability of electric power was not a concrete issue, no inventories could be held, allowing the capacity of response to cover the short position in forward contracts traded in this market; therefore, the first basic approach cannot be applied. Thus, the second basic approach is suitable taking into account conditions of the market on study. Secondly, Geman and Eydeland (1999) agree with the fact that the no-arbitrage approach does not apply to electric power and noticed that there were differences between forward prices and expected spot prices, which implied presence of the Forward Premia, suggesting that the Forward Premia represent compensation for bearing risk. A similar approach to that of Bessembinder and Lemmon (2002) and Longstaff and Wang (2004) is used in this paper. Foremost, this research has found that expected Forward Risk Premia depend on factors such as variations in expected spot prices, due to the climatic conditions generated by the Oceanic Niño Index (ONI), and implications by variations in demanded electricity. The case of study is the Colombian market as an example of the emerging markets.

The data for this study consists of the hourly spot and forward non-delivery contracts from an unregulated market segment in the Wholesale Electric Power Market in Colombia (WPMC) since January 1, 2000 to December 31, 2008. The sample has 24 series from a total of 3287 days. The series included the daily spot price and settlement forward prices determined for one year, in which the delivery is to be made one year forward.

Those series of data offer an almost ideal way to study the properties of electric power prices. In order to find whether the mean-reverting and serial correlation conditions are present in the series on study; the unit-root tests[1] and portmanteau test "RW1" were applied. These tests essentially determine if the predictability condition is presented in the explicative variables. Results showed that series are stationary and autocorrelation should be zero. (See pag. 20 Appendix 1)

Particularly, economic effects, that are not visible with daily and monthly-level data studied at an hourly level, could be identified. In addition, the data included the first data set of the ONI series from January 2000 to December 2008 in a monthly pattern from the National Oceanic & Atmospheric Administration US. Thus, there is a significant correlation between the climatic variable and the volatility of electric power prices. Figure 1.3 shows the relationship between the logarithmic average of daily electric power prices and the El niño phenomenon.

The ONI is based on Sea Surface Temperatures (SST) departures from average in the Niño 3.4 region, which is the principal measure for monitoring, assessing, and predicting the NIÑO phenomenon. Three-month running-mean SST departures in the Niño 3.4 region are defined and departures are based on a set of improved homogeneous historical SST analyses (Extended Reconstructed SST. v2)[2]. pag. 21 Appendix 2 shows i.) the el Niño and la Niña phenomena presented in the average surface temperature shown in the ONI, ii.) the climatic effects on spot price due to el Niño behaviour, and iii.) the climatic effects on spot prices due to la Niña behavior.

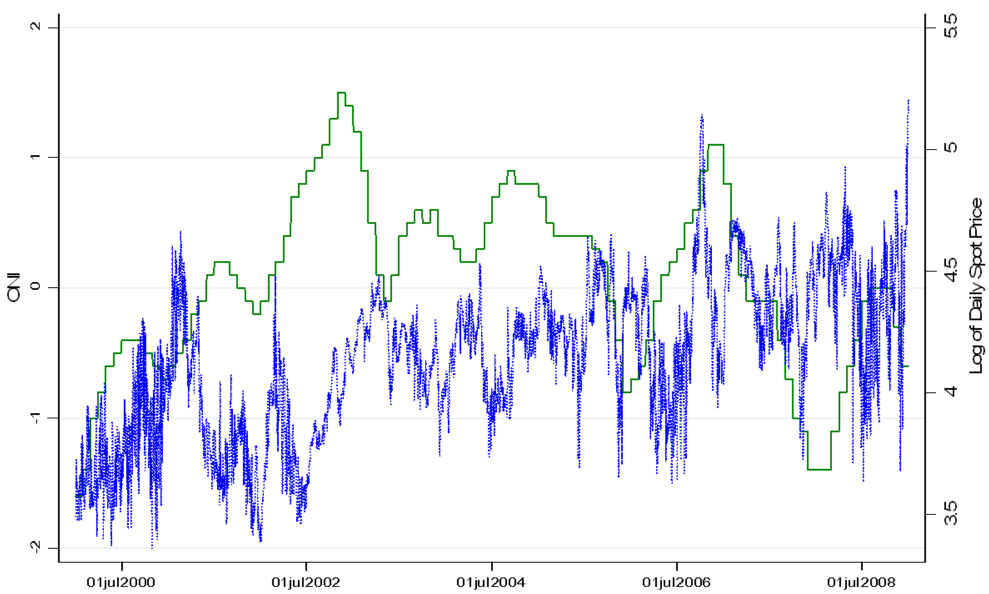

Figure 1.3: Logarithmic Average of Daily Electric Power Prices (blue) in COP[3] per Kilowatts from WPMC[4] in Colombia since January 1, 2000 to December 31, 2008 and ONI (green) from NOAA. U.S, since January 01, 2000 to December 31, 2008.

[1] Augmented Dickey-Fuller, Phillips-Perron and Kwiatkowski-Phillips-Schmidt-Shin tests statistic
[2] The methodology is described in Smith and Reynolds, 2003, J. Climate, 16, 1495-1510
[3] COP: International Code of Currency from Colombian Peso
[4] In Spanish: M.E.M ("Mercado de Energía Mayorista")

Several el niño studies (e.g., Barnston and Glantz (1999); Barnston and Chelliah (1997); Barnston and Ropelewski (1992)) have shown significant effects on a number of economically critical climate variables in North and South America, climatic effects producing drought periods, and rainfall along the Pacific and Atlantic Coasts. The correlation between the volatility prices and the ONI is explained by the fact that tropical countries such as Colombia have coastal areas at least in one of both oceans and the electric power production system is hydraulic dependent. On the other hand, the hourly pattern shows differences throughout the day between the expected spot price and the forward prices. Moreover, expected spot prices are higher than forward prices, which is consistent with classical literature (e.g., Hicks (1939), Cootner (1960)) and recent studies by Bessembinder and Lemmon (2002), and Longstaff and Wang (2004).

In this chapter, particular features about forward premia in the WPMC were found, which are consistent with the aforementioned classical literature. For instance, in this work, expected forward premia of unregulated markets show twenty one significant values. According to statistics, values also vary throughout the day and display a minimum of -9.97% early in the morning and a maximum of 26.77% in the evening. Positive forward premia reach high values between 2.36% and the maximum percentage, which means that the expected spot prices are greater than the forward prices. On average, forward premia across the day are near to 2.13% (see pag. 26Appendix7).

Nonetheless, the electric power prices show a particular behavior: both mean and median of spot prices for a 24 hour period, except from 18:00 to 21:00, when they are lower than the mean of forward prices. The relationship between spot and forward price behavior, which allows seeing that the spot price in a daily pattern is volatile and the forward price exhibits smooth (Fig. 1.4). Thus, it could be hypothesized that agents try to determine forward prices rationally as a risk-averse action. Then, forward premia represent compensation by bearing risk due to electric power prices, which are subject to sudden sharp upward jumps because of several conditions. Longstaff and Wang (2004) and Bessembinder and Lemmon (2002) reached the same conclusions about Forward Premia; on the wholesale electric power market of Pennsylvania, New Jersey, and Maryland “PJM”.

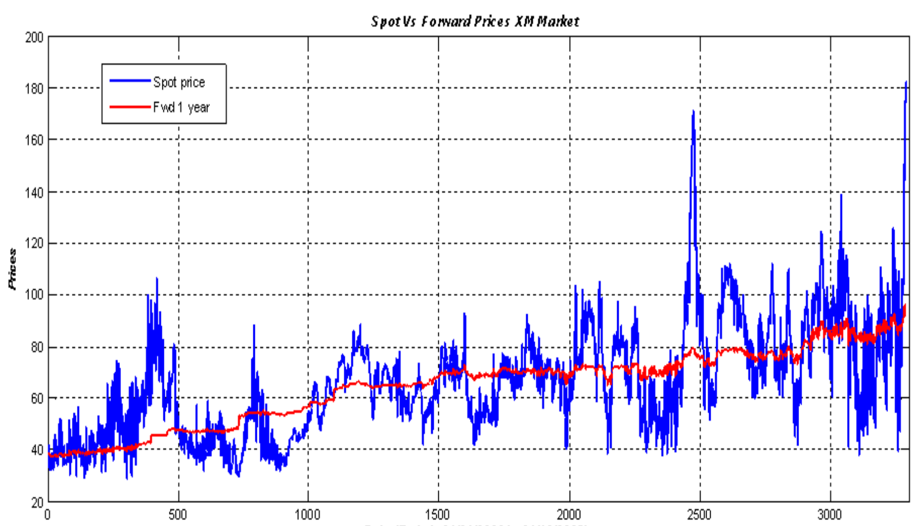

Figure 1.4: Average Daily Electric Power Prices (blue) in COP[5] per Kilowatts from WPM[6], and Forward Prices (red), since January 1, 2000 to December 31, 2008.

Longstaff and Wang (2004) found that there are significant risk premia in electric power forward prices and suggest that risk-averse economic agents determine forward prices rationally. Bessembinder and Lemmon (2002) present an equilibrium model of electric power spot and forward prices. Other papers focusing on energy contracts include Routledge, Seppi, and Spatt (2000), Geman and Eydeland (1999), Pirrong and Jermakyan (1999), Kellerhals (2001), and Lucia and Schwartz(2002). This research shows that, in emerging markets as well as in the Colombian WPMC, Forward premia for electric power prices have peculiar conditions related to their own characteristics and conditions.

This chapter is organized as follows.In section 2 describes the wholesale electric power market in Colombia and the interaction between physical production of electricity and the financial contracts to trade it. Section 3 presents the data used in the empirical work. Section 4 discusses the empirical work and section 5 concludes.

2 WHOLESALE ELECTRIC POWER MARKET

The wholesale electric power market started in Colombia in the late nineteenth century and its development was the result of a private investment initiative that aimed at the generation, distribution, and commercialization of electric power. In the mid twentieth century this scheme started changing and, the privately owned companies were nationalized. In the 1990s, the electric power sector was in bankruptcy due to poor operational and financial management, which resulted in national electric rationing from 1991 to 1992. Since the approval of the new Colombian Constitution in 1991, new regulations have been established for the entry of private investors to the electric generation business. The government was authorized to make decisions related to the construction of new electric generation plants and their guarantees. Thus, the government authorized the involvement of governmental agencies to sign buying and selling contracts of electricity in the long term with the companies selected for that aim.

The Regulatory Commission for Gas and Electricity (CREG) was created by statutes, and its function is to regulate the entrepreneurial, commercial, technical, and operational aspects of this new structure of the electric power sector. This includes the generation, transmission, and distribution/commercialization of electric power. Regulation in the WPMC also created the figure of the “pure marketer”, which is an intermediary agent whose purpose is to make competition dynamic and to provide the final customers with different ways to access competitive prices in the electric market of wholesalers. Regulation in Colombian market allows these agents to sell electric power to their customers through contracts that have no “steady electric power to endorse”, that is, that endorsed by the electric generators to guarantee supply to these users. Moreover, these agents can take endless risks and, in the case of bankruptcy, they do not have assets to lose.

Electric power generators have warned of the risk that the existence of certain agents, who have agreed on long-term contracts without a real electric endorsement and used the electric financial market as an instrument to comply with their contractual obligations, could have on the future feasibility of the electric wholesale market. In times of low prices, the "pure marketers" have probably not had any difficulty to comply with their obligations with the electric financial market and signed contracts. On the contrary, in times of high prices caused by phenomena such as el niño or a poor equilibrium between supply and demand,

[5] COP: International Code of Currency from Colombian Peso

[6] In Spanish: M.E.M (“Mercado de Energía Mayorista”)

these agents would be surely facing financial difficulties due to their losses and would opt out of businesses regardless of the purchases already made to the wholesale market and their customer supply. In 2008 the installed capacity of electric power generation in Colombia (13.4 GW) was 67% hydro (including small hydro), 27% natural gas, 5% coal, and 0.3% wind and cogeneration.

Colombia's electricity supply system is severely affected by adverse weather systems. Drought periods due to El Niño events can last for many months. El Niño periods between 1951 and 2007 are shown in Table 2.1, with the most severe periods of the last 20 years highlighted in blue. The effect of these droughts on water availability for electric power generation is demonstrated in Figure 2.5 for the 2002-2003 events, when waters levels in hydroelectricity dams fell much more than the normal seasonal fluctuation.

Table 2.1: El NIÑO events in Colombia, 1951-2007.

Start	Finish	Months	Start	Finish	Months
01/07/1951	01/01/1952	6	01/04/1982	01/07/1983	15
01/03/1957	01/07/1958	14	01/07/1986	01/03/1988	20
01/06/1963	01/02/1964	8	**01/04/1991**	**01/07/1992**	**15**
01/05/1965	01/05/1966	13	01/02/1993	00/01/1900	6
01/10/1968	01/06/1969	8	**01/03/1994**	**01/04/1995**	**13**
01/08/1969	01/02/1970	6	**01/04/1997**	**01/05/1998**	**13**
01/04/1972	01/02/1973	10	**01/04/2002**	**01/04/2003**	**12**
01/08/1976	01/03/1977	7	01/01/2004	01/03/2005	8
01/08/1977	01/02/1978	6	01/08/2006	01/02/2007	6

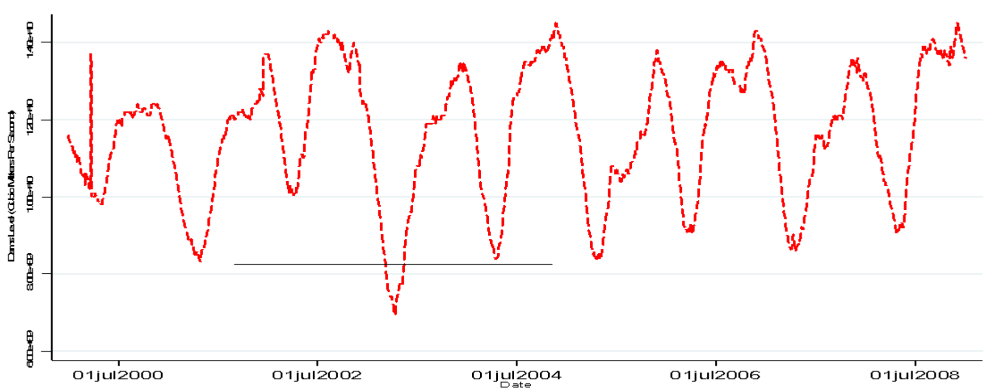

Figure 2.5: Water levels (M^3/sec) in hydroelectricity dams in Colombia, 2002 - 2008.

In the Colombian system, the electric power generators would be obliged to supply electric power by the regulatory institution even if they were impaired by agents who did not comply with the secondary markets. These generators cannot assume this additional risk, because it is not part of their activity. If this situation were to occur, it would not only impair them financially, but it would also make them turn to the Justice system to determine who must assume liability for the resulting impairments. Hence, one of the Authority's demands for electric generators is that they must insure themselves against price volatility risks. This opens up the opportunity of using derivative instruments in the Colombian electric sector in the foreseen future.

By Decree 055/2005, CREG had promoted the project to create a futures and options market. In 2007, the clearing house was created and started functioning in September 2008 in cooperation with The Colombian Stock Market to offer Treasury Bonds future contracts. In October 2010, the first future contracts of electric energy were offered. This is the beginning of this kind of contract and it will reinforce the need to create a system of margin risk analysis, which basically depends on the forward prices of electricity, volatility, and interest rates, among other factors. Thus, it is necessary to determine which volatility model adjusts best to the characteristics of electric prices throughout time for the Colombian electric market.

3 DATA SET

The main data for this work consists of the hourly spot and forward non-delivery contracts from an unregulated market segment in the WPMC since January 1, 2000 to December 31, 2008. The sample has 24 series from a total of 3287 days. The series included the daily spot price and settlement forward prices determined for one year, in which the delivery has to be made one year forward.

The results of this study show that the mean of electric power spot prices varies throughout the day; the lowest price is $56.43 COP/KWh obtained in the morning (04:00) and $87.05 COP/KWh the highest price in the evening (20:00). Pag. 22Appendix 3 shows the pattern of hourly spot series for the most representative hours, particularly during peak demand hours, for instance from 19:00 to 22:00. The statistics summary and figure shown in pag. 22Appendix 3 allow observing that there is time series variation in the spot price for the peak hours, and the distribution of electric power spot prices is highly right-skewed[7] For instance, the maximum spot price during peak hours is around $440 COP/KWh and the standard deviation for the spot prices for the same hours exceeds $34 COP/KWh, which is nearly 50% of the mean value.

Pag. 23 Appendix 4 shows a summary of the statistics for the electric power forward prices, which are expressed in COP/KWh in the same units as spot prices. The forward prices patterns exhibit smoothness which is also supported by the fact that the average forward prices are comparable in magnitude to the average spot prices. The standard deviations of the forward prices are uniformly lower than the corresponding standard deviation for the spot prices, which are volatile. The average skewness of the forward prices shows that there is no presence of the extreme variations or peaks, such as spot prices that show right skewness, and that spot prices tend to display greater prices than forward prices.

A less volatile pattern can be observed in forward prices, whereas spot prices show erratic behavior, which is verifiable with the standard deviation pattern. On the other hand, maximum forward prices are lower than the maximum spot prices. In addition to the basic data, the data on electric power load, spot price and the weather variable (dam level variation) were included and used to construct a set of explanatory variables in order to forecast the changes in the expected spot price and the changes in the expected load, throughout Vector Auto-Regression (VAR).

In order to measure the risk of unexpected price and load quantity changes, the VAR framework (Sims (1980)) combined with GARCH model (Engle (1982)) were adopted and followed. Firstly, the set of ex-

[7] This dominant feature of Electric power spot prices was established by Routledge, Seppi, and Spatt (2000), Longstaff and Wang (2004).

planatory variables including the spot price, load quantities and the Dam levels for the wholesale power market in Colombia were used. Secondly, VAR provides forecast price change in the spot price from day $t\ to\ t+365$ using information available on day t. Then, conditional volatility of unexpected spot price changes was estimated using the GARCH $(1,1)$ model. Thirdly, the VAR framework to forecast the expected load quantity from day $t\ to\ t+365$ was used again to obtain a measure of volumetric risk, taking into account information available on day t from the Neon system of the wholesale electric power market in Colombia. Subsequently, a GARCH $(1,1)$ model was fitted to obtain the innovations of unexpected changes in load. Fourthly, the dummy variable was also included to capture the climatic impact during the drought periods due to the el Niño phenomenon, which are highly related to the Dam levels. Finally, GARCH variables and the dummy ONI variable were used to capture the impact on unexpected quantity and price changes. The constructed set of forecast series was used to predict the FRP throughout OLS Model. pag. 24Appendix 5 reports the R^2 for the VAR Model which forecasts changes price and load quantity.

4 EMPIRICAL WORK

Following the general approach used in the literature in which the model of forward prices is based on equilibrium (e.g.,Hicks (1939); Cootner (1960); Bessembinder and Lemmon (2002); Longstaff and Wang (2004)) this chapter is focused on the existence of FRP, or the difference between forward prices and expected spot prices. The Forward Risk Premia could be seen such as a ratio or percentage forward premia (French (1986); Fama and French (1987); Longstaff and Wang (2004)). From this work it is evident that market behavior depends on whether FRP are positive or negative. Positive premia are referred to as normal backwardation; negative premia are referred to as contango. Throughout this paper, $FRP_{j,t}$ will represent the FRP obtained in the hour "j" of the day "t". $SP_{j,t+365}$, will stand for the forward price in the hour j of day t, for delivery during day $t+365$. Therefore, the FRP are defined by:

$$FRP_{j,t} = E_t\left[\frac{SP_{j,t+365} - FW_{j,t}}{FW_{j,t}}\right] \tag{4.1}$$

In order to construct an empirical analysis, the FRP were studied to determine their existence, and, if so, the kind of characteristics could take at an unconditional level of economic risk measure in the electric power market. The analysis of the expected FRP at conditional level are presented, and the impact of the drought periods due to ONI as a quantity risk measure is explored.

In order to verify if FRP are zero on average, the mean of the FRP were calculated hour by hour and it was found that the percentage forward premia (2.14%) out of 24 are positive. Results are consistent with the classical literature that expected spot prices should be higher than forward prices, and this is consistent with Longstaff and Wang (2004) results. Thus, the FRP vary systematically throughout the day with significant behavior related to an hourly demand. Equation (4.1) shows that the forward risk premia can be expressed as a conditional form (Longstaff and Wang (2004)), which includes the unexpected component of the realized FRP presented in equation (4.2) and denoted by $\xi_{j,t+365}$.

$$\frac{SP_{j,t+365} - FW_{j,t}}{FW_{j,t}} = E\left[\frac{SP_{j,t+365} - FW_{j,t}}{FW_{j,t}}\right] + \xi_{j,t+365} \tag{4.2}$$

Based on equation (4.2), the regression model to estimate the forward risk premia for the $t+365$ period could be established hour-by-hour as well as ex-post occurrence of Conditional Forward premia in electric power markets. This estimate of forward risk premia depend on risk measures that capture unexpected changes on prices, demands and risk preferences.

Using the time series from WPMC, Neon System, registered hour by hour, the spot prices and settlement forward prices of bilateral contracts on day t to be delivered on day Using the time series from WPMC, Neon System, registered hour by hour, the spot prices and settlement forward prices of bilateral contracts on day t to be delivered on day $t+365$ at the same hour. Three variables were included; firstly, the conditional volatility of expected changes on the spot prices, expressed by $CV_{j,t}$ or conditional volatility in the hour j of the day t. Secondly, the innovations of expected changes in load quantities, expressed by $IIN_{j,t}$,or innovations in the hour j of the day t. Finally, the dummy ONI variable is denoted by $\delta\_\text{Niño}_{j,t}$. The ONI variable was used as a dummy variable that takes a value of one for drought periods and zero in all other cases. The first and second variables were obtained using GARCH $(1,1)$ model to estimate expected spot prices according to volatility and expected changes in load. The third variable, due to technical conditions of electric power markets, is strongly related to climatic effects on load, which implies effects on prices. These explanatory variables permit forecasting Forward Risk Premia (FRP), using the Ordinary Least Square OLS regression model. pag. 22 Appendix 3 presents statistics for three statistically significant risk factor.

$$FRP_{j,t} = \beta_{0j} + \beta_{1j}CV_{j,t} + \beta_{2j}IIN_{j,t} + \beta_{3j}\delta\_\text{Niño}_{j,t} + \xi_{j,t+365} \quad (4.3)$$

Pag. 23 Appendix 4 shows that the economic risk factors are highly significant in statistics. In the Colombian Wholesale Electric Power Market, the volatility also varies directly with FRP every hour throughout the day, except for a few hours early in the morning. In addition, the risk premia is positive from 9:00 to 22:00, which are the hours with medium or high demand, and strongly positive from 19.00 to 21.00, which are hours with high demand. Therefore, when peak demand increases at those hours, long positions on forward markets obtain higher premia but the sensitivity of volatility in those hours is negative. pag. 25 Appendix 6 shows the summarized statistics of the FRP, including the Breusch-Pagan and Cook-Weisberg test for Heteroskedasticity.

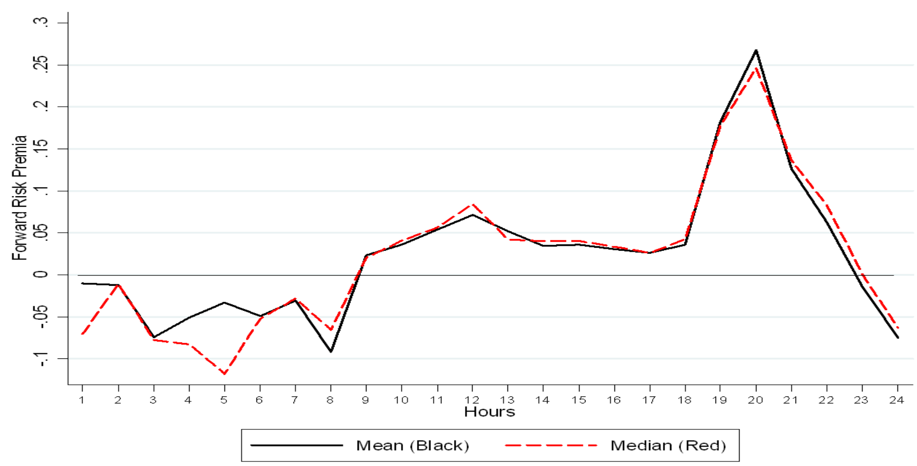

Figure 4.6: Expected Forward Risk Premia for Electric Power Market Using climatic effects

5 CONCLUSIONS

This chapter presents new evidence about Forward Premia and includes new significant conditions that are relevant in wholesale electric power markets (WPMC). These results are perceptible in electric power spot prices and derivative prices for markets in tropical countries, where there are no seasons but climatic conditions such as el Niño phenomenon. Using the hourly spot and forward non-delivery contracts data set, it could be verified that the FRP take place and has the characteristics of an economic risk measure of the market agents.

This chapter focused on the percentage FRP as shown in French (1986), Fama and French (1987), Bessembinder and Lemmon (2002) and Longstaff and Wang (2004). According to empirical results, the average expected forward risk premia can have a positive behavior in seventeen out of twenty-four hours, with a 2.14% on average for all the hours in a range from -9.97% to 26.77%. These results represent the equilibrium compensation for bearing the price risk of the electric power for one year. The median or Typical Forward premia is positive and very near to the average forward premia (2.12%) this is an opposite result to that of Longstaff and Wang (2004), who found that median is negative. Our results suggest that the forward premia represents compensation for bearing the risk in the Colombian case.

In the Colombian electric power market the risk taker is the marketer, particularly in the unregulated market segment, because they are assuming the price risk in the long-term negotiations. The marketer represented by this demand, tries to insure their future revenues and so they sacrifice their premia. It is relevant for further studies to evaluate the efficiency of this market, and the characteristics which determine why the marketer is willing to pay FRP and why the generator is in a better position to receive this bonus.

From the above, it applies that sellers assume the FRP for unregulated markets and so they have to pay for long-term contracts in order to guarantee future sales. For the unregulated market segment, the natural sellers are the marketers who pay the premia and assume the risk as well. The WPMC shows normal backwardation behaviour, probably due to the large opportunity cost related to electric power markets, which are characterized by high hydraulic dependence.

It was examined whether the FRP reflect compensation for risk taking by market agents throughout several risk measures. One way to obtain these measures is suggested by very interesting works proposed by Longstaff and Wang (2004) and Bessembinder and Lemmon (2002). These include volatilities of unexpected spot changes, volatilities of unexpected load changes and the weather variable, to capture price uncertainty, quantity uncertainty and climatic effects. Such variables play a significant role in explaining the FRP in the Colombian Market.

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Appendix 1:Unit Root Tests Statistic

This table presents the summarized statistics for unit-root test (Augmented Dickey and Fuller (1982), Phillips and Perron (1988) and Kwiatkowski-Phillips-Schmidt-Shin (1992) tests statistic). Critical values for Augmented Dickey and Fuller (1982) and Phillips and Perron (1988) tests statistic(-2.86209) are used at the 95% level, Asymptotic critical value for Kwiatkowski-Phillips-Schmidt-Shin (1992) test statistic is 0.463 at the 95% level.

	Spot Price					*Forward Price*				
	ADF Test Statistic		Phillips-Perron test		KPSS	ADF Test Statistic		Phillips-Perron test		KPSS
	t-Stat.	Prob*	t-Stat.	Prob*	LM-Stat**	t-Stat.	Prob*	t-Stat.	Prob*	LM-Stat**
H1	-5,2548	0,0000	-13,0662	0,0000	0,0393	-55,7414	0,0001	-92,2246	0,0001	0,4363
H2	-9,1834	0,0000	-11,9576	0,0000	0,1498	-47,5081	0,0001	-128,0060	0,0001	0,1010
H3	-5,1699	0,0000	-37,5253	0,0000	0,0921	-54,4905	0,0001	-86,4832	0,0001	0,4079
H4	-10,2669	0,0000	-27,8396	0,0000	0,1763	-54,4115	0,0001	-87,6790	0,0001	0,4598
H5	-3,9322	0,0018	-4,0825	0,0010	0,0255	-34,4695	0,0000	-92,5141	0,0001	0,3680
H6	-10,1957	0,0000	-23,3397	0,0000	0,3690	-26,5682	0,0000	-90,7813	0,0001	0,4348
H7	-6,4108	0,0000	-5,3315	0,0000	0,0141	-42,8137	0,0000	-84,6452	0,0001	0,4247
H8	-6,0487	0,0000	-4,7742	0,0001	0,0202	-23,5522	0,0000	-92,4667	0,0001	0,2927
H9	-4,5490	0,0002	-4,4020	0,0003	0,4206	-39,7447	0,0000	-100,2260	0,0001	0,2198
H10	-3,1761	0,0215	-9,7161	0,0000	0,4875	-22,4594	0,0000	-89,3568	0,0001	0,3224
H11	-8,3349	0,0000	-36,6288	0,0000	0,0188	-24,6599	0,0000	-76,3195	0,0001	0,0881
H12	-6,2378	0,0000	-21,3233	0,0000	0,0161	-38,3103	0,0000	-95,4690	0,0001	0,1719
H13	-5,0330	0,0000	-4,8319	0,0000	0,0117	-45,0071	0,0001	-90,0516	0,0001	0,1112
H14	-11,4594	0,0000	-21,6266	0,0000	0,3488	-28,6575	0,0000	-114,3660	0,0001	0,0929
H15	-5,9315	0,0000	-5,1431	0,0000	0,0121	-26,8293	0,0000	-92,3401	0,0001	0,3401
H16	-8,1143	0,0000	-7,3344	0,0000	0,0126	-23,4427	0,0000	-95,7354	0,0001	0,4503
H17	-6,2247	0,0000	-13,6101	0,0000	0,2978	-23,3098	0,0000	-93,7481	0,0001	0,2499
H18	-5,1921	0,0000	-4,7783	0,0001	0,2412	-17,7194	0,0000	-106,3290	0,0001	0,3756
H19	-5,7636	0,0000	-5,4114	0,0000	0,3341	-22,1013	0,0000	-98,7503	0,0001	0,3871
H20	-4,3895	0,0008	-5,1662	0,0000	0,2970	-32,8136	0,0000	-111,0040	0,0001	0,0768
H21	-5,0906	0,0000	-4,7480	0,0001	0,1346	-34,7996	0,0000	-105,1100	0,0001	0,2070
H22	-19,6602	0,0000	-23,3454	0,0000	0,0574	-56,0568	0,0001	-87,2489	0,0001	0,2332
H23	-12,4294	0,0000	-33,6373	0,0000	0,0157	-52,2075	0,0001	-81,4285	0,0001	0,4427
H24	-11,0617	0,0000	-40,5991	0,0000	0,3235	-55,7192	0,0001	-90,1016	0,0001	0,4551

* MacKinnon (1996) one-sided p-values.

** Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Appendix 2:Oceanic Niño Index - ONI

The ONI is based on SST departures from average in El Niño 3.4 region, which is a principal measure for monitoring, assessing, and predicting El Niño phenomenon. It is defined as the three-month running-mean SST departures in the Niño 3.4 region; departures are based on a set of improved homogeneous historical SST analyses (Extended Reconstructed SST - ERSST.v2). The methodology described by Smith and Reynolds (1999) is used to place current conditions in historical perspective. NOAA's operational definitions of El Niño and La Niña are key to the ONI index. El Niño strengthened during December 2006, with above-average sea surface temperatures (SST) encompassing the central and eastern equatorial Pacific Ocean (Fig. 5.7).

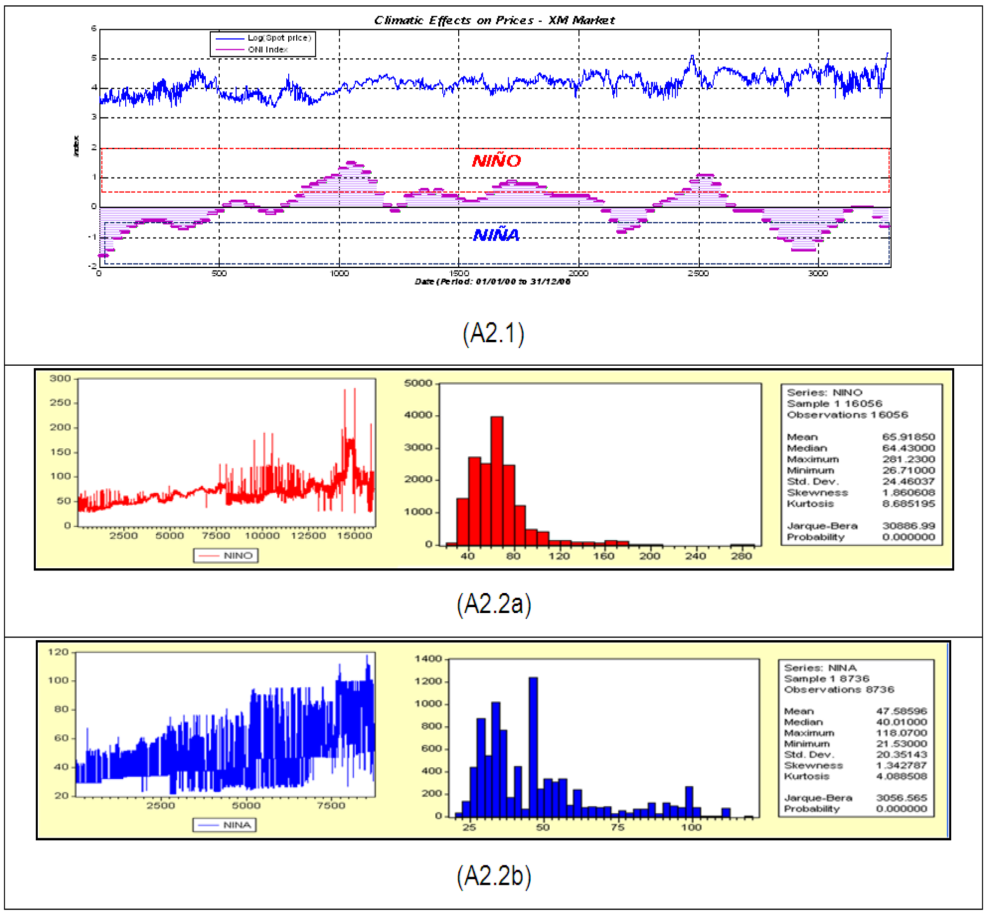


Figure 5.7: Climatic effects on prices, average sea surface temperature (SST) anomalies ($°C$) 2000-2008. Fig A2.1, Log of Spot price and ONI index. Fig A2.2a, price behavior during El Niño phenomenon. Fig A2.2b, Price behavior during La Niña phenomenon. Source: Author (NOAA/National Weather Service Data)

Appendix 3:Hourly Spot Prices

This table presents summarized statistics for hourly electric power spot prices reported by WPMC, prices are reported in COP's per Kilowatt/Hour. The sample consists of daily observations for each of the twenty-four hourly spot prices from January 01, 2000 to December 31, 2008.

Spot Prices	Mean	Median	Minimum	Maximum	Stand Dev	Skewness	Kurtosis
H1	58,65	57,47	24,95	170,23	20,84	0,91	1,80
H2	57,26	55,54	24,95	170,23	20,56	0,92	1,85
H3	56,66	54,40	24,49	170,23	20,43	0,92	1,83
H4	56,43	54,26	24,49	169,23	20,31	0,91	1,78
H5	57,83	56,58	24,49	169,23	20,55	0,93	2,03
H6	61,36	60,46	24,35	169,23	21,17	0,89	1,79
H7	62,32	61,96	21,53	188,37	22,33	0,74	1,53
H8	64,03	63,41	21,53	188,57	22,38	0,77	1,61
H9	66,48	65,75	21,53	188,57	22,84	0,79	1,55
H10	67,49	66,49	21,53	195,37	23,22	0,79	1,61
H11	68,98	67,49	21,53	200,37	23,97	0,94	2,24
H12	70,37	68,35	21,53	210,40	24,47	1,04	2,66
H13	68,78	67,41	21,53	200,37	23,45	0,95	2,27
H14	67,24	66,08	21,53	200,37	23,52	0,88	2,01
H15	67,26	66,11	21,53	200,37	24,22	0,91	2,17
H16	66,81	65,87	21,53	200,37	23,95	0,83	1,79
H17	66,56	65,79	21,53	188,57	23,45	0,75	1,38
H18	67,69	66,59	21,53	190,37	23,59	0,88	2,05
H19	80,73	75,43	23,10	438,84	31,90	2,12	10,25
H20	87,05	80,26	25,95	316,75	33,48	1,68	4,77
H21	76,18	73,30	23,10	233,42	25,31	1,16	3,36
H22	70,09	68,77	21,53	206,37	23,16	0,84	2,31
H23	64,61	63,45	24,95	190,37	21,80	0,95	2,08
H24	60,13	59,00	23,27	188,57	21,13	0,88	1,93
Overall	**66,29**	**64,59**	**22,83**	**206,03**	**23,42**	**0,97**	**2,44**

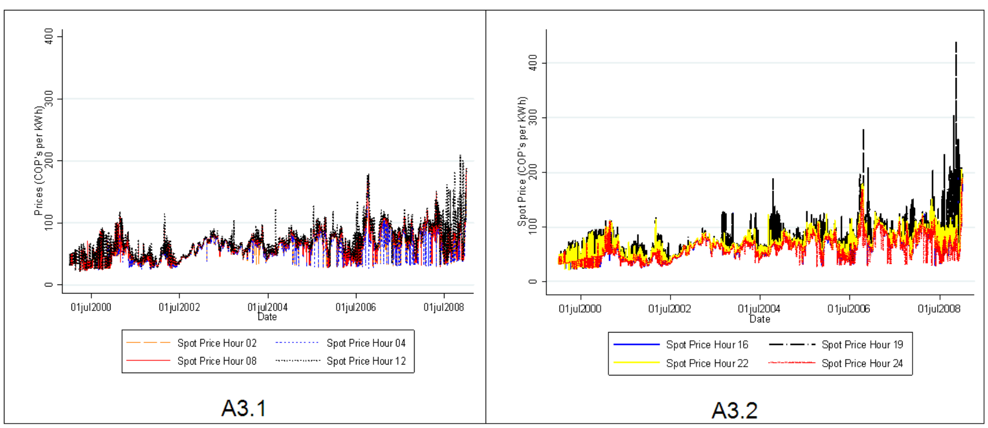


Figure 5.8: Electric power spot prices reported by WPMC, prices are reported in COP's per Kilowatt/Hour from January 01, 2000 to December 31, 2008. Fig A3.1, Spot price plot including the most representative hours in this market; such as 2:00, 4:00, 8:00, 12:00, Fig A3.2, Hours 16:00, 19:00, 22:00 and 24:00.

Appendix 4:Hourly Forward Prices

This table presents summary statistics for the hourly day-ahead Forward prices reported by WPMC. Prices are reported in COP's per kilowatt-hour. The sample consists of daily observations for each of the twenty-four hourly forward prices since January 1, 2000 to December 31, 2008.

Forward Prices	Mean	Median	Minimum	Maximum	Stand Dev	Skewness	Kurtosis
H1	64,40	67,98	36,85	99,22	14,61	-0,26	-0,73
H2	64,27	67,72	36,85	99,22	14,54	-0,25	-0,71
H3	64,21	67,69	36,85	99,22	14,50	-0,26	-0,71
H4	64,21	67,69	36,85	99,22	14,53	-0,25	-0,70
H5	64,51	67,97	36,85	99,34	14,55	-0,24	-0,69
H6	64,65	67,87	36,82	98,37	14,60	-0,23	-0,71
H7	64,05	67,84	36,95	94,49	13,78	-0,44	-0,81
H8	64,19	68,09	36,95	94,49	13,79	-0,44	-0,83
H9	64,38	68,37	36,95	94,49	13,84	-0,43	-0,85
H10	64,61	68,77	37,06	93,55	13,89	-0,47	-0,83
H11	64,72	68,92	37,06	93,55	13,92	-0,46	-0,84
H12	64,79	69,07	37,06	93,06	13,91	-0,47	-0,85
H13	64,54	68,72	37,07	93,03	13,83	-0,45	-0,88
H14	64,43	68,53	37,04	94,00	13,86	-0,45	-0,86
H15	64,42	68,54	37,04	94,00	13,89	-0,45	-0,86
H16	64,39	68,54	37,03	94,00	13,89	-0,46	-0,86
H17	64,36	68,46	36,95	94,00	13,85	-0,46	-0,85
H18	64,59	68,87	36,95	93,03	13,91	-0,47	-0,88
H19	66,49	70,56	37,03	98,31	14,77	-0,32	-0,77
H20	66,74	70,97	37,03	98,40	14,76	-0,35	-0,81
H21	66,33	70,56	37,03	98,31	14,72	-0,32	-0,74
H22	65,52	69,64	36,78	98,90	14,97	-0,26	-0,77
H23	65,01	68,72	36,85	98,90	14,64	-0,29	-0,76
H24	64,50	68,20	36,85	98,74	14,62	-0,29	-0,76
Overall	**64,76**	**68,68**	**36,95**	**96,33**	**14,26**	**-0,37**	**-0,79**

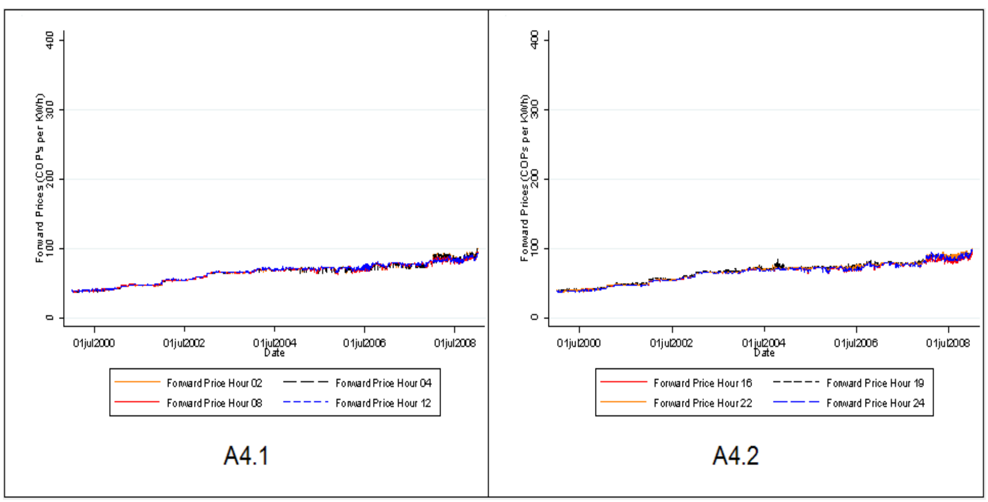


Figure 5.9: Electric power Forward prices reported by WPMC, prices are reported in COP's per Kilo-watt/Hour from January 01, 2000 to December 31, 2008. Fig A4.1, Forward prices plots including the most representative hours in this market; such as 2:00, 4:00, 8:00, 12:00, Fig A4.2, Hours 16:00, 19:00, 22:00 and 24:00.

Appendix 5: Values of R^2 for VAR forecasting price and load quantity changes

This table reports the R^2 from the VAR model used to forecast the hourly spot electric power prices and load quantities. The VARs forecast $t+360$ for spot price and quantity includes the spot and quantity at time t and their lags, and the weather variable that represents hydrology of the market on study, called dam variations.

$$Sp_{i,t+365} = \gamma_0 + \gamma_1 \sum_{i=1}^{24} Q_{i,t} + \gamma_2 \sum_{i=1}^{24} Sp_{i,t-1} + \gamma_3 \sum_{i=1}^{21} Sp_{i,t-2} + \gamma_4 \sum_{i=1}^{22} Q_{i,t-2} + \gamma_5 VarDl_t + u_{t+360}$$

$$Q_{i,t+365} = \gamma_0 + \gamma_1 \sum_{i=1}^{24} Q_{i,t} + \gamma_2 \sum_{i=1}^{24} Sp_{i,t-1} + \gamma_3 \sum_{i=1}^{21} Sp_{i,t-2} + \gamma_4 \sum_{i=1}^{22} Q_{i,t-2} + \gamma_5 VarDl_t + v_{t+360} \quad (5.4)$$

Table A5: R^2 from the VAR model used to forecast the hourly spot electric power prices and load quantity.

Hour	*Exp_Spotprice_VAR (R^2)*	*Exp_Load_VAR (R^2)*
1	0,5494	0,8947
2	0,5452	0,8949
3	0,5236	0,875
4	0,5114	0,8736
5	0,4942	0,8825
6	0,6251	0,8934
7	0,6516	0,8858
8	0,6141	0,8708
9	0,6335	0,7885
10	0,6427	0,7727
11	0,4591	0,7695
12	0,4262	0,7788
13	0,5117	0,86
14	0,532	0,7746
15	0,4583	0,7789
16	0,4394	0,7876
17	0,5215	0,778
18	0,5313	0,8802
19	0,3836	0,8868
20	0,3915	0,8883
21	0,5243	0,8892
22	0,537	0,8912
23	0,5487	0,8938
24	0,6233	0,8943

Appendix 6:Results from Regressions of Realized Percentage Forward premia

This table presents summarized statistics from a regression model from individual hourly time-series including three variables: i) the conditional volatility of unexpected spot price changes (expressed by $CV_{j,t}$ or conditional volatility on hour j of day t); ii) the innovations or GARCH standardized returns of unexpected changes in load (denoted by $INN_{j,t}$, innovations on hour j of day t), and finally iii) the ONI, expressed by the dummy variable $\delta\_\text{Niño}_{j,t}$. The first and second variable were obtained using GARCH $(1,1)$ model from VAR expected spot price and VAR expected Load.

$$FRP_{j,t} = \beta_{0j} + \beta_{1j} CV_{j,t} + \beta_{2j} IIN_{j,t} + \beta_{3j} \delta\_\text{Niño}_{j,t} + \xi_{j,t+365} \quad (5.5)$$

Table A6: **Statistics from a regression model from individual hourly time-series**

Hour	Const ßo	CV_E(Sp) ß1	INN_E(Load) ß2	Dummy_Niño ß3	t(ßo)	t(ß1)	t(ß2)	t(ß3)	P > t(ßo)	P > t(ß1)	P > t(ß2)	P > t(ß3)	R²
H1	0,0107395	-0,9158818	-0,0266856	0,0198636	0,29	-3,09	-3,24	1,93	0,002	0,001	0,024	0,008	0,800%
H2	0,0128536	0,1171866	0,0081618	0,0104576	-3,45	-7,07	-3,28	1,49	0,000	0,001	0,014	0,001	2,370%
H3	-0,0432498	-0,9429345	-0,0155768	0,0171551	-3,35	-8,02	-3,34	1,63	0,000	0,001	0,010	0,001	2,910%
H4	-0,1089033	-0,2834784	-0,012405	0,0260782	-8,71	-2,72	-2,7	2,37	0,007	0,007	0,018	0,000	0,910%
H5	-0,0742418	-0,4249733	-0,0038628	0,0141781	-5,92	-4,28	-0,6	1,31	0,000	0,055	0,019	0,000	0,860%
H6	-0,0012556	-0,6094195	0,0202965	-0,0247922	-0,12	-5,99	4,16	-2,17	0,000	0,000	0,030	0,019	1,800%
H7	-0,0720809	0,4484124	0,0346783	-0,0072274	-5,26	3,92	6,99	-0,6	0,000	0,000	0,035	0,000	2,290%
H8	0,0272609	-0,7598599	0,0278942	-0,2916654	1,76	-5,68	3,76	-16,62	0,000	0,000	0,000	0,028	9,120%
H9	-0,0347073	0,6983237	0,0296342	-0,0284896	-2,65	6,19	5,86	-2,34	0,000	0,000	0,020	0,008	3,060%
H10	0,1295084	-0,8288422	0,0299871	-0,0794249	9,48	-6,25	5,81	-6,48	0,000	0,000	0,000	0,000	3,260%
H11	0,0269341	0,5052315	0,0363241	-0,0561816	2,31	4,57	6,98	-4,54	0,000	0,000	0,000	0,021	3,450%
H12	0,0932019	-0,0105613	0,0411952	-0,0796484	6,96	-0,08	7,83	-6,38	0,009	0,000	0,000	0,000	3,440%
H13	-0,0308206	1,055768	0,0242053	-0,0309772	-2,59	10,18	4,76	-2,59	0,000	0,000	0,010	0,010	5,150%
H14	0,1318952	-0,7793383	0,0278557	-0,0735821	9,45	-6,41	5,64	-6,26	0,000	0,000	0,000	0,000	3,140%
H15	0,0731175	-0,2012569	0,0341165	-0,0611366	6,95	-2,21	6,75	-5,12	0,027	0,000	0,000	0,000	2,410%
H16	0,0951943	-0,4946276	0,0367734	-0,065363	8,74	-5,06	7,37	-5,57	0,000	0,000	0,000	0,000	3,280%
H17	0,1201225	-0,8072349	0,0368666	-0,0684546	9,74	-7,05	7,53	-5,91	0,000	0,000	0,000	0,000	4,030%
H18	0,0573564	-0,0564242	0,0368993	-0,0462608	4,9	-0,59	7,46	-3,92	0,025	0,000	0,000	0,000	2,390%
H19	0,1123254	0,9900856	0,0569398	-0,0517618	6,67	6,06	8,98	-3,43	0,000	0,000	0,001	0,000	4,710%
H20	0,3744301	-0,352121	0,0790559	-0,1377334	2,69	-0,49	12,17	-9,18	0,033	0,000	0,000	0,007	7,400%
H21	0,1224132	0,2541385	0,0324085	-0,0815469	8,79	2,27	6,16	-6,48	0,023	0,000	0,000	0,000	3,230%
H22	0,0994232	-0,2966335	0,0098494	-0,05901	4,05	-0,9	1,96	-5,06	0,037	0,020	0,000	0,000	1,000%
H23	0,1345179	-1,508127	-0,0007551	-0,0572712	11,33	-13,43	-0,16	-5,18	0,000	0,038	0,000	0,000	5,990%
H24	0,0839348	-2,243182	-0,0064028	-0,0009298	3,8	-7,73	-0,73	-0,09	0,000	0,046	0,048	0,000	2,030%

Appendix 7:Results from regressions of realized percentage forward premia based on economic risk measures

Table A7: Descriptive statistics from regressions of realized percentage forward premia based on economic risk measures.

Hour	Mean	Median	St_Dev	Error	F(1 , 2919)*	Prob > F
H1	-0,09971346	-0,0702916	0,02162549	0,00040013	8,33	0,004
H2	-0,011924	-0,0106195	0,03716698	0,00068769	30,93	0,000
H3	-0,07421089	-0,0775082	0,04143761	0,00076671	36,2	0,000
H4	-0,05066861	-0,0824066	0,02324258	0,00043005	35,11	0,000
H5	-0,0331075	-0,1176352	0,02270227	0,00042005	16,13	0,000
H6	-0,04866039	-0,0521937	0,03507915	0,00064906	108,75	0,000
H7	-0,03031701	-0,0280901	0,04073718	0,00075375	22,35	0,000
H8	-0,09143834	-0,0625202	0,12634721	0,00233776	205,28	0,000
H9	0,02357925	0,0207474	0,04837227	0,00089502	58,46	0,000
H10	0,03625197	0,0407245	0,05108624	0,00094523	7,02	0,008
H11	0,05401677	0,0565708	0,05319233	0,0009842	95,27	0,000
H12	0,07133653	0,0845366	0,05382473	0,0009959	8,13	0,004
H13	0,05216946	0,0419459	0,0626297	0,00115882	299,04	0,000
H14	0,03482814	0,040293	0,04815507	0,000891	16,68	0,000
H15	0,03614242	0,0405704	0,04267022	0,00078951	8,97	0,003
H16	0,03080555	0,0335401	0,04925607	0,00091137	11,4	0,001
H17	0,02658233	0,0261566	0,05381165	0,00099566	8,48	0,004
H18	0,03596371	0,0425609	0,04148958	0,00076767	8,700	0,004
H19	0,18207874	0,1777844	0,07625334	0,00141089	68,22	0,000
H20	0,26769463	0,2466361	0,09927693	0,00183689	37,97	0,000
H21	0,1264317	0,136639	0,05202542	0,00096261	18,98	0,000
H22	0,06261725	0,0826715	0,02739005	0,00050679	8,3	0,004
H23	-0,01349949	0,0013267	0,06384115	0,00118123	40,71	0,000
H24	-0,07445258	-0,0631964	0,03467353	0,00064155	5,4	0,020
Overall	2,1354%	2,1177%	0,0503	0,00092998		

*Breusch-Pagan / Cook-Weisberg test for heteroskedasticity

Chapter II

Optimal Static Hedging of Energy Price and Volume Risk: Closed-Form Results

Abstract

As an extension of the VaR-constrained hedging, we propose a closed- form solution to the problem of optimizing portfolios, based on price and weather. For electric power companies, price and quantity are volatile, and in hydro-electricity generation quantity can be related to weather conditions. An optimum portfolio is derived from expected utility maximization problem, including weather indices to minimize losses. The main approach to this problem is shown, the independence case, in which price and weather index are uncorrelated. Due to electric power features, agents in this market are facing price and volumetric risks, the difficulty to storage efficiently electric power cannot permit to mitigate volumetric risk and alternatively weather instruments can be used in order to hedge unexpected changes in weather; the purpose of weather derivatives is to smooth out the temporal fluctuations in the company's revenues. For electric power companies price and quantity are volatile, and quantity is correlated to the weather conditions. Moreover, exposures to price and volume risks make necessary the inclusion of the weather pay-off. Thus, we derive the optimal portfolio from the expected utility maximization problem including vanilla and weather derivatives whose payoffs will minimize losses.

Keywords: Static Hedging, Energy Risk Mitigation,Volumetric Hedging, Incomplete Markets.

JEL Classification: G0, G13, C32.

1 INTRODUCTION

Electric power markets are going through an infancy period compared with other more developed markets such as fixed income securities, stocks and currencies. In addition, the energy market is a special market case given that it has some added complexities. Electric power needs real time balancing between supply and demand because electricity is consumed at the same time as it is produced; inventories cannot be held to compensate price and quantity fluctuations. Electricity is unlike other financial products. The technological inability to store electric power efficiently and marginal production costs create jumps in the spot price, so that arbitrage arguments have been difficult to deal with. All these specifications make classical dynamic hedging theory impossible to apply. It is clearly not possible to dynamically hedge an asset when you cannot know whether the quantity that you had contracted is the capacity that you have. Thus, because of the "virtual" storability of electricity, agents face volumetric risk.

Furthermore, the market participants, who can be generators, marketers or load serving entities (LSE), who are not the end-users of electricity, have to sell or buy electricity at a price set by the supply and demand equilibrium when the final users consume the electricity at a fixed regulated price. In addition, the regulated demand is inelastic and residential customers must be serviced at all times. A LSE unit has the obligation to deliver electricity on demand at a fixed price without fail, whatever it costs.

The difficulty of storing electric power efficiently does not allow mitigation of volumetric risk. Weather derivatives can be used in order to hedge unexpected changes in weather. Weather derivatives are based on indexes of temperature, such as the indexes from Chicago Mercantile Exchange (CME), such as Cooling-Degree-Days (CDD), or Heating-Degree-Days (HDD). Sometimes insurance companies trying to transfer their climate-related risk to capital markets need to transform non-tradable risk into tradable financial securities such as weather derivatives.

Weather derivatives were first launched in 1996 in the United States as a mechanism of protection against weather anomalies. The purpose of weather derivatives is to smooth out the temporal fluctuations in the company's revenues. There are a number of financial and commercial reasons why this is beneficial (Jewson (2004)). Companies hedge their portfolios against unexpected weather variations using contracts that are not correlated with classical financial assets. For instance, the El Niño phenomenon was responsible for weather anomalies that took place over thirteen months between April 1997 and May 1998 and over one year between April 2002 and April 2003 in South and North America. Chicago Mercantile Exchange Anon CME. (2005) started offering the first standardized weather derivatives in September 1999, with the purpose of increasing liquidity and accessibility on this kind of contract. The market was accepted this and grew quickly.

CME offers weather futures and options. Contract specifications include: type, contract size, product description, tick size, period and the settlement procedure (Anon CME. (2005) 2010). The daily average temperature T_j is defined as the arithmetic average of the maximum and minimum temperature recorded between 12:01 a.m. and 12:00 a.m. midnight as reported by MacDonald Dettwiler and Associates (MDA) information System, Inc.

$$T_j = \frac{T_j^{max} + T_j^{min}}{2} \tag{1.1}$$

For each day during winter, Heating-Degree-Days (HDD) is the maximum between zero and 65 degrees Fahrenheit ($\sim$ 18 degrees Celsius) minus the daily average temperature T_j. For each day during summer, Cooling-Degree-Days (CDD) is the maximum between the daily average temperature T_j minus 65 degrees Fahrenheit ($\sim$ 18 degrees Celsius) and zero (Anon CME. (2005)). Weather derivatives are basically a speculative security because those indexes are not a tradable commodity or a delivery asset. Due incomplete characterization, the weather derivatives market still does not have an effective pricing model.

Several authors have proposed pricing models for weather derivatives in continuous time framework. Richards et al (2004) presented an equilibrium pricing model based on temperature processes of a mean-reverting Brownian motion. Chaumont et al (2005) considered that under an equilibrium condition, the market price of risk is uniquely determined by a backward stochastic differential equation, and they translate these stochastic equations into semi-linear partial differential equations. They then choose two simple models for sea surface temperature. Lee and Oren (2009) derived an equilibrium pricing model for weather derivatives and measured risk hedging, including weather derivatives, in a volumetric hedging strategy.

Volumetric risk in electric power markets has significant dimensions when quantity is affected by weather conditions; in countries with seasons, random movements in temperature affect electric power demand. Some tropical countries are also affected by hydrological conditions and the correlation between the load volatility and the weather variable. In general, power generation is affected by hydrological variables when production system uses hydro generation. It has been empirically shown that the most important factor affecting the quantity of power generation is the climatic conditions, and load is correlated to the weather. Economic earnings obtained by industries which are weather-sensitive are affected by weather anomalies which is the case of energy industries (Dutton (2002)). The volumetric risk faced by electric power companies is correlated with unexpected changes in weather or hydrology which cause demand and price fluctuations.

As an extension of the VaR-constrained hedging introduced in Oum and Oren (2008), this chapter proposes a new way to hedge a LSE's profit based on the constitution of an optimal portfolio composed by two claims: standard contracts on price and weather derivatives. The most important risks faced by the market participants are price risk and quantity risk. Variations in weather conditions affect quantity; price risk is caused by extreme high volatility, and the volumetric risk is determined by the uncertainty of final consumption.

The main purpose of this chapter is to derive the hedging portfolio based on two claims: price and volumetric hedging instruments. We derive the optimal portfolio from the expected utility maximization problem using vanilla and weather derivatives whose payoffs will minimize losses. This proposal is based on the independence assumption between both claims that means that price and weather are not correlated. In this case we derived the optimal payoff functions, and found evidence that the inclusion of two payoffs generates incremental improvements over agent's revenues and minimizes risk measures. Evidence shows that the profit after hedging is significantly better than before hedging, increasing the mean of revenue and minimizing risk measure.

This chapter is organized as follows. In Section 2 we provide an overview of the problem statement and Section 3 we show the theoretical results. In Section 4 we illustrate these results, and Section 5 concludes.

2 Problem Statement

LSE has to provide electric power on demand at a fixed price but faces uncertainty about the quantity of electric power to supply and the price it will pay. Hedging strategy allows the agents throughout the derivatives-contracts payoffs to mitigate losses caused by unexpected changes in price and quantity. We derive the hedging portfolio including the weather derivatives whose payoffs will minimize these losses. The portfolio construction problem follows Markowitz's (1952) model, where an investor's goal defines the portfolio construction in order to maximize expected future returns given a certain level of risk. The Markowitz model establishes that the volatility of portfolio returns measures the risk. Campbell et al (2001) introduced a similar portfolio allocation problem using VaR as a risk measure. In the electric power literature, several authors follow Markowitz's methodology to address hedging strategy using vanilla derivatives. Nasakkala and Keppo (2005), and Woo et al (2004),studied the interaction between stochastic consumption volumes and electricity prices, and proposed a mean-variance type model to determine optimal hedging strategies. Vehvilainen and Keppo (2006) optimized hedging strategies taking into account the Value at Risk as risk measure. Huisman (2007) introduced a one-period framework to determine optimal positions in peak and off-peak contracts in order to purchase future consumption volume. In this framework, hedging strategy is assumed to minimize expected costs relating to an ex-ante risk limit defined in terms of Value at Risk.

The main assumption about VaR techniques is that of normality, which is strongly accepted in the case of financial markets. Specific characteristics of electric power markets are price and load spikes, and the corresponding effect is Fat Tails distribution than normal, except for very large number of periods in the planning horizonHull and White (1998). However, VaR captures the case of normality but other cases such as Student-t, Weibull, among other distributions.

The Markowitz (1952)concept of efficient frontier also applies to electricity, but previous authors did not consider the effect of volumetric risk exposure in their optimization solutions. Volumetric risk exposure can be a potent component of portfolio losses due to adverse movements in quantity in the electric power market. Authors cited above have tried to solve the Markowitz problem, but the portfolio is only composed in order to hedge price risk exposure. Oum and Oren (2008) developed a self-financed hedging portfolio consisting of derivatives contracts, and they obtained the optimal hedging strategy in order to hedge price, and volumetric risk maximizing the expected utility of hedge profit for the Load Serving Entities (LSE). But this work shows that their hedging position can be improved including weather derivatives in order to complete the hedging over volumetric risk exposure.

3 Theoretical Results

This chapter was developed under an independence assumption in which there is no correlation between price and the weather index, but quantity and the weather index are strongly correlated, this assumption will be proof statistically in Chapter III using real data from electric power markets in US and Colombia. Thus, let $y(p,q)$ be the LSE's profit from serving the customers' demand q at the fixed retail rate r at time $T, x(p)$ is a function of the Spot price at time $T, z(\iota)$ is a function of the weather at time T and Y is the overall profit. The hedged profit

$$Y(p,q,x(p),z(\iota)) = y(p,q) + x(p) + z(\iota) \tag{3.2}$$

$$\text{Where, } y(p,q) = (r-p)q$$

This portfolio considers that an LSE has to provide electric power on demand at the difference between the fixed price r and the spot price on the wholesale market p. The LSE's preference utility is characterized by a concave utility function U defined over the total profit $Y(p,q,x(p),z(\iota))$ at time 1. Let $f(p,q)$ be the joint density function for positive p and q defined on the probability measure **P** which represents the beliefs on the realization of p and q. let **Q** be a risk neutral probability measure which is not unique since the electric power market is incomplete and $g(p)$ the density function of p under **Q**. Then the problem can formulated as follows:

$$\max_{x(p),z(\iota)} E[U(y(p,q),x(p),z(\iota))] \tag{3.3}$$

$$s.t\ E^Q[x(p)] = 0$$

$$E^Q[z(\iota)] = 0$$

VaR constraint could be expressed such as:

$$VaR_y(Y(x^*(p),z^*(\iota))) \leq V_0$$

It costs zero to construct a portfolio at time 0, where $E[.]$ and $E^Q[.]$ denote expectations under the probability measure **P** and **Q**, respectively.

3.1 Optimal pay-offs of the hedging strategy

Here we give an explicit solution to the optimization problem showed in (3.3). We obtain an optimal pay-off of the hedging strategy which depends on the utility function that describes the LSE's preferences.
The LSE's hedging problem of price and volume risk under VaR criteria has been considered by Oum and Oren (2008), Kleindorfer and Li (2005), Woo et al (2004), and Wagner et al (2003). VaR defined as a maximum possible loss with $(1-\gamma)$ percent confidence, is considered such as risk measure in practice. Furthermore, the optimization problems with the VaR risk measure are hard to solve analytically without very restrictive assumptions more in the case of both price and quantity are volatile.

Optimality condition

Let $x(p)$ the pay-off of the hedging strategy against price risk, $z(\iota)$ the pay-off of the hedging strategy against volumetric risk, and U is the utility function that describes the LSE's preferences. Thus, the optimal pay-offs of the hedging strategy against price and volumetric risk is the solution of the following optimization problem:

$$\max_{x(p),z(\iota)} E[U(y(p,q),x(p),z(\iota))]$$

$$s.t\ E^Q[x(p)] = 0$$

$$E^Q[z(\iota)] = 0$$

$$VaR_y(Y(x^*(p), z^*(\iota))) \leq V_0$$

The optimal pay-offs $x^*(p)$ and $z^*(\iota)$ are:

$$E\big(U'(Y(p,q,x^*(p),z(\iota))|p)\big) = \lambda_x^* \frac{g_x(p)}{f_x(p)}$$

$$E\big(U'(Y(p,q,x^*(p),z(\iota))|\iota)\big) = \lambda_z^* \frac{g_z(\iota)}{f_z(\iota)}$$

Where λ is the Lagrange multiplier, and for an agent who maximizes mean-variance expected utility of profit,

$$U(Y) = Y - \frac{1}{2}a(Y^* - E[Y^*]^2)$$

Theorem 1. *Based on Kleindorfer and Li (2005) and Oum and Oren (2008), the assumption in this part is that $VaR(Y(x,z))$ is determined by $PrX_t \geq -VaR = \gamma$, where X_t denotes the typical daily cash flow. Therefore, $VaR_t = z(\gamma)\sigma_t - \mu_t$, where $z(\gamma)$ is the z-score of a standardized normal random variable. There exists a continuous function $\eta : (E, \Sigma, \gamma) \to \Re$, and that the function is strictly increasing in σ and where $Var_\gamma(\mu, \sigma, \gamma) = \eta(\mu, \sigma, \gamma) - \mu$ is non-increasing in μ, then:*

$$P\big(Y(x,z) \leq \mu_\gamma - \eta(\mu, \sigma, \gamma)\big) \equiv 1 - \gamma$$

If $Y(x,z)$ is normally distributed, then the risk aversion assumption is satisfied with $\eta(\mu, \sigma, \gamma) = Z(\gamma)\sigma$, where $Z(\gamma)$ is the standard z-score at the confidence level. Where $\eta(\mu, \sigma, \gamma)$ is continuous and increasing in σ and the VaR function $VaR_\gamma(\mu, \sigma, \gamma)$ is non-increasing in μ for $\mu = E[Y(x,z)]$ and $\sigma^2 = var(Y(x,z))$. Therefore, if $x^*(p) + z^*(\iota)$ solves the problem (4), then it can hold that:

i. If $(x^*(p), z^*(i))$ is on efficient frontier of the $(E - V)$ space, then it can hold that any feasible pair $(x(p), z(i))$ is mapped to a corresponding point $(V(Y(x,z)), E[Y(x,z)])$.

ii. We can assume that fixed $a \geq 0$, let $Y(x,z) = Y(x^a, z^a)$ be the portfolio obtained by maximizing $(E - aV)$, therefore $Y(x^a, z^a)$ is on the border of the feasible set in $(E - VaR_\gamma)$ space, and for any feasible portfolio $Y'(x,z)$ for which $E[Y'(x,z)] = E[Y(x^a, z^a)]$ and $VaR[Y'(x,z)] \geq E[Y(x^a, z^a)]$, there exists $a \geq 0$ such that $(x^*(p), z^*(\iota))$ solves $\max_{x(p)\in x(p), z(\iota)} E[Y(x,z)] - \frac{1}{2}a * var(Y(x,z))$.

Proof of Theorem 1.

The proof of Theorem 1, is provided in pag. 49 appendix 1.

Proposition 1. *Based on Oum and Oren (2008) we will show how the solution to the mean-variance problem can be used to approximate the solution to the VaR-constrained problem*

$$(x^a(p), z^a(\iota)) = argmax_{x(p)\in X(p), z(\iota)\in Z(\iota)} E[Y(x,z)] - \frac{1}{2} a * var(Y(x,z))$$
$$s.t.\ E^Q[x(p)] = 0$$
$$E^Q[z(\iota)] = 0$$

Then $E[Y(x^a(p) + z^a(\iota))]$ *and* $var(Y(x^a(p) + z^a(\iota)))$ *are monotonically non-increasing in a*

Proof of Proposition 1.

Proof. Let $a_2 > a_1 > 0$ and specify that $Y(x^{a_i} + z^{a_i}) = Y_i \quad fori = 1,2$, then

$$E(Y_1) - a_1 var(Y_1) \geq E[Y_2] - a_1 var(Y_2)$$

$$E(Y_2) - a_2 var(Y_2) \geq E[Y_1] - a_2 var(Y_1)$$

Adding the last two expressions gives

$$(a_2 - a_1) var(Y_1) \geq (a_2 - a_1) var(Y_2)$$

Then $var(Y_1) \geq var(Y_2)$

We can hold that $E[Y_1] - E[Y_2] \geq a_1(var(Y_1) - var < (Y_2)) \geq 0$ □

Proposition 2. ***:Closed-Form Results for the Independence Case*** *Based on Id Brik (2011) closed-form results for the independence case is derived, the Independence case is the case in which there is no correlation between price and the weather index, but quantity and weather index are strongly correlated.*
Maximizing the mean-variance utility function on profit,

$$E[U(Y)] = E[Y(x,z)] - \frac{1}{2} a * var(Y(x,z))$$

For maximizing mean-variance expected utility the optimal solution $x^*(p)$ *and* $z^*(\iota)$ *to problem*

$$\max_{x(p), z(\iota)} E[U(y(p,q), x(p), z(\iota))]$$

$$s.t.\ E^Q[x(p)] = 0$$
$$E^Q[z(\iota)] = 0$$

That is given by:

$$x^* = \frac{1}{a} - E[y(p,q)|p] - E[z^*|p] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_x(p)}{f_x(p)} + (E[x^*] + E[z^*])\frac{g_x(p)}{f_x(p)}$$

$$z^* = \frac{1}{a} - E[y(p,q)|\iota] - E[x^*|\iota] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_z(\iota)}{f_z(\iota)} + (E[x^*] + E[z^*])\frac{g_z(\iota)}{f_z(\iota)}$$

Under the Independence assumption that p, and ι are uncorrelated then we can establish that:

$$E[z^*|p] = E[z^*]$$
$$E[x^*|\iota] = E[x^*]$$

And, finally we have:

$$x^* = \left[\frac{1}{a} - E[y(p,q)|p] + \left(E[y(p,q)] - \frac{1}{2}\right)\frac{g_x(p)}{f_x(p)}\right] + E[x^*]\left[\frac{g_x(p)}{f_x(p)}\right] + E[z^*]\left[\frac{g_x(p)}{f_x(p)} - 1\right] \tag{3.4}$$

$$z^* = \left[\frac{1}{a} - E[y(p,q)|\iota] + \left(E[y(p,q)] - \frac{1}{2}\right)\frac{g_x(\iota)}{f_x(\iota)}\right] + E[x^*]\left[\frac{g_x(\iota)}{f_x(\iota)}\right] + E[z^*]\left[\frac{g_x(\iota)}{f_x(\iota)} - 1\right] \tag{3.5}$$

Where,

$$\begin{aligned} E[x^*] \quad = \quad & \frac{\left[\frac{1}{a} - E\left[y(p,q)|p\right] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_x(p)}{f_x(p)}\right]\frac{g_z(\iota)}{f_z(\iota)}}{\left[\frac{g_z(\iota)}{f_z(\iota)} - 1\right]\left[\frac{g_x(p)}{f_x(p)} - 1\right] - \left[\frac{g_x(p)}{f_x(p)}\right]\left[\frac{g_z(\iota)}{f_z(\iota)}\right]} \\ & - \frac{\left[\frac{1}{a} - E\left[y(p,q)|\iota\right] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_z(\iota)}{f_z(\iota)}\right]\left[\frac{g_x(p)}{f_x(p)} - 1\right]}{\left[\frac{g_z(\iota)}{f_z(\iota)} - 1\right]\left[\frac{g_x(p)}{f_x(p)} - 1\right] - \left[\frac{g_x(p)}{f_x(p)}\right]\left[\frac{g_z(\iota)}{f_z(\iota)}\right]} \end{aligned} \tag{3.6}$$

$$\begin{aligned} E[z^*] \quad = \quad & \frac{\left[\frac{1}{a} - E\left[y(p,q)|\iota\right] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_z(\iota)}{f_z(\iota)}\right] + \left[\frac{g_z(\iota)}{f_z(\iota)} - 1\right]}{\left[\frac{g_z(\iota)}{f_z(\iota)}\right]\left(\left[\frac{g_z(\iota)}{f_z(\iota)} - 1\right]\left[\frac{g_x(p)}{f_x(p)} - 1\right] - \left[\frac{g_x(p)}{f_x(p)}\right]\left[\frac{g_z(\iota)}{f_z(\iota)}\right]\right)} \\ & \left(\left[\frac{1}{a} - E\left[y(p,q)|p\right] + \frac{\left(E[y(p,q)] - \frac{1}{a}\right)g_x(p)}{f_x(p)}\right]E^Q\left[\frac{g_z(\iota)}{f_z(\iota)}\right]\right. \\ & \left. - \left[\frac{1}{a} - E\left[y(p,q)|\iota\right] + \frac{\left(E[y(p,q)] - \frac{1}{a}\right)g_z(\iota)}{f_z(\iota)}\right]\left[\frac{g_x(\iota)}{f_x(\iota)} - 1\right]\right). \end{aligned} \tag{3.7}$$

Proof of Proposition 2.

The proof of Proposition 2, is provided in pag. 51 appendix 2.

Proposition 3

Let $(p;q)$ and $(\iota;q)$ be each a 2-dimensional random vector. p is the price the LSE pays when it buys electricity and ι is the weather index used to optimize hedging. q is the quantity of electricity purchased, if $(p;q)$and $(\iota;q)$ follow a log-normal / normal distribution where,

$$(\log p, q) \sim N(\mu_{p,q}, \Sigma_{p,q})$$

Under assumption that (p,q) are correlated, the density function of q, is given by:

$$\mu_{pq} = \begin{pmatrix} \mu_p \\ \mu_q \end{pmatrix},$$

$$\sum_{pq} = \begin{bmatrix} \sigma_p^2 & \rho_{p,q}\sigma_p\sigma_q \\ \rho_{p,q}\sigma_p\sigma_q & \sigma_q^2 \end{bmatrix}$$

$$(\log p, q) \sim N(\mu_p, \mu_q, \sigma_p^2, \sigma_q^2, \rho_{p,q})$$

$$q|p \sim N\left(\mu_q + \rho_{p,q}\frac{\sigma_q}{\sigma_p}(\ln p - \mu_p), \sigma_q^2(1-\rho_{p,q}^2)\right)$$

The Independence case is special case of this expression and we can establish that when (p,q) are independent the density function of q, is given by:

$$q \sim N(\mu_q, \sigma_p)$$

And in the case of (ι,q) they are correlated so that

$$\mu_{\iota q} = \begin{pmatrix} \mu_\iota \\ \mu_q \end{pmatrix},$$

$$\sum_{\iota q} = \begin{bmatrix} \sigma_\iota^2 & \rho_{\iota,q}\sigma_\iota\sigma_q \\ \rho_{\iota,q}\sigma_\iota\sigma_q & \sigma_q^2 \end{bmatrix}$$

$$(\log \iota, q) \sim N(\mu_\iota, \mu_q, \sigma_\iota^2, \sigma_q^2, \rho_{\iota,q})$$

$$q|p \sim N\left(\mu_q + \rho_{\iota,q}\frac{\sigma_q}{\sigma_\iota}(\ln \iota - \mu_\iota), \sigma_q^2(1-\rho_{\iota,q}^2)\right)$$

Then the density function of q knowing ι is given by:

Proof of Proposition 3.

Proof. The density function of an n-dimensional normal vector, whose mean is μ and variance-covariance matrix is $\sum$, is given by:

$$f(\mathbf{x}) = \frac{1}{(2\pi)^{\frac{n}{2}}} \frac{1}{\sqrt{\det(\Sigma)}} e^{-\frac{1}{2}(\mathbf{x}-\mu)'\Sigma(\mathbf{x}-\mu)}$$

We consider a 2-dimensional normal vector (u,v), but, the density function of u knowing v is equal to the joint density of (u,v) divided by the marginal density function of v,

$$f_{u|v}(u|v) = \frac{f_{uv}(u,v)}{f_v(v)}$$

In the case of ι, because (ι,q) are correlated, the variance-covariance matrix is:

$$\Sigma = \begin{bmatrix} \sigma_\iota^2 & \rho_{\iota,q}\sigma_\iota\sigma_q \\ \rho_{\iota,q}\sigma_\iota\sigma_q & \sigma_q^2 \end{bmatrix}$$

The determinant is:

$$\det\Sigma = \sigma_\iota^2\sigma_q^2 - \rho_{\iota,q}^2\sigma_\iota^2\sigma_q^2 = \sigma_\iota^2\sigma_q^2(1-\rho_{\iota,q}^2)$$

Hence, the inverse of variance-covariance matrix is given by:

$$\Sigma^{-1} = \frac{1}{(1-\rho_{\iota,q}^2)\sigma_\iota^2\sigma_q^2} \begin{bmatrix} \sigma_q^2 & -\rho_{\iota,q}\sigma_\iota\sigma_q \\ -\rho_{\iota,q}\sigma_\iota\sigma_q & \sigma_\iota^2 \end{bmatrix}$$

The joint density of $(\ln(p);q)$ is defined by:

$$f(\mathbf{x}) = \frac{1}{2\pi} \frac{1}{\sigma_\iota\sigma_q\sqrt{1-\rho_{\iota,q}^2}} e^{-\frac{1}{2}\frac{1}{\sigma_\iota^2\sigma_q^2(1-\rho_{i,q}^2)}M}$$

Where M is can be formulated such as:

$$M = \begin{bmatrix} \log\iota - \mu_\iota \\ q-\mu_q \end{bmatrix}' \begin{bmatrix} \sigma_q^2 & -\rho_{\iota,q}\sigma_\iota\sigma_q \\ -\rho_{\iota,q}\sigma_\iota\sigma_q & \sigma_\iota^2 \end{bmatrix} \begin{bmatrix} \log\iota - \mu_\iota \\ q-\mu_q \end{bmatrix}$$

We also have the marginal density of $\ln(\iota)$:

$$f_{(\ln\iota)}(\ln\iota) = \frac{1}{2\pi}\frac{1}{\sigma_\iota} e^{-\frac{1}{2}\left(\frac{\log\iota-\mu_\iota}{\sigma_\iota}\right)^2}$$

Then we deduce the density function of q knowing $\ln(\iota)$:

$$f_{q|\ln(\iota)}(q|\ln(\iota)) = \frac{\frac{1}{2\pi}\frac{1}{\sigma_\iota\sigma_q\sqrt{1-\rho_{\iota,q}^2}}e^{-\frac{1}{2}\frac{1}{\sigma_\iota^2\sigma_q^2(1-\rho_{\iota,q}^2)}M}}{\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_\iota}e^{-\frac{1}{2}\left(\frac{\log\iota-\mu_\iota}{\sigma_\iota}\right)^2}}$$

Therefore

$$f_{q|\ln(p)}(q|\ln(p)) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_q\sqrt{1-\rho_{\iota,q}^2}}e^{-\frac{1}{2}N}$$

Where N is defined by:

$$\begin{aligned} N &= \frac{M}{\sigma_\iota^2\sigma_q^2(1-\rho_{\iota,q}^2)} - \left(\frac{\log\iota-\mu_\iota}{\sigma_\iota}\right)^2 \\ &= \frac{1}{\sigma_q^2(1-\rho_{\iota,q}^2)}\left[q-\left(\mu_q+\rho_{\iota,q}\frac{\sigma_q}{\sigma_\iota}(\ln\iota-\mu_\iota)\right)\right]^2 \end{aligned}$$

We finally obtain:

$$f_q^{\{\ln\iota\}}(q) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_q\sqrt{1-\rho_{\iota,q}^2}}e^{-\frac{1}{2}\frac{\left[q-\left(\mu_q+\rho_{\iota,q}\frac{\sigma_q}{\sigma_\iota}(\ln\iota-\mu_\iota)\right)\right]^2}{\sigma_q^2(1-\rho_{\iota,q}^2)}}$$

In other words,

$$q_{|\iota} \sim N\left(\mu_q+\rho_{\iota,q}\frac{\sigma_q}{\sigma_\iota}(\ln\iota-\mu_\iota), \sigma_q^2(1-\rho_{\iota,q}^2)\right)$$

□

Finally the marginal distribution of p,ι and q are as follows:

Under P:

$$\begin{aligned} \ln p &\sim N(\mu 1_p, \sigma_P^2) \\ q &\sim N(\mu_q, \sigma_q^2) \\ \ln\iota &\sim N(\mu 1_\iota, \sigma_\iota^2) \\ Corr(\ln p, q) &= \rho_{p,q} \\ Corr(\ln\iota, q) &= \rho_{\iota,q} \end{aligned}$$

Under Q:

$$\begin{aligned}\ln p &\sim N(\mu 2_p, \sigma_P^2)\\ \ln \iota &\sim N(\mu 2_\iota, \sigma_\iota^2)\end{aligned}$$

From a density function of log-normal distribution, we have:

$$\begin{aligned}\frac{g_x(p)}{f_x(p)} &= e^{\frac{\mu 2_p - \mu 1_p}{\sigma_P^2}\ln p + \frac{\mu 1_p^2 - \mu 2_p^2}{\sigma_P^2}}\\ \frac{g_z(\iota)}{f_z(\iota)} &= e^{\frac{\mu 2_\iota - \mu 1_\iota}{\sigma_\iota^2}\ln p + \frac{\mu 2_i^2 - \mu 1_\iota^2}{\sigma_\iota^2}}\end{aligned}$$

$$\begin{aligned}E^Q\left[\frac{g_x(p)}{f_x(p)}\right] &= e^{\left(\frac{\mu 2_p - \mu 1_p}{\sigma_P}\right)^2}\\ E^Q\left[\frac{g_z(\iota)}{f_z(\iota)}\right] &= e^{\left(\frac{\mu 2_i - \mu 1_\iota}{\sigma_\iota}\right)^2}\end{aligned}$$

where,

$$\begin{aligned}\frac{g_x(p)}{f_x(p)} &= \frac{\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_p}e^{-\frac{1}{2}\left(\frac{\log p - \mu 2_p}{\sigma p}\right)^2}}{\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_p}e^{-\frac{1}{2}\left(\frac{\log p - \mu 1_p}{\sigma p}\right)^2}}\\ &= e^{\frac{1}{2}\left(\frac{\log p - \mu 1_p}{\sigma p}\right)^2 - \frac{1}{2}\left(\frac{\log p - \mu 2_p}{\sigma p}\right)^2}\\ &= e^{\frac{\mu 2_p - \mu 1_p}{\sigma p}\log p - \frac{1}{2}\frac{(\mu 2_p)^2 - (\mu 1_p)^2}{\sigma^2 p}}\end{aligned}$$

Under Q,

$$\frac{\mu 2_p - \mu 1_p}{\sigma p}\log p - \frac{1}{2}\frac{(\mu 1_p)^2 - (\mu 2_p)^2}{\sigma^2 p} \sim N\left(\frac{\mu 2_p - \mu 1_p}{\sigma p}\mu 2_p + \frac{1}{2}\frac{(\mu 1_p)^2 - (\mu 2_p)^2}{\sigma^2 p}, \left(\frac{\mu 2_p - \mu 1_p}{\sigma p^2}\right)^2 \sigma p^2\right)$$

Then

$$\begin{aligned}E^Q\left[\frac{g_x(p)}{f_x(p)}\right] &= e^{\frac{\mu 2_p - \mu 1_p}{\sigma p}\mu 2_p + \frac{1}{2}\frac{(\mu 1_p)^2 - (\mu 2_p)^2}{\sigma^2 p} + \frac{1}{2}\left(\frac{\mu 2_p - \mu 1_p}{\sigma p^2}\right)^2 \sigma p^2}\\ &= e^{\left(\frac{\mu 2_p - \mu 1_p}{\sigma p}\right)^2}\end{aligned}$$

Then, under a bivariate log-normal distribution, we can compute the next mathematical means:

$$E[y(p,q)|\iota] = (r-p)E(q|\iota) = (r-p)\left(\mu_q + \rho_{\iota,q}\frac{\sigma_q}{\sigma_\iota}(\ln\iota - \mu_\iota)\right)$$

$$\begin{aligned} E[y(p,q)] &= E[(r-p)q] \\ &= r\mu_q - E[pq] \\ &= r\mu_q - \mu_q e^{\mu_P + \frac{1}{2}\sigma_p^2} \end{aligned}$$

Hence,

$$E^Q[E[y(p,q)]] = \mu_q\left(r - e^{\mu 2_P + \frac{1}{2}\sigma_p^2}\right)$$

$$\begin{aligned} E^Q\left[E[y(p,q)|\iota]\right] &= \left(r - \exp\left(\mu 2_\iota + \frac{1}{2}\sigma_\iota^2\right)\right)\left(\mu_q + \rho_{\iota,q}\frac{\sigma_q}{\sigma_\iota}\mu 1_\iota\right) \\ &\quad + \rho_{\iota,q}\frac{\sigma_q}{\sigma_\iota}\left(r\mu 2_\iota - (\mu 2_\iota + \sigma_\iota^2)\exp\left(\mu 2_\iota + \frac{1}{2}\sigma_\iota^2\right)\right) \end{aligned}$$

3.2 The replication of pay-offs

Carr and Madan (2001) showed that any continuously differentiable functions $x(p)$and $z(\iota)$ can be written in the following form: for an arbitrary positive s,

$$x(p) = [x(s) - x'(s)s] + x'(s)p + \int_0^s x''(K)(K-p)^+ dK + \int_s^\infty x''(K)(p-K)^+ dK$$

$$z(\iota) = [z(s) - z'(s)s] + z'(s)\iota$$

In this case, if F_p is the forward price of electricity and F_ι is the forward weather-related claim, the property proved by Carr and Madan (2001) has the next interpretation:
$f(p,q) = (r-p)q + x(p) + z(\iota)$,

$$x(p) = x(F_p).1 + x'(F_p)(p-F_p) + \int_0^{F_p} x''(K)(K-p)^+ dK + \int_{F_p}^\infty x''(K)(p-K)^+ dK$$

$$z(\iota) = z'(F_\iota)(\iota - F_\iota) + \int_0^{F_p} x''(K)(K-p)^+ dK + \int_{F_p}^\infty x''(K)(p-K)^+ dK$$

$$z(\iota) = z'(f_\iota)(\iota - f_\iota) \tag{3.8}$$

To replicate in continuous time a hedging strategy against price risk and quantity risk, the LSE should have a position on:

- $x(F_p)$ units of bonds
- $x'(F_p)$ units of forward price
- $z'(F_\iota)$ units of forward weather-related claim
- $x''(K)dK$ units of put options with strike K for $K < F_p$
- $x''(K)dK$ units of call options with strike K for $K > F_p$

In practice, we do not have a continuous set of strike prices and we need to work in discrete time. Thus, by assuming we have n strike prices for put options and m strike prices for call options such that $0 < K_1 < K_n < F_p < K'_1 < K'_2 < \cdots < K'_m$, replicating the hedging strategy should require a position on:

- $x(F_p)$ units of bonds
- $x'(F_p)$ units of forward price
- $z'(F_i)$ units of forward weather-related claim
- $\frac{1}{2}(x''(K_{i+1}) - x''(K_{i-1}))$ units of put options with strike $(K_i, i = 1, \cdots n)$
- $\frac{1}{2}(x''(K'_{i+1}) - x''(K'_{i-1}))$ units of call options with strike $(K'_i, i = 1, \cdots n)$

In this approximation scheme, the error will be small if $x''(p)$ is a constant in each interval between two consecutive strike prices, and when price realizations p are close to the discrete strike prices.

4 Empirical Work

4.1 Implementation Algorithm

The issue is to know how many forwards and options at a given strike price the LSE should purchase. Note that the hedging portfolio also includes money market accounts, letting the LSEs borrow money to finance hedging instruments. It is a one-period model where the hedging portfolio is built at time 0 for a delivery at time 1. The feasible set of the VaR-constrained problem is restricted to the solution of mean-variance problem for varying a (see Theorem 1 and Proposition 1). Thus, the solution to VaR-constrained optimization problem can be obtained in the next algorithm:

i Fix parameters including range for a (min, max and steps).

ii Fix number of simulations num_trab "large".

iii Generate random price p, load q and weather variable w, using a multivariate normal distribution.

iv Compute the payoff $x^*(p)$ and $z^*(\iota)$ (Equations 4 and 5).

v We can obtain $(x^a(p),\ z^a(\iota))$ that maximizes:

$$E[Y(x,z)] - \frac{1}{2}a * var(Y(x,z))$$

vi For each a, calculate associated $VaR(a) \equiv VaR(Y(x^a, z^a))$ such that $P\{Y(x^a, z^a) \geq -VaR(a)\} = \gamma$

vii Find smallest $a(a_{opt})$ such that $VaR_\gamma(Y(x^a, z^a)) \leq V_0$

viii Using a_{opt} in order to find $Y(x^(a_{opt}),\ z^(a_{opt}))$. $Y(\cdot)$, be the profit distribution of the expected utility maximizing solution, under Optimal Static Hedging including the weather claim (see figure 4.3)

ix Using the payoff functions $x(a_{opt})$, and $z^(a_{opt})$, and based on Carr and Madan (2001) we can define the replication of payoff (Equation (3.8)), (see figure 4.4).

This algorithm above permits us obtain the optimal static hedging profit distribution using two claims and the concerning replication payoff function.

4.2 Empirical Result

Computing an approximate optimal VaR-constrained volumetric hedging problem according to the above development, we will show two groups of results: results under the independence assumption, and also under the general case. In both we will present the comparison of different possibilities, which are: without-hedge, hedging using $x^*(p)$ following Oum and Oren's model, and my proposal using $[x^*(p) + z'^*(\iota)]$; note that $x^*(p) \neq x'^*(p)$, because $x^*(p)$ corresponds to Oum and Oren model. Following the same application made by Oum and Oren (2008), the hedging strategy for an LSE that maximizes the expected pay-off with VaR constraint of -$60.000 is composed by a hypothetical LSE that charges a flat retail rate of $120 per MWh. The spot price p at which the agent has to buy electric power, the weather-index ι and the quantity q is the load at which the LSE supplies in a fixed interval; the three variables, price, temperature and quantity are volatile and these variations affect the agents' revenues; that is the problem that agents will try to solve using an optimal static hedging solution. In order to obtain the solution of the mean-variance problem for varying a we assume that P and Q distributions are different. All of three variables are distributed according to a bivariate distribution in log price and quantity, and the log weather-index and quantity, as follows:

Independence Case:

Under P: $lnp \sim N(4, 0.7^2)$ $q \sim N(3000, 650^2)$ $logi \sim N(2.2, 0.0821^2)$
$Corr(lnp, q) = 0$ $Corr(ln\iota, q) = 0.5$
Under Q:$ln\iota \sim N(4, 0.7^2)$ $log\iota \sim N(2.1, 0.0821^2)$

Taking in account the last parameters, and the normal bivariate probability distribution, we fitted Monte-Carlo simulation technique to generate spot price, load and weather index patterns. Figure 4.1 shows the spot price, load and weather index patterns. Using the normality assumption on the volume of consumption and on the logarithm of spot prices, we go easy with the volatility exerted on these two variables. Thus, despite an average of 67.86 U.S. $/MWh in the spot price, it can reach heights up to 58 U.S. $/MWh, which is actually quite well the reality of the market. The same applies to the trading volume that varies between 10 MWh and 1818 MWh with an average of 516 MWh.

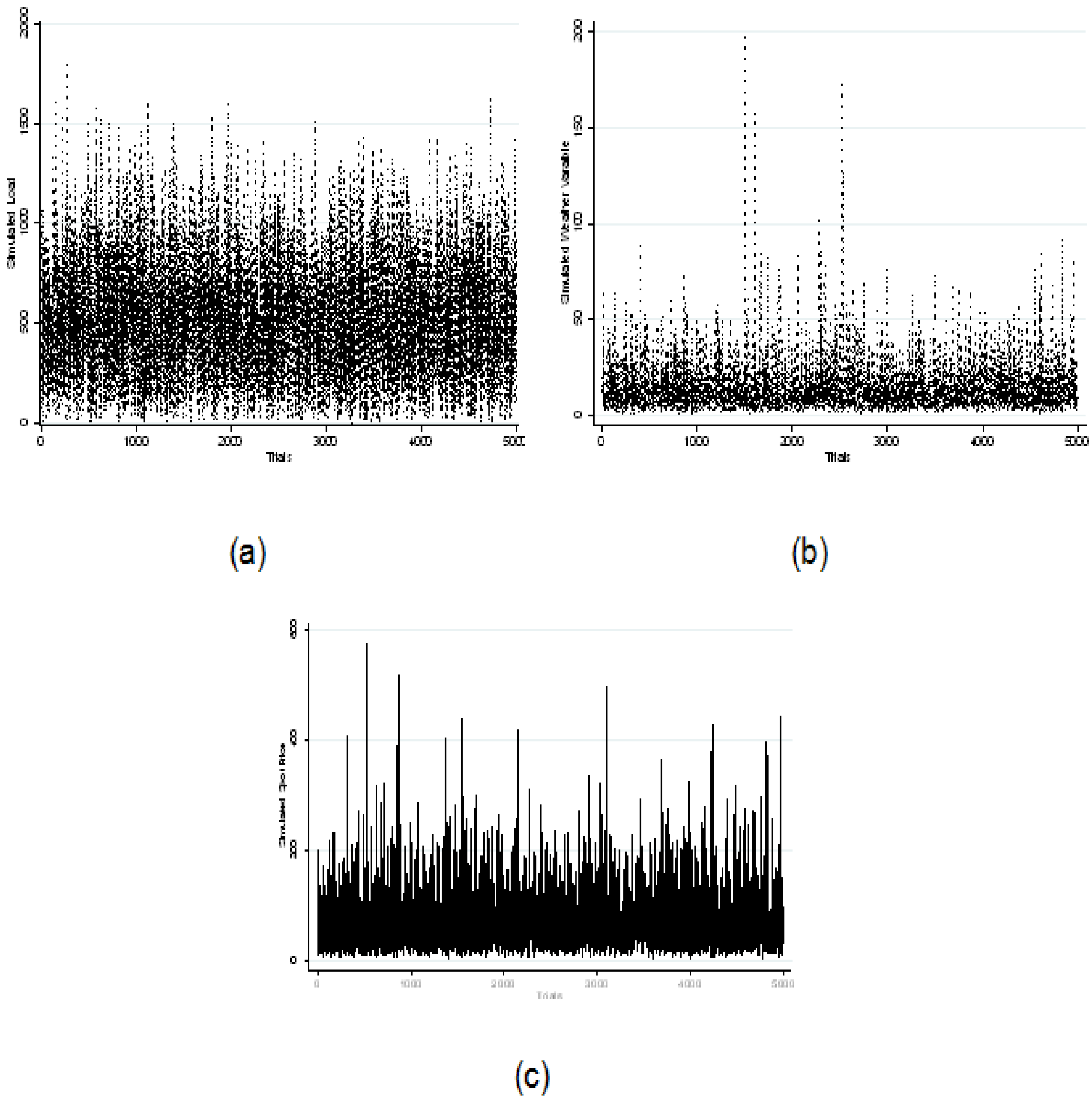
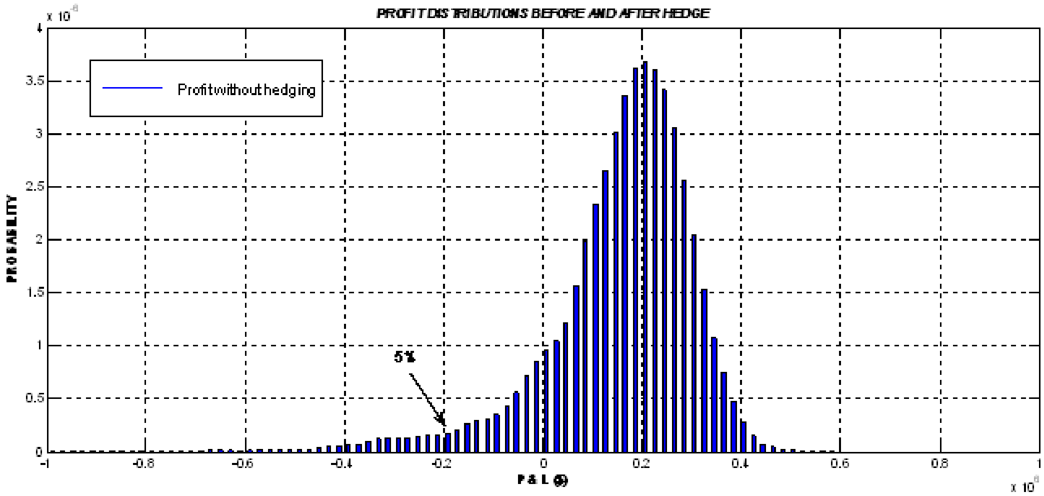
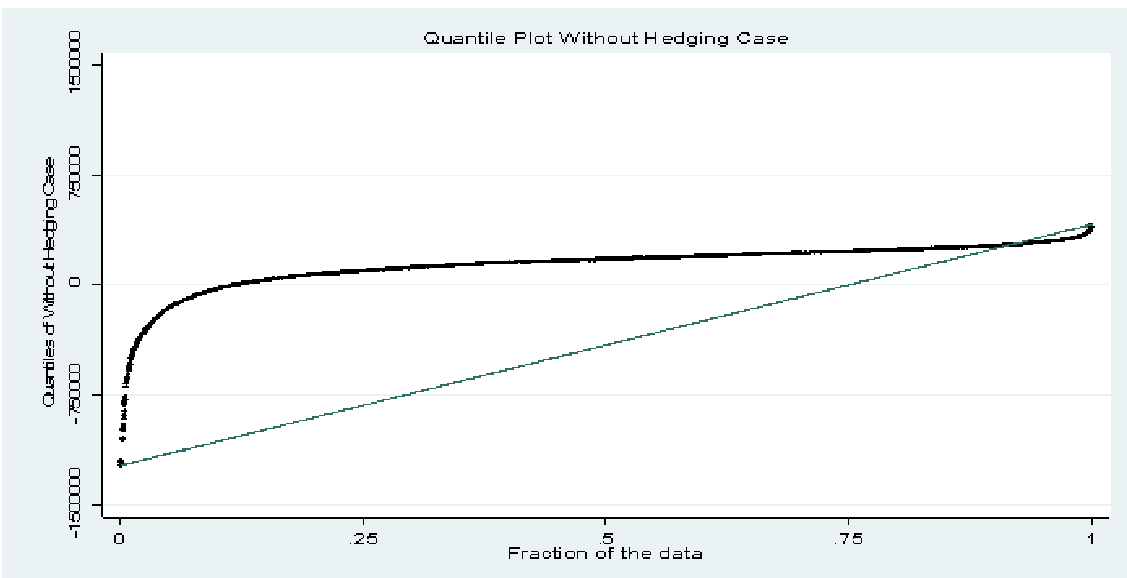


Figure 4.1: Simulated patterns using Oum and Oren's parameters for independence case. (a) Load, (b) Weather-index, (c) Spot Price.



(a)



(b)

Figure 4.2: Distribution of profit without hedging y(p,q)=(r-p)q assuming r=$120/MWh (a) Normal bivariate distribution of profit. (b) Quantile plot without hedging.

Figure 4.2 shows the basis of the problem; profit distribution without hedging, considering the distribution of parameters. The profit without hedging only considers the LSE fixed rate, the spot price and quantity denoted by $y(p,q) = (r-p)q$. Without hedging, 1205 simulations of 10000 are negative. So there is a probability of 12.10% of have a net loss for the LSE. Moreover, the VaR is estimated at 5%, -1.78E05. In the end, the average of simulations equals to $-1.49E05$.

Due to P distribution being different from Q, for various levels of risk aversion a there exists a mean-variance problem solution. We restrict the set of solutions using the VaR-constrained problem (see theorem 1) in order to find the optimal one.

Figure 4.3 shows the comparison of different possibilities, which are: without-hedge, hedging using $x*(p)$ following Oum and Oren model, and my proposal using $[x'^*(p) + z'^*(\iota)]$, for the independence case.

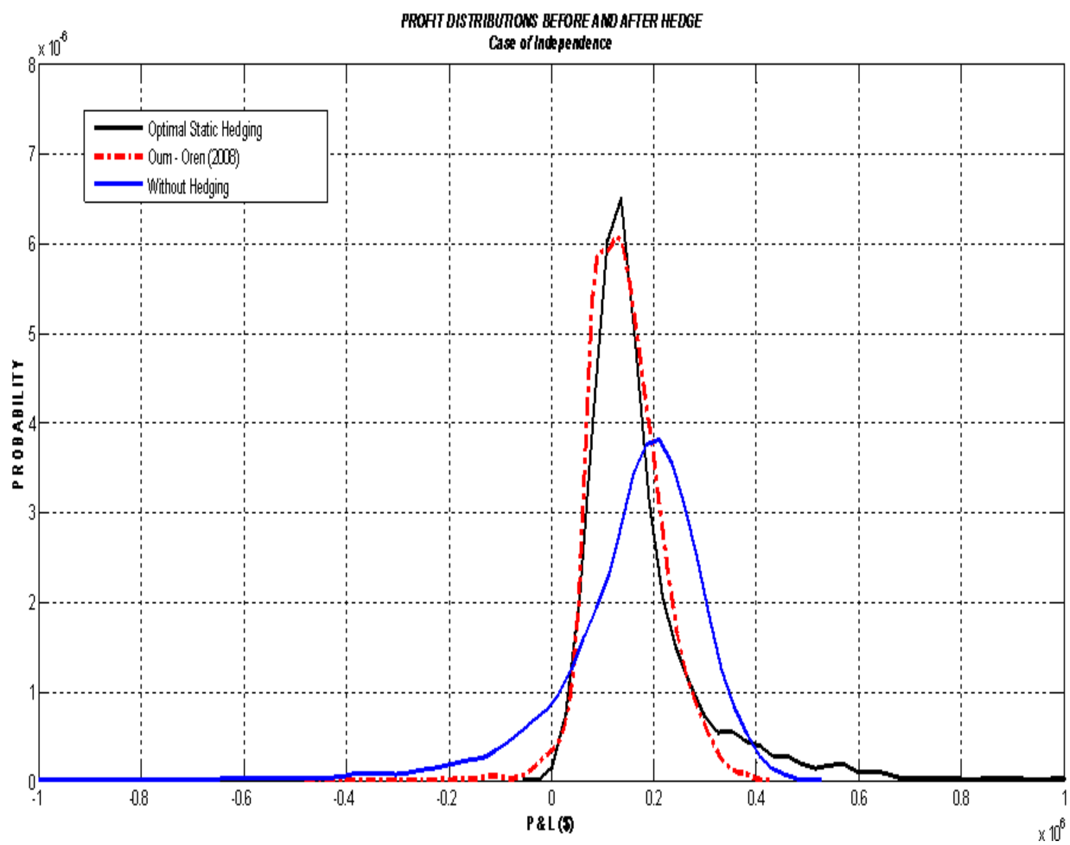


Figure 4.3: Profit distributions under three cases: without-hedge, Oum and Oren results and my proposal $[x'^*(p) + z'^*(\iota)]$.

Figure 4.3 compares profit distributions under different scenarios of hedging. I could notice that Oum and Oren profit distribution improves in terms of mean, standard deviations and the VaR measure the scenario without hedging, and the independence case profit distribution improved by mean and standard deviation what had been already achieved by Oum and Oren, (see Table 4.1).

Figure 4.4 shows the optimal mean-variance hedging strategy corresponding to optimal a^*. I show the optimal payoff function obtained as an approximation for VaR-constrained problem (Equation (3.8))

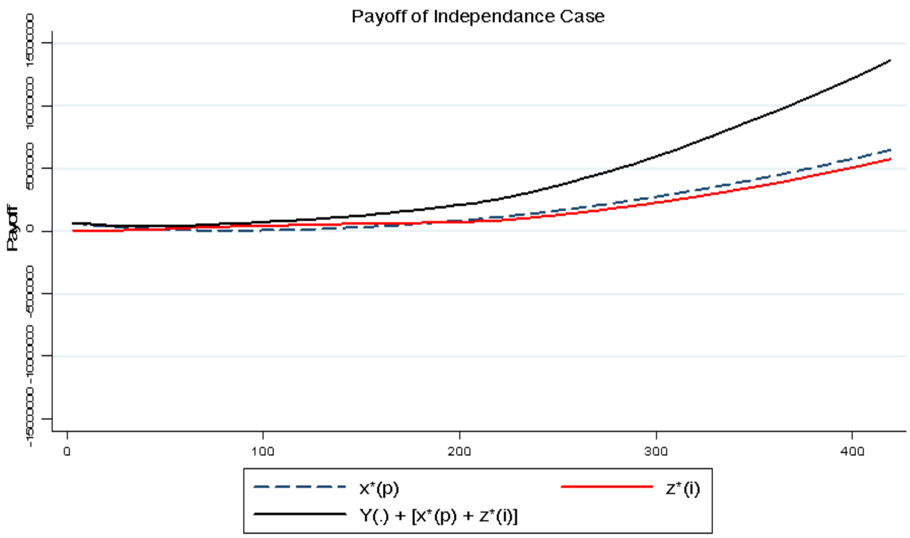


Figure 4.4: Hedging Strategy for an LSE that maximizes the expected payoff with VaR constraint. Black line represents the hedging position; dashed line represents the payoff linear in price, and the red line exhibit the weather payoff.

Figure 4.4 shows the expected payoff functions according to different claims to hedge. We could notice that the sum of payoffs $x^*(p) and z^*(\iota)$ result improves in terms of value the individual payoffs result, and the independence case profit plus the sum of payoffs improved by mean and standard deviation what had been already achieved by the scenario without hedging.

Table 4.1 shows the percentiles for the cases shown in the figure 4.4.

Table 4.1: Percentiles when fewer than three cases occur: without-hedge, Oum and Oren results and my proposal $[x'^*(p) + z'^*(\iota)]$ for independence.

Percentiles			
	Without Hedging	**Oum-Oren Case Parameters**	**Independence Case**
1%	-5.18E+05	-4.81E+04	2.74E+04
5%	-1.49E+05	3.61E+04	3.57E+04
10%	-2.21E+04	5.87E+04	2.31E+04
25%	1.01E+05	8.44E+04	9.36E+04
50%	1.80E+05	1.27E+05	1.34E+05
75%	2.32E+05	1.75E+05	1.64E+05
90%	2.73E+05	2.20E+05	2.14E+05
95%	2.96E+05	2.49E+05	2.43E+05
99%	3.40E+05	3.04E+05	3.39E+05
Mean	1.41E+05	1.26E+05	1.83E+05
Std. Dev	1.62E+05	6.93E+04	6.51E+04
Skewness	-2.99E+00	-1.06E+01	4.65E+00
Kurtosis	1.71E+01	4.75E+00	1.39E+01

Table 4.1 shows diverse percentiles that compare the different scenarios of hedging. We could note that Oum and Oren methodology sharply the scenario without hedging; whoever the independence case improved by mean and standard deviation what had been already achieved by Oum and Oren. For instance, the independence case shows a mean of $1.83E+05$ whereas Oum and Oren is $1.26E+05$. Regarding the standard deviation, the independence case is $4.2E+03$ below Oum and Oren.

5 Conclusions

Transfer of climatic risk exposure to capital markets allows transforming of non-tradable risk into financial assets which are, of course, tradable. Using forward contracts over weather offers to agents the chance to hedge their volumetric risk exposure in electric power markets. While the optimal electric power portfolio is an open problem in stating specific conditions to define the payoff structure of portfolios according to the agents' exposure, this chapter presents closed-form results that permit the second claim to complete the market.

As an extension of Oum and Oren (2008), this chapter proposes a method to mitigate price and volumetric risk, exploiting significant correlation among price, quantity and weather-index. I developed the optimization problem of portfolios composed of two claims, price and weather, according factors featured in electric power markets such as price volatility, price spikes, and climatic conditions that influence quantity volatility. Our results arose due to the inclusion of the weather variable, and the hedging position was improved by minimizing the risk and increasing mean according to positive correlation among price, quantity, and the weather variable. For the electric power market, wholesale spot price and quantity are volatile, and the latter is correlated with weather conditions. Results confirm that the weather payoff allows adjustment of hedge strategy with the price payoff in order to hedge the double exposure of the agents. Table 4.1 shows statistics of all of the cases and also confirms that general case hedging is better than the no-hedging scenario and Oum-Oren model. Limiting the problem using a VaR-constrained solution permits to address the solution against the non-linearity condition of the hedging strategy. The hedging portfolio is solved using the price and weather payoff functions that represent the payoff of electric power derivatives and the payoff of the forward weather-related index, solving those payoffs we obtain a hedging portfolio in realistic conditions.

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Appendix 1:Proof of Theorem 1.

I am taking into account, as an assumption, that the portfolio once composed, won't be changed during the whole horizon-time, and under the Risk Aversion assumption[1] on the left-tail of the $Y(x,z)$ distribution, let $Y(x,z)$ be a random variable (representing the profit of the agents) whose probability distribution function is continuous with mean μ, and standard deviation σ.

Proof. i. $(x^*(p),z^*(\iota))$ is the optimal solution to (3.3) and is on the efficient frontier of $(E-VaR_\gamma)$ plane. Then considering the alternative $(x^*(p),z^*(\iota)) \in X(p),Z(\iota)$ that reduce the variance without reducing the mean of the $Y(x(p),z(\iota))$ distribution, then $\mu \geq \mu^*$ where $\mu = E[Y(x(p),z(\iota))]$ and $\mu^* = E[Y(x^*(p),z^*(\iota))]$ and $\sigma^2 < \sigma^{*2}$ where $\sigma^2 = V[Y(x(p),z(\iota))]$ and $\sigma^{*2} = V[Y(x^*(p),z^*(\iota))]$ then $\eta(\mu,\sigma,\gamma)$ which is increasing in σ and non-increasing in μ.

$$\begin{aligned} &VaR_\gamma(Y(x(p),z(\iota))) = \eta(\mu,\sigma,\gamma) \\ &\leq \eta(\mu^*,\sigma,\gamma) < \eta(\mu^*,\sigma^*,\gamma) \\ &= VaR_\gamma(Y(x^*(p),z^*(\iota))) \end{aligned}$$

Thus, the statement shows before contradicts the assumption that $(x^*(p),z^*(\iota))$ is on the efficient frontier in the $(E-VaR_\gamma)$ plane. This implies that for a fixed γ a feasible perturbation on $(x^*(p),z^*(\iota))$ that solves (3.3) cannot reduce the variance of the $Y(x(p),z(\iota))$ distribution without increasing the mean. Hence, $(x^*(p),z^*(\iota))$ is also on the efficient frontier in the $(E-V)$ plane.

ii. Let $Y(x(p),z(i))$ be an electric power portfolio on the efficient frontier in $(E-V)$ space; the equation $(E = aV + c)$ defines a straight line for any constant c.
Thus, maximizing $E\{Y(x(p),z(\iota))\} - aV\{Y(x(p),z(\iota))\}$ is equivalent to maximizing $(E-aV)$. Then, any $Y(x(p),z(\iota))$ maximizing $E\{Y(x(p),z(\iota))\} - aV\{Y(x(p),z(\iota))\}$ must be on the efficient frontier in $(E-V)$ space. This same $Y(x(p),z(\iota))$ must clearly also be on the efficient frontier in $(E-\sigma)$ space, due to any portfolio $Y'(x(p),z(\iota))$ with the same or equal expected payoff and smaller variance, having smaller standard deviation: if $Y(x(p),z(\iota))$ has expected profit μ_1 and standard deviation μ_1. Whether there is a portfolio with expected profit and VaR, say μ_2, and VaR_2 such that $\mu_1 = \mu_2$ and $VaR_2 < VaR_1$, thus $\eta(\mu_2,\sigma_2,\gamma) - \mu_2 < \eta(\mu_1,\sigma_1,\gamma) - \mu_1$ and hence we have $\eta(\mu_2,\sigma_2,\gamma) < \eta(\mu_1,\sigma_1,\gamma)$ by the which the monotonicity of η in σ implies $\sigma_2 < \sigma_1$. Which is impossible since $Y(x(p),z(\iota))$ was assumed to be on the $E-\sigma$ frontier. Then, $Y(x(p),z(\iota))$ be on the left border of the feasible set in $(E-VaR)$ space.
Sharpe (2000) establishes that taking in account the linear constraints, the efficient frontier in $(E-\sigma)$ space is concave. Furthermore, if for any portfolio ι we have (E_i,σ_i),$(E_{i+1}$,$\sigma_{i+1})$ and (E_{i+2},σ_{i+2}) are on the efficient frontier and $E_{i+2} = \delta E_i + (1-\delta)E_{i+1}$ for some δ, with $0 < \delta < 1$, then $\sigma_{i+2} \leq \delta\sigma_i + (1-\delta)\sigma_{i+1}$. We can see that the fronties in $(E-V)$ space is also concave. That is, for the same portfolios we show $\sigma^2_{i+2} \leq \delta\sigma^2_i + (1-\delta)\sigma^2_{i+1}$ then $\sigma^2_{i+2} \leq \delta^2\sigma^2_i + (1-\delta)^2\sigma^2_{i+1} + 2\delta(1-\delta)\sigma_i\sigma_{i+1}$
Hence, $\sigma^2_{i+2} - [\delta\sigma^2_i + (1-\delta)\sigma^2_{i+1}] \leq \delta^2\sigma^2_i + (1-\delta)^2\sigma^2_{i+1} + 2\delta(1-\delta)\sigma_i\sigma_{i+1} - [\delta\sigma^2_i + (1-\delta)\sigma^2_{i+1}] = (\delta^2-\delta)(\sigma_i - \sigma_{i+1})^2 < 0$
Therefore, $\sigma^2_{i+2} < \delta\sigma^2_i + (1-\delta)\sigma^2_{i+1}$ from the concavity of efficient frontier in $(E-V)$ space, we can see that if $Y(x(p),z(i))$ is on efficient frontier in $(E-V)$ space, there will be a straight-line tangent to

[1] See Sharpe (2000) and Kleindorfer and Li (2005)

the frontier curve at $Y(x(p),z(i))$. Choosing a as the slope of this line, and maximizing $(E-aV)$ will result in the $(E-V)$ of the portfolio $Y(x(p),z(i))$

$\square$

Appendix 2:Proof of Proposition 2

The Lagrange function for the constrained optimal problem is given by,

$$\partial(x,z) = \iint_{R^2} U\left(Y|p,\iota\right) f_{p,\iota}(p,\iota) dp d\iota - \lambda_x \int_{R^2} x(p) g_x(p) dp - \lambda_x \int_{R^2} z(\iota) g_z(\iota) d\iota$$

$$\overrightarrow{grad}\left(\mathcal{L}(x,z)\right) = \vec{0}$$

With the Lagrangian multipliers λ_x, λ_z and the marginal density functions $f_x(p)$ of p and $f_z(\iota)$ of ι under P, by differentiation of $\mathcal{L}(x(p))$ with respect to $x(p)$ and $\mathcal{L}(z(\iota))$ with respect to $z(\iota)$ results in

$$\frac{\partial \mathcal{L}}{\partial x} = E\left[\frac{\partial Y}{\partial x} U'(Y)|p\right] f_x(p) - \lambda_x g_x(p) = 0 \tag{5.1}$$

$$\frac{\partial \mathcal{L}}{\partial z} = E\left[\frac{\partial Y}{\partial z} U'(Y)|\iota\right] f_z(\iota) - \lambda_z g_z(\iota) = 0$$

By the Euler equation from (1) and substituting $\frac{\partial Y}{\partial x} = 1$ and $\frac{\partial Y}{\partial z} = 1$ from (1) yields the first order conditions for the optimal solutions $x^*(p)$ and $z^*(\iota)$ as follows:

$$\begin{aligned} E\left[U'(Y(p,q,x^*(p),z^*(\iota))|p\right] &= \lambda_x^* \frac{g_x(p)}{f_x(p)} \\ E\left[U'(Y(p,q,x^*(p),z^*(\iota))|\iota\right] &= \lambda_z^* \frac{g_z(\iota)}{f_z(\iota)} \end{aligned}$$

For an agent who maximizes mean-variance expected utility of profit,

$$U(Y) = Y - \frac{1}{2} a(Y^* - E[Y^*]^2)$$

Then, by substituting $U' = (1 - aY^*)$, the optimal condition is given by:

$$\begin{aligned} 1 - aE\left[Y^*|p\right] &= \lambda_x^* \frac{g_x(p)}{f_x(p)} \\ 1 - aE\left[Y^*|\iota\right] &= \lambda_z^* \frac{g_z(\iota)}{f_z(\iota)} \end{aligned}$$

Equivalently,

$$\begin{aligned} f_x(p) - aE\left[Y^*|p\right] f_x(p) &= \lambda_x^* g_x(p) \\ f_z(\iota) - aE\left[Y^*|\iota\right] f_z(\iota) &= \lambda_z^* g_z(\iota) \end{aligned}$$

Integrating both sides with respect to p and ι from $-\infty$ to ∞, we obtain $\lambda_x^* = 1 - aE[Y^*]$ and $\lambda_z^* = 1 - aE[Y^*]$ by substituting λ_x^*, λ_z^* and $Y^* = y(p,q) + x^*(p) + z^*(\iota)$ gives,

$$\begin{aligned}
f_x(p) - a\left(E[Y^*|p] + E[x^*|p] + E[z^*|p]\right) f_x(p) &= [1 - aE[Y^*]]g_x(p) - a\left(E[x^*] + E[z^*]\right) g_x(p) \\
f_z(\iota) - a\left(E[Y^*|\iota] + E[x^*|\iota] + E[z^*|\iota]\right) f_x(\iota) &= [1 - aE[Y^*]]g_z(\iota) - a\left(E[x^*] + E[z^*]\right) g_z(\iota)
\end{aligned}$$

Then,

$$\begin{aligned}
f_x(p) - a\left(E[y(p,q)|p] + x^*(p) + E[z^*|p]\right) f_x(p) &= \left(1 - aE[y(p,q)] - a\left(E[x^*] + E[z^*]\right)\right) g_x(p) \\
f_z(\iota) - a\left(E[y(p,q)|\iota] + z^*(\iota) + E[x^*|\iota]\right) f_z(\iota) &= \left([1 - aE[y(p,q)]] - a(E[x^*] + E[z^*])\right) g_z(\iota)
\end{aligned}$$

By rearranging we obtain:

$$\begin{aligned}
x^* &= \frac{1}{a} - E\left[y(p,q)|p\right] - E[z^*|p] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_x(p)}{f_x(p)} + \left(E[x^*] + E[z^*]\right)\frac{g_x(p)}{f_x(p)} \\
z^* &= \frac{1}{a} - E\left[y(p,q)|\iota\right] - E[x^*|\iota] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_z(\iota)}{f_z(\iota)} + \left(E[x^*] + E[z^*]\right)\frac{g_z(\iota)}{f_z(\iota)}
\end{aligned}$$

If p and ι are uncorrelated then we can establish that:

$$\begin{aligned}
E[z^*|p] &= E[z^*] \\
E[x^*|\iota] &= E[x^*]
\end{aligned}$$

Finally we have:

$$\begin{aligned}
x^* &= \left[\frac{1}{a} - E\left[y(p,q)|p\right] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_x(p)}{f_x(p)}\right] + E[x^*]\frac{g_x(p)}{f_x(p)} + E[z^*]\left[\frac{g_x(p)}{f_x(p)} - 1\right] \\
z^* &= \left[\frac{1}{a} - E\left[y(p,q)|\iota\right] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_z(\iota)}{f_z(\iota)}\right] + E[z^*]\frac{g_z(\iota)}{f_z(\iota)} + E[x^*]\left[\frac{g_z(\iota)}{f_z(\iota)} - 1\right]
\end{aligned}$$

In order to obtain the final formula for the optimal payoff function under mean-variance utility the next system of equations could be utilized:

$$\begin{aligned}
x^* &= b_1(p) + a_{11}(p)E[x^*] + a_{12}(p)E[z^*] \\
z^* &= b_2(\iota) + a_{21}(\iota)E[x^*] + a_{22}(i)E[z^*]
\end{aligned}$$

We take expectation under Q

$$0 = E^{Q}[b_1(p)] + E^{Q}[a_{11}(p)]E[x^*] + E^{Q}[a_{12}(p)]E[z^*] \tag{5.2}$$

$$0 = E^{Q}[b_2(\iota)] + E^{Q}[a_{21}(\iota)]E[x^*] + E^{Q}[a_{22}(\iota)]E[z^*] \tag{5.3}$$

And subtract Eq.(5.3)*$E^{Q}[a_{12}(p)]$ from Eq.(5.2)*$E^{Q}[a_{22}(\iota)]$

$$\begin{aligned}0 = & E^{Q}[b_1(p)]E^{Q}[a_{22}(\iota)] - E^{Q}[b_2(\iota)]E^{Q}[a_{12}(p)] \\ & + \left[E^{Q}[a_{11}(p)]E^{Q}[a_{22}(\iota)] - E^{Q}[a_{21}(\iota)]E^{Q}[a_{12}(p)]\right] E[x^*]\end{aligned}$$

Where,

$$E[x^*] = \frac{E^{Q}[b_1(p)]E^{Q}[a_{22}(\iota)] - E^{Q}[b_2(\iota)]E^{Q}[a_{12}(p)]}{E^{Q}[a_{21}(\iota)]E^{Q}[a_{12}(p)] - E^{Q}[a_{11}(p)]E^{Q}[a_{22}(\iota)]} \tag{5.4}$$

By substituting $E[x^*]$ in Eq. (5.3) we obtain,

$$E[z^*] = -\frac{E^{Q}[b_2(\iota)] + E^{Q}[a_{21}(\iota)]\frac{E^{Q}[b_1(p)]E^{Q}[a_{22}(\iota)] - E^{Q}[b_2(\iota)]E^{Q}[a_{12}(p)]}{E^{Q}[a_{21}(\iota)]E^{Q}[a_{12}(p)] - E^{Q}[a_{11}(p)]E^{Q}[a_{22}(\iota)]}}{E^{Q}[a_{22}(\iota)]} \tag{5.5}$$

Moreover, Eq.(5.4) and Eq.(5.5) could be expressed as follows:

$$\begin{aligned}E[x^*] \quad = \quad & \frac{\left[\frac{1}{a} - E\left[y(p,q)|p\right] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_x(p)}{f_x(p)}\right]\frac{g_z(\iota)}{f_z(\iota)}}{\left[\frac{g_z(\iota)}{f_z(\iota)} - 1\right]\left[\frac{g_x(p)}{f_x(p)} - 1\right] - \left[\frac{g_x(p)}{f_x(p)}\right]\left[\frac{g_z(\iota)}{f_z(\iota)}\right]} \\ & - \frac{\left[\frac{1}{a} - E\left[y(p,q)|\iota\right] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_z(\iota)}{f_z(\iota)}\right]\left[\frac{g_x(p)}{f_x(p)} - 1\right]}{\left[\frac{g_z(\iota)}{f_z(\iota)} - 1\right]\left[\frac{g_x(p)}{f_x(p)} - 1\right] - \left[\frac{g_x(p)}{f_x(p)}\right]\left[\frac{g_z(\iota)}{f_z(\iota)}\right]}\end{aligned} \tag{5.6}$$

$$\begin{aligned}E[z^*] \quad = \quad & \frac{\left[\frac{1}{a} - E\left[y(p,q)|\iota\right] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_z(\iota)}{f_z(\iota)}\right] + \left[\frac{g_z(\iota)}{f_z(\iota)} - 1\right]}{\left[\frac{g_z(\iota)}{f_z(\iota)}\right]\left(\left[\frac{g_z(\iota)}{f_z(\iota)} - 1\right]\left[\frac{g_x(p)}{f_x(p)} - 1\right] - \left[\frac{g_x(p)}{f_x(p)}\right]\left[\frac{g_z(\iota)}{f_z(\iota)}\right]\right)} \\ & \left(\left[\frac{1}{a} - E\left[y(p,q)|p\right] + \frac{\left(E[y(p,q)] - \frac{1}{a}\right)g_x(p)}{f_x(p)}\right]E^{Q}\left[\frac{g_z(\iota)}{f_z(\iota)}\right]\right. \\ & \left. - \left[\frac{1}{a} - E\left[y(p,q)|\iota\right] + \frac{\left(E[y(p,q)] - \frac{1}{a}\right)g_z(\iota)}{f_z(\iota)}\right]\left[\frac{g_x(\iota)}{f_x(\iota)} - 1\right]\right).\end{aligned} \tag{5.7}$$

For maximizing mean-variance expected utility the optimal solution $x^*(p)$ and $z^*(\iota)$ to problem (5.4) is given as:

$$
\begin{aligned}
x^* &= \left[\frac{1}{a} - E\left[y(p,q)|p\right] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_x(p)}{f_x(p)}\right] + E[x^*]\left[\frac{g_x(p)}{f_x(p)}\right] + E[z^*]\left[\frac{g_x(p)}{f_x(p)} - 1\right] \\
z^* &= \left[\frac{1}{a} - E\left[y(p,q)|p\right] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_z(\iota)}{f_z(\iota)}\right] + E[z^*]\left[\frac{g_z(\iota)}{f_z(\iota)}\right] + E[x^*]\left[\frac{g_z(\iota)}{f_z(\iota)} - 1\right]
\end{aligned}
$$

Chapter III

Applications of Optimal Static Hedging of Energy Price and Volume Risk to markets in the US and Colombia

Abstract

The optimal hedging strategy proposed in Chapter II is applied to the real markets, to the Pennsylvania, New Jersey and Maryland system (PJM) in United States and the Wholesale Power Market in Colombia (WPMC). Parameters for price and quantity were obtained from each market, weather indexes for the PJM market were obtained from the Chicago Mercantile Exchange (CME) and hydrological indexes were obtained from Colombian market. The hydrological index is based on the hydrological contributions of the rivers to dam levels. El Niño and La Niña have also influenced the quantity variations, and the agents in those markets are exposed to both price and quantity volatilities.

Keywords: Static Hedging, Hydrological Indexes, Volumetric Hedging, PJM System, Wholesale Electric Power Market in Colombia.

JEL Classification: G0, G13, C32.

1 Introduction

Weather affects the habits, preferences and consumption of resources by people so that some industries are affected by and become sensitive to unexpected variations in the weather. Industries such as energy, agriculture, and insurance are examples of weather sensitive industries. They may lose trillions of dollars annually due to weather (Dutton (2002)). Some productive sectors are exposed to weather and volumetric risks such as hydro-generated electric power, and uncertainty caused by climate variables is the main source of the volumetric risk exposure which affects the agents' revenues.

The profit function for electric power agents has two random components, spot price and quantity, which imply that they are facing two types of risk, price risk in the spot markets and volumetric risk caused by quantity fluctuations. Together, price and quantity uncertainty are a potent combination for the electric power markets even when hydro plants do not contribute a large proportion of a region's electricity generation. In the energy sector, volumetric risk is becoming increasingly important given the nature of electric power and the strong links between physical production of electricity and the financial operation market. Physical production is affected by variations due to climatic conditions.

The electric power sector is highly dependent on weather conditions; therefore, price, quantity and weather are all correlated; thus, unexpected changes in any of the three variables which affects one or all of others. For instance, the effect of the weather can be seen adverse movements in price and volume which result in the presence of severe price and volumetric risk exposures. This volume exposure is perhaps the most complicated kind of exposure. However, agents in the market misjudge their volume risk exposure, possibly leading to unexpected losses not backed by generation.

Evidence from the Colombian and USA markets show that climate variation affects spot price and quantity behavior. The wholesale spot price in Colombia exhibits volatile behaviour, for instance in November 2010 average daily price per KWh was 22.7 COP less than in November 2009 due to La Niña phenomenon (138 COPs per KWh in November 2010, and 160.7 COPs per KWh in November 2009). About generation, there was 19.8% more capacity in 2010 after the El Niño effect of 2009 had diminished (75.2% in 2010 and 55.4% in 2009). The spot price in PJM Market reached $246 per MWh in the fall of 2006. At that time the normal price range was around $ 23-$ 101. In the winter of 2002 load in the PJM system was 183200 MW. At that time the normal load range was around 40000 MW - 138000 MW.

In the United States the Chicago Mercantile Exchange (CME) offers standardized weather derivatives with the purpose of increasing liquidity and accessibility in these kinds of contracts. Information about atmospheric conditions is provided by the National Oceanic and Atmospheric Administration (NOAA), which conducts research and gathers data about the oceans, atmosphere, space and sun and applies this knowledge to the service of the markets. NOAA provides the Heating-Degree-Day-Data, Cooling-Degree-Day-Data, and Oceanic Niño Index Data (ONI). Temperature indexes reflect the sum of the US averages from the start of the year until the current week. Cumulative indexes are normally calculated from January 1 to December 31. In the Colombian market, there are no weather derivatives, which the hedging strategy makes complicated. Since, the most important generation system is hydro plants, hydrology is the variable that needs attention in order to understand the quantity uncertainty.

Weather derivatives based on indexes of temperature such as Cooling-Degree-Days (CDD) and Heating-Degree-Days (HDD) from CME could be used in order to hedge unexpected changes in the weather transferring climatic uncertainties to the capital markets. Financial securities such as weather futures and options allow compensation for volumetric risk. The purpose of the weather derivatives is to smooth out the temporal fluctuations in expected earnings obtained by the agents. The hedging of portfolios against unexpected weather variations could be done by financial contracts that allow compensation for losses. Climatic factors such as El Niño and La Niña are responsible for anomalies that affect the price and quantity volatilities and, consequently, the agents' revenues.

Here we show that the model presented in Chapter II can be applied by using real parameters from the electric power markets PJM in US, and WPMC in Colombia. Secondly, we show that this model, optimal static hedging general case, can accurately capture the correlation structure and allow hedging of price and volumetric risk. Thirdly, we present results of the application the model including weather derivatives and compare the performance in two different scenarios: without hedging and the hedging strategy for each of PJM and WPMC. For the Colombian case we had to establish a hydrology index based on river contributions to dam levels in the Colombian hydro-electrical system.

In this Chapter, Section 2 describes the problem statement and solution algorithm, Section 3 presents the market characteristics and parameters, Section 4 shows empirical results, and section 5 concludes

2 Problem Statement

This case study aims to discuss the implications of the volumetric and price exposures caused by the transaction of electricity generated by water when the agents try to ensure future revenues. We illustrate the applications of the optimal static hedging strategy when investors deal their portfolio composed by price and weather derivatives, and when countries such as Colombia are exposed to fluctuations of dam levels due to El Niño and La Niña phenomena, in the same way countries such as a US are exposed to temperature variations. We demonstrate the hedging technique proposed in Chapter II of this thesis.

The problem of optimal static hedging was solved in Chapter II using derivation of payoff functions for the case of independence, in which price is not correlated with the weather variable. In this chapter we propose the application of the optimal static hedging, based on the development proposed in Chapter II, on the real markets in two different countries, conditions and features; the electric power markets PJM in US, and WPMC in Colombia. Thus, the traditional hedging strategy using vanilla derivatives becomes completed using weather derivatives which can be used as a measure to mitigate volumetric risk exposure.

In modern finance, the portfolio construction problem follows Markowitz (1952) model which provides specific solution in order to maximize expected future returns given a certain level of risk. Campbell et al (2001) introduced a similar portfolio allocation problem using VaR as a risk measure. In the electric power literature, several authors follow Markowitz's methodology to address hedging strategy using vanilla derivatives. Nasakkala and Keppo (2005), and Woo et al (2004) studied the interaction between stochastic consumption volumes and electricity prices, and proposed a mean-variance type model to determine optimal hedging strategies. Vehvilainen and Keppo (2006) optimized hedging strategies taking into account the Value at Risk as risk measure. Huisman (2007) introduced a one-period framework to determine optimal positions in peak and off-peak contracts in order to purchase future consumption volume. In this framework, hedging strategy is assumed to minimize expected costs relating to an ex-ante risk limit defined in terms of Value at Risk. Oum and Oren (2008) developed a self-financed hedging portfolio consisting of derivatives contracts, and they obtained the optimal hedging strategy in order to hedge price, and volumetric risk maximizing the expected utility of hedge profit for the Load Serving Entities (LSE).

Finally, Chapter II of this thesis shows that Oum and Oren solution could be improved including weather derivatives in order to complete the hedging over volumetric risk exposure, proposing the Optimal static hedging model for the independence case assumption.

Proposition 2 in Chapter II established that electric power agent's problem could be summarized as follows:

Maximizing the mean-variance utility function on profit,

$$E[U(Y)] = E[Y(x,z)] - \frac{1}{2}a * var(Y(x,z))$$

For maximizing mean-variance expected utility the optimal solution $x^*(p)$ and $z^*(\iota)$ to the problem

$$\max_{x(p),z(\iota)} E[U(y(p,q),x(p),z(\iota))] \tag{2.1}$$

$$s.t\ E^Q[x(p)] = 0$$

$$E^Q[z(\iota)] = 0$$

2.1 Model and Solution Algorithm

In order to improve optimal static hedging solution and address price and volumetric risk mitigation, Chapter II proposes a new way to hedge a LSE's profit based on the constitution of an optimal portfolio composed by two claims: standard contracts on price and weather derivatives. Closed-form results proposed in Chapter II define the follow equations:

$$x^* = \frac{1}{a} - E[y(p,q)|p] - E[z^*|p] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_x(p)}{f_x(p)} + (E[x^*] + E[z^*])\frac{g_x(p)}{f_x(p)} \tag{2.2}$$

$$z^* = \frac{1}{a} - E[y(p,q)|\iota] - E[x^*|\iota] + \left(E[y(p,q)] - \frac{1}{a}\right)\frac{g_z(\iota)}{f_z(\iota)} + (E[x^*] + E[z^*])\frac{g_z(\iota)}{f_z(\iota)} \tag{2.3}$$

More precisely optimal static hedging implementation procedure requires following the next algorithm:

i Collecting historical data of markets on study.

ii Computing historical and simulated parameters from PJM-system and WPMC-system.

iii Computing descriptive statistics of variables from PJM and WPMC markets compare to the corresponding average value of real market over the value of simulated trials.

iv Constructing a hydrological index that characterizes hydrological contributions of the rivers during a period of time.

v Fix parameters including range for $a(\min, \max andsteps)$.

vi Fix number of simulations num_{trab} "large".

vii Generate random price p, load q and weather variable w, using a multivariate normal distribution.

viii Compute the payoff $x^*(p)$ and $z^*(\iota)$ (Equations (2.1) and (2.3)).

ix We can obtain $(x^a(p), z^a(\iota))$ that maximizes:

$$E[Y(x,z)] - \frac{1}{2}a * var(Y(x,z))$$

x For each a, calculate associated $VaR(a) \equiv VaR(Y(x^a, z^a))$ such that $P\{Y(x^a, z^a) \geq -VaR(a)\} = \gamma$

xi Find smallest $a(a_{opt}) such that VaR_\gamma(Y(x^a, z^a)) \leq V_0$

xii Using a_{opt} in order to find $Y(x^(a_{opt}), z^(a_{opt})).Y(.)$, which is the profit distribution of the expected utility maximizing solution, under Optimal Static Hedging including the weather claim (see Figures 8 and 9)

xiii Using the payoff functions $x^{(a_{opt})}$, and $z^{(a_{opt})}$, and based on Carr and Madan (2001) we can define the replication of payoff using Equation (2.4) (see Figures 4.10 and 4.11).

$f(p,q) = (r-p)q + x(p) + z(\iota)$,

$$\begin{aligned} x(p) &= x(F_p).1 + x'(F_p)(p - F_p) \\ &\quad + \int_0^{F_p} x''(k)(k-p)^+ dk + \int_{F_p}^{\infty} x''(k)(p-k)^+ dk \\ &\quad z(\iota) = z'(F_\iota)(\iota - F_\iota) \end{aligned} \tag{2.4}$$

The algorithm above permits us obtain the optimal static hedging profit distribution using two claims and the concerning replication payoff function for each market on study.

3 Market Characteristics

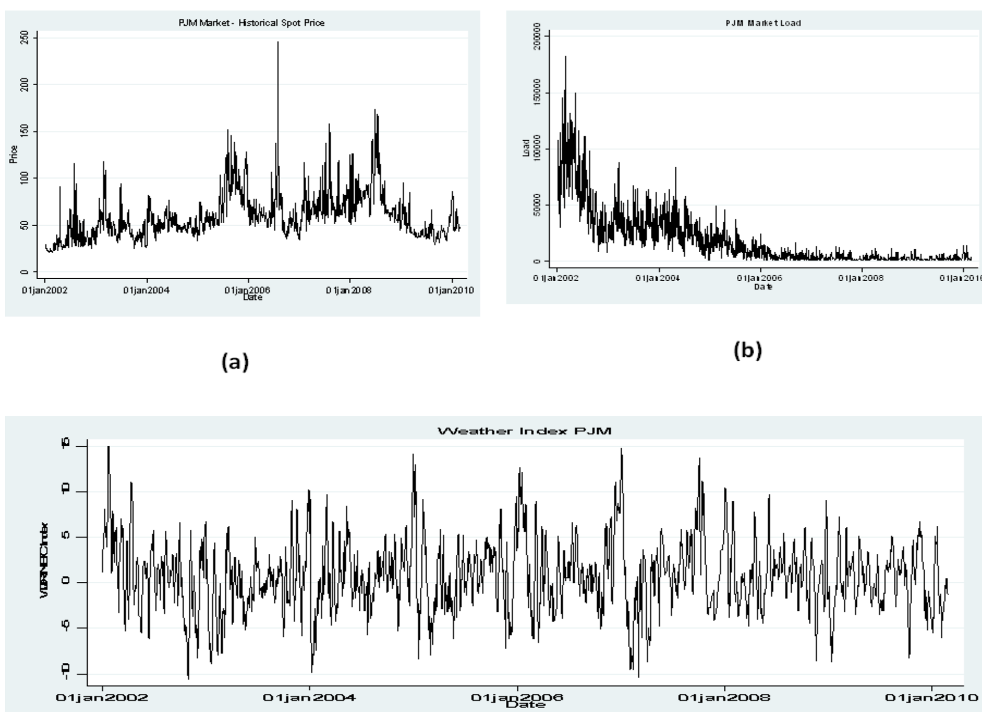
This chapter aims to present an application of the Optimal Static Hedging Model in the real market from two different zones and conditions. PJM market in US use the CME to trade energy throughout contracts and allows hedging of price and volume volatility. WPMC in Colombia is less developed than PJM. The Colombian market will be studied further as an example of an emerging market. In this section, we present the features of PJM and WPMC to see why optimal static hedging strategy is possible in the context.

3.1 PJM Interconnection System

PJM Interconnection system coordinates the continuous buying, selling and delivery of wholesale electricity through the energy market. In its role as market operator, PJM balances the needs of suppliers, wholesale customers and other market participants and monitors market activities to ensure open, fair and equitable access. The PJM system has since expanded into the deregulated electric power markets in the world. PJM has generation capacity for nearly 167,454 Megawatts per year, and an all-time peak demand of 144,644 Megawatts. Its main fuel types are coal (74%), natural gas (22%) and others (4%), (2009 data). The PJM system covers a large region in US: Pennsylvania, New Jersey, Delaware, Maryland, Virginia, Ohio, West Virginia, New York, and Washington D.C.

PJM's energy market operates much like a stock exchange, with market participants establishing a price for electricity by matching supply and demand. The market uses locational marginal pricing (LMP), that reflects the value of the energy at the specific location and time it is delivered. If the lowest-priced electricity can reach all locations, prices are the same across the entire grid. When there is transmission congestion, energy cannot flow freely to certain locations. In that case, more-expensive electricity is ordered to meet that demand. As a result, the locational marginal price is higher in those locations. The energy market consists of Day-Ahead and Real-Time markets. The Day-Ahead Market is a forward market in which hourly LMPs are calculated for the next operating day based on generation offers, demand bids and scheduled bilateral transactions.

Since, the wholesale spot price is referred to a locational marginal price, location influences price behavior due to the cost of transportation as well as other commodity markets. This feature adds volatility to the wholesale prices. The Real-Time Market is a spot market in which current LMPs are calculated at five-minute intervals based on actual grid operating conditions. Real-time prices are available. PJM settles transactions hourly and issues invoices to market participants monthly.



(c)

Figure 3.1: PJM market variables. (a) Wholesale spot price, (b) PJM load, (c) Weather index for PJM, source: Data from PJM system (Bloomberg).

Figure 3.1 shows patterns of behavior of the whole variables trajectories and the market features that establish prices and loads experienced in several events where peaks are identifiable.

Optimal static hedging strategy assumes the independence assumption. Thus, we show that for the PJM-system case, price and the weather index are uncorrelated. Using the Granger test for causality (Granger (1969)), we also test that price and weather index are not related. We found that the test accepts the null hypothesis and then, both price and weather index in the PJM market is no related or weather index doesn't cause price. Furthermore, correlation between price and hydrological index is $Corr(p,\iota) = -0.0116$. Figure 3.2 shows correlation between price and weather index in US.

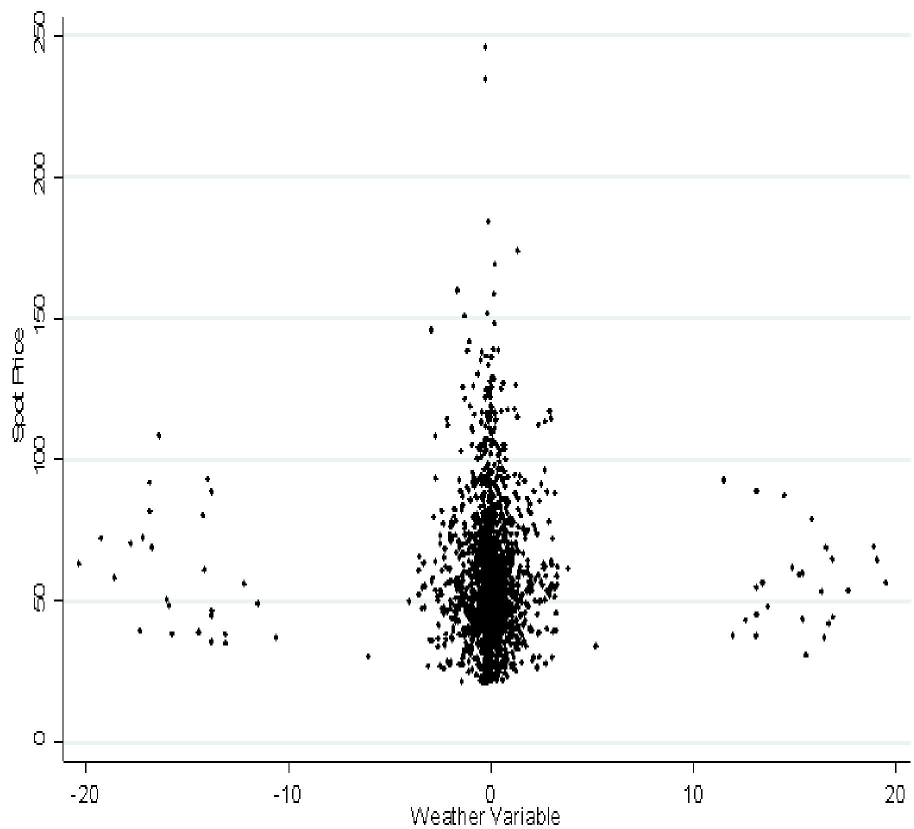


Figure 3.2: Market correlations between price and weather index, US market.

Market parameters

The data set used consisted of 3765 observations of spot prices, loads and weather index from PJM system and NOAA from January 1, 2000 to February 22, 2010. Seven descriptive statistics of daily pattern were calculated over fous simulated sample paths. Table 3.1 shows the empirical average, median, standard deviation, skewness, kurtosis, minimum and maximum calculated for the real and simulated data.

Table 3.1: Percentiles when fewer than two cases occur: without-hedge, and optimal static hedging application $[x'^*(p) + z'^*(\iota)]$ for PJM-system and WPMC-system.

	Spot Price	Sim Sp P.	Load	Sim Load	Log P.	Sim Log Price	Weather Index	Sim Windx	Log Widx	Sim Log Widx
Mean	58,38	58,4	2053	2039,6	3,994	3,996	2,409	2,7	0,53	0,396
Median	53,44	54,1	1040	2040,9	3,979	3,991	1,7	1,491	0,4	0,399
S. Deviat.	23,69	23,2	539,8	440,74	0,378	0,378	2,173	4,238	1,099	1,094
Skeewness	1,61	1,37	0,212	0,028	0,13	0,066	1,501	0,891	-0,626	-0,017
kurtosis	5,47	3,73	5,382	-0,114	0,181	-0,01	2,597	16,28	-0,126	2,846
Min	21,14	15,4	800	474,48	3,051	2,732	0,1	0,042	-2,303	-3,179
Max	246	231	18320	3632	5,505	5,443	12,5	11,52	2,526	4,747

3.2 Wholesale Power Market in Colombia WPMC

Colombia has a rich endowment of energy sources: natural gas, coal, oil and a hydroelectricity potential. Its hydropower capacity of approx 67% of the total capacity represents 13.5 GW. Other sources are natural gas 27%, coal 5%, and 0.3 per cent other sources.Thus, this market is mainly hydro and climatic events such as El Niño can have a large effect. The total power demand in 2009 was 9 GW, meaning that capacity was around 30 per cent greater than demand (UPME (2009)). Regulation, dispatch system and an unbundled scheme generated an excellent environment making the Colombian electricity market mature very quickly.

The Regulatory Commission for Gas and Electricity (CREG) was created by statutes and its function is to regulate the entrepreneurial, commercial, technical, and operational aspects of the present structure of the electric power sector. This includes the generation, transmission, and distribution/commercialization of electric power. Regulation in the WPMC also created the figure of the “pure marketer”, which is an intermediary agent whose purpose is to make competition dynamic and to provide the final customers with different ways to access competitive prices in the electric market of wholesalers.

Regulations in the Colombian market allow these agents to sell electric power to their customers through contracts that have no “steady electric power to endorse”, that is, that endorsed by the electric generators to guarantee supply to these users. Moreover, these agents can take endless risks, and, in the case of bankruptcy, they do not have assets to lose, discarding their obligations effortlessly.

In the Colombian system, the electric power generators are obliged to supply electric power by the regulatory institution even if they were impaired by the agents who did not comply with the secondary markets. These generators lack valid arguments to assume this additional risk, because it is not part of their activity. If this situation was to occur, it would not only impair them financially, but it would also make them turn to the Justice system to determine who must assume liability for the resulting impairments. Hence, one of the authority’s demands for electric generators is that they must insure themselves against price volatility risks. This opens up the opportunity of using derivatives instruments in the Colombian electric sector in the foreseen future.

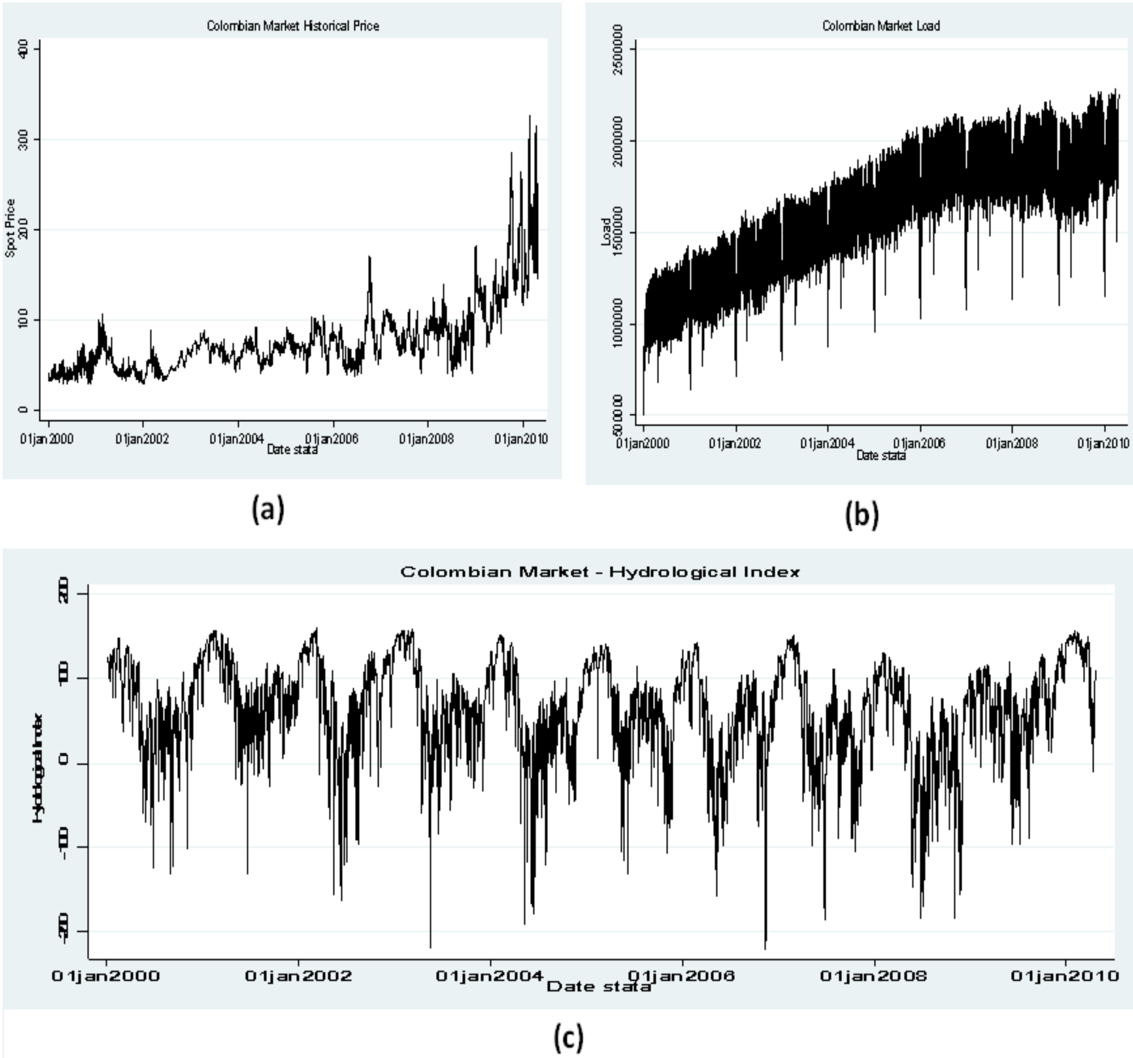


Figure 3.3: the whole variables trajectories and the market features. Fig. 2(a) shows price behavior in the last years from January 1, 2000 to February 22, 2010. Fig. 2(b) exhibits load pattern in the same period of prices and shows us convergence to the minimum load and negative correlation between prices and loads. Fig. 2(c) presents the hydrology-index pattern that was used to complement hedging strategy, source: Derived from Neon system.

Figure 3.3 shows market variables of the collecting data from Colombian market from January 1, 2000 to February 22, 2010. Data was obtained from Neon-system of Experts on Markets (XM) in Colombia.

Optimal static hedging strategy assumes the independence assumption. So, we show that for the WPMC-system case, price and the hydrological index are uncorrelated. Using the Granger test for causality (Granger, 1969), we also test that price and hydrological variable are not related. We found that the test accepts the null hypothesis and then, both price and hydrological index in the WPMC market is no related or hydrological index doesn't cause price. Furthermore, correlation between price and hydrological index is $Corr(p,\iota) = 0.0074$. Figure 3.4 shows correlation between price and weather index in Colombia.

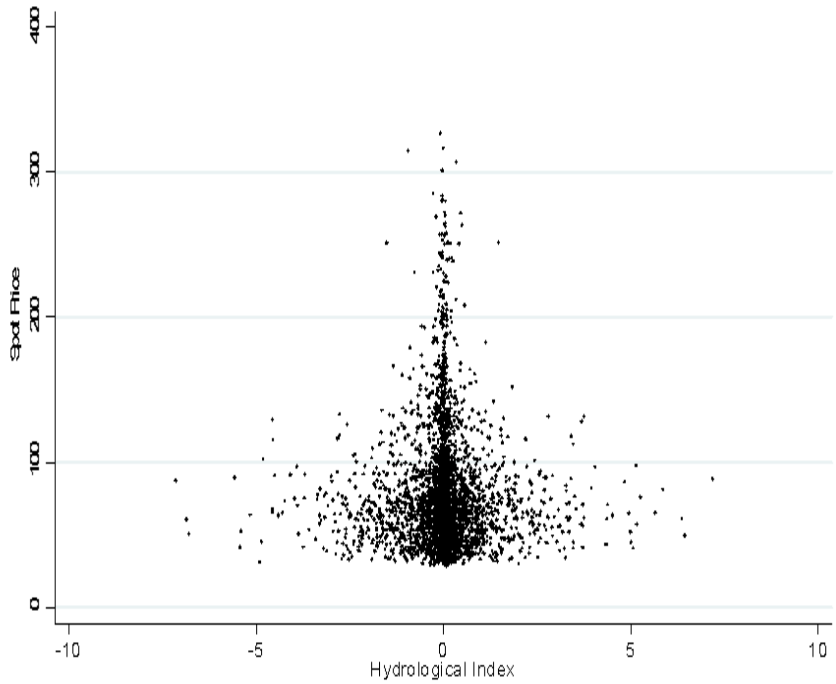


Figure 3.4: Market correlations between price and weather index, Colombian market.

Market parameters

We dispose also full data set of quoted spot prices, loads and hydrological contributions from Colombian power market; starting from January 1, 2000 to February 22, 2010. The data set comprises 3765 observations exhibiting varying path properties over time.

The data set used consisted of 3765 observations of spot prices, loads and hydrological contributions from WPMC system from January 1, 2000 to February 22, 2010. Seven descriptive statistics of daily pattern were calculated over fous simulated sample paths. Table 3.2 shows the empirical average, median, standard deviation, skewness, kurtosis, minimum and maximum calculated for the real and simulated data.

Table 3.2: Descriptive statistics of variables from Colombian market compare to the corresponding average value of real market over a number of simulated trials.

	Spot Price	**Simul Spot P**	**Load**	**Simul_Load**	**Log_Price**	**Simul_Log_P**
Mean	77.07	74.650	1685	1697.773	4.25	4.223
Median	68.31	67.359	1692	1698.000	4.22	4.210
S. Deviat	38.66	32.741	345	349.813	0.42	0.425
Skewness	2.21	1.185	-0.38	-0.035	0.58	-0.002
Kurtosis	6.87	1.974	-0.76	0.078	0.47	-0.196
Min	28.84	16.860	502	528.700	3.36	2.825
Max	326.77	241.050	2288	2956.900	5.79	5.485

3.3 Hydrological index for Colombian Market

There are several reasons that will be required for structuring a contract over hydrology or climate. Furthermore, the index represents fluctuations of the observed variables and the profit function (Caballero, et al. 2002).In Colombia, We needed to construct a hydrological index that characterizes hydrological contributions of the rivers during a period of time called "time of impact", and this index could be used in order to structure the optimal static hedging strategy. The hydrological index known as **Hyidx** and is related to rainfall levels that determine the contributions, drought periods and periods of excessive rainfalls. Following the same methodology as CME, the index will be discriminated for each case based on the average level of contributions for a period of average or typical behavior of the Colombian hydrology. To define a period of a typical performance in hydrology, we have used the Oceanic Niño Index (ONI), which measures ocean temperatures to define drought periods or El Niño events or rainfall periods or La Niña events (data for this index were obtained from NOAA in November, 2010). The average performance period was the period from January 2004 to December 2006. Figure 3.5 shows the behavior of the ONI index Pacific 3.4 which is most appropriated for Colombian market (Vergara et al., (2010)).

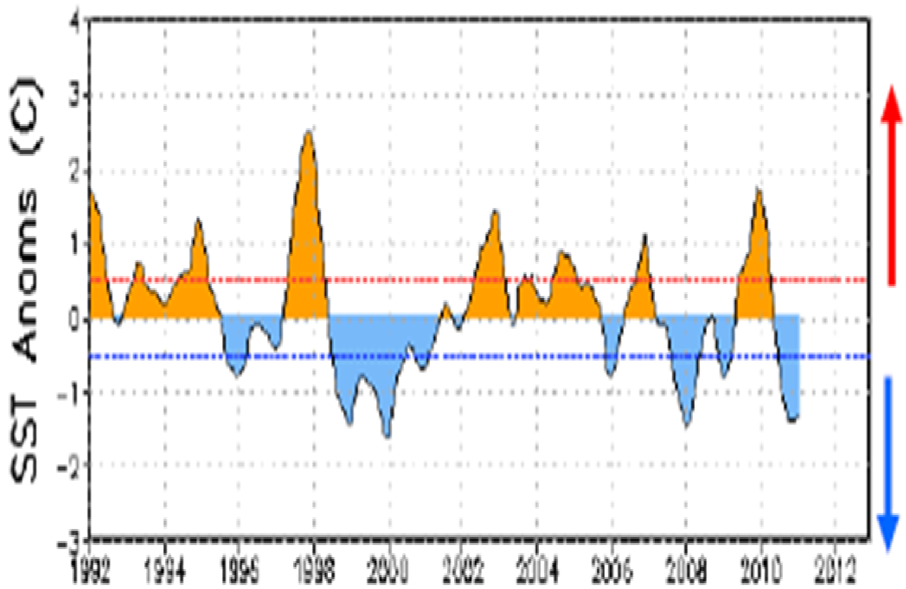


Figure 3.5: Oceanic Niño Index ONI pacific 3.4, base period 1992-2011, source: Plot were obtained from NOAA in February, 2011 .

The hydrological index is normally an accumulation of the Drought Period-Days **DPDs** or Rainfall Period-Days **RPDs** during the time of the event. For El Niño events, the hydrological contributions will be lower than the average performance denoted by **hycont***, thus, the function for drought periods can be defined as follows:

$$DPD_i = \max(hycont^* - hyC_i, 0) \tag{3.5}$$

Where hyC_i denoted the average hydrological contribution for day i and it is defined as:

$$hyC_i = \frac{hyC_i^{\max} - hyC_i^{\min}}{2} \tag{3.6}$$

From Equation 3.6 and the average of typical performance **hycont***, RPD_i for day i is given by:

$$RPD_i = \max(hyC_i - hycont^*, 0) \tag{3.7}$$

The index defines the total number of *DPDs* or *RPDs* over the impact period for the *n* days.

$$I^d = \sum_{i=1}^{n} DPD_i \quad and \quad I^r = \sum_{i=1}^{n} RPD_i \tag{3.8}$$

4 Empirical Results

In Chapter II of this dissertation, we suggest one possible way to solve the constrained optimization problem. In this chapter we apply the optimal static hedging closed-form solution using two examples of the different characterized real markets. We consider different conditions, different locations and levels of development, the PJM and WPMC markets. The first of these is in the US, highly developed and supported by CME, one of the most developed, mainly commodities markets in the world. The second is located in Colombia, and at the end of 2010 the first standardized contract over electric power was offered: Futures contracts. This market is considered as an emerging market case, and the main power source is water which implies specific characteristics and high hydrologic dependence.

We illustrate the optimal static hedging with numerical examples based on the VaR-constrained mean-variance utility function, and we apply the model to two cases; first, the US market characterized by presence of seasons which means that the temperature is the climatic variable that produces anomalies and, consequently, volumetric risk exposure. Second, the Colombian market is one of the most interesting electric power markets in South America, characterized by high hydro-electric generation, and it is affected climatically by drought or rainfall periods defined by El Niño and La Niña. High hydro-dependence implies that hydrology is the critical variable that produces anomalies that affect the agent's revenues.

Taking in account that weather derivatives are not available in Colombia, this chapter designed a hydrology-related index that is used as the underlying asset of the forward contracts that allows to the agents to perform the hedging strategy over its exposure to unexpected climate changes from events related to El Niño and La Niña. Equations 3.5 and 3.7 allow calculating, respectively, the rates for drought and rainfall daily events during the impact-period. Equation 3.6 derived the average hydrological contributions for day *i*, and it will be compared against the average or typical performance denoted by **hycont*** which characteristics the neutral hydrological-period in which there is no presence of hydrological phenomena (El Niño or La Niña). Equation 3.8 allows calculation of hydrological indices which are normally calculated as the accumulation of days in drought and rainfall day periods.

Table 4.3 shows the average o typical performance denoted by **hycont*** determined between January 2004 and December 2006.

Table 4.3: Average of typical behavior of hydrological contributions.

Descriptive Statistics	Average Typical Behavior 01/012004 - 01/12/2006	
	KWh	**MWh**
Min	33535200	33.535200
Max	406477000	406.477000
Average	186,470,900	186.4709

In Table 4.4 we show the historical and simulated statistics for the hydrological index from WPMC. Five descriptive statistics of daily pattern were calculated for two simulated sample paths.

Table 4.4: Descriptive statistics of hydrological index from Colombian market compared to the corresponding average value of real market over a number of simulated trials.

	Hyd_Index	**Simul_Hyindx**	**Log_HyIndex**	**Simul_Log_WIdx**
Mean	63.018454	63.198	4.018218427	4.376
Median	70.522	65.861	4.305453318	4.385
S. Deviat	58.903	63.851	0.995413685	0.504
Skewness	-0.8710621	1.523	-2.149440435	-0.076
Kurtosis	1.0250601	3.944	6.900005491	-0.133

Figure 4.6 shows indexes calculated from WPMC system, determined between January 2004 and December 2006. This index was used in order to complete the optimal hedging strategy for the Colombian market.

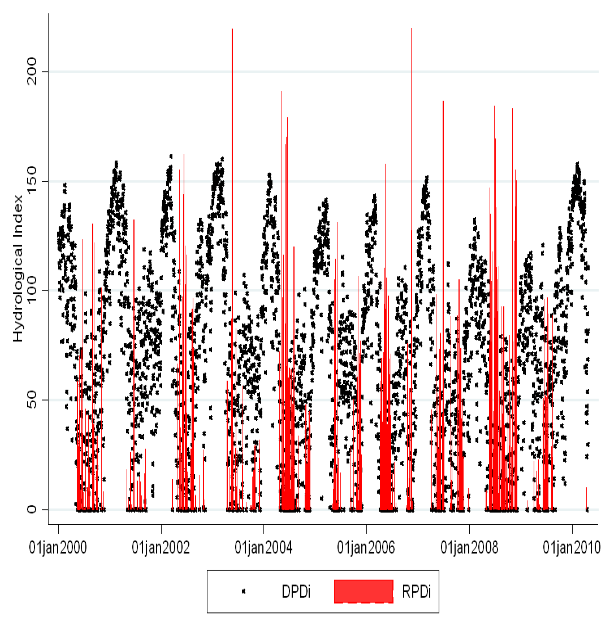


Figure 4.6: Hydrological Indexes DPDs and RPDs for Colombian market.

We consider that the variables price, quantity and the weather index exhibit correlation among them, which is that (p,q) and (ι,q) are correlated, for the PJM market (p,q) exhibits negative correlation, and positive for (ι,q). This behavior is taking into account that the period of study is from January 2002 to April 2010. For the Colombian market case, the same period is evaluated and exhibits a significant positive correlation between price and quantity and negative for quantity and hydrological index.

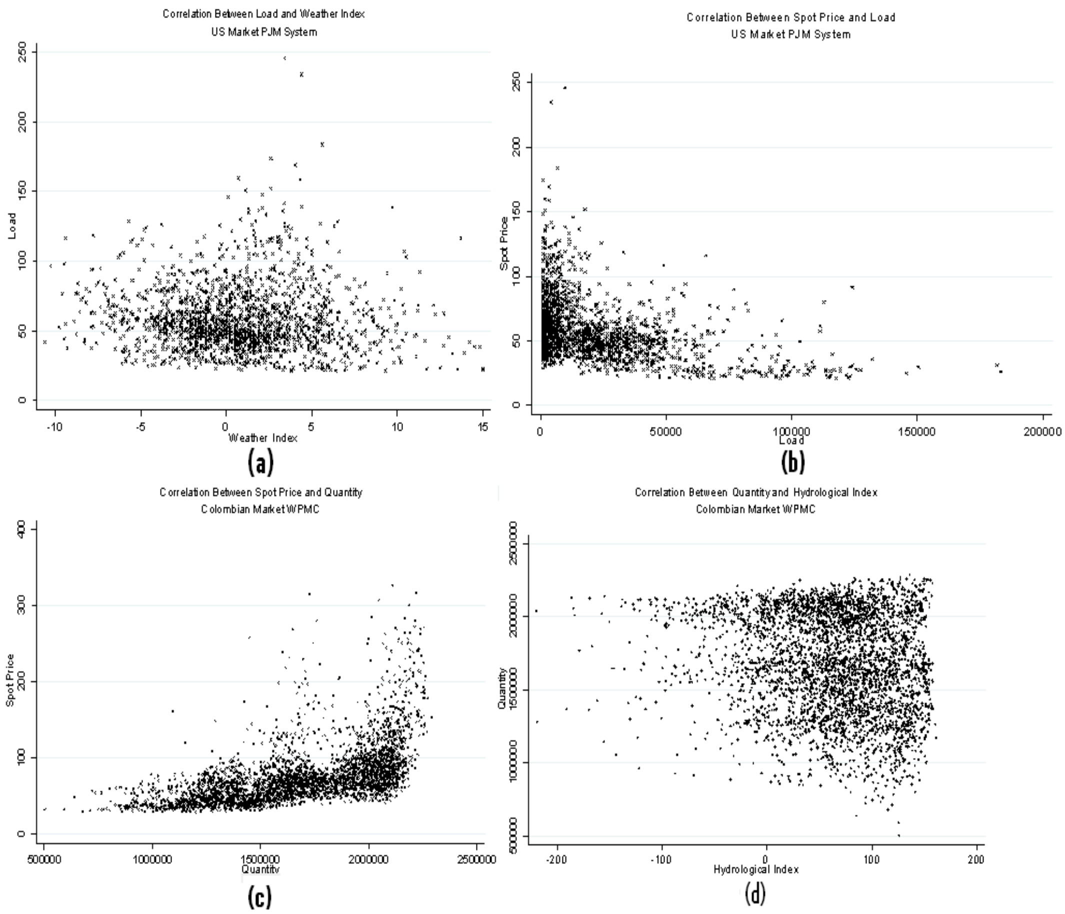


Figure 4.7: Market correlations between price, quantity and climatic index. (a) Correlation spot price and load PJM system, (b) Correlation load and weather index PJM system, (c) Correlation spot price and load WPMC, (d) Correlation load and hydrological index WPMC, source: Developed using data from PJM system, and WPMC system.

Figure 4.7 shows the relationship among variables for each market on study, we could note that for the PJM market correlation between spot price and load strongly the relationship between load and weather variable. The same result exhibit the variables correlation for the Colombian market which it could explain due to the independence assumption that states independence between price and weather variable, which was showed in Figure 3.4.

The spot price p at which the agent has to buy electric power, the weather-index ι is the climatic variable, and the quantity q is the load at which the LSE supplies power in a fixed interval. The three variables, price, temperature and quantity, are volatile and variations affect the agent's revenues. This is the problem that agents will try to solve using the optimal static hedging solution. In order to obtain the solution of the mean-variance problem for varying a we assume that P and Q distributions are different. All three variables are distributed according to a bivariate distribution in log price and quantity, and the log weather-index and quantity, as follows:

4.1 Market Probability Distribution Parameters

PJM-system

Under P:$\ln p \sim N(3.99, 0.38^2)$ $\quad q \sim N(2053, 540^2)$ $\quad \log\iota \sim N(0.53, 1.10^2)$
$Corr(p,\iota) = -0.0116$ $\quad Corr(\ln\iota, q) = 0.24$
Under Q:$\ln p \sim N(3.98, 0.38^2)$ $\quad \ln\iota \sim N(0.40, 1.10^2)$

WPMC-system

Under P: $\ln p \sim N(4.25, 0.42^2)$ $\quad q \sim N(1685, 345^2)$
$Corr(p,\iota) = -0.0074$ $\quad Corr(\ln\iota, q) = -0.26$
Under Q: $\ln p \sim N(4.22, 0.42^2)$ $\quad \ln\iota \sim N(4.39, 0.5^2)$

Table 4.5: Percentiles when fewer than two cases occur: without-hedge, and optimal static hedging application $[x'^*(p) + z'^*(\iota)]$ for PJM-system and WPMC-system.

Percentiles				
	Without Hedging PJM-System	**PJM-System Optimal Hedging**	**Without Hedging WPMC system**	**WPMC-system Optimal Hedging**
1%	-2.1E+04	-2.5E+04	-5.4E+05	-7.6E+04
5%	4.6E+04	5.5E+04	-1.5E+05	-4.0E+04
10%	7.1E+04	7.1E+04	-2.4E+04	-2.4E+04
25%	1.0E+05	9.5E+04	1.1E+05	-6.0E+02
50%	1.3E+05	1.4E+05	1.8E+05	2.3E+04
75%	1.5E+05	1.9E+05	2.2E+05	4.7E+04
90%	1.7E+05	2.4E+05	2.6E+05	6.9E+04
95%	1.9E+05	2.7E+05	2.8E+05	8.1E+04
99%	2.1E+05	3.2E+05	3.1E+05	1.1E+05
Mean	1.2E+05	1.4E+05	1.4E+05	1.2E+05
Std. Dev	4.5E+04	4.0E+04	1.7E+05	3.8E+04
Skewness	-9.9E-01	-8.2E-02	-4.5E+00	-4.5E-01
Kurtosis	6.2E+00	6.0E+00	3.9E+01	5.2E+00

Table 4.5 shows diverse percentiles that compare the different scenarios of hedging for both markets PJM and WPMC. We could note that for the PJM market optimal static hedging methodology sharply the scenario

without hedging; whoever the hedged scenario improved by mean and standard deviation what had been already achieved by Chapter II in this thesis. For instance, the hedged scenario shows a mean of $1.4E+05$ whereas without hedging is $1.2E+05$. Regarding the standard deviation, the hedging scenario is $5.0E+04$ below without hedging. For the Colombian market hedging scenario improves by standard deviation but not by mean. Regarding mean and standard deviation, the hedged scenario is $2.0E+04$ below without hedging in mean and $1.32E+05$ in standard deviation.

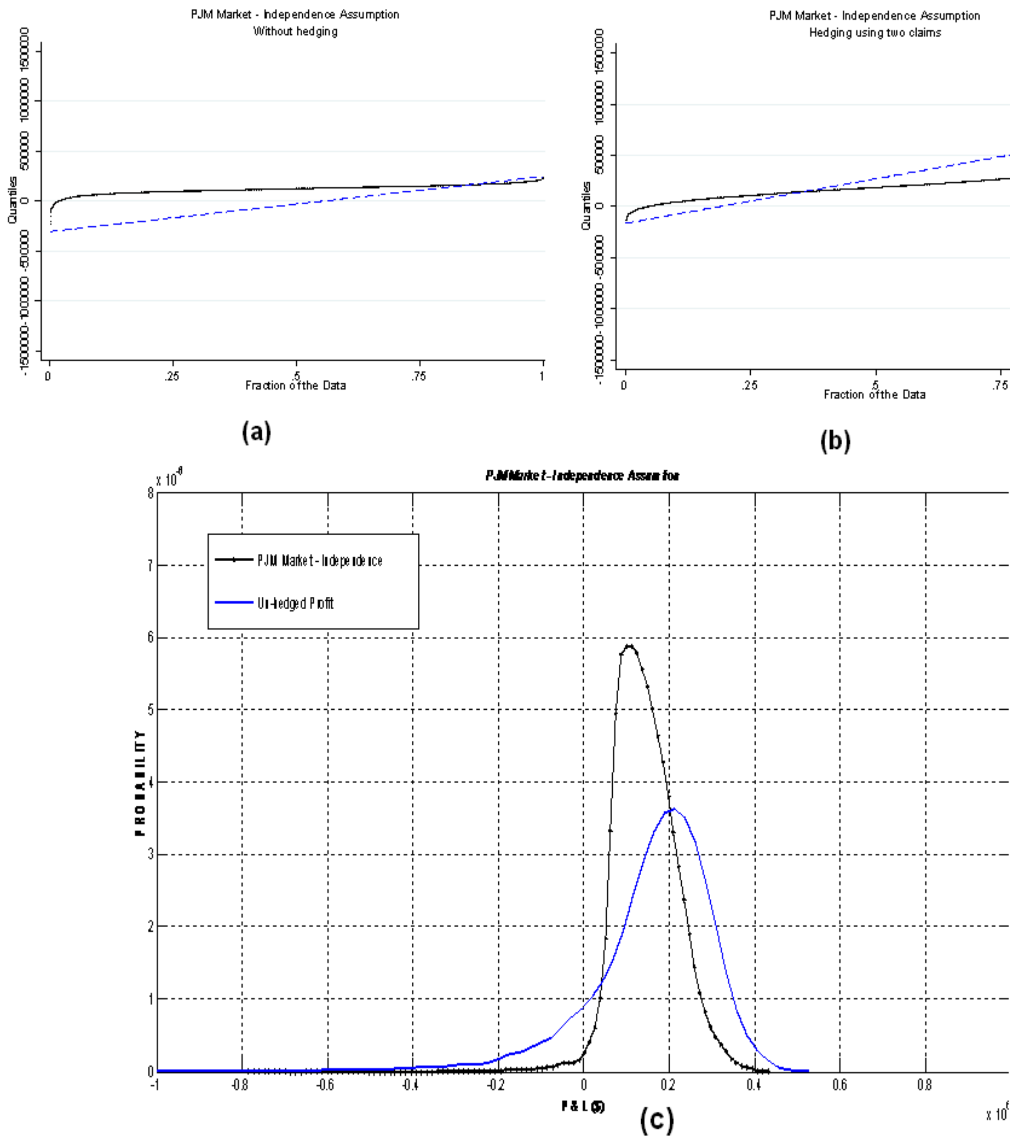


Figure 4.8: PJM-system profit distributions under two cases: without-hedge, and application of optimal static hedging model$[x'^{*}(p)+z'^{*}(\iota)]$. (a) Quantile - without hedging. (b) Quantile -Optimal hedging. (c) Profit distribution.

Figure 4.8 compares profit distributions under two scenarios of hedging for the PJM market. We could notice that optimal static hedging profit distribution improves in terms of mean, standard deviations and the VaR measure the scenario without hedging what had been already achieved by Chapter II in this thesis, (see Table

4.5).

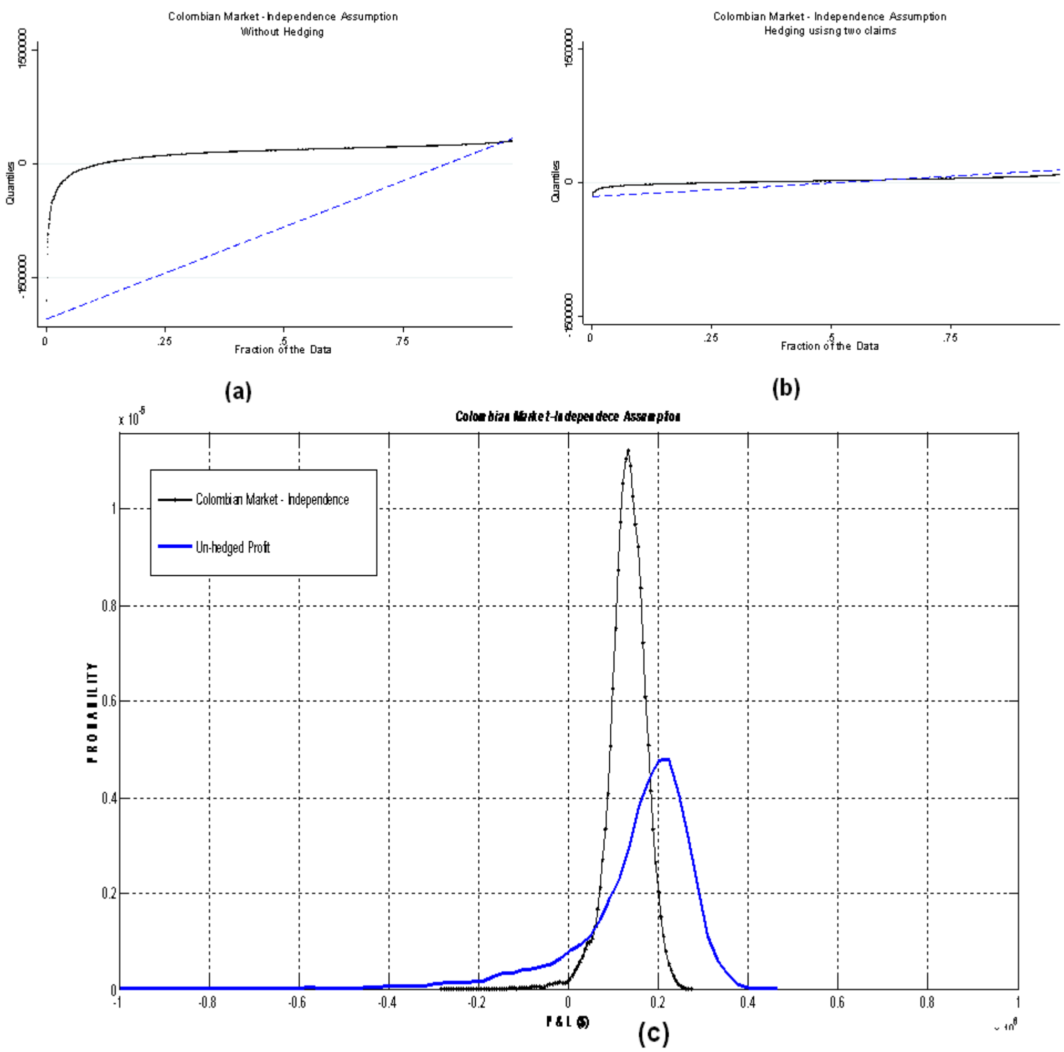
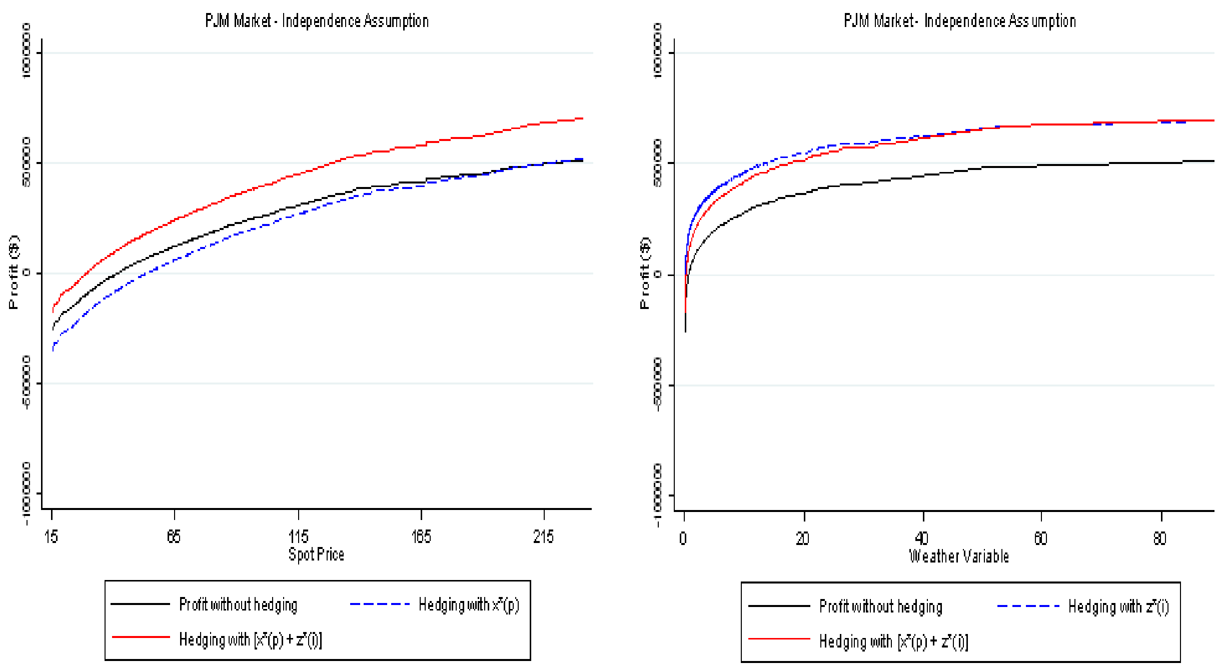


Figure 4.9: WPMC-system profit distributions under two cases: without-hedge, and application of optimal static hedging model $[x'^{*}(p) + z'^{*}(\iota)]$. Quantile - without hedging. (b) Quantile -Optimal hedging. (c) Profit distribution.

The WPMC system's profit distribution function is shown without and with hedging in Figure 4.9. By using two claims (price and hydrological variable), the closed-form solution proposed in Chapter II allows completion of hedging so that agents can reduce variance and improve the mean of the utility function.

Figure 4.9 compares profit distributions under two scenarios of hedging for the Colombian market. We could notice that optimal static hedging profit distribution improves in terms standard deviations and the VaR measure the scenario without hedging but not in terms of mean, (see Table 4.5).



(a) (b)

Figure 4.10: Optimal payoff $[x^*(p)+z^*(\iota)]$ of the Optimal Static Hedging Portfolio. (a) PJM-system optimal payoff function for the independence case respect to spot price; dashed line represents the profit without hedging, blue line represents the hedging payoff of price claim and red line represents the payoff as a sum of price and weather. (b) PJM-system optimal payoff function for the independence case respect to weather variable; dashed line represents the profit without hedging, blue line represents the hedging payoff of the weather claim and red line represents the payoff as a sum of price and weather.

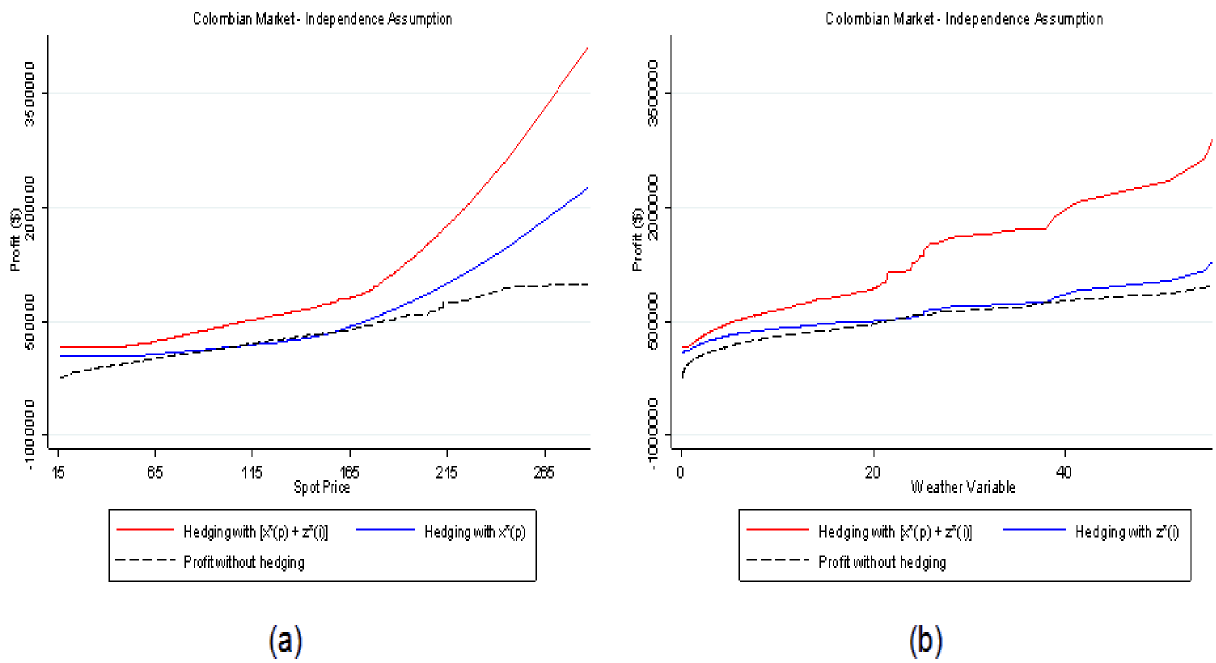


Figure 4.11: Optimal payoff $[x^*(p) + z^*(\iota)]$ of the Optimal Static Hedging Portfolio. (a) WPMC-system optimal payoff function for the independence case respect to spot price; dashed line represents the profit without hedging, blue line represents the hedging payoff of price claim and red line represents the payoff as a sum of price and weather. (b) WPMC-system optimal payoff function for the independence case respect to weather variable; dashed line represents the profit without hedging, blue line represents the hedging payoff of the weather claim and red line represents the payoff as a sum of price and weather.

Figures 4.10 and 4.11 show the optimal payoff function of the portfolios composed by two claims for the Colombian market system under independence assumption. We can show that for different values of the claim for instance spot price payoff improves the hedging position of the agents that use electricity and weather derivatives to hedge their positions.

5 Conclusions

Under VaR-constrained mean-variance utility function, in this chapter we aim to apply the optimal static hedging, closed-form results presented in Chapter II to two different types of electric power markets, one in the US and the second in Colombia. The first case is one of the most representative electric power markets that uses a secondary market, Chicago Mercantile Exchange (CME). Furthermore, United States of America is a seasonal country that is affected by the temperature variations which implies that temperature is a critical variable at the hour of measurement of volumetric risk exposure. The second case, the Colombian market, is characterized by being highly hydroelectrically dependent due to the main source of generation being water, and, only two seasons can clearly be identified, drought and rainfall. The aforementioned descriptions define two especial cases to be applied in the model.

Constructing the hydrological index for the Colombian market is an important contribution and this index could be use by the agents in order to complete the hedging strategy.

Numerical developments using Monte-Carlo simulation including specific characteristics of each market in the study shows that the optimal payoff functions allow completion of a hedging scheme under correlations among price, weather and quantity. Furthermore, it can be shown through numerical development that hedging strategy improves the agents' profit distributions for both PJM and WPMC. According to the market features, can be structured optimal payoff functions under specific conditions defined by the mean-variance utility function for the market agents that permit them to improve through weather instruments. Applications to real markets allow verification that optimal static hedging allows hedging of volumetric exposure and diversify capability.

Taking into account the differences between the PJM system and WPMC system, results confirm that the weather index and, in the Colombian case, hydrological index allow completion of the market and adjustment of the hedging strategy, improving the hedging agent's position under independence assumption.

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Chapter IV

Final Discussion

The original question aimed to find out whether the energy power market takes into account the risk factor in diverse transactions, which could be indicated by the existence of a forward risk premium; this is assumed by generators and marketers and reflects the tradable energy quantity taking on the price volatility fluctuations. The greatest problem that agents have to face in this type of market is the capacity to compensate for possible losses when taking short and long positions on the future through financial contracts, whose settlement would generate a profit equivalent to the loss caused in such an operation.

In the Colombian market, the risk taker is the marketer, specifically in the unregulated market segment, because they are assuming the price risk in the long-term negotiations. The marketer represented by this demand tries to insure their future revenues, and so they sacrifice their premia. It is relevant for further studies to evaluate the efficiency of this market, and the characteristics to determine why the marketer is willing to pay forward risk premia (FRP) and why the generator is in a better position to receive this bonus.

In the simplest case, we may think of an electric power agent (generator or marketer) that could take positions over the financial contracts against unusual climatic behavior during a certain period of exposure, and the payoff function could be entitled to a cash payment which could be negatively correlated to the agents' income. Therefore, agents' risk exposure can be reduced by trading the weather derivative contract, which means transferring climatic risk exposure to capital markets or secondary markets.

Beyond the financial markets, this dissertation aims to encode the interrelations generated between the physical production of electricity and the financial contracts to trade. We aimed to find how an agent, either generator or marketer, can utilize financial contracts in order to transfer their risk exposure to the financial markets. Volumetric price risk exposures state specific conditions to built portfolio's payoff structure according to the agent' features. Results from modeling forward risk premium and optimal static hedging and its applications allow understanding and offer a solution to the mean-variance utility function problem.

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