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FORECASTING CONSUMER PRICE INDEX IN UKRAINE WITH REGRESSION MODELS AND ADAPTIVE KALMAN FILTER

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The paper considers the problem of short term forecasting of consumer price index using regression models and adaptive Kalman filter. The main purpose of the study is constructing of high quality model for forecasting of consumer price index and application of Kalman filter for computing optimal estimates of states for the process under investigation. The basic results of the study are as follows: two modifications of the Kalman filter (ordinary and adaptive), directed towards estimation of covariances for stochastic state disturbances and measurement errors. Alternative short term forecasts are generated with regression models and Kalman filters. A comparative analysis of results achieved is given. The necessary statistical data was taken from Ukrainian economy in transition.

INTRODUCTION

The problem of high quality forecasting for economic and financial processes requires development and application of new modern techniques that are based on systemic approach to development of respective computational software. Most often this software is implemented in the form of modern decision support systems (DSS) that become popular as a solution instrument for many practical problems. One of the simplest definitions of DSS is as follows: DSS is computer based information processing system that provides decision making user with any help relevant to data collecting and storing, preliminary data processing, constructing necessary mathematical models, generating alternatives, selection of the best solution, convenient presentation of intermediate and final results etc [1].

To improve substantially quality of forecasts for macroeconomic processes in the frames of DSS it is useful to construct adaptive computing schemes for a model and parameter estimation [2]. According to this scheme each step of data processing is controlled by appropriate set of statistical parameters each of which characterizes specific features of data, model as a whole, model parameters and quality of forecasts estimates. A substantial help in forecasting linear and nonlinear nonstationary processes can be provided by application of adaptive Kalman filter that is useful for estimation of covariances for external stochastic disturbances and measurement noise (errors) [3]. The filter also provides a possibility for computing optimal estimates for state vector of a system under study and quality short term forecasts.

The paper considers the possibilities for modeling selected financial and economic processes with regression analysis approach and adaptive Kalman filter using statistical data from Ukrainian economy in transition.

PROBLEM STATEMENT

The purpose of the study is as follows:

- to construct mathematical models for selected financial and economic processes providing acceptable quality of short term forecasts;
- to transform the models into state space representation and apply adaptive Kalman filter to generate optimal state vector estimates and short term forecasts;
- to perform comparison of the results achieved;
- to develop software necessary for performing computational experiments that incorporates possibilities for adaptive model structure and parameters estimation.

PROPOSED APPROACH TO FORECASTING

We propose adaptive computing scheme that is distinguished with several possibilities for adaptation using complex quality criterion. The statistical data collected should be correctly prepared for model structure and parameter estimation. The model structure and parameter estimation is a key issue for reaching necessary quality of forecasts. It is proposed to define model structure as follows: $S = \{r, p, m, n, d, z, l\}$, where r is model dimensionality (number of equations); p is model order (maximum order of differential or difference equation in a model); m is a number of independent variables in the right hand side; n is a type of nonlinearity; d is a lag or output reaction delay time; z is external disturbance and its type; l are possible restrictions for variables. To perform automatic search for the "best" model it is proposed to use the following criterion:

$$V_N(\theta, D_N) = e^{|1-R^2|} + \ln\left(1 + \frac{SSE}{N}\right) + e^{|2-DW|} + \ln(1 + MSE) + \ln(MAPE) + e^{U},$$

where θ is a vector of model parameters; N is a power of time series used; R^2 is a determination coefficient; DW is Durbin-Watson statistic; MSE is mean squared error; $MAPE$ is mean absolute percentage error; U is Theil coefficient. The power of the criterion was tested experimentally with a wide set of linear and pseudolinear models with positive results.

There are several possibilities for adaptive model structure estimation: (1) automatic analysis of partial autocorrelation for determining autoregression order; (2) automatic search for the exogeneous variables lag estimate (detection of leading indicators); (3) automatic analysis of residual properties; (4) analysis of data distribution type and its use for selecting correct model parameters estimation method; (5) adaptive model parameter estimation with hiring extra data; (6) optimal selection of weighting coefficients for exponential smoothing, nearest-neighbor interpolation and some other techniques; (7) the use of adaptive approach to model type selection. The use of specific adaptation scheme depends on volume and quality of data, specific problem statement, requirements to forecast estimates, etc. In some cases it is possible to use logistic regression together with linear regression to describe the data with nonlinearities. Application of the adaptive concept described provides the following advantages: (1) automatic search for the «best» model reduces the search time for many times; (2) it is possible to analyze much wider set of candidate models than manually; (3) the search is optimized thanks to the use of complex quality criterion; (4) in the frames of computer system developed it is possible to integrate ideologically different methods of modeling and forecasting and compute combined forecasts estimates that are distinguished with better quality.

FILTERING ALGORITHM

Most of modern macroeconomic and financial processes on the post soviet territory are influenced by a substantial number of stochastic disturbances. Among them are the following: low qualification of managerial staff; high dependence on energy import; inconsistent and unstable laws; outdated technologies in industry and agriculture, unstable and outdated education system, high inflation (hyperinflation in 90s) etc. It was also revealed that data taken from different sources very often contain substantially different values for the same variables. It means that the measurement errors are available that also should be taken into consideration when a process model is developed. To take into consideration these disturbances and to reduce influence of measurement errors linear optimal Kalman filter is applied. The filtering procedure for scalar case is given below [3].

Step 1. Mathematical model of linear dynamic system:

$$x(k) = F(k)\,x(k-1) + C(k)\,u(k-1) + w(k-1),$$

where $x(k) = y(k)$ — process state vector (selected variable); $F(k) = \alpha_1$ — coefficient characterizing system dynamics; $C(k)\,u(k-1) = \beta_1 x(k-1)$ — influence of possible control actions (regressor); $w(k) \sim N(0,\, Q0,\, Q(k))$ — external stochastic disturbances; $z(k) = H(k)\; x(k) + v(k)$ — measurement equation; $H(k) = 1$ — measurement matrix; $v(k) \sim N(0, R(k))$ — measurement errors; initial conditions: $E\{x_0\} = \hat{x}_0$, $E\{\hat{x}_0,\, \hat{x}_0^{\mathrm{T}}\} = P_0 = P_0'$; $P(k)$ — posterior covariance matrix for state vector estimation errors; $P'(k)$ — prior covariance matrix for state vector estimation errors; $K(k)$ — optimal Kalman filter coefficient; initial covariances:

$$E\{w(k),\, v^{\mathrm{T}}(k)\} = 0, \quad E\{w(k),\, x_0^{\mathrm{T}}(0)\} = 0, \quad E\{v(k),\, x_0^{\mathrm{T}}\} = 0.$$

Step 2. State vector extrapolation:

$$\hat{x}(k) = F(k)\hat{x}(k-1) + C(k)u(k-1) = \alpha_1 y(k-1) + \beta_1 x(k-1).$$

Step 3. Extrapolation for covariance matrix of estimation errors:

$$P'(k) = F(k)P(k-1)F^{\mathrm{T}}(k) + Q(k-1) = \alpha_1^2 P(k-1) + Q(k-1).$$

Step 4. Scalar coefficient of the filter is recalculated adaptively using covariance of measurement errors as root mean squared for filtering errors: $R(k) = \sqrt{E\{\hat{x}_0, \hat{x}_0^{\mathrm{T}}\}}$. To compute the covariances an adaptive KF with moving window (AKFMW) is proposed:

$$K(k) = P'(k)H^{\mathrm{T}}(k)\{H(k)P'(k)H^{\mathrm{T}}(k) + R(k)\}^{-1} = \frac{P'(k)}{P'(k) + R(k)}.$$

Step 5. Computing of state vector estimate using the last measurement received $z(k)$:

$$\hat{x}(k) = \hat{x}(k) + K(k)\left[z(k) - H(k)\,\hat{x}(k)\right].$$

Step 6. Computing the posterior matrix for state vector estimation errors:

$$P(k) = [I - K(k)H(k)]P'(k).$$

Step 7. Go to step 2.

The statistical data for consumer price index (CPI) was taken from the state statistic tables for the period from 01.01.2000 to 01.08.2014 [4] (Fig. 1). The computations were performed using developed software [5].

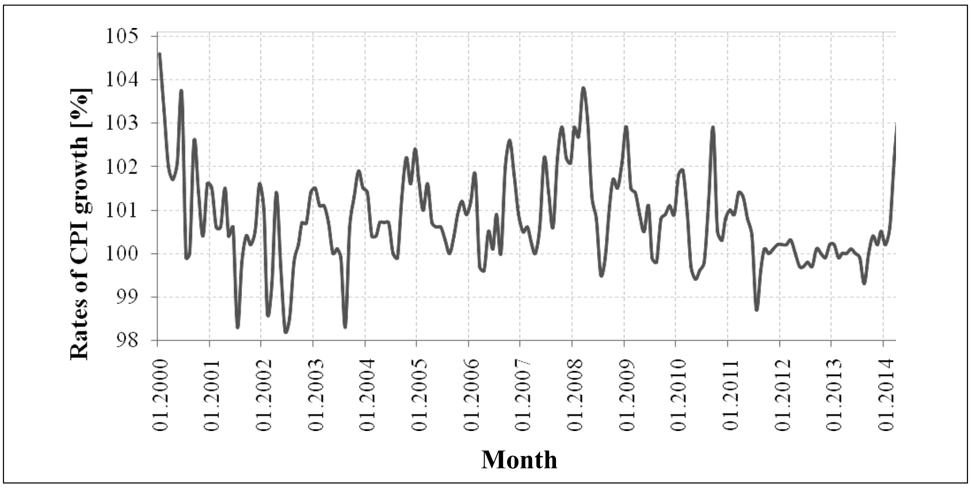

Fig. 1. Consumer price index

Constructed autoregression AR(2) model without constant is as follows:

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + \varepsilon(k),$$

where $y(k)$ — volume of imported natural gas; $x(k)$ — volume of consumed gas. After estimation of the model parameters we have the equation:

$$y(k) = 0{,}92 \cdot y(k-1) + 0{,}8 \cdot y(k-2).$$

Now test the distribution law for model residuals; respective histogram is given in Fig. 2. According to statistics χ^2

$$\sum_{k=1}^{r} \frac{(n_k - np_k)^2}{np_k} = 3{,}17 < 3{,}84 = \chi^2_{\alpha,1} = \chi^2_{0.05,1}$$

— the normality hypothesis is accepted.

In this expression n is a number of observations; r is a number of data intervals; n_k — is a number of measurements that are in a specific $k-th$ interval; p_k is a probability of appearing random value in $k-th$ interval of normal distribution; α is a level of significance.

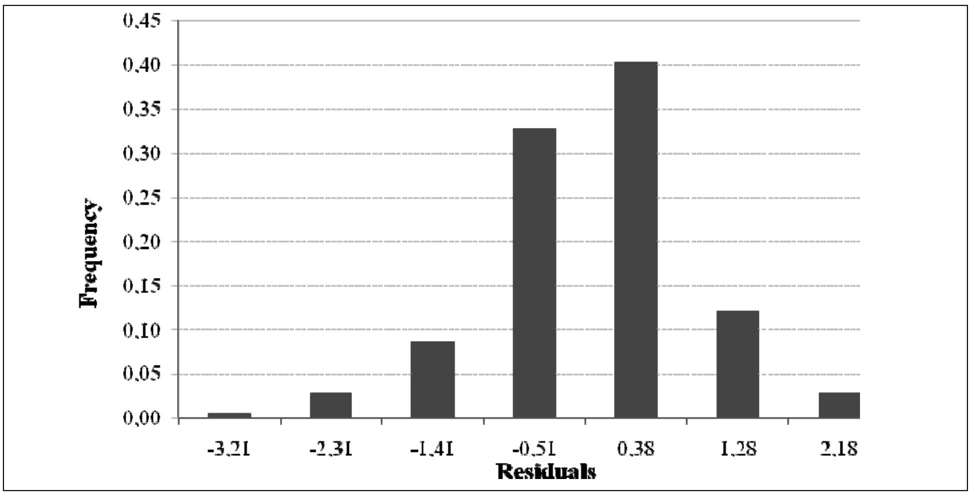

Fig. 2. Residuals histogram for CPI model AR(2)

Model quality is characterized with the following statistics: $R^2 = 0,21$; $DW = 2,07$; $e^2 = 148,01$ (Table 1). Three steps forecasts quality is given in Table 2.

T a b l e 1 . Quality of the forecasts

Statistics / Model type	MSE	MAPE	*U*	*K*
AR(2)	148,07	0,66	0,01	–
KF ($i = 0,5$)	36,39	0,36	0,01	0,73
KF ($r = 1$)	**22,04**	**0,26**	**0,01**	**0,61**
AKF	24,49	0,27	0,01	0,60
AKFMW 100	24,01	0,27	0,01	0,61
AKFMW 50	23,15	0,26	0,01	0,62
AKFMW 20	22,90	0,26	0,01	0,64
AKFMW 10	24,15	0,26	0,01	0,65
AKFMW 5	27,76	0,26	0,01	0,68
AKFMW 3	30,86	0,26	0,01	0,70

T a b l e 2 . Three-step forecast

Model type	June 2014		July 2014		August 2014	
	Value	%	Value	%	Value	%
Data	101,00		100,40		100,80	
Model	103,74	2,71	103,72	3,31	103,70	2,88
KF ($r = 0,5$)	**102,60**	**1,59**	**102,58**	**2,18**	**102,57**	**1,75**
KF ($i = 1$)	103,22	2,19	103,20	2,79	103,18	2,36
AKF	103,26	2,23	103,24	2,83	103,22	2,40
AKFMW 100	103,29	2,26	103,27	2,86	103,25	2,43
AKFMW 50	103,32	2,30	103,31	2,90	103,29	2,47
AKFMW 20	103,39	2,37	103,38	2,97	103,36	2,54

Table 2 (*continued*)

AKFMW 10	103,30	2,27	103,28	2,87	103,26	2,44
AKFMW 5	103,25	2,22	103,23	2,82	103,21	2,39
AKFMW 3	103,47	2,44	103,46	3,04	103,44	2,61

The Kalman filter characteristics of functioning are provided in Figs. 3–5.

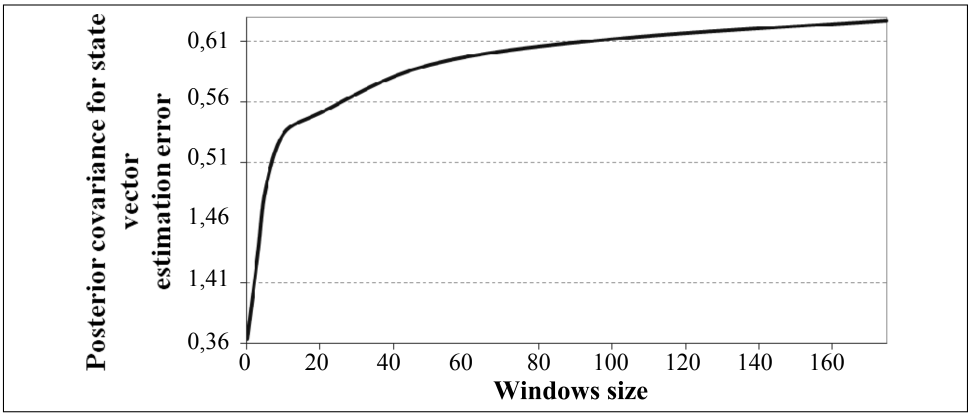

Fig. 3 Posterior covariance for state vector estimation error versus AKFMW window size

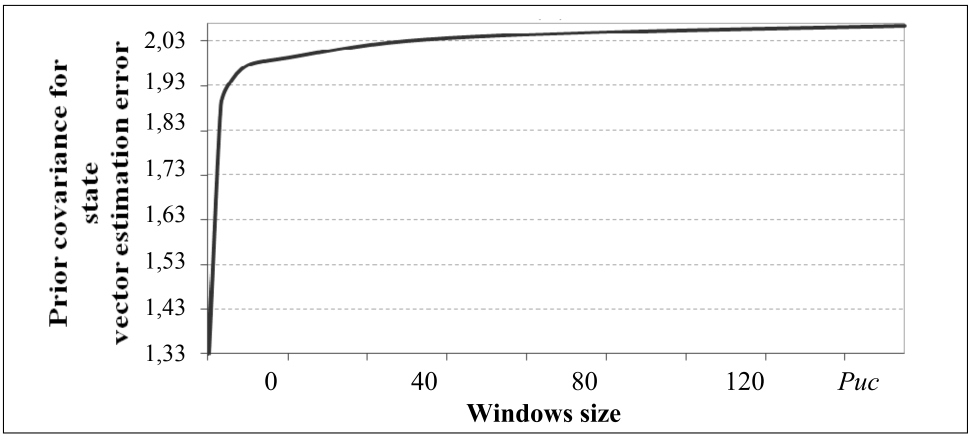

Fig. 4. Prior covariance for state vector estimation error versus AKFMW window size

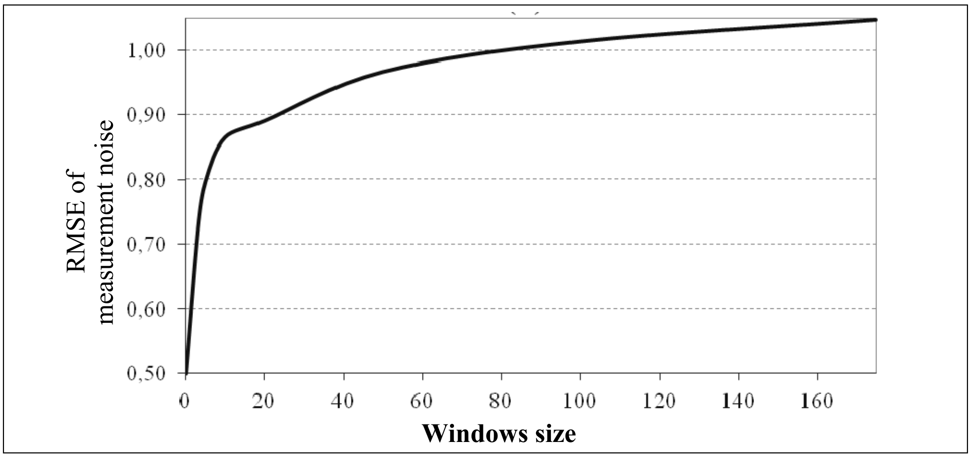

Fig. 5. RMSE of measurement noise versus AKFMW

Thus, the computational experiments performed support the idea that application of optimal filtering provides a possibility for improvement of short term forecasts using state space models. Different versions of optimal KF usually show somewhat different results though in general final result of forecasting is quite acceptable.

CONCLUSIONS

An adaptive modeling concept is proposed based on application of simultaneous model structure and parameters estimation strategy and adaptive Kalman filter. Using statistical data for the consumer price index in Ukraine an autoregression AR(2) model was constructed for short term forecasting. To compute optimal state vector estimates two versions of Kalman filter have been applied: ordinary optimal KF and adaptive KF. The latter version was constructed with hiring moving data windows for estimation of measurement noise covariances. The moving window size has been varied in a wide range from 3 to 100. The best forecasting results were received with ordinary optimal KF with noise variance $R = 0{,}5$; and lower results were received with model AR(2) without filtering procedure.

The best one step ahead forecast was received with KF that uses $R = 1{,}0$, what can be explained by higher information quality at this period of time or on purpose decreasing of CPI. Somewhat lower results of forecasting with adaptive filter could be explained by the process stationarity at the period of time examined. During such periods the adaptation is not required.

Testing of the forecasting system with wide set of macroeconomic and stock price data showed that it is possible easy to reach a value of mean absolute percentage error of about 3–4 % for short term forecasting. The use of dynamic and static estimates allows for generating necessary forecasts estimates depending on specific problem statement.

In the future studies it will be reasonable to incorporate into the software developed new forecasting techniques such as neural networks, probabilistic Bayesian networks, and immune algorithms that cover a wide class of nonlinear processes. Further steps towards automation of a model development and selection procedure are also necessary.

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