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NEW WAYS OF TRANSITION TO DETERMINISTIC CHAOS IN NONIDEAL OSCILLATING SYSTEMS

We considered nonideal dynamical system with a five-dimensional phase space. Questions of occurrence of deterministic chaos in such systems are investigated. By using the previously developed by the authors the technique of computer simulation of deterministic chaos, we discovered and described a number of new scenarios of transition to deterministic chaos. In this research analyzed in detail the phase portraits, signatures of spectrum of Lyapunov characteristic exponents and distribution of the invariant measure in various regular and chaotic attractors. In particular, there is found the transition to chaos by the scenario of generalized intermittency with two laminar phases. Succeed to identify the transition to chaos, which begins by the Feigenbaum scenario and ends through intermittency. The role of symmetry of attractors in such transitions is explored. Identified the transitions to chaos, by scenario of the generalized intermittency, with two coarse-grained laminar phases. Also managed to find the scenario of generalized intermittency, in which taken place the transition from hyper-attractor of one type to another type of hyper-attractor.

Keywords: nonideal dynamical system, chaotic attractor, scenarios of transitions to chaos.

Introduction

Despite the infinite variety of the dynamic systems, it is possible to divide all scenarios of transitions to deterministic chaos into three basic groups [1, 2]. To the first group belongs Feigenbaum scenario of transition to chaos through an infinite cascade of period-doubling bifurcations of limit cycles. To the second group of scenarios belong transitions to chaos through an intermittency by Pomeau–Manneville of various types. At last, to the third group of scenarios belong transitions to chaos through destruction of quasi-periodic attractors (invariant toruses). However, the description of the scenarios of transition to chaos in dynamical systems is found in initial stages of the study. Many questions about the relationship between the different scenarios of transition to chaos remain unexplored [1].

Statement of the problem

All oscillatory dynamic systems consist of two main parts, a source of vibration excitation and vibrational loads. If the power of the excitation source is comparable to the power consumed by the vibrational load, then such a system is called non-ideal by Sommerfeld–Kononenko [3]. If the power of the excitation source considerably exceeds the the power consumption of the vibrational load, then such a system is called an ideal by Sommerfeld–Kononenko.

Objectives of the global energy saving, forced to the maximum to minimize the power of various sources of vibration excitation. Therefore, the vast majority, the real modern oscillatory systems are nonideal. In mathematical modeling of nonideal systems is necessary to consider the influence of vibrational loads on the functioning of the source of vibration excitation. Neglecting the influence of loads on excitation source may lead to serious errors in the description of the dynamic behavior of the system. In particularly, may be completely lost the information about real existing deterministic vibrational regimes [2].

Consider the following nonlinear system of differential equations:

$$\frac{dp_1}{d\tau} = \alpha_1 p_1 - \left[\beta + \frac{A}{2}(p_1^2 + q_1^2 + p_2^2 + q_2^2)\right] q_1 + B(p_1 q_2 - p_2 q_1) p_2;$$

$$\frac{dq_1}{d\tau} = \alpha_1 q_1 + \left[\beta + \frac{A}{2}(p_1^2 + q_1^2 + p_2^2 + q_2^2)\right] p_1 + B(p_1 q_2 - p_2 q_1) q_2 + 1;$$

$$\frac{d\beta}{d\tau} = N_3 + N_1\beta - \mu_1 q_1; \qquad (1)$$

$$\frac{dp_2}{d\tau} = \alpha_1 p_2 - [\beta + \frac{A}{2}(p_1^2 + q_1^2 + p_2^2 + q_2^2)] q_2 - B(p_1 q_2 - p_2 q_1) p_1;$$

$$\frac{dq_2}{d\tau} = \alpha_1 q_2 + [\beta + \frac{A}{2}(p_1^2 + q_1^2 + p_2^2 + q_2^2)] p_2 - B(p_1 q_2 - p_2 q_1) q_1,$$

where $p_1, q_1, \beta, p_2, q_2$ – phase coordinates, τ – time, $A, B, \alpha_1, \mu_1, N_1, N_3$ – some parameters. As was established in works [2, 4, 5] the system of equations (1) is used to describe the fluid oscillations in cylindrical tanks, for the study of pendulum systems with a vibrating suspension point, for the simulation of vibrations of thin shells and a number other topical problems of nonlinear dynamics.

The parameters $A, B, \alpha_1, \mu_1, N_1, N_3$ have one or another physical or geometrical meaning depending on the considered application task. Phase variables p_1, q_1, p_2, q_2 are the generalized coordinates of the vibrational subsystem and phase variable β describes the operation of the source of excitation of oscillations.

System of equations (1) is a nonlinear deterministic dynamical system with a five-dimensional phase space. In papers [2, 6, 7] it is esteblished that there are several types of regular and chaotic attractors of this system and show that the transition to deterministic chaos can be carried out on all three groups of scenarios. The aim of this work is the identification and description of new scenarios of transition to deterministic chaos in dynamical systems of the form (1). We shall investigate, as the scenarios of transition from regular attractors to chaotic attractors, as the scenarios of transitions between different types of chaotic attractors.

Identification and description different scenarios of transition to chaos

Since the system of equations (1) is a nonlinear system of ordinary differential equations of the fifth order, the construction of attractors of such a system is only possible through the application of different numerical methods. Large series computer simulation of the system of equations (1) was carried out in the space of the parameters $(\alpha_1, A, B, N_1, N_3, \mu_1)$ of this system. The methodology of such a computer simulation is described in detail in the works [2, 6]. As a result of such a computer simulation was able to identify and describe some of the new scenarios of transition to chaos.

Let the parameters of the system (1) takes the value: $A = 1{,}12$; $B = -1{,}531$; $\mu_1 = 0{,}5$; $N_1 = -1$; $\alpha_1 = -0{,}3$. As bifurcation parameter we choose

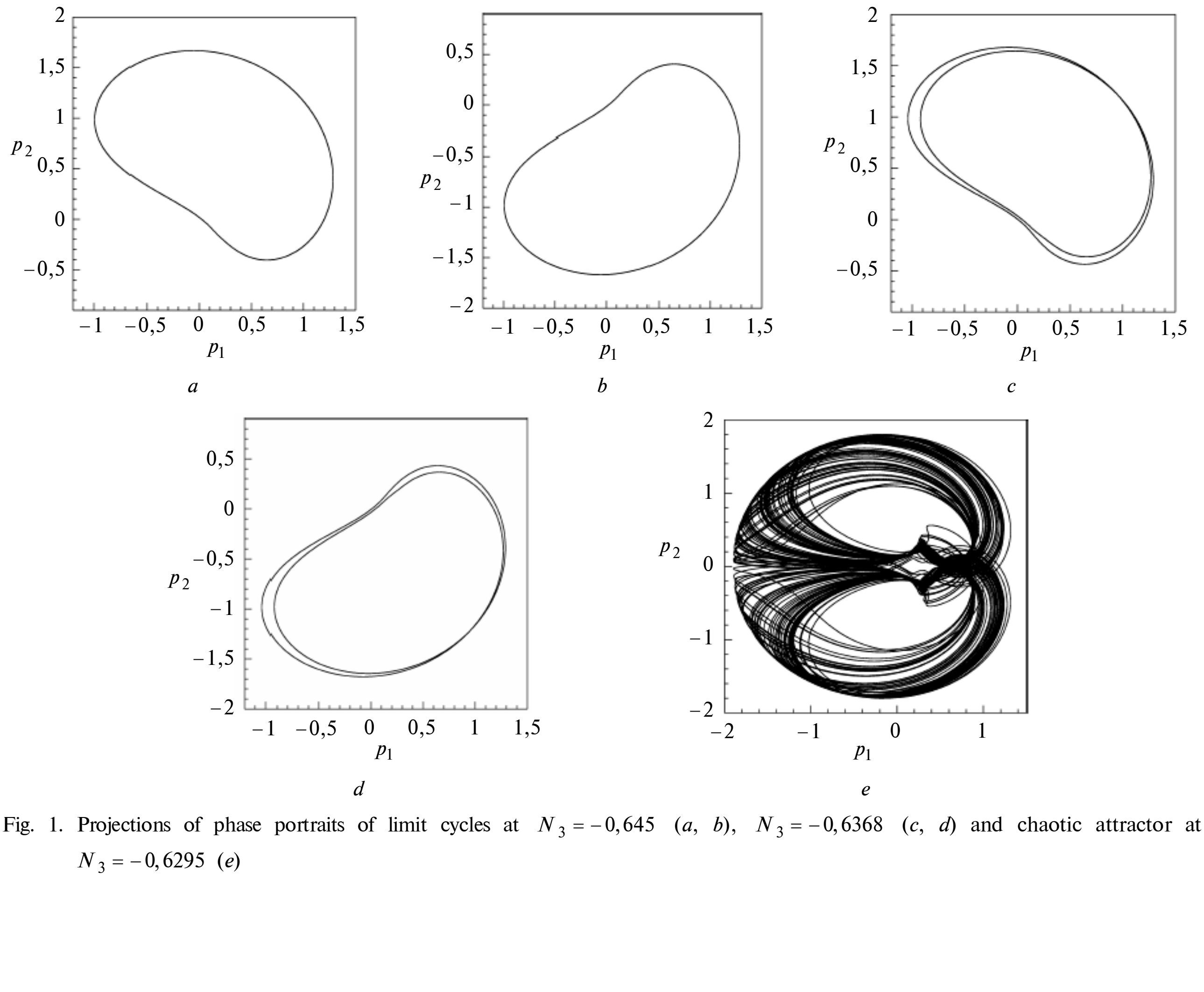

Fig. 1. Projections of phase portraits of limit cycles at $N_3 = -0{,}645$ (*a*, *b*), $N_3 = -0{,}6368$ (*c*, *d*) and chaotic attractor at $N_3 = -0{,}6295$ (*e*)

the parameter N_3. When N_3 changing in limits $-0,6526 < N_3 < -0,6302$ in system exists two single-stroke stable limit cycles. The projections of the phase portraits of such limit cycles, built at $N_3 = -0,645$ are shown in Fig. 1, *a*, *b*. Their projections are symmetrical relative to the axis of abscissa $p_2 = 0$. Signatures of the spectrum of the Lyapunov characteristic exponents (LCE) of such cycles are of the form $\langle 0,-,-,-,- \rangle$, which is typical for periodic regimes [1]. At increasing the value of the parameter N_3, there are simultaneous period doubling of the existing limit cycles. The projections of phase portraits arising 2-stroke cycles, built at $N_3 = -0,6368$ are shown in Fig. 1, *c*, *d*. Their projections are also symmetrical relative to the axis of abscissa. With further increasing the value of the parameter N_3 going infinite cascade of period-doubling of the existing limit cycles. This cascade ends with the emergence of a chaotic attractor (Fig. 1, *e*). The projection of this attractor consists of two symmetrical about a horizontal axis parts, and occupies the region of phase space in which symmetrical limit cycles exist.

The signature of the LCE spectrum of chaotic attractor has the form $\langle +,0,-,-,- \rangle$. In this case, unlike the regular attractors, chaotic attractor has several important distinguishing features. Maximal Lyapunov exponent of the chaotic attractor is positive, all the trajectories of the attractor unstable by Lyapunov and moment of time of the Poincarй returns of trajectories are unpredictable. The motion of the representative point of the trajectory on a chaotic attractor can be divided into two phases. In a first phase, a trajectory commits chaotic wandering along the top or bottom turns of the attractor. At unpredictable times trajectory moves in the opposite symmetric part of the attractor, and there continue a chaotic wandering in this part of chaotic attractor. This process is repeated an infinite number of times. Thus, in this case, the transition to chaos united the peculiarities of Feigenbaum scenario (infinite cascade of period doubling bifurcations of limit cycles), and the peculiarities of Pomeau—Manneville scenario (unpredictable intermittency between the upper and lower parts of chaotic attractor).

Consider the features of the transition to deterministic chaos at the exiting of the parameter N_3 out of the boundaries of the interval $-1,2105 < N_3 < -1,1829$. At $N_3 \in (-1,2105;-1,1829)$ attractors of the system (1) will be symmetrical, relative to the axis of abscissa, limit cycles. The projections of the phase portraits of a typical pair of limit cycles, constructed for $N_3 = -1,183$, are shown in Fig. 2, *a*, *b*. These cycles have more complex structure than previously considered in Fig. 1, *a*, *b*. If the increasing (or decreasing) of parameter N_3 leads to exit the value of this parameter out of the boundaries of the interval $-1,2105 < N_3 < -1,1829$ both limit cycles are shown in Fig. 2, *a*, *b* are vanishing. Full-blown chaotic attractor arise in system. Typical projection of the phase portrait of the chaotic attractor of this form is shown in Fig. 2, *c*. In Fig. 2, *d* is shown the distribution of the invariant measure by the phase portrait of the chaotic attractor.

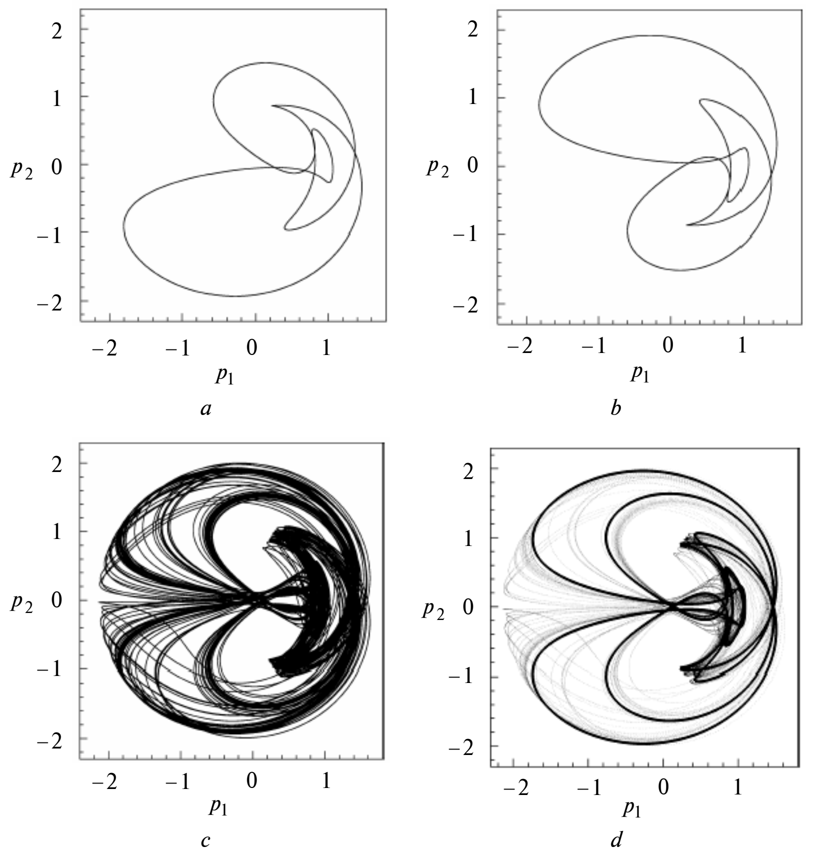

Fig. 2. Projections of phase portraits of limit cycles at $N_3 = -1,183$ (*a*, *b*), the projection of the phase portrait (*c*) and and the distribution of the invariant measure (*d*) of the chaotic attractor at $N_3 = -1,182$

Heavily drawn area in Fig. 2, *d* by the form reminiscent of the “splice” symmetric limit cycles. The distribution of the invariant measure by the phase portrait of the chaotic attractor clarifies the mechanism of its occurrence. The emergence of chaos in this case has the characteristic features of intermittency. Typical motion of trajectories by the attractor consists three phases, two laminar and turbulent. In each laminar phase the trajectory commits a quasiperiodic motion in a small neighborhood of “upper” or “lower” of vanishing limit cycles. At unpredictable moments of time a turbulent burst occurs and the trajectory goes to remote regions of phase space. Turbulent phase of motion correspond paler areas of distribution of the invariant measure in Fig. 2, *d*. Process of motion of trajectory by attractor consists following phases: “one of the laminar phases — turbulent phase — one of the laminar phases”. This process repeated endlessly. Moments of “burst” of the trajectory in turbulent phase and the “switches” between the two laminar phases are unpredictable and represent a chaotic sequence of moments of times. In this case, the transition to chaos reminiscent of the classic scenario intermittency of Pomeau—Manneville. However, in this case, the transition to deterministic chaos consists not one, but two laminar phases.

At computer modeling and numerical analysis of the existing system of dynamic regimes, it was found that when $-0{,}3916 < N_3 < -0{,}3893$ in system exist symmetric stable limit cycles (Fig. 3, *a*, *b*). At increasing of parameter N_3, namely when $N_3 = -0{,}3892$, the simultaneous period doubling bifurcation of existing cycles take place (Fig. 3, *c*, *d*). Further increase value of the parameter N_3 generates an infinite cascade of period doubling of symmetric cycles, which ends the appearance of symmetric chaotic attractors at (Fig. 3, *e*, *g*). Arising of each of the symmetric chaotic attractors occurs by Feigenbaum scenario. It is worth noting that the obtained chaotic attractors (Fig. 3, *d*, *e*) are separate and have different basins of attraction. In this case there is no phenomenon of “splice” chaotic attractors, which was observed in the event of chaotic attractors in Fig. 1, *e* and Fig. 2, *c*.

Let us analyze the scenario of transition to chaos at decreasing N_3 and exit it through the left border of interval $-0{,}3916 < N_3 < -0{,}3873$. As mentioned above, for values of the parameter $-0{,}3916 < N_3 < -0{,}3893$ in the system exist stable symmetric limit cycles (Fig. 3, *a*, *b*). At $N_3 = -0{,}39161$ the symmetric limit cycles disappear and in system arise two symmetric chaotic attractors (Fig. 3, *e*, *f*). Transition from regular regime to chaotic one for both symmetric chaotic attractors occur by classical scenario of intermittency by Pomeau — Manneville. This fact clearly illustrated the distrubution of invariant measure by the phase portrait of the chaotic attractors (Fig. 3, *g*, *h*). Heavily drawn areas in these figures which are located in a neighbourhood of disappearing symmetrical limit cycles, correspond to laminar phase of an intermittency.

In this phase a trajectories of chaotic attractors carry out quasi-periodic motions in a small neighbourhood of disappearing limit cycles. More pale parts of distributions (Fig. 3, *g*, *h*) correspond to a turbulent phase of intermittency. Arising symmetrical chaotic attractors exist in small interval of a changing of parameter N_3.

At further decrease of N_3 at $N_3 = -0{,}397$ in system occurs bifurcation “chaos—chaos”. The symmetric chaotic attractors (Fig. 3, *e*, *f*) are disappeared after transiting of a point of a bifurcation. In the system (1) arise the chaotic attractor of other type with more complicated structure that shown in Fig. 3, *k*. The chaotic attractor of such type represents “splice” of disappearing symmetrical chaotic attractors (Fig. 3, *e*, *f*). Its structure is similar to the chaotic attractors considered above (Fig. 1, *e*, Fig. 2, *c*). Transition to chaos in this case happens absolutely under other scenario which considerably differs from the previous scenarios.

The distribution of invariant measure for complicated chaotic attractor is shown in Fig. 3, *l*. The two more dark symmetrical areas of this distribution have the geometrical form similar to corresponding geometrical form symmetric chaotic attractors from Fig. 3, *e*, *f*. Analysis the dynamics of process shown in Fig. 3, *l* allows to state that transition to chaos here happens under the scenario of the generalised intermittency which is described for the first time in [2, 6]. Moving of a typical trajectory in attractor consists of three phases. Two of them are termed coarse-grained laminar [2, 6, 8]. In these phases trajectory carry out chaotic motions in one of the dark areas of Fig. 3, *l*, where were located the disappeared chaotic attractors of the previous type. In the third phase — turbulent, attractor trajectories leave these areas and move to remote areas of a phase space. The more pale parts in Fig. 3, *l* correspond to turbulent phase. After that the trajectories return to

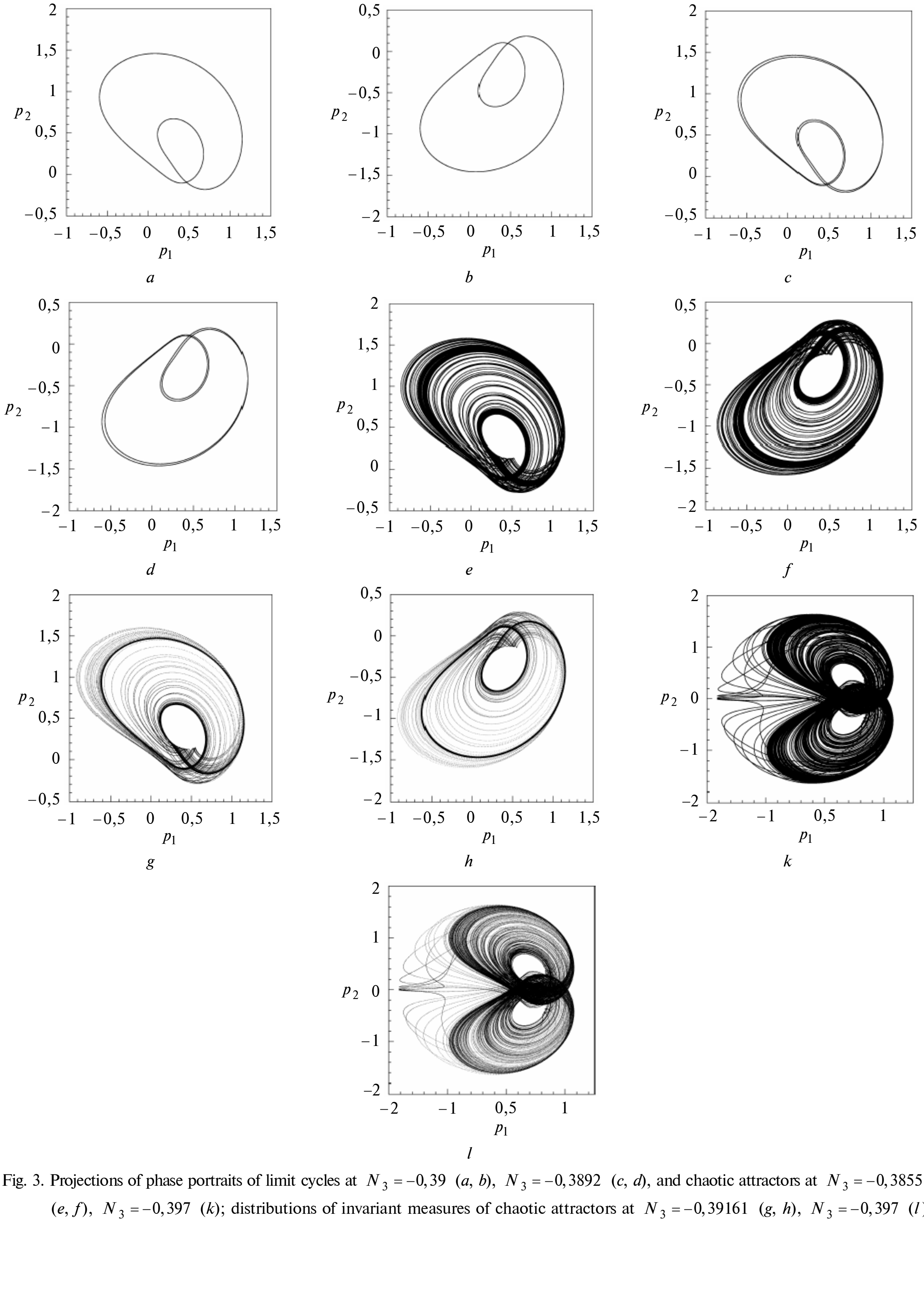

Fig. 3. Projections of phase portraits of limit cycles at $N_3 = -0,39$ (*a*, *b*), $N_3 = -0,3892$ (*c*, *d*), and chaotic attractors at $N_3 = -0,3855$ (*e*, *f*), $N_3 = -0,397$ (*k*); distributions of invariant measures of chaotic attractors at $N_3 = -0,39161$ (*g*, *h*), $N_3 = -0,397$ (*l*)

areas of coarse-grained laminar phases. Transition of a trajectory from one of coarse-grained laminar phases to turbulent phase is unpredictable in regard to a time and repeats the infinite number of times. Also are possible unpredictable direct transitions from one of coarse-grained laminar phase to other coarse-grained laminar phase. It is necessary to notice that here, unlike described in [2, 6, 8], generalised intermittency consist of two coarse-grained laminar phases.

At last we will describe scenarios of transitions "regular regime–chaos" and "chaos–other chaos" at following values of parameters of system (1): $A = 1{,}12$; $B = -1{,}531$; $N_3 = -1$; $\mu_1 = 4{,}125$; $N_1 = -1$. As bifurcation parameter we will choose parameter α_1. Computer simulation of the system (1) revealed that at values $-0{,}0254 < \alpha_1 < -0{,}0102$ in system exists a single-stroke stable limit cycle. At $\alpha_1 = -0{,}0254$ existing limit cycle loses stability and the system, as a result of bifurcation Neimark, quasi-periodic attractor arises with the signature spectrum LCE $\langle 0, 0, -, -, - \rangle$. Presence in spectrum ЛХП of two zero exponents testifies about quasiperiodicity of attractor [1]. In fig. 4, *b* is shown the projection of a phase portrait of such qua-

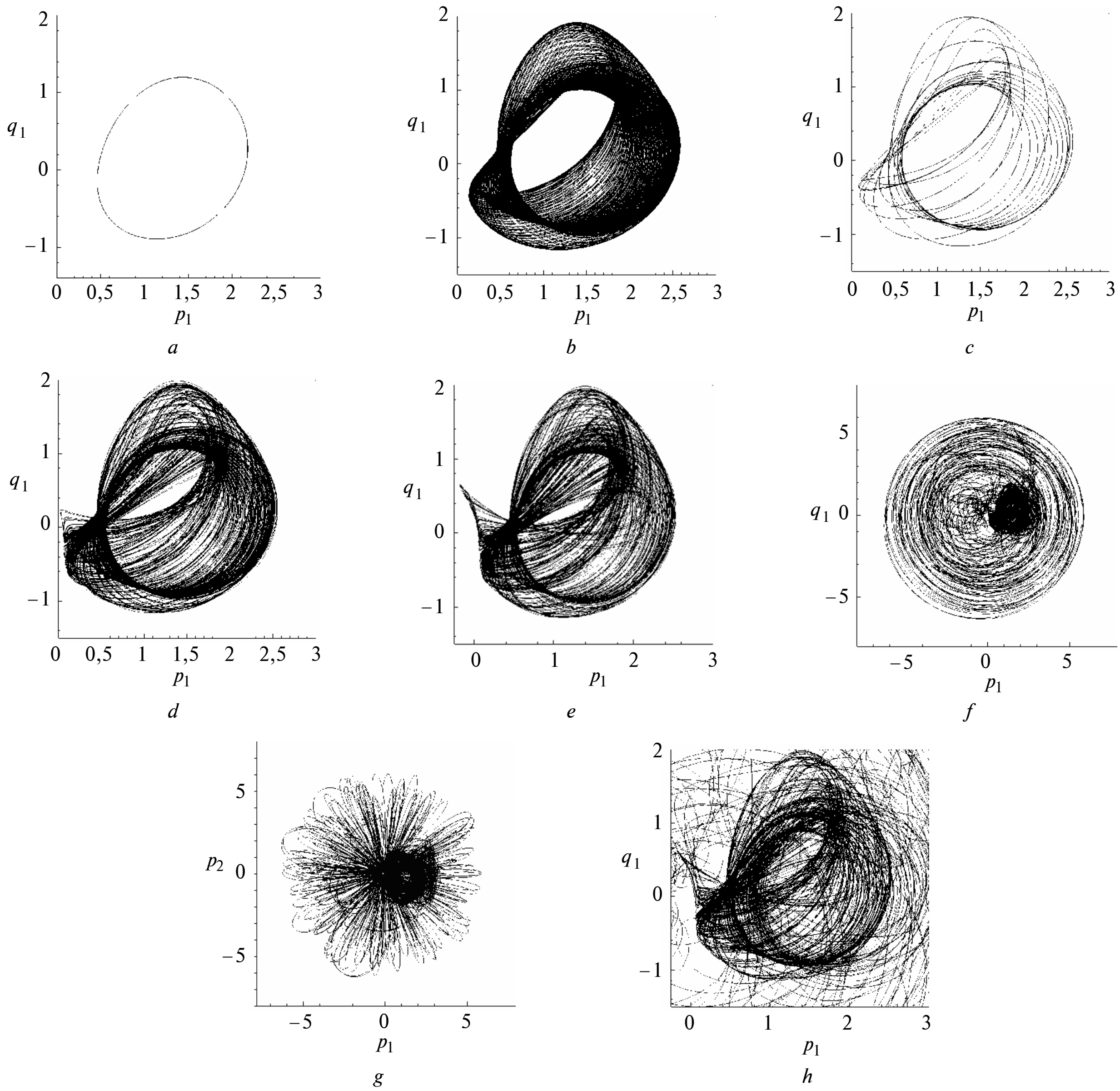

Fig. 4. Projections of phase portraits of limit cycles at $\alpha_1 = -0{,}025$ (*a*), $\alpha_1 = -0{,}0352$ (*c*); a quasiperiodic attractor $\alpha_1 = -0{,}03$ (*b*); a chaotic attractor at $\alpha_1 = -0{,}0382$ (*d*); "small" $\alpha_1 = -0{,}0388$ (*e*) and "large" $\alpha_1 = -0{,}0402$ (*f, g, h*) hyperchaotic attractors

siperiodic attractor built at $\alpha_1 = -0,03$. Trajectories of quasiperiodic attractor everywhere dense coat toroidal-shaped surface of attractor and are always returned in arbitrary small neighbourhood of attractor with some almost period.

At the further decrease of value of parameter, namely at $\alpha_1 = -0,0352$ there occurs destruction of torus. Trajectory of system carry out finite number of turnovers in torus and becomes closed, thus in system arise a resonance limit cycle in torus (Fig. 4, *c*) [1]. At $\alpha_1 = -0,03825$ a resonance limit cycle is disappear and in the system (1) arises a chaotic attractor. The projection of the phase portrait of the chaotic attractor of such type built at $\alpha_1 = -0,0382$ is shown in Fig. 4, *d*. In this case the scenario of transition to chaos through destroy quasiperiodic regimes [1] is realised.

At the value of parameter $\alpha_1 = -0,0388$ bifurcation "chaos-hyperchaos" takes place in system (1). As result of this bifurcation arise the hyper-chaotic attractor (Fig. 4, *e*) with the signature of spectrum LCE ⟨+, +, 0, –, –⟩. Phase portrait of an originating hyperchaotic attractor is similar to phase portrait of adisappearing chaotic attractor. However hyper-chaotic attractor has two positive Lyapunov's exponents, therefore close phase trajectories of a hyper-chaotic attractor diverge in two directions of a phase space. While close phase trajectories of a chaotic attractor diverge only in one direction of a phase space.

Extremely interesting bifurcation occurs in the system at $\alpha_1 = -0,0402$. Existing at $\alpha_1 < -0,0402$ hyper-chaotic attractor disappears and in the system appears hyper-chaotic attractor of an entirely different type. The different projections of the phase portrait of the hyper- chaotic attractor of this type, built at $\alpha_1 = -0,0402$, are shown in Fig. 4, *f*, *g*. This attractor is localized in a much larger volume of phase space than the attractor that shown in Fig. 4, *e*. In Fig. 4, *g* is shown an enlarged fragment of the central part of Fig. 4, *f*. Therefore, the attractor which shown in Fig. 4, *e*, is called called "small", and in Fig. 4, *f*, *g* is called "big". In [2, 6, 8] has been described a scenario of generalized intermittency in transitions "chaos–chaos". A careful study of Fig. 4, *f*, *g*, *h* shows that in this case is realized scenario of generalized intermittency for transition "hyper-chaos–hyper-chaos". Coarse-grained laminar phase of such intermittency are motion trajectories of "big" hyper-chaotic attractor in the neighborhood of disappeared "small" hyper-chaotic attractor. Accordingly, the turbulent phase of generalized intermittency is the departure of trajectories in remote areas of phase space. It should be noted, that the realization of the scenario of generalized intermittency for transition "hyper-chaos–hyper-chaos" for the system (1) is detected for the first time.

Conclusions

Thus in the work are identified and described the new scenarios of transition to deterministic chaos in some, important for applications, nonideal dynamical system. In further research is planned detection of such new scenarios for other types of dynamical systems. Also great interest calls the problem of finding the boundaries of the basins of attraction of considered regular and chaotic attractors.

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