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A MULTI-OBJECTIVE MIXED INTEGER PROGRAMMING MODEL FOR MULTI ECHELON SUPPLY CHAIN NETWORK DESIGN AND OPTIMIZATION

TURAN PAKSOY, EREN ÖZCEYLAN, GERHARD-WILHELM WEBER

This paper applies a mixed integer linear programming to designing a multi echelon supply chain network (SCN) via optimizing commodity transportation and distribution of a SCN. Proposed model attempts to aim multi objectives of SCN by considering total transportation costs and capacities of all echelons. The model composed of three different objective functions. The first one is minimizing the total transportation costs between all echelons. Second one is minimizing of holding and ordering costs in distribution centers (DCs) and the last objective function is minimizing the unnecessary and unused capacity of plants and DCs.

1. INTRODUCTION

Supply chain management has been a hot topic in the management arena in the recent years. The term «supply chain» conjures up images of products, or supplies, moving from manufacturers to distributors to retailers to customers, along a chain, in order to fulfill a customer request (Gong et al., 2008).

Supply chain management (SCM) explicitly recognizes interdependencies and requires effective relationship management between chains. The challenge in global SCM is the development of decision-making frameworks that accommodate diverse concerns of multiple entities across the supply chain. Considerable efforts have been expended in developing decision models for supply chain problems (Narasimhan and Mahapatra, 2004).

Enterprises have to satisfy customers with a high service level during standing high transportation, raw material and distribution costs. In traditional supply chains, purchasing, production, distribution, planning and other logistics functions are handled independently by decision makers although supply chains have different objectives. To overcome global risks in related markets, decision makers are obliged to fix a mechanism which different objective functions (minimizing transportation/production, backorder, holding, purchasing costs and maximizing profit and customer service level etc.) can be integrated together. Illustration of a supply chain network includes suppliers, plants, DCs and customers in Fig. 1 (Syarif et al., 2002).

The design of SC networks is a difficult task because of the intrinsic complexity of the major subsystems of these networks and the many interactions among these subsystems, as well as external factors such as the considerable multi objective functions (Gumus et al., 2009). In the past, this complexity has forced much of the research in this area to focus on individual components of supply chain networks. Recently, however, attention has increasingly been placed on the performance, design, and analysis of the supply chain as a whole.

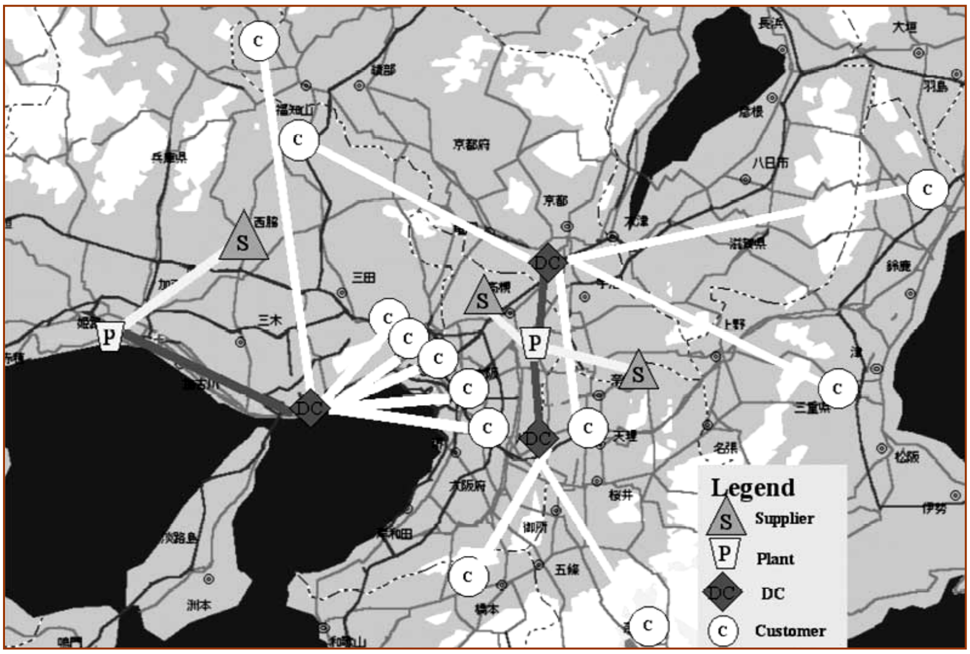

Fig. 1. Illustration of a Supply Chain Network (Syarif et al. 2002)

Supply chains performance measures are categorized as qualitative and quantitative. Customer satisfaction, flexibility, and effective risk management belong to qualitative performance measures. Quantitative performance measures are also categorized by: (1) objectives that are based directly on cost or profit such as cost minimization, sales maximization, profit maximization, etc. and (2) objectives that are based on some measure of customer responsiveness such as fill rate maximization, customer response time minimization, lead time minimization, etc (Altiparmak et al., 2006).

However, the SCM design and planning is usually involving trade-offs among different goals. In this study, we developed a mixed integer linear programming model to design and optimize a supply chain network via providing multi objective functions mentioned above together. We considered three objectives for SCM problem: (1) minimization of total transportation costs between suppliers-manufacturers-distribution centers and distribution costs between distribution centers and customers, (2) minimization of holding and ordering costs in DCs based EOQ (economic order quantity) and (3) providing equity of the capacity utilization ratio of manufacturers and DCs.

In this field, numerous researches are conducted. (Williams, 1981), developed seven heuristic algorithms to minimize distribution and production costs in supply chain. (Cohen and Lee, 1989), present a deterministic, mixed integer, nonlinear programming with economic order quantity technique to develop global supply chain plan. (Pyke and Cohen), 1993, developed a mathematical programming model by using stochastic sub-models to design an integrated supply chain involves manufacturers, warehouses and retailers. (Özdamar and Yazgaç, 1997), developed a distribution/production system involves a manufacturer center and its warehouses. They try to minimize total costs such as inventory; transportation costs etc under production capacity and inventory equilibrium constraints. (Petrovic et al., 1999), modeled supply chain behaviors under fuzzy constraints. Their model showed that, uncertain customer demands and deliveries play a big role about behaviors. (Syarif et al., 2002), developed a new algorithm based genetic algorithm to design a supply chain distribution network under capacity constraints for each echelon. (Yan et al., 2003), tried to contrive a network which involves suppliers, manufacturers, distribution centers and customers via a mixed integer programming under logic and material requirements constraints. (Yılmaz, 2004), handled a strategic planning problem for three echelon supply chain involves suppliers, manufacturers and distribution centers to minimize transportation, distribution, production costs. (Chen and Lee, 2004), developed a multi-product, multi-stage, and multi-period scheduling model to deal with multiple incommensurable goals for a multi-echelon supply chain network with uncertain market demands and product prices. The uncertain market demands are modeled as a number of discrete scenarios with known probabilities, and the fuzzy sets are used for describing the sellers' and buyers' incompatible preference on product prices. The supply chain scheduling model is constructed as a mixed-integer nonlinear programming problem to satisfy several conflict objectives, such as fair profit distribution among all participants, safe inventory levels, maximum customer service levels, and robustness of decision to uncertain product demands, therein the compromised preference levels on product prices from the sellers and buyers point of view are simultaneously taken into account. (Nagurney and Toyasaki, 2005), try to balance e-cycling in multi tiered supply chain process. (Gen and Syarif, 2005), developed a hybrid genetic algorithm for a multi period multi product supply chain network design. (Paksoy, 2005), developed a mixed integer linear programming to design a multi echelon supply chain network under material requirement constraints. (Lin et al., 2007), compared flexible supply chains and traditional supply chains with a hybrid genetic algorithm and mentioned advantages of flexible ones. (Wang, 2007), explained the imbalance between echelons with peccant supply chain by changing chain's perfect balanced. He used ant colony technique to minimize costs in peccant imbalanced supply chains. (Azaron et al., 2008), developed a multi-objective stochastic programming approach for supply chain design under uncertainty. Demands, supplies, processing, transportation, shortage and capacity expansion costs are all considered as the uncertain parameters. Their multi-objective model includes (i) the minimization of the sum of current investment costs and the expected future processing, transportation, shortage and capacity expansion costs, (ii) the minimization of the variance of the total cost and (iii) the minimization of the financial risk or the probability of not meeting a certain budget. (You and Grossmann, 2008), addressed the optimization of supply chain design and planning under responsive criterion and economic criterion with the presence of demand uncertainty. By using a probabilistic model for stock-out, the expected lead time is proposed as the quantitative measure of supply chain responsiveness. (Schütz et al., 2008), presented a supply chain design problem modeled as a sequence of splitting and combining processes. They formulated the problem as a two-stage stochastic program. The first-stage decisions are strategic location decisions, whereas the second stage consists of operational decisions. The objective is to minimize the sum of investment costs and expected costs of operating the supply chain. (Tuzkaya and Önüt, 2009), developed a model to minimize holding inventory and penalty cost for suppliers, warehouse and manufacturers based a holononic approach. (Sourirajan et al., 2009), considered a two-stage supply chain with a production facility that replenishes a single product at retailers. The objective is to locate distribution centers in the network such that the sum of facility location, pipeline inventory, and safety stock costs is minimized. They use genetic algorithms to solve the model and compare their performance to that of a Lagrangian heuristic developed in earlier work. (Ahumada and Villalobos, 2009), reviewed the main contributions in the field of production and distribution planning for agri-foods based on agricultural crops. Through their analysis of the current state of the research, they diagnosed some of the future requirements for modeling the supply chain of agri-foods. (Gunasekaran and Ngai, 2009), have developed a unified framework for modeling and analyzing BTO-SCM and suggest some future research directions. (Xu and Nozick, 2009), formulated a two-stage stochastic program and a solution procedure to optimize supplier selection to hedge against disruptions. Their model allows for the effective quantitative exploration of the trade-off between cost and risks to support improved decision-making in global supply chain design. (Shin et al., 2009), provided buying firms with a useful sourcing policy decision tool to help them determine an optimum set of suppliers when a number of sourcing alternatives exist. They proposed a probabilistic cost model in which suppliers' quality performance is measured by inconformity of the end product measurements and delivery performance is estimated based on the suppliers' expected delivery earliness and tardiness.

After giving the introduction and the relevant literature, At the second section, the proposed model which is a multi objective mixed integer linear programming model is presented. We tested the novel model with a numerical example and discussed the results obtained by LINGO package programmer at the last section.

2. PROBLEM STATEMENT

Here, the constituted model represents three echelons, multi supplier, multi manufacturer, multi DC, and multi customer problem. Decision maker wishes to design of SC network for the end product, select suppliers, determine the manufacturers and DCs and design the distribution network strategy that will satisfy all capacities and demand requirement for the product imposed via customers. The problem is a single-product, multi-stage SCN design problem. Considering company managers' objectives, we formulated the SCN design problem as a multi-objective mixed-integer non-linear programming model. The objectives are minimization of the total cost of supply chain, minimization holding and ordering costs in DCs, and maximization of capacity utilization balance for DCs (i.e. equity on utilization ratios). The assumptions used in this problem are: (1) the number of customers and suppliers and their demand and capacities are known, (2) the number of plants and DCs and their maximum capacities are known, (3) customers are supplied product from a single DC. Fig. 2 presents a simple network of three-stages in supply chain network.

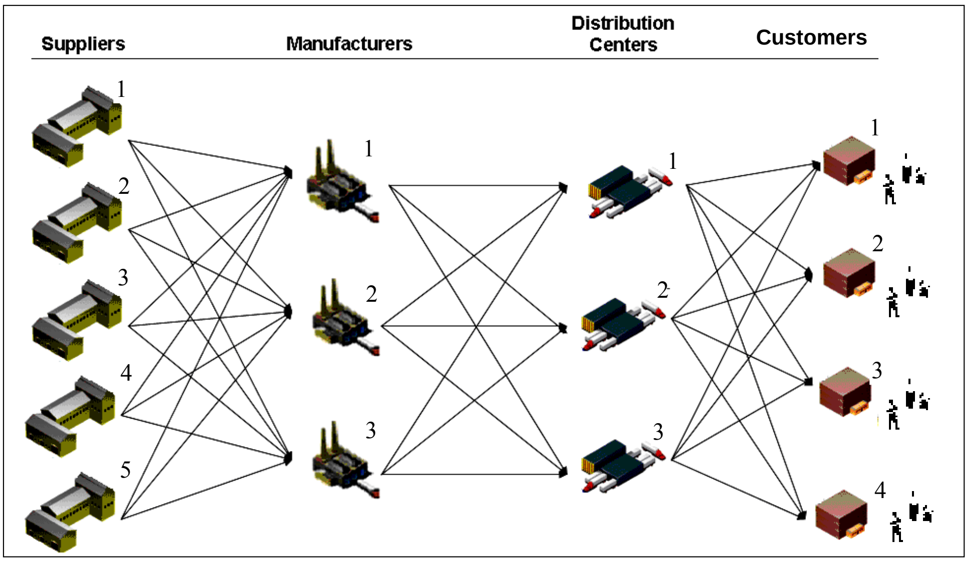

Fig. 2. Supply Chain Network of Proposed Model

2.1 Model Variables and Parameters.

i is an index for customers ($i \in I$),

j is an index for DCs ($j \in J$),

k is an index for manufacturing plants ($k \in K$), s is an index for suppliers ($s \in S$),

b_{sk} is the quantity of raw material shipped from supplier s to plant k,

f_{kj} is the quantity of the product shipped from plant k to DC j, q_{ji} is the quantity of the product shipped from DC j to customer i,

$$y_{ji} = \begin{cases} 1, & \text{if DC is serves customer } i, \\ 0, & \text{otherwise,} \end{cases}$$

D_k is the capacity of plant k,

$\sup_s$ is the capacity of supplier s for raw material, W_j is distribution capacity of DC j, d_i is the demand for the product at customer i, c_{ji} is the unit transportation cost for the product from DC j to customer i, a_{kj} is the unit transportation cost for the product from plant k to DC j, t_{sk} is the unit transportation and purchasing cost for the raw material from supplier s to plant c, c_h is the holding cost per year at DC j, S is ordering cost to manufacturer k from each of DCs.

2.2 Objective Function, Constraints.

f_1 is the total cost of SCN. It includes the variable costs of transportation raw material from suppliers to manufacturers and the transportation the product from plants to customers through DCs.

f_2 is annual holding and ordering cost of products in DCs according to the economic order quantity (EOQ) model.

f_3 is the equity of the capacity utilization ratio for manufacturers and DCs, and it is measured by mean square error (MSE) of capacity utilization ratios. The smaller value is, the closer the capacity utilization ratio for every manufacturer and DC is, thus ensuring the demand are fairly distributed among the DCs and manufacturers, and so it maximizes the capacity utilization balance.

$$\text{Minimize } f_1 = \sum_s \sum_k t_{sk} b_{sk} + \sum_k \sum_j a_{kj} f_{kj} + \sum_j \sum_i c_{ji} q_{ji} \ ; \quad (1)$$

$$\text{Minimize } f_2 = \sum_j \left[\frac{S \sum_k f_{kj}}{\sqrt{\dfrac{\sum_k 2S f_{kj}}{c_h}}} + \frac{c_h \sqrt{\dfrac{\sum_k 2S f_{kj}}{c_h}}}{2} \right]; \quad (2)$$

$$\text{Minimize } f_3 = \sqrt{\frac{\sum_k [(\sum_j f_{kj} / D_k) - (\sum_k \sum_j f_{kj} / \sum_k D_k)]^2}{\sum k}} +$$

$$+ \sqrt{\frac{\sum_j [(\sum_i q_{ji} / W_j) - (\sum_j \sum_i q_{ji} / \sum_i W_i)]^2}{\sum j}} \ ; \quad (3)$$

$$\sum_j y_{ji} = 1 \quad \forall i \,, \quad (4)$$

$$\sum_j d_i y_{ji} \le W_j \quad \forall j \,, \quad (5)$$

$$q_{ji} = d_i y_{ji} \quad \forall i, j \,, \quad (6)$$

$$\sum_k f_{kj} = \sum_i q_{ji} = \quad \forall j \,, \quad (7)$$

$$\sum_k b_{sk} \le \sup_s \quad \forall k \,, \quad (8)$$

$$\sum_j f_{kj} \le \sum_s b_{sk} \quad \forall k \,, \quad (9)$$

$$\sum_j f_{kj} \le D_k \quad \forall k \,, \quad (10)$$

$$y_{ji} = \{0,1\} \quad \forall i, j \,, \quad (11)$$

$$b_{sk} f_{kj} q_{ji} \ge 0 \quad \forall i, j, k, s \,. \quad (12)$$

The model is composed of three objective functions (Eq. 1–3). The first objective function (Eq. 1) defines minimizing shipment costs between suppliers, manufacturers, DCs and customers. The second objective function is minimizing holding and ordering costs in DCs using economic order quantity model (Eq. 2). Equation 3 (third objective) minimizes equity of the capacity utilization ratio of manufacturers and DCs.

Constraint (Eq. 4) represents the unique assignment of a DC to a customer, (Eq. 5) is the capacity constraint for DCs, (Eq. 6) and (Eq. 7) gives the satisfaction of customer and DCs demands for the product, (Eq. 8) gives the supplier capacity constraint, (Eq. 9) describes the raw material supply restriction, (Eq. 10) is the manufacturer production capacity constraint. Finally, constraints (Eq. 11) and (Eq. 12) are integrality constraints.

3. NUMERICAL EXAMPLE

In this section we present a numerical example to illustrate the proposed model mentioned in previous section. The application of the model is performed for a logical data which was inspired from related cases in the real world. The considered supply chain network includes five suppliers which are located different places, three manufacturers, three distribution centers and four customers (Fig. 2). The network is structured to supply raw materials and transport products from suppliers to end-users is constituted from multi echelon and capacitated elements of network considering minimizing the total transportation costs between all echelons (suppliers, manufacturers, distribution centers (DCs) and customers, holding and ordering costs in DCs and unnecessary and unused capacity of plants and DCs via decreasing variance of transported amounts between echelons. Numerical data used in example are given below, respectively. Table 1 and 2 gives the priorities of objectives obtained by Expert Choice 11.5 program to find rate of purposes according to AHP methodology.

Table 1. Relatives of Objective Functions (AHP)

	f_1	f_2	f_3
f_1	1	2	3
f_2	1/2	1	3/2
f_3	1/3	2/3	1
Sum	1.83	3.67	5.5

Table 2. Normalized AHP Matrix

	f_1	f_2	f_3
f_1	0.545	0.545	0.545
f_2	0.273	0.273	0.273
f_3	0.182	0.182	0.182

According to Table 2, weight of each objective function is 0.542, 0.273 and 0.182 respectively. Because of matrix consistency < 0.1, this matrix will be accepted. Parameters: Number of Total Suppliers: 5; Number of Total Customers: 4; Number of Total Manufacturers: 3; Number of Total Distribution Centers: 3; S 20 tl; C_h 1,5 tl.

Table 3. Unit transportation costs values between suppliers and manufacturers (TL)

Manufacturers	Suppliers				
	1	2	3	4	5
1	0.5	0.3	0.4	0.4	0.5
2	0.6	0.4	0.5	0.6	0.6
3	0.5	0.5	0.6	0.6	0.4

Table 4. Unit transportation costs values between manufacturers and DCs (TL)

DCs	Manufacturers		
	1	2	3
1	1.4	1.1	1.1
2	1.1	1.2	0.8
3	1.3	1.4	0.9

Table 5. Unit transportation costs values between DCs and Customers (TL)

Customers	DCs		
	1	2	3
1	0.9	0.9	0.7
2	0.7	0.6	0.6
3	0.8	0.5	0.7
4	0.6	0.9	0.8

Table 6. Capacities of Suppliers, Manufacturers, DCs and Demands of Customers (unit)

	Suppliers	Manufacturers	DCs	Customers
1	5000	7000	6300	3100
2	5500	6500	6700	3100
3	5250	6500	6000	3100
4	4750	–	–	3100
5	4500	–	–	–

Table 7. The results obtained by LINGO package program

Decision	Value	Decision	Value
$X_{1,3}$	2000	$Y_{3,2}$	3400
$X_{2,1}$	2400	$Y_{3,3}$	3100
$X_{2,2}$	3100	$Z_{1,4}$	3100
$X_{3,1}$	400	$Z_{2,2}$	3100
$X_{5,3}$	4500	$Z_{2,3}$	3100
$Y_{1,2}$	2800	$Z_{3,1}$	3100
$Y_{2,1}$	3100	Objective (tl)	13678

According to data obtained LINGO package program, results are given above table 7. Under capacity constraint and transportation costs, decision maker purchased raw materials from all suppliers except fourth. 2000 units from first suppliers, 5500 units from second supplier, 400 units from third and 4500 units from fifth supplier, are transported to manufacturers. 2800 units which come from second and third supplier are shipped to second DC from first manufacturer. Also 3100 units of product are transported from second manufacturer to first DC. 3400 units to second DC and 3100 units to third DC totally 4460 units of product shipped from third manufacturer. Supporting equation 4 constraint, each customer provided their demand only one DC via providing a better balanced distribution. All customers' demand is supplied from DCs as 3100, 6200 and 3100 units respectively (Fig. 3). At three echelons, all transportation costs and holding/ordering costs in DCs (first and second objective functions) calculated about 13678tl. Providing the third objective, the unnecessary and unused capacity of plants and DCs are minimized via decreasing variance of transported amounts between second and third echelons. When we examined the second and third echelons' distribution, it's seen that the transportation between manufacturers-DCs-customers come and go from 2800 units to 3400 units considering balancing distribution.

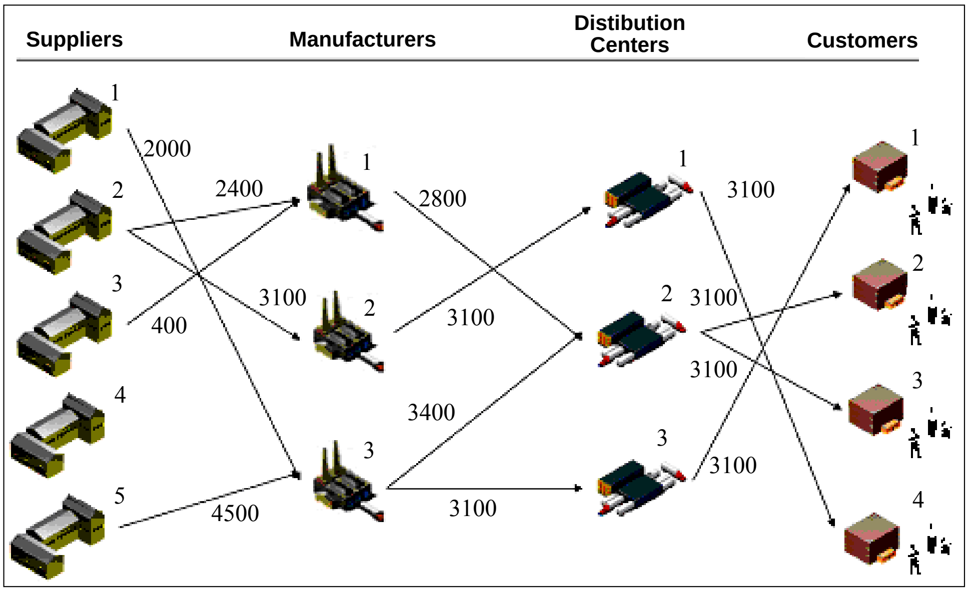

Fig. 3. Raw Material and Product flow

4. CONCLUSION

In this study, a mixed integer non-linear programming model is developed to design a supply chain network by combining three different objectives. Considered three objectives: (1) minimization of total transportation cost of plants and distribution centers (DCs), inbound and outbound distribution costs, (2) minimization of holding and ordering costs via EOQ method (3) maximization of capacity utilization balance for DCs (i.e. equity on utilization ratios). We used the developed model to determine from which suppliers, manufacturers, DCs and how much amounts will be transported to answer customers demand. We developed binary variables to provide a DC for a customer. So we have prevented unbalanced distributions between DCs and customers.

In future, new solution methodology based on tabu search or heuristic methods can be developed to obtain new optimal solutions for the multi-objective SCN design problem, and the effectiveness of the solution methodology can be investigated. Additionally, uncertainty of costs and demands can be considered in the model and new solution methodologies including uncertainty can be developed via fuzzy models.

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