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SYNTHESIS OF EXPONENTIAL SPLINES BASED ON MODELS OF LINEAR ELECTRICALAI CIRCUITS

Introduction

In most important practical cases the measuring task is formulated as a determination of *R*, *L*, *C*-parameters of the linear passive bipolar electrical circuits (BEC) [1]. The important problem is the control of electrical and radio elements as a part of electrical schemes [2, 3]. The control of radio elements is carried out without physical discontinuity of electrical circuit by creating a special mode in the difficult multipolar electrical circuit, that allows to detach separate sections in the form of multi-element electrical bipolars [4]. It is important to provide the high-speed measuring transformation in the manufacture. The usage of test signals (TS) in special form and usage of transitive processes in investigated circuits for the realization of selective transformation allows to determine the bipolar electrical circuits parameters by comparably simple device and provides the minimum time of measuring parameters [4, 5].

The principle of selective transformation by synthesis of the special form of TS is used in measuring parameters of BEC according to the zeros-and-poles method (ZPM) [6, 7]. TS, created by the combination of exponents, image of which is represented in the form of fractional-rational function of the complex variable, is used Zeros and poles of TS compensate accordingly poles and zeros of impedance function of BEC. In this case response reduction to the set form, comfortable for analysis, occurs. According to the creation of TS inverse model of impedance function of BEC is used. Zeros-and-poles values of TS during the control are taken equally for the nominal values of poles and zeros of an impedance function. According to the deflection of response from the set response the tolerance R, L, C – parameters' control is carried out.

The formulation of a problem

The usage of the high-powered measuring methods and the control of R, L, C-parameters of BEC demands the solution of some urgent tasks, in particular tasks of discrete synthesis of exponential TS [6, 7]. A class and properties of system of the approximating functions [8, 9], used in the synthesis, determine qualify of reproduced exponential dependences which are represented as an electrical signal. The demand of TS permanence and its derivatives are made accordingly.

In such way, precondition for the usage of the exponential splines method (ES) in problem of measuring are:

– class of the exponential TS that are used in measurement and control of R, L, C-parameters of BEC;

– the requirement of TS smoothness (there is a possibility of signal differentiation in the investigated electrical circuit during measurements);

– necessity in the matching method of the signal analytical description, with the character of signals, formed in electrical circuit;

– during measuring of BEC parameters, using ZPM, the temporal approach is important (the form and instantaneous values of transient process in investigated circuit are informative [6, 7], accordingly the temporal approach must be when the synthesis of TS occurs).

Nowadays the method of ES is generally known as a method of spline function in Computational Mathematics [10]. The close analogy takes place in measurements. In the problem of measurements the method of ES actually means approximation of investigated processes for creating their analytical model and its usage for treatment of measuring information and the usage of splines for electrical signals generation and their ("approximant") usage as test signals for the implementation of measuring transformation [11].

Accordingly the properties "approximant" determine accuracy of receiving information about parameters of electrical circuits during the process of measuring. A discrete character of splines successfully coordinates with discrete way of signals formation using digital-analog device. In this case the development of methodology of TS is formed on the basis of ES. This will allow to use given signals in the tasks of measuring parameters.

Theoretical positions

Let's look into the following aspect of splines. Let system of nodes (grid) $\Delta_N : a = t_0 < t_1 < ... < t_N = b$ be set on segment $[a, b]$. Let's mark the set of $(m–v)$-times continuous differentiable functions on the segment $[a, b]$ as $C^{m-v}[a, b]$.

Definition 1. *m–ordered exponential spline (ES) of C^{m-v} class ($1 \le v \le m$) with nodes on the grid Δ_N is represented as a segmented function $sf_{Gm}(t)$, which is **(a)** – is a solution to some linear inhomogeneous differential equation (LIDE) with constant coefficients on every segment* $[t_i, t_{i+1}]$, $i = 0,1,..., N-1$

$$\sum_{j=0}^{I} a_j \cdot D^{l-j} s_i(t) = f_i(t);\; D^j = \left(\frac{d}{dt}\right)^j \qquad (1)$$

with such right part: $f_i(t) = P_{k,i}(t) \cdot e^{\lambda_i t}$, $P_{k,i}(t)$ *– is a k-ordered polynom, its Laplas transformation gives fractionally—rational function;* ***(b)*** *– in this case* $sf_{Gm}(t) \in C^{m-v}[a, b]$.

Exponential splines can be presented using the system of basic splines that are not equal to zero in finite interval.

Definition 2. *Basic exponential splines (BES) or m – ordered G – splines* of $C^{m-v}[a, b]$ class are functions $G_{m,i}(t) \in sf_{Gm}(t)$, which $G_{m,i}(t) \neq 0$ *on the interval* $t \in [t_i, t_{i+m}]$ *and* $G_{m,i}(t) \equiv 0$ *out off the interval* $t \in [t_i, t_{i+m}]$.

The order of splines $m = l + k + 1$ is "order" of functions forming a spline, which is understood to be an order of LIDE, determined by number of roots of the characteristic polynom $Q_m(p)$. This polynom can be received for the solutions of LIDE taking for the consideration of the right part. An interval, where G–spline is not equal to zero, is minimum and uniquely determined by the order of LIDE. The functions $G_{m,i}(t)$, $i = \overline{0, N-1}$ are linearly independent and form basis in space of splines $sf_{Gm}(t)$.

Let's look into uniform interval of discretization. Let's inject the variable $\bar{t} = i + \varepsilon$, which means relative time connected with current $\bar{t} = t / h$, where h – is the interval of discretization; $i = 0, 1, 2, \ldots\ 0 \leq \varepsilon \leq 1$. Let's consider G – spline of the third order as an example

$$G_3(\bar{t}) = \begin{cases} \dfrac{1}{\alpha(1-e^{-\alpha})}[-1 + \alpha\varepsilon + e^{-\alpha\varepsilon}], \bar{t} \in [0,1]; \\ \dfrac{1}{\alpha(1-e^{-\alpha})}[1 + \alpha + e^{-\alpha} - (1 + e^{-\alpha})\alpha\varepsilon - 2e^{-\alpha\varepsilon}], \bar{t} \in [1,2]; \\ \dfrac{1}{\alpha(1-e^{-\alpha})} e^{-\alpha}\left[-1 + \alpha(\varepsilon - 1) + e^{-\alpha(\varepsilon-1)}\right], \bar{t} \in [2,3]; \\ 0, \bar{t} < 0, \bar{t} > 3. \end{cases} \qquad (2)$$

The spline-function looks like

$$\begin{aligned} sf_{G3}(\bar{t}) = \frac{1}{\alpha(1-e^{-\alpha})}\{&f[i+1] \cdot (-1 + \alpha\varepsilon + e^{-\alpha\varepsilon}) + \\ &+ f[i][1 + \alpha + e^{-\alpha} - (1 + e^{-\alpha})\alpha\varepsilon - 2e^{-\alpha\varepsilon}] + \\ &+ f[i-1]e^{-\alpha}\left[\alpha(\varepsilon - 1) - 1 + e^{-\alpha(\varepsilon-1)}\right]\}, (\bar{t}) = i + \varepsilon. \end{aligned} \qquad (3)$$

Mentioned splines are formed on each interval $\bar{t} \in [i, i + 1]$ by solutions of LIDE as:

$$h^2 \cdot [D^2 + \alpha \cdot D] s_i(\bar{t}) = d_i \cdot 1(\bar{t}) \quad (4)$$

The solution of LIDE on each interval as the Koshi task gives:

$$s_i(\bar{t}) = \frac{h^2}{\alpha^2}\{\alpha S_i(0) + \alpha S_i'(0) \cdot (1 - e^{-\alpha\varepsilon}) + \\ + [-1 + \alpha\varepsilon + e^{-\alpha\varepsilon}] \cdot d_i\}, \bar{t} = i + \varepsilon; \quad (5)$$

Solutions have three "free" parameters $S_i(0)$, $S_i'(0)$ and d_i. Parameters $S_i(0)$ and $S_i'(0)$ can be set according to the meaning at the end of previous interval considering their continuity in i-th node: $S_i(0) = S_{i-1}(1)$, $S_i'(0) = S_{i-1}'(1)$. Parameter d_i on each interval is set by external action (right part of LIDE) and can be defined from an interpolating in the $(i + l)$-th node.

Let's look into to the synthesis of splines that are based on the modal of linear electrical circuits. The spline-function can be represented as a sum of some finite functions shifted in time:

$$sf_{G3}(\bar{t}) = \sum_{j=0}^{\infty} f[j] \cdot G_3(\bar{t} - j), \bar{t} = i + \varepsilon$$

Thus

$$G_3(\bar{t}) = \sum_{j=0}^{\infty} \left(\frac{h^2}{\alpha^2}\right) \left[-1 + \alpha(\bar{t} - k) + e^{-\alpha(\bar{t}-k)}\right]_+ \cdot d_k, \quad (6)$$

$$\bar{t} = i + \varepsilon,$$

where

$$\mathrm{g}(\bar{t}) = \left(\frac{h^2}{\alpha^2}\right) \left[-1 + \alpha \cdot \bar{t} + e^{-\alpha \cdot \bar{t}}\right]_+, \mathrm{g}(\bar{t})_- = \begin{cases} \mathrm{g}(\bar{t}), \bar{t} > 0 \\ 0, \bar{t} \le 0; \end{cases}$$

is the solution of LIDE (4) within zero entry conditions, for which during the moments of "inclusion" in the nodes $g(0_+) = 0$; $g'(0_+) = 0$. The values of function and its first derivative during the moment of "inclusion" can be determined as

$L\{\mathrm{g}(t)\} = \frac{1}{p^2(p + \alpha_T)}$, where $\alpha = \alpha_t h, L -$ is the operator of Laplas transformation for continuous functions. Image $W(p) = \frac{1}{p^2(p + \alpha_T)}$ can be determined as reduced transmittive function of some linear forming electrical circuit, for which $\lim_{p \to \infty} \frac{1}{p^2(p + \alpha_T)} \times p = 0$ and

$\lim\limits_{p\to\infty} \dfrac{1}{p^2(p+\alpha_T)} \times p^2 = 0$. That means that function $G_3(\bar{t})$ will be continuous and will have a continuous first derivative. In this way, continuity is determined by the choice of LIDE or transmittive function of the forming circuit *W(p)*.

Coefficients d_i in the equation (6) can be determined from the condition $G_3(\bar{t}) \neq 0;\ \bar{t} \in [0, 3]$ and $G_3(\bar{t}) \equiv 0;\ \bar{t} \notin [0, 3]$. Let's look into the following of these conditions. If we apply discrete Laplas transformation [12] in relation to the shifted gridded functions (SGF) in both parts (6), we'll get

$$G_3^*(q,\varepsilon) = H(e^{-q}) \cdot W^*(q,\varepsilon) = H(e^{-q}) \cdot \frac{\frac{h^2}{\alpha^2} \cdot \sum_{k=0}^{m-1} b_k(\varepsilon) \cdot e^{q(m-k)}}{\sum_{j=0}^{m} a_j \cdot e^{q \cdot j}},$$

where

$$q = ph;\ m = 3;\ b_0(\varepsilon) = -1 + \alpha\varepsilon + e^{-\alpha\varepsilon};\ 0 \le \varepsilon \le 1;$$

$$b_1(\varepsilon) = 1 + \alpha + e^{-\alpha} - (1 + e^{-\alpha}) \cdot \alpha\varepsilon - 2e^{-\alpha\varepsilon};$$

$$b_2 = e^{-\alpha}\,[\alpha(\varepsilon - 1) - 1 + e^{-\alpha(\varepsilon-1)}];$$

$$H(e^{-q}) = A(\alpha, h) \cdot (1 + d_1 \cdot e^{-q} + d_2 \cdot e^{-2q} + d_3 \cdot e^{-3q});$$

$$\sum_{j=0}^{3} a_j \cdot e^{q \cdot j} = (e^q - e^{-\alpha})(e^q - 1)^2.$$

If

$$H(e^{-q}) = A(\alpha, h) \cdot \sum_{j=0}^{m} a_j \cdot e^{q(j-m)},$$

then

$$G_3^*(q,\varepsilon) = H(e^{-q}) \cdot W^*(q,\varepsilon)$$

$$= A(\alpha,h) \cdot \frac{h^2}{\alpha^2} \cdot e^{-mq} \cdot \sum_{k=0}^{m-1} b_k(\varepsilon) \cdot e^{q(m-k)}, \qquad (7)$$

which means expression $G_3^*(q,\varepsilon)$ will contain only zeros. The expression (7) in time domain is matched to the pulse function $g^*(\bar{t}) \neq 0$ on the interval $\bar{t} \in [0, m]$ and $g^*(\bar{t}) = 0$ out of this interval [12]. Thus, G–spline, taking into account the normalizing multiplier, can be presented.

$$G_m(\alpha,\bar{t}) = A(\alpha,h) \cdot g^*(\alpha,\bar{t}) =$$

$$= A(\alpha,h) \cdot \frac{h^2}{\alpha^2} \cdot \mathbf{D}^{-1}\left\{e^{-mq} \cdot \sum_{k=0}^{m-1} b_k(\varepsilon) \cdot e^{q(m-k)}\right\} \qquad (8)$$

where $A(\alpha,h)$ is the normalizing multiplier, such that

$A(\alpha, h) = \frac{\alpha^2}{h^2} \Big/ \left| \sum_{k=0}^{m-1} b_k(\varepsilon)_{/\varepsilon=1} \right|$; $\mathbf{D}^{-1}$ – is the operator of inverse discrete Laplas transformation of the SGF. Variable ε is a real parameter for discrete Laplas transformation. That causes a simple transition to time domain (according to the expression (2)).

The expression of spline-function (3) is received by combination of $G_3(\bar{t})$. The change of coefficients $f[i]$ does not influence the form of functions $G_3(\bar{t})$, that is a resultant function $sf_{G3}(\bar{t}) \in C^1[a,b]$ too.

The form of representation (3) is suitable for ES because the coefficients $f[i]$ are directly (straight) represented as counts, feeding to the entrance port of some spline–approximating filter, which has the transmittive function that looks like $G_3^*(q, \varepsilon)$.

Let's show that received segmented-polynomial function $sf_{G3}(\bar{t})$ of a form (3) is a spline. By definition, function on the i - th interval is $sf_{G3}(\bar{t})$ = $sf_{G3}(i, \varepsilon)$, where $\varepsilon \in [0,1]$. The values of the function in nodes are determined when ε = 0 and ε = 1. The values for spline–function at the end of previous interval $sf_{G3}(i-1, \varepsilon)$ are determined when ε = l. For each i we have $sf_{G3}(i-1, 1)$ = $sf_{G3}(i, 0)$. That means that the condition of function's continuity in nodes is being satisfied. The expression of the first spline–function's derivative:

$$sf'_{G3}(i,\varepsilon) = \frac{1}{(1-e^{-\alpha})} \cdot \{f[i+1] \cdot [1 - e^{-\alpha\varepsilon}] +$$
$$+ f[i] \cdot [-1 - e^{-\alpha} + 2 \cdot e^{-\alpha\varepsilon}] + f[i-1] \cdot e^{-\alpha}[1 - e^{-\alpha(\varepsilon-1)}]\}$$

The values of derivative in the end–points on the i–th interval for ε = 0 and ε = 1 are received in the same way. The value of derivative at the end of previous interval $sf'_{G3}(i-1, \varepsilon)$ is determined when ε = l. Thus $sf'_{G3}(i-1,\ 1)$ = $sf'_{G3}(i, 0)$. That means that the conditions of the first derivative's continuity in nodes are satisfied.

The expression (3), when ε = 0 and ε = 1 means that the values of spline–function and the restored signal, which assigned by its discrete counts $f[i]$, in nodes mismatch in times of discretization. Thus, function (3) should be considered as approximating. It is easy to show that certain Shenberg splines $B_n(\bar{t})$ [8] when $n < m$ are the particular variant of splines $G_m(\bar{t})$ and can be received from $G_m(\bar{t})$ as some limit when $\alpha \to 0$. Parameter α differed from $B_n(\bar{t})$ makes it possible to change the form of spline.

Conclusion

ES creates "real" basis of approximate function for mentioned class of test signals. An advantage of basis is the possibility of generation in real electrical circuits. The resulted continuous part of forming circuit defines a kind of splines, providing a continuity of the function and its derivatives. The discrete part provides ended duration of basic functions.

Using the models of linear electrical circuits many various models of ES can be constructed. It is also obvious that the usage of real signals takes off accuracy limitation for reproduction of basic functions because of ended high–speed performance of used element basis, in particular because of ended coefficient of amplification of operating amplifiers, on the base of which integrators are constructed, in a strip of frequency.

There is principle possibility to improve accuracy of reproduction of the set form TS and their parameters, if there are limitation of dimensions of the basis for the class of exponential signals, described by the same functions as the splines. This allows to improve the accuracy of measurements.

It's expedient to continue further development of exponential spline method in the field of research metrological aspects of applying test signals, formed on basis of exponential spline models, in tasks of measuring.

Bibliography

1. *Кнеллер В.Ю., Боровских Л.П.* Определение параметров многоэлементных двухполюсников. – М. : Энергоатомиздат, 1986. – 144 с.
2. *Байда Н.П., Кузьмин И.В., Шпилевой В.Т.* Микропроцессорные системы поэлементного диагностирования РЭА. – М. : Радио и Связь, 1987. – 256 с.
3. *Лихтциндер Б.Я.* Автоматизация поэлементного контроля многополюсных электрических цепей // Измерение, контроль, автоматизация. – М., 1983. – Вып. 3(47). – С. 14–24.
4. *Мартяшин А.И., Куликовский К.Л., Куроедов С.К., Орлова Л.В.* Основы инвариантного преобразования параметров электрических цепей / Под ред. А.И. Мартяшина. – М. : Энергоатомиздат, 1990. – 216 с.
5. *Боровских Л.П., Павлов А.М.* О преобразовании параметров многоэлементных двухполюсников при импульсном питании // Приборы и системы управления. – 1989. – № 2. – С. 24–25.

6. *Осадченко В.П.* Электрические цепи в условиях воздействий, форма которых совпадает с формой свободной составляющей переходного процесса в цепи // Технология приборостроения : научн-техн. сб., Сер. XII. – М. : ЦНТИ “Поиск”. – 1984. – Вып. 2. – С. 103–115.
7. *Туз Ю.М., Осадченко В.П.* Некоторые особенности контроля параметров электрических цепей по методу нулей и полюсов // Структурные методы повышения точности, чувствительности и быстродействия измерительных приборов и систем. Респ. науч.-техн. конф., Вып. 1. – Киев, 1985. – С. 127, 128.
8. *Алберг Дж., Нельсон Э., Уолт Дж.* Теория сплайнов и ее приложения. –М. : Мир, 1972. – 320 с.
9. *Завьялов Ю. С, Квасов Б.И., Мирошниченко В.Л.* Методы сплайн–функций. – М. : Наука, 1980. – 352 с.
10. *Маккартин Б.Дж.* Применение экспоненциальных сплайнов в вычислительной гидродинамике // Аэрокосмическая техника. – 1984. – № 4 (т. 2). – С. 13–20.
11. *Туз Ю.М., Шумков Ю. С.* Метод экспоненциальной сплайн–аппроксимации в задачах измерения и моделирования // Науково-технічна конф. "Памяті академіка М.П. Кравчука" (до 100-річчя з дня народження). – Київ, 1992. – С. 20–21.
12. *Цыпкин Я.З.* Теория линейных импульсных систем. – М. : Физматгиз, 1963. – 968 с.
13. *Улахович Д.А.* Основы теории линейных электрических цепей. – СПб. : БХВ – Петербург, 2009. – 816 с.