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STANDARD CONVERTER OF SINE VOLTAGE INTO TIME INTERVAL FOR MEASURING THERMOELECTRIC TRANSDUCER ERROR δ_{acdc}

Introduction

One of the most important elements of state metrological assurance is an AC voltage standard. The most widely-spread method that is used in such references is the thermal comparison method. This method has such serious defect as δ_{acdc}. This method consists in the next: RMS of AC voltage is regarded as equal to some DC voltage, if being by turns connected up to the same thermal transducer these voltages create equal thermal effects. This error is evoked by the fact, that during current passage through the thermoelectric transducer not only Joule heat is emitted, but also the heat induced by Thomson effect and Peltier effect, which changes the distribution of heat in space depending on current direction. To take this error into account in metrological performance of standard thermal transducers we must have an instrument measuring δ_{acdc}.

If AC voltage has a pure sine form, its amplitude is $\sqrt{2}$ times more than its RMS. It follows that equivalent DC voltage must be

$$U^{=} = \frac{U_a}{\sqrt{2}} \qquad (1)$$

For sine voltage this ratio correspond to cutoff angel of 90°. Hence we can build a system for measuring δ_{acdc} error by defining the difference between cutoff angle and 90°.

Functional system for propagation of AC voltage unit was developed and created in research institute of Experimental Research Automation in NTUU “KPI”. The core of this system is a synthesizer of ultrapure sine voltage and converter of its amplitude into time period with errors that do not exceed millionth parts (ppm – parts per million) of set values [1]. Such high accuracy was reached thanks to absolutely new approach protected by patent of Ukraine [2]. The main idea of this approach is illustrated on figure 1.

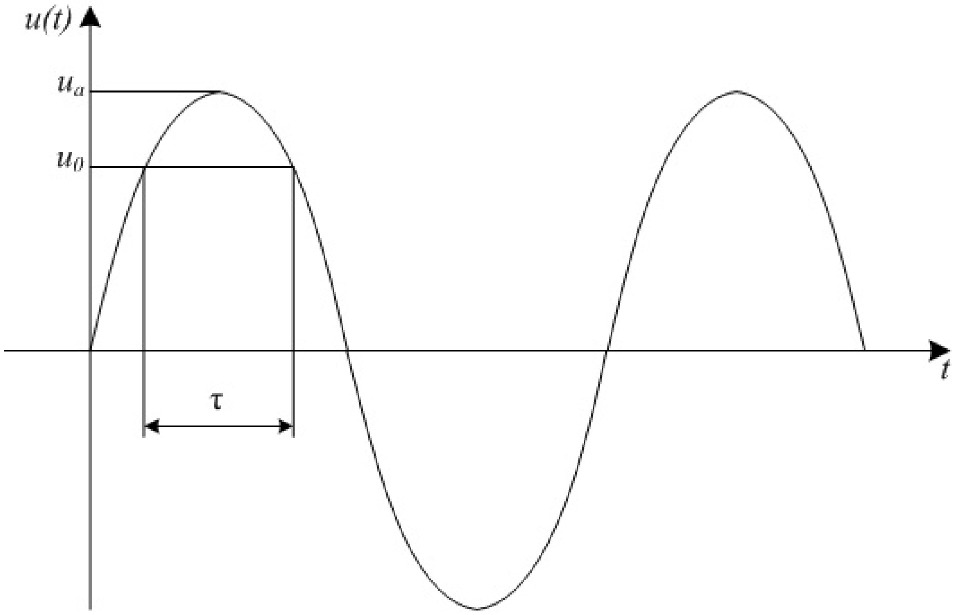

Fig. 1. Conversion of amplitude value of sine voltage into time period

Amplitude value of sine voltage u_a and value of reference voltage u_0 are related as:

$$u_a = \frac{u_0}{\cos \pi f \tau} \tag{2}$$

The dependence of u_a on τ is nonlinear. That’s why it is important to define an optimal relation of the product $f \cdot \tau$, which will give the minimal error

$$\xi_{u_a} = \frac{\Delta u_a}{u_a} \tag{3}$$

Let’s designate an absolute error of measuring as Δτ,

$$\gamma_\tau = \frac{\Delta\tau}{\tau} \tag{4}$$

– is a relative error of time interval measuring, Δτ has additive and multiplicative components.

The relative component is defined by counting discreteness of pulses of reference frequency, by which interval τ is being filled. This error is a random error with uniform low.

Multiplicative component is defined by multiplicative error of quantizing pulses following with frequency f_0.

If signal frequency (or period) is defined by the same frequency f_0 of quantizing pulses as the interval τ, the multiplicative component of error in product $f \cdot \tau$ will be cancelled.

Thus, it is reasonable to synthesize the sine signal with clock rate f_0 so, that its period is an integer function $\tau_0 = 1/f_0$. In this case measurement error of $f \cdot \tau$ won't have multiplicative component and will be defined only by its additive component, which is equal to $\pm 1 \cdot \tau_0$.

Object statement

The aim of this research is to create a theoretic base for designing of standard converter of AC voltage amplitude into time interval for definition of standard thermal transducer δ_{acdc} error during thermal comparison. To reach this goal we need to study next questions:

- define, how much does the time interval measurement error affect final result of δ_{acdc} measurement by converting of sine voltage into time interval. According to this, define an optimal calibration frequency and base voltage to get the maximal accuracy using potentialities of modern electronic components;
- study possible affects of quasi-sinusoidal voltage nonlinear distortions and develop methods to remove them;
- define and reason key requirements to future instrument to realize the suggested method;
- design a structural scheme and algorithm of standard converter.

1. Influence of time interval measurement error on final result of δ_{acdc} measurement

Let's estimate error ξ_{u_a} in case of small γ_τ error:

$$\Delta u_a \approx \frac{\partial u_a}{\partial \tau} \cdot \Delta\tau = \frac{u_0 \pi f \sin \pi f \tau}{\cos^2 \pi f \tau} \cdot \Delta\tau \quad , \tag{5}$$

or

$$\xi_{u_a} = \frac{u_0}{u_a} \cdot \frac{\pi f \tau \sin \pi f \tau}{\cos^2 \pi f \tau} \cdot \gamma_\tau \tag{6}$$

In consideration of (2) and (4)

$$\xi_{u_a} = \pi f \tau \cdot tg \pi f \tau \cdot \gamma_\tau = \pi f \tau \cdot tg \pi f \tau \cdot \frac{\Delta\tau}{\tau} \tag{7}$$

Finally

$$\xi_{u_a} = \pi f \Delta\tau \cdot tg\pi f\tau \tag{8}$$

As we can see from (7) influence coefficient of time interval measurement error is equal to

$$\chi_\tau = \pi f\tau \cdot tg\pi f\tau \tag{9}$$

Hereby the voltage measurement error ξ_{u_a} depends on $f \cdot \tau$ and time interval measurement error $\Delta\tau$. That is to say, to reduce this error we should reduce $f \cdot \tau$ or increase the precision of time interval measurement error.

As we can see from fig. 1 product $f \cdot \tau$ can reach values from 0 to 0.5, herewith value of ξ_{u_a} will alter from 0 to ∞.

The dependences of ξ_{u_a} on $f \cdot \tau$ with different $\Delta\tau$ are shown on fig.2 and its values are given in table 1.

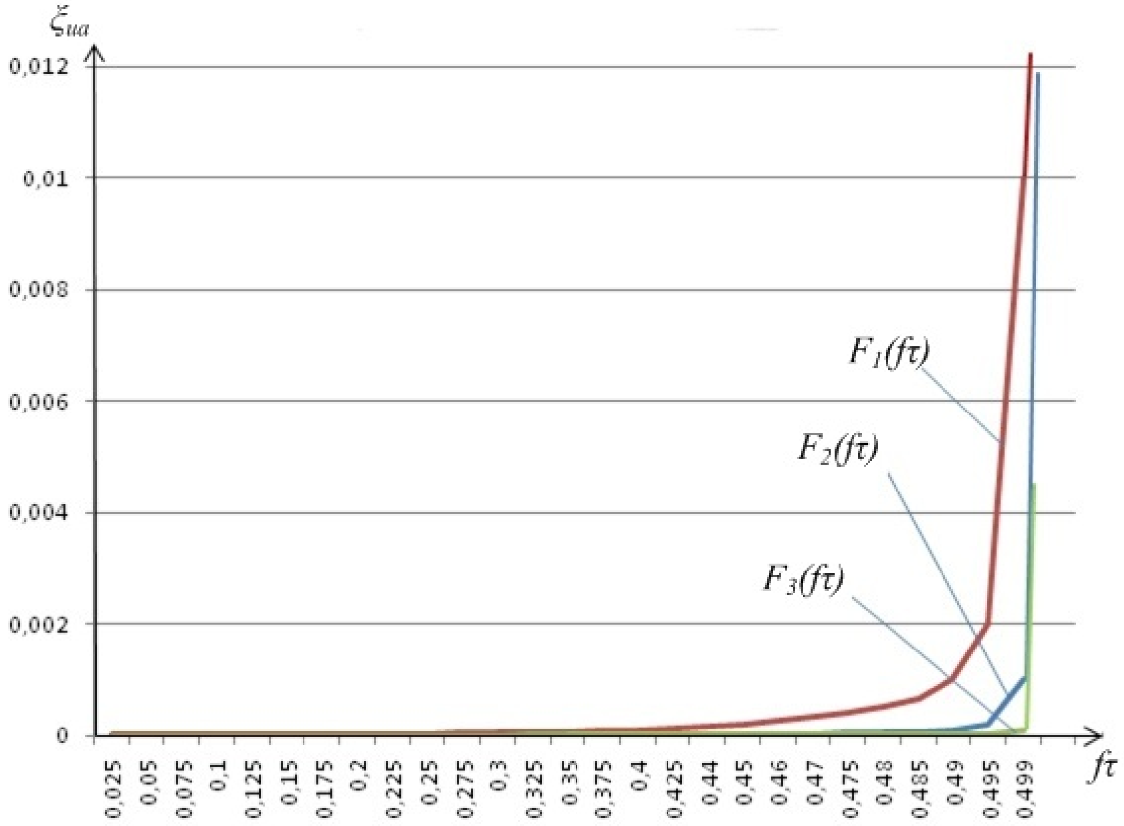

Fig. 2. $F_1(f\tau)$ – ξ_{u_a} when $\Delta\tau = 10^{-7} s$, $F_2(f\tau)$ – ξ_{u_a} when $\Delta\tau = 10^{-8} s$, $F_3(f\tau)$ – ξ_{u_a} when $\Delta\tau = 10^{-9} s$

Table 1.

Values of ξ_{u_a} error with depending on some values of $f \cdot \tau$ and $\Delta\tau$

$f \cdot \tau$	$f = 100 \; Hz$

	ξ_{u_a} when $\Delta\tau = 10^{-7}$, [ppm]	ξ_{u_a} when $\Delta\tau = 10^{-8}$, [ppm]	ξ_{u_a} when $\Delta\tau = 10^{-9}$, [ppm]
0	0	0	0
0,025	2,47	0,247	0,0247
0,1	1,02	1,02	0,102
0,2	22,8	2,28	0,228
0,3	43,2	4,32	0,432
0,4	96,7	9,67	0,967
0,45	198	19,8	1,98
0,495	1999,84	199,984	19,9984
0,5	∞	∞	∞

From formula (8) and fig. 2 we can see, that the requirements to time interval measurement are the least, when $\tau \to 0$ $u_a/u_0 \to 1$, i.e. just the fact of equality $u_a = u_0$ is registered. Such principle underlies the diode compensating voltmeters [3], in which time interval measurement is not carried out. This is possible, because influence coefficient χ_τ by $f\tau \to 0$ also tends to zero: $\lim\limits_{f\tau \to 0} \chi_\tau = 0$. But in practice this method is very hard to realize. As a proof of this– the best diode compensating voltmeter V3-49 has an error of 0,05%.

According to the method of converting of sine voltage amplitude into time interval, the base voltage for comparison U_0 and DC voltage for thermal comparison $U^{=}$ must be equal to $\frac{U_a}{\sqrt{2}}$. On frequency of 100 Hz the time interval that corresponds to this ratio is:

$$\tau = \frac{\frac{\pi}{4}}{\pi \cdot f} = \frac{1}{4 \cdot 100} = 2{,}5\,\text{мс}$$

2. Influence of higher harmonics of quasi-sinusoidal voltage and methods of their removal

Calculation of signal amplitude and RMS through measured time interval is possible only in case of pure sine form of signal. That’s why one of the main demands on this method is pure spectral composition of sine voltage. To remove higher harmonics low-pass filter is used, which totally removes harmonics evoked by usage of step-sine voltage

$$k = pn \pm 1, \; p = 1, 2, \ldots, m \tag{10}$$

where k – number of harmonic evoked by stepping-sine voltage;

n – number of sampling periods.

Each higher harmonic will be introduced by the number of samples that is inversely proportional to sequence number of this harmonic. Let's say number of sampling periods on main harmonic is $n = 1000$ with signal frequency $f = 100$ Hz. The nearest harmonics evoked by such discreteness will appear on frequency:

$$f_h = f \cdot k = 100 \cdot 1000 \pm 1 = 100000 \pm 1$$

Then, for example, the fifth harmonic will be introduced by $\frac{n}{5} = 200$ sampling period. It will evoke an appearance of harmonics on frequency

$$f_{h5} = f_5 \cdot k_5 = 500 \cdot 200 \pm 1 = 100000 \pm 1$$

So the appearance of higher harmonics evoked by discreteness of quasi-sinusoidal voltage will depend only on the number of samples on main harmonic.

Method of addition of harmonics in antiphase by DAC code generation is used to remove lower harmonics (2-7). The essence of the method is next: initially we generate the DAC code, which corresponds to quasi-sinusoidal voltage with one harmonic; then we measure amplitude and phase spectrum of signal on filter output with the help of external. After this we add into code the harmonic components that have the same amplitudes as harmonics in output spectrum, but phases of corresponding harmonics differ in π. Thus, having the DAC calibrated (possibly after several iterations) we can get an ultrapure sine voltage synthesizer.

3. Requirements to future instrument for realization of suggested method

To implement the thermal comparison method for δ_{acdc} measurement a few more terms except of pure sine form of AC voltage must be fulfilled:

- $|U^{+}| = |U^{-}| = U^{=}$, to prove this correct we must have a high resolution DC voltage source, and carry out the measurement of positive and negative voltage with high-precision multimeter.
- We must reach the equality of thermal e. m. f. on the thermal-electric transducer output $E^{\sim} = E_c^{=}$ by smooth selection of sine voltage amplitude.

$$E_c^{=} = \frac{E^{+} + E^{-}}{2} \tag{11}$$

$$E^{+} = K^{+}\left(U^{+}\right)^2 \tag{12}$$

$$E^{-} = K^{-}\left(U^{-}\right)^2 \tag{13}$$

$$E_C = \frac{E^{+} + E^{-}}{2} = \frac{K^{+}\left(U^{+}\right)^2 + K^{-}\left(U^{-}\right)^2}{2}. \tag{14}$$

If

$$\left|U^{+}\right| = \left|U^{-}\right| = U^{=} \tag{15}$$

$$E_C = \left(U^{=}\right)^2 \cdot \frac{K^{+} + K^{-}}{2} \tag{16}$$

Let’s implement zero thermal comparison, that is to say we convey such AC voltage that

$$E^{\sim} = E_c^{=} \tag{17}$$

$E^{\sim}$ - thermal e. m. f. by effect of AC voltage.

$$E^{\sim} = K^{\sim}(U^{\sim})^2 \tag{18}$$

After substituting (16) and (18) into (17) we’ll get:

$$K^{\sim}\left(U^{\sim}\right)^2 = \left(U^{=}\right)^2 \cdot \frac{K^{+} + K^{-}}{2} \tag{19}$$

Hence

$$U^{\sim} = U^{=}\sqrt{\frac{K^{+} + K^{-}}{2K^{\sim}}} \tag{20}$$

$$\delta_{acdc} = \frac{U^{\sim} - U^{=}}{U^{=}} = \frac{U^{\sim}}{U^{=}} - 1 \tag{21}$$

Substituting (20) into (21) we’ll get

$$\delta_{acdc} = \sqrt{\frac{K^{+} + K^{-}}{2K^{\sim}}} - 1 \tag{22}$$

Due to (2) we can write:

$$\delta_{acdc} = \frac{1}{2\cos(\pi f \tau)}\sqrt{\frac{E^{+} + E^{-}}{E^{\sim}}} - 1 \tag{23}$$

In consideration of (17) we get:

$$\delta_{acdc} = \frac{1}{\sqrt{2}\cos(\pi f \tau)} - 1 \tag{24}$$

Thus, the precision of δ_{acdc} defining depends only on precision of measurement of frequency f and time interval τ.

4. Structural scheme and algorithm of standard converter of sine voltage into time interval

In existing system of AC voltage reference this method is realized on several instruments. Among them there are commercial high precision instruments (multimeter HP3458A, nanovoltmeter HP34420A, industrial computer NI PXI-1042Q and digital frequency meter Ch3-64/1) and unit designed and produced in research institute of Experimental Research Automation as well.

All above-named instruments are pretty expensive, that's why it is not reasonable to use them just for several of their potentialities. Hence the need has appeared to design and create an ad-hoc and cheaper instrument, which could provide the same accuracy as the existing system. This problem became the subject of this research.

After consideration of a number of existing structural and circuit solutions, and potentialities of modern electronic components as well a structural scheme of instrument was designed (Fig.3).

The principle of the scheme shown on fig. 3 is next:

DC voltage U^{+} with value ca 1 V is generated on DAC1; an exact value of generated voltage is measured with ADC1 (SW1 is in position a); this voltage is conveyed to output Out1 (SW2 is also in position 2); to which a thermal transducer is connected. Then we measure thermal e. m. f. E^{+} on the output of thermocouple by switching SW1 into position b. The gotten value is registered in MC memory. After this with the help of DAC1 and ADC1 we try to achieve the negative voltage U^{-} on the output of DAC1, that is modulo equal to U^{+}. Similarly to the first case we measure E^{-} and put it into MC memory. Then we programmatically calculate $E_{c}^{=}$ according to formula (11).

After this with the help of the part of the scheme that consists of digital-to-analog converters DAC2 and DAC3, scaling block SB, filter F and two amplifiers A1 and A2 a pure sine voltage $U^{\sim}$ is generated. To obtain the precision of generation on level of 1 ppm we need to have a 20-bit DAC. The existing today components with such digit capacity don't provide sufficient stability and linearity. That's why a scheme of parallel connection of two 16-bit DACs and switching of 20-bit code between them with further scaling and summation of output voltage is suggested.

Switch SW2 is in position b and voltage $U^{\sim}$ goes to the output Out1. On the output of thermocouple thermal e. m. f. $E^{\sim}$ is generated. We try to obtain equality $E^{\sim} = E_c^{=}$ by changing amplification factor of A1.

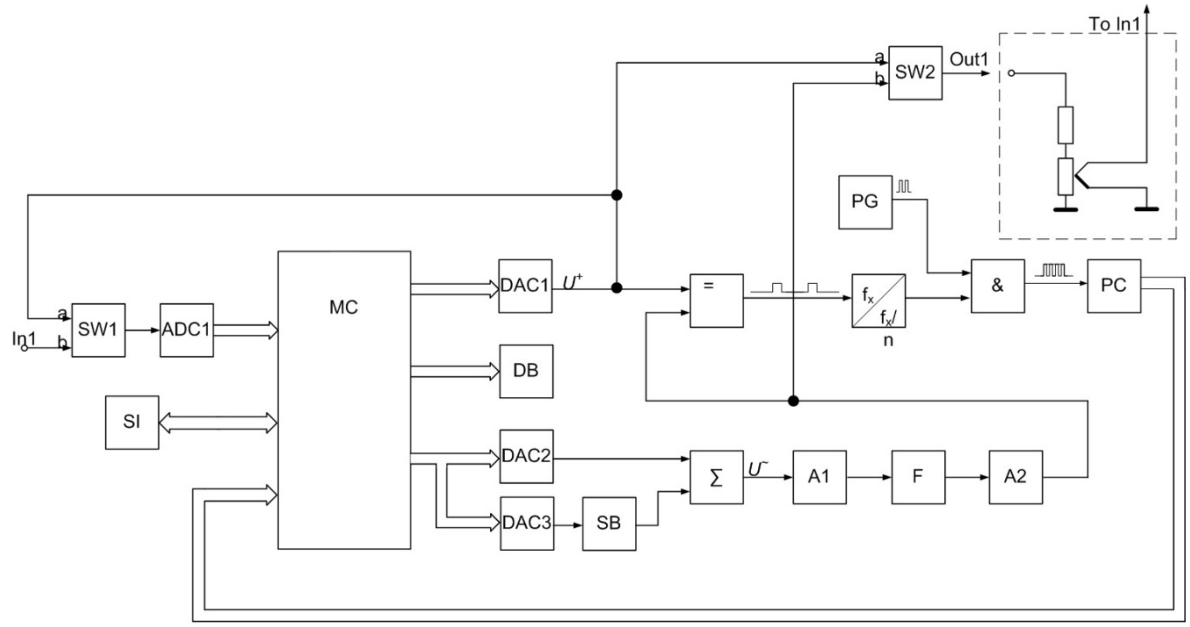

Fig. 3. Structural scheme of reference converter of sine voltage into time interval

On this scheme:

- MC – microcontroller;
- SW1, SW2 – switches;
- ADC1 – analog-to-digital converter;
- DAC1, DAC2, DAC3 – digital-to-analog converter;
- DB – display block;
- SI – system interface;
- A1, A2 – amplifiers;
- F – filter;
- SB – scaling block;
- PG – pulse generator;
- PC – pulse counter.

This equality means that thermal effect in thermal transducer during alternating current passage is equal to thermal effect during direct current passage. Hence, having found out how much does RMS of $U^{\sim}$ differ from value of U^{+} we can define δ_{acdc} error of this thermal converter.

Voltage $U^{\sim}$ comes to one of the comparator inputs, to the other input DC voltage U^{+} is conveyed. Thus, on the output of comparator A-quad-B pulses are generated with duration τ (fig. 1). Sequence of pulses comes to controllable frequency divider, that allows to carry out measurement of several periods. After this the pulses come to one of the inputs of logic gate “AND”, where time intervals are created, that are filled with clock pulses from pulse generator PG coming to another input of “AND” gate. During the period which is proportional to τ the clock pulses come to input of pulse counter PC. Counting result is recorded into MC memory. If PG frequency is stable and is more than $4 \cdot 10^{6}$ times greater than frequency of signal $U^{\sim}$, we can measure the time interval τ with precision of 1 ppm. That means that with the signal frequency $f = 1000\ \text{Hz}$ the clock frequency must be equal to $4\ \text{GHz}$. Analysis of modern electronic components from leading producers has shown that flip-flops working on such frequencies are too expensive and stable comparators with propagation delay less than $2{,}5\ \text{ns}$ don’t exist at all. That’s why it is reasonable to build the converter of sine voltage into time interval with signal frequency $f = 100\ \text{Hz}$. In this case frequency of the most quick-operating components will be ca $400\ \text{MHz}$, which is absolutely reachable with modern components. Thus, we can get the values of time interval τ and frequency f with precision of 1 ppm. As long as accuracy of δ_{acdc} measurement, as it was said on the top of this paper, depends only on accuracy of τ and f measurement, with the help of this instrument we can measure δ_{acdc} error with precision of a couple of ppm.

Conclusion

Using the method of converting of sine voltage amplitude into time interval we can implement AC voltage measurement with precision of a couple of ppm. Combining this method with the method of ultrapure sine voltage generation using two DACs and program spectrum control we can create a instrument for measuring of standard thermal transducer δ_{acdc} error.

This instrument is supposed to become a part of AC voltage standard system and provide a high-precision measurement of δ_{acdc} error in reference thermal transducers.

References

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