UDC 629.735.051-52(045)

A. A. Tunik, M. M. Komnatska

AN LMI – DESIGN OF FLIGHT CONTROL SYSTEMS WITH FUZZY CONTROLLERS

Introduction

Recently, the usage of unmanned aerial vehicles (UAVs) has gained a great attention from the control system society, since these vehicles are able to perform different tasks starting from civil missions as agriculture, ecological and metrological, to military operations. Moreover, such UAVs are subjected to various disturbances within the flight envelope. These perturbations could be internal and/or external as well as structured and/or unstructured. Furthermore, to reduce the weight of the UAV only partial components of the state space vector are possible to measure. Therefore, the remaining challenge in this area is to design a control system with law cost and less power consumption without compromising the flight mission.

To satisfy the aforementioned requirements, the robust control theory could be applied. The designer of such control law needs to take into account several objectives to meet the desired performance and robustness of the closed loop system. Nowadays a variety of concepts of robust flight control system design are discussed in literature. Among them it is possible to mark out works related to the robust PD controller design developed in [1]. In [2–4] the flight control structure is supplemented with fuzzy control. The usage of fuzzy control in a loop permits to enhance the UAVs performance within the flight envelope under the different external and/or internal perturbations.

As it was mentioned above, the control law for such UAVs should be simple enough to be implement on board computer with restricted abilities. These circumstances lead to the problem of a static output feedback (SOF) controller design. The main advantage of SOF design is that it requires only available signals from the plant to be controlled. Thus, the restricted – measurement static output design is extremely important in the area of small UAV flight control. The SOF problem concerns finding a static or feedback gain to achieve certain desired closed-loop characteristics. At the same time, the output feedback problem is much more difficult to solve as a state feedback control problem. A survey devoted to this problem is presented in [5].

Moreover, the flight control systems with combined hard and soft controllers have received a wide application in the area of UAVs in the last years [2–4]. It is justified by the fact of high level of flight control system flexibility, safety and performance.

Problem Statement

This paper is addressed to the SOF design with γ – disturbance attenuation for small UAV under LMI approach. Notice, that proposed algorithm doesn't need an initial stabilizing gain, since it uses Riccati inequality solution. Furthermore, the structure of the flight control system includes a crisp inner controller and a fuzzy outer loop controller. The outer loop is represented with Parallel Distributed Compensation (PDC) controller basing on the idea of Tanaka-Sugeno fuzzy model [6]. The FCS with combined structure is given on Fig. 1.

The main feature of this paper implies that LMI approach used for crisp inner loop feedback controller synthesis is extended to the outer loop SOF design for Takagi-Sugeno (T-S) Fuzzy System. Thus, a formal procedure for the inner and the outer loops controllers construction composing complete FCS under the unified algorithmic approach via LMIs is created.

In order, to prove the efficiency of the proposed technique, the lateral channel of UAV in coordinate turn is used as a case study.

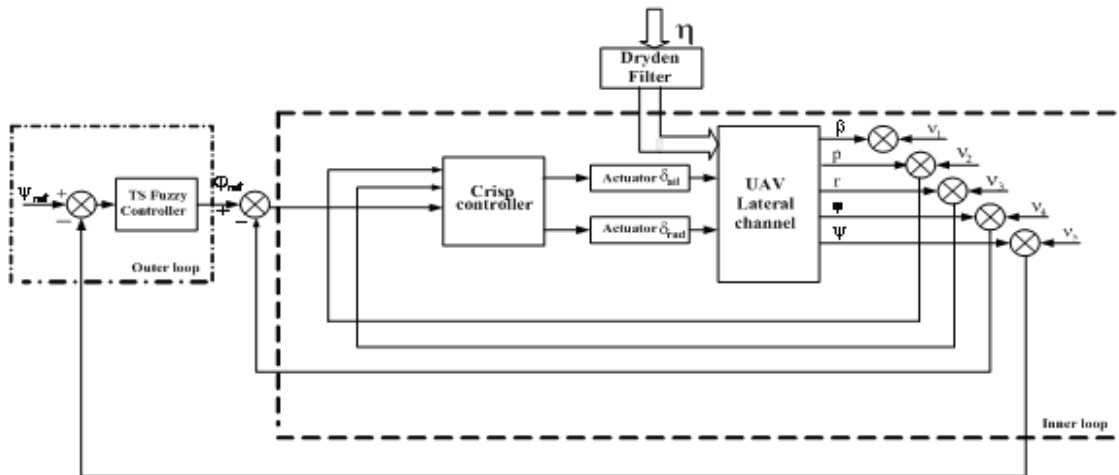*Notation:* The symbol "*" represents symmetric term in a block matrix.

Fig. 1. Block diagram of the overall closed loop system

H-infinity Static Output Feedback Design Approach

This section presents the H_∞ static output feedback (SOF) controller design with disturbance attenuation. The controller design is formulated in the context of the convex analysis via Linear Matrix Inequalities (LMI) [7]. The LMI approach permits to obtain a constant OPFB gain K for a set of linear models received by the linearization of the nonlinear model for different operating conditions.

System Description

The model of the controlled plant could be represented as follows:

$$\begin{cases} \dot{x} = Ax + B_u u + B_d d \\ y = Cx \end{cases},$$

(1)

where $x \in R^n$ is the state space vector, $u \in R^m$ is the control vector, $y \in R^p$ and $d \in R^n$ is a disturbance vector. Besides that, the state space matrices of the controlled plant have the following dimension $A \in R^{n \times n}$, $B \in R^{m \times n}$, $C \in R^{p \times n}$. Thus, It could be seen, that number of measuring variables p is less than number of all phase coordinate n. Hence, our control law is designed taking into account only variables that are available for measurement.

The control law is given by:

$$u(t) = -Ky(t) = -KCx,$$

(2)

where K is a constant output feedback gain, that minimizes performance index (3):

$$J = \int_0^\infty \|z(t)\|^2 = \int_0^\infty \left(x^T Q x + u^T R u \right) dt,$$

(3)

where $Q \geq 0$ and $R > 0$ are diagonal matrices, weighting each state and control variables, respectively. Output signal $z(t)$ used for performance evaluation is defined as follows:

$$z = \begin{bmatrix} \sqrt{Q} & 0 \\ 0 & \sqrt{R} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}.$$

Bounded L_2 Gain Design Problem

The system L_2 gain is said to be bounded or attenuated by γ if [7–9]:

$$\frac{\int_0^\infty \|z(t)\|^2 dt}{\int_0^\infty \|d(t)\|^2 dt} = \frac{\int_0^\infty \left(x^T Q x + u^T R u \right) dt}{\int_0^\infty \left(d^T d \right) dt} \leq \gamma^2.$$

(4)

Therefore, it is necessary to find constant output feedback gain matrix K that stabilizes the control plant such that the infinity norm of the transfer function referring exogenous input to performance output $z(t)$ approaches minimum. The minimum gain is denoted by γ^*.

In order to find constant output feedback gain K the following theorem is needed.

Theorem: Necessary and Sufficient Conditions for H_∞ Static OPFB Control Design Assume that $Q \geq 0$ and $\left(A, \sqrt{Q}\right)$ is detectable. Then the system defined by equations (1) is output-feedback stabilizable with L_2 gain bounded by γ, if and only if (A, B) is stabilizable and (A, C) is detectable. Therefore, such matrices K^* and L exist that

$$K^* C = R^{-1}\left(B^T P + L\right),$$ (5)

where $P > 0, P^T = P$ is a solution of the following equality:

$$PA + A^T P + Q + \frac{1}{\gamma^2} PB_d B_d^T P - PBR^{-1}B^T P + L^T R^{-1}L = 0.$$ (6)

The Proof see in the reference [8].

Notice that according to the above stated definition the pair (A, B) is said to be stabilizable if there exists a real matrix K such that $(A - BK)$ is (asymptotically) stable. The pair (A, C) is said to be detectable if there exists a real matrix L such that $(A - LC)$ is stable. The system (2) is said to be OPFB stabilizable if there exists a real matrix K such that $A - BKC$ is stable.

Our goal is to find SOF controller that simultaneously stabilizes a set of autonomous system. The LMI technique permits to solve this problem [7]. Thus, it is possible to transform the equality (6) into the LMI form. On the next stage, we have used the following change of variable $X = P^{-1}$. Pre-multiplying and post-multiplying right and left sides of the equality (6) by X, taking into account it's transformed form, and basin on Schur's complement, we obtain:

$$\begin{bmatrix} A_i X + X A_i^T & B_i & B_{di} & XQ^{1/2} & XL^T \\ B_i^T & -R & 0 & 0 & 0 \\ B_{di}^T & 0 & -\gamma^2 I & 0 & 0 \\ Q^{1/2} X & 0 & 0 & -I & 0 \\ LX & 0 & 0 & 0 & -R \end{bmatrix} \leq 0,$$ (7)

where $i = 1, \ldots, N$ in (7) denotes a set of models associated with certain operating conditions within the flight envelope.

SOF Design Algorithm

In the following the steps of SOF design are given. This is the algorithm of H_∞ SOF design, which uses the solution of Riccati equation in contrast to Lyapunov equation, at each step to solve the H_∞ problem for a specified admissible disturbance attenuation.

1. Initialization: set $n=0, L_0=0$, specify γ, Q, R.

2. solve for P_n the following inequality:

$$\begin{bmatrix} P_n A_i + A_i^T P_n + Q & P_n B_i & P_n B_{di} & L_n^T \\ B_i^T P_n & -R & 0 & 0 \\ B_{di}^T P_n & 0 & -\gamma^2 I & 0 \\ L_n & 0 & 0 & -R \end{bmatrix} < 0$$

Update K

$$K_{n+1} = R^{-1}\left(B^T P_n + L_n\right)C^T\left(CC^T\right)^{-1}$$

Update L

$$L_{n+1} = R K_{n+1} C - B^T P_n$$

3. Check convergence: if $\|K_n - K_{n+1}\| \le \varepsilon$, namely if K_{n+1} and K_n are close enough to each other, go to 4, otherwise set $n=n+1$ and go to step 2.

4. Terminate: set $K=K_{n+1}$.

For the Proof of Lemma and convergence condition see [8–9].

Outer Loop Static Output Feedback Design for Fuzzy Systems

The intensive growth of interest to fuzzy control opens new and alternative ways to the control of complex systems, where the knowledge and expertise of human are consulted for the controller design [10]. Owing to the complexity and nonlinearity of the fuzzy rules, it is very difficult to develop a general analysis and design theory for fuzzy control. In order to resolve this problem, the idea of Takagi-Sugeno (T-S) fuzzy model was proposed, where a linear model is adopted as a consequent part of the fuzzy rule [6]. Indeed, the T-S fuzzy model local dynamics in different state space regions are represented by local linear models. Thus, the overall model is obtained by "blending" together these linear models with a nonlinear membership functions. Once the fuzzy model is obtained, the control design is performed based on the fuzzy models via the PDC approach. Behind this approach lies a concept that for each local linear

model, there is an associated linear feedback control. The overall controller, which is nonlinear in general, is the fuzzy blending of each individual linear controller.

In this section, the outer loop static output controller design is considered. The outer loop is represented with T-S fuzzy system. It is commonly known, that T-S fuzzy model is represented by IF-THEN rules [6]. Consider the T-S fuzzy model with disturbance which is described by the rules of the following form:

Plant rule i: **IF** $z_1(t)$ is M_{i1} and …and $z_p(t)$ is M_{ip},

   THEN
$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + B_{di} d(t), \\ y(t) = C_i x(t). \end{cases}$$

Here, $i=1,...,r$, r is the number of fuzzy rules; $z_k(t)$ are the premise variables that may be functions of the state variables, external disturbances and/or time; M_{ik} is the fuzzy set, $k=1,...,p$, p is the number of premise variables; $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^l$, $d(t) \in R^q$ are denote state space, control, output and disturbance vectors, respectively. Matrices $A_i \in R^{nxn}$, $B_i \in R^{nxm}$, $C_i \in R^{lxn}$ are of appropriate dimensions.

Given a pair of $(x(t), u(t))$, the final output of the fuzzy system is inferred as follows

$$\begin{cases} \dot{x}(t) = \dfrac{\sum\limits_{i=1}^{r} \omega_i(z(t))\{A_i x(t) + B_i u(t) + B_{di} d(t)\}}{\sum\limits_{i=1}^{r} \omega_i(z(t))} = \sum\limits_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t) + B_{di} d(t)\} \\ y(t) = \dfrac{\sum\limits_{i=1}^{r} \omega_i(z(t))C_i x(t)}{\sum\limits_{i=1}^{r} \omega_i(z(t))} = \sum\limits_{i=1}^{r} h_i(z(t))C_i x(t), \end{cases}$$

(8)

where $\omega_i(z(t)) = \prod\limits_{k=1}^{p} M_{ik}(z_k(t))$, $h_i(z(t)) = \dfrac{\omega_i(z(t))}{\sum\limits_{i=1}^{r} \omega_i(z(t))}$.

$M_{ik}(z_k(t))$ is the grade of membership of $z_k(t)$ in M_{ik}.

Since $\begin{cases} \sum\limits_{i=1}^{r} \omega_i(z(t)) > 0, \\ \omega_i(z(t)) \geq 0, \quad i=1,2,...,r \end{cases}$, we have $\begin{cases} \sum\limits_{i=1}^{r} h_i(z(t)) = 1, \\ h_i(z(t)) \geq 0, \quad i=1,2,...,r \end{cases}$.

For notational convenience, it is possible to rewrite expression (8) in the following way:

$$\begin{cases} \dot{x}(t) = \bar{A}(h)x(t) + \bar{B}(h)u(t) + \bar{B}_{di}(h)d(t), \\ y(t) = \bar{C}(h)x(t) \end{cases} \tag{9}$$

where $\bar{A}(h) = \sum_{i=1}^{r} h_i(z(t))A_i$, $\bar{B}(h) = \sum_{i=1}^{r} h_i(z(t))B_i$, $\bar{C}(h) = \sum_{i=1}^{r} h_i(z(t))C_i$,

$\bar{B}_d(h) = \sum_{i=1}^{r} h_i(z(t))B_{di}$. Note that the model (8), (9) is nonlinear in general.

Definition 1. Given a real number $\gamma > 0$, the T-S fuzzy system (9) is said to be asymptotically stable with an H_∞ performance γ for any $d(t) \in L_2[0,\infty)$ if the following condition is satisfied:

$$\int_0^\infty z^T(t)z(t)dt \le \gamma^2 \int_0^\infty d^T(t)d(t)dt$$

It is considered the static output feedback controller of the following defuzzified form:

$$u(t) = \bar{K}(h)y(t) = \sum_{i,j=1}^{r} h_i(z(t))h_j(z(t))K_iC_jx(t), \tag{10}$$

where K_i are the local controller gain to be determined. By substituting (10) in (9) the closed-loop fuzzy system can be represented as

$$\dot{x}(t) = (\bar{A}(h) - \bar{B}(h)\bar{K}(h)\bar{C}(h))x(t) + \bar{B}_d(h)d(t). \tag{11}$$

The objective of this paper is to obtain K_i, $i=1,...,r$ such that the closed-loop fuzzy system (11) is asymptotically stable with γ – disturbance attenuation. In order to determine the outer loop controllers and designer convenience a user friendly unificated iterative algorithm represented above is exploited. The theorem 1 reflects the LMIs conditions for designing SOF with γ – disturbance attenuation.

Theorem 1 The continuous T-S fuzzy system (10) is asymptotically stable with γ - disturbance attenuation by means of static output feedback if there exist symmetric matrix $P > 0$ and matrices K_i, $i=1,...,r$ and a free parameter matrix L satisfying the following LMIs:

$$\begin{bmatrix} PA_i + A_i^T P + Q & PB_i & PB_{di} & L_i^T \\ B_i^T P & -R & 0 & 0 \\ B_{di}^T P & 0 & -\gamma^2 I & 0 \\ L_i & 0 & 0 & -R \end{bmatrix} < 0 \quad (i=1,...,r) \tag{12}$$

$$\begin{bmatrix} \frac{1}{2}\left(PA_i + A_i^T P + PA_j + A_j^T P + Q\right) & \frac{1}{2}\left(PB_i + PB_j\right) & \frac{1}{2}\left(PB_{di} + PB_{dj}\right) & \frac{1}{2}\left(L_i + L_j\right)^T \\ \frac{1}{2}\left(PB_i + PB_j\right)^T & -R & 0 & 0 \\ \frac{1}{2}\left(PB_{di} + PB_{dj}\right)^T & 0 & -\gamma^2 I & 0 \\ \frac{1}{2}\left(L_i + L_j\right) & 0 & 0 & -R \end{bmatrix} \le 0$$

$$(i \ne j), \qquad (13)$$

Proof. Basing on the technique in [8–9], the proof is easily obtained and omitted.

Case study

To demonstrate the efficiency of the proposed approach a lateral channel of the Unmanned Aerial Vehicle (Aerosonde UAV) is used as a case study. The state space vector of the lateral channel is $X = \begin{bmatrix} v, & p, & r, & \phi, & \psi \end{bmatrix}$, where v is the lateral velocity component, r is the yaw rate, p is the roll rate, ϕ is the bank angle and ψ is the heading angle. The control input vector $U = \begin{bmatrix} \delta_a & \delta_r \end{bmatrix}$ is represented by ailerons and rudder deflections, respectively. The nonlinear model of the Aerosonde is linearized for three operating conditions to form a nominal model at true airspeed of 26 m/sec and two parametrically perturbed models at 23 m/sec and 30 m/sec. The linearized state space models are represented by matrices A, B, C. Disturbance d is affecting the lateral speed component v, the yaw rate r and the roll rate p, so that $d = \begin{bmatrix} v & p & r \end{bmatrix}^T$. The state space matrices of the nominal model and the matrix of external disturbances are given below:

$$A_n = \begin{bmatrix} -0.72 & 1.07 & -25.98 & 9.81 & 0 \\ -4.73 & -23.31 & 11.22 & 0 & 0 \\ 0.77 & -3.02 & -1.17 & 0 & 0 \\ 0 & 1 & 0.04 & 0 & 0 \\ 0 & 0 & 1.0009 & 0 & 0 \end{bmatrix}; \quad B_n = \begin{bmatrix} -1.59 & 4.08 \\ -140.33 & 2.52 \\ -5.53 & -25.78 \\ 0 & 0 \\ 0 & 0 \end{bmatrix};$$

$$B_{dn} = \begin{bmatrix} 0.72 & -1.07 & 25.98 \\ 4.73 & 23.31 & -11.22 \\ -0.77 & 3.02 & 1.17 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

The measured variables for the inner loop are $\overline{X} = [p, \quad r, \quad \phi]$, hence the observation matrix is given as follows:

$$C = \begin{bmatrix} 0 & 1 & 0 & 0; 0 & 0 & 1 & 0; 0 & 0 & 0 & 1 \end{bmatrix}^T$$

The atmospheric turbulence used in the simulation is given by Dryden filter [11]. The attenuation level γ is chosen to be equal to 0.92. The stabilizable static output feedback gain matrix of the inner loop obtained with the help of the proposed algorithm is found as follows:

$$K_{inner} = \begin{bmatrix} -0.0371 & 0.1303 & -0.3142 \\ 0.0165 & -0.1342 & 0.0103 \end{bmatrix}.$$

Moreover, the synthesized inner loop static output feedback controller attenuates disturbances along with the established level, namely

$$\|H\|_{n\infty} = 0.6230 < 0.92,$$
$$\|H\|_{p1\infty} = 0.5911 < 0.92,$$
$$\|H\|_{p2\infty} = 0.6566 < 0.92$$

As stated above, the outer loop controller is designed using T-S FC for yaw angle hold mode at the reference signal. Three Gaussian shaped membership functions are utilized to represent the "crisp" value on the universe of discourse and represent the premise part of fuzz rules. The initial position of the membership functions are chosen to be uniformly distributed; with the centers c^i and spreads σ^i, where the centers have been located at: [-1.2 0 1.2] and the spreads have been chosen equal to $\sigma^i = 2.68$, respectively [10]. The total number of control rules is 3. The observation matrix of the outer loop is given as follows:

$$C_1 = C_2 = C_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The obtained static output feedback gain matrices are given below:

$$K_1 = \begin{bmatrix} -0.0140 \\ -0.0268 \end{bmatrix}; \quad K_2 = \begin{bmatrix} -0.0030 \\ -0.0268 \end{bmatrix}; \quad K_3 = \begin{bmatrix} -0.0140 \\ -0.0268 \end{bmatrix}$$

Table reflects standard deviations of the UAV outputs of the nominal and parametrically perturbed models under the atmospheric disturbances.
The following figures demonstrate the results of simulation with the yaw angle reference signal (Fig. 2).

Table

Standard deviations of the nominal and perturbed closed loop systems with combined structure under the atmospheric disturbances

Plant	Standard deviation						
	σ_v, °	σ_p, deg/sec	σ_r, deg/sec	σ_φ, °	σ_ψ, °	σ_{ail}, °	σ_{rud}, °

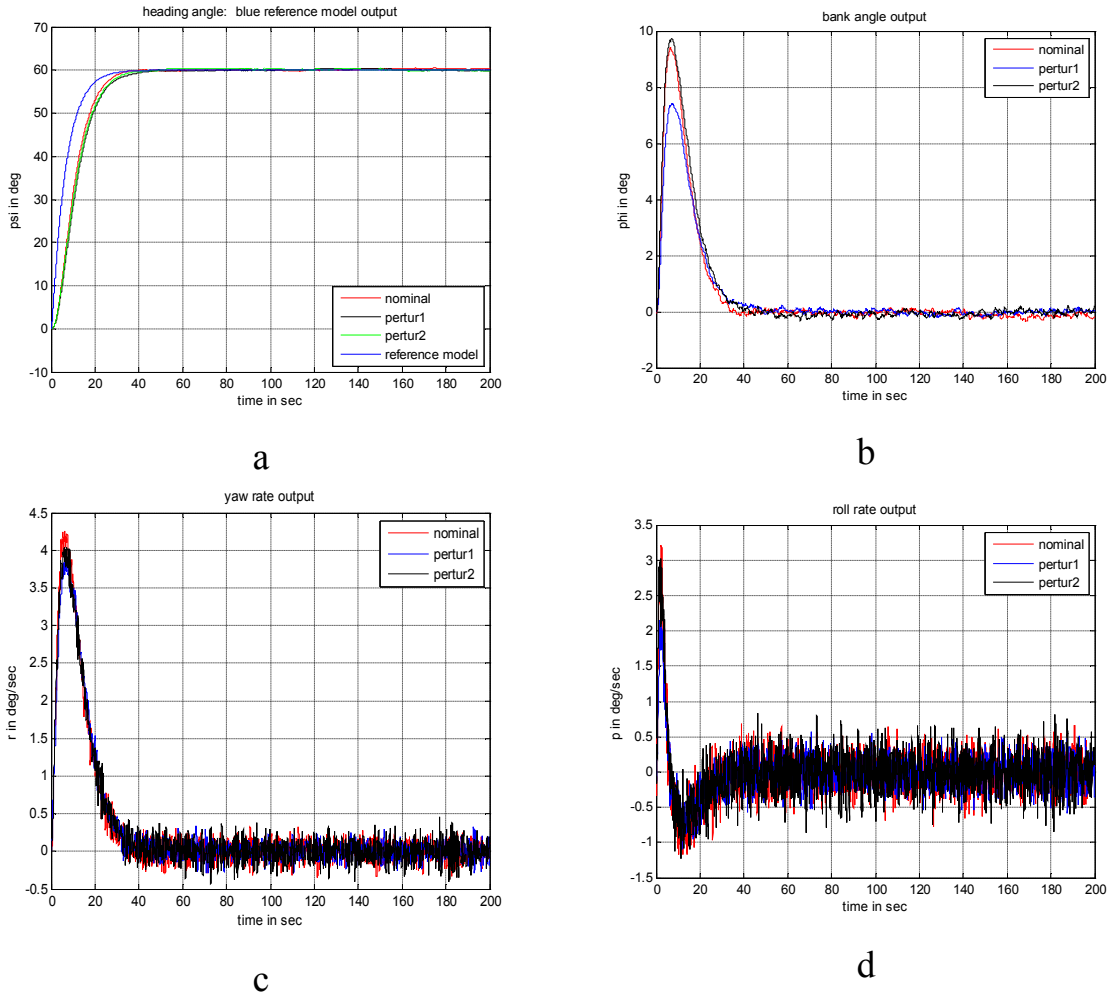| Nom.	0.0579	0.2227	0.2280	.4571	1.2568	0.0014	0.0010
Pert.1	0.0818	0.1952	0.2404	0.4736	1.6207	0.0020	0.0016
Pert.2	0.0721	0.2540	0.2502	0.5636	1.5243	0.0018	0.0015

a

b

c

d

Fig. 2. Simulation results of motion nominal and perturbed models in the presence of turbulence: a – heading angle in deg;

b –bank angle in deg;

c – yaw rate in deg/sec;

d – roll rate in deg/sec;

Conclusion

The simulation results of the lateral channel of the UAV prove the effectiveness of the proposed control method. The required flight performances are respected as well as the robustness of the closed loop system. It can be seen that the

handling quality of the nominal and the perturbed models are satisfied. The heading of the UAV is held at the reference and the other angle deflections for such UAV are respected within the flight envelope.

References

1. *Tunik A.A.* Parametric Robust Optimization of Digital Flight Control Systems / A.A. Tunik, E.A. Abramovich // Proceedings of the National Aviation University.– 2003.– №2. – pp. 31 – 37.
2. *Tunik A.A* Combination of Fuzzy and Crisp Control in Airborne Robust Multivariable Systems / A.A. Tunik, M.A. Touat, M.M. Komnatska // Proceedings of the Third World Congress in the XXI – st Century: "Safety in Aviation and Space technology" .– Kyiv, Ukraine.–2008.– Vol.2 September 22-24.– pp. 33.38 – 33.47.
3. *Tunik A A..* Combination of Fuzzy and "Crisp" Principles in Modern Multivariable Robust Flight Control / A. A. Tunik, M.A. Touat // Системні технології. Регіональний міжвузівський збірник наукових праць. – Випуск 6 (59). – Дніпропетровськ, 2008.– с. 10 – 26.
4. *Komnatska M.M.* Robust Stabilization and Optimization of Flight Control System with State Feedback and Fuzzy Logics / M.M. Komnatska, A.A. Tunik // NAU Proceedings. – 2009. – № 4. – pp. 33 – 41.
5. *Syrmos V.L.* Static Output Feedback: a survey / V.L. Syrmos, C. Abdallah and P. Dorato // Proceedings of 33rd IEEE Conference on Decision and Control, Orlando, FL. – pp. 837 – 842.
6. *Tanaka K.* Fuzzy Control System Design and Analysis, A Linear Matrix Inequality Approach / K. Tanaka, H.O. Wang.– New York.: John Wiley& Sons, Inc,2001.– 305p
7. *Boyd S.* Linear Matrix Inequalities in System and Control Theory / S. Boyd, L. El Ghaoui, E. Feron, V. Balakrishnan.– Philadelphia, PA SIAM, 1994.– 416 p.
8. *Gadewadikar J.* Necessary and Sufficient Conditions for H-infinity Static Output Feedback Control / J. Gadewadikar, F. Lewis, M. Abu-Khalaf // Journal of Guidance, Control and Dynamics.– 2006.– Vol. 29.– pp. 915 – 921.
9. *Gadewadikar J.* Aircraft Flight Controller Tracking Design Using H-Infinity Static Output-Feedback / J. Gadewadikar, F. Lewis // Transactions of the Institute of Measurement and Control.– 2006.–Vol. 28.– No.8.– pp. 429 – 440.
10. *Passino K. M* Fuzzy Control / K.M Passino, S. Yurkovich.– Addison – Wesley.– Menlo Park, Reading, Harlow, Berkley, Sidney, Bonn, Amsterdam, 1998.–P. 502.
11. *McLean D.* Automatic Flight Control Systems / D. McLean. Prentice Hall Inc., Englewood Cliffs,– 1990.– 593 p.