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TANKER ROUTING PROBLEM WITH FUZZY DEMANDS OF SERVED SHIPS

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The routing problem for tanker-refuellers is considered. The tankers start at the bunkering company and must serve several ships in different ports. In principle the modeling and algorithmic approaches for capacitated vehicle routing problems can be used. Since the demands of the ships are uncertain and vague they are modeled using fuzzy sets. The first compromise solution can interactively be modified to meet the decision makers requirements with respect to the different criteria. The results are demonstrated using a small example.

1. INTRODUCTION

This paper deals with a very important routing problem in marine practice: the routing of tanker-refuellers (in the following called tankers) which provide bunkering (transportation and unloading) operations for various ships. These ordering ships are located at different ports or, in general, at different points offshore. The importance of transportation is continuously growing due to the growing internationalization of this business. Optimization models and methods are very appropriate for transportation and distribution planning. This problem can be modeled by the so called capacitated vehicle routing problem (CVRP).

The development of efficient models and algorithms to solve routing problems is an important contribution of operations research to theory and practice. A lot of different approaches have been suggested and are applied to handle a wide spectrum of real world problems. An overview can be found in [4]. In particular, vehicle routing problems (VRP) with various modifications and different approaches are considered [16]. Some of these approaches use methods to solve the traveling salesperson problem (TSP) as an integral part. A survey on TSP is given in [15]. Most of the VRP techniques can be applied successfully also to ship routing problems [3, 9, 17, 19]. If the demand of the customers is not precisely known in advance deterministic and crisp models are not appropriate for decision support. The VRP with stochastic demands of nodes is described in such publications as [1, 2, 5, 6, 8] but the efficient solution of SVRP (Stochastic Vehicle Routing Problem) presumes precise information about probability functions of the demands. To handle imprecise and vague information a fuzzy theory based approach fits well.

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In marine practice information about customers' demands to be served can be uncertain too. Such kind of uncertain information characterizes the bunkering process (BP) by which ships are supplied with ordered fuel [14]. Usually the ship-owner sends an order for fuel supplying to the bunkering company using such uncertain terms as "approximately VALUE", "about VALUE", "between VALUE_1 and VALUE_2", "at least VALUE", "not less then VALUE", "not more then VALUE". The efficiency of the bunkering operations can be evaluated by the possibility to serve the orders with a minimum of consumption of own tanker's fuel during delivery which depends on the total distance traveled by all tankers.

A fuzzy set theoretical approach to the vehicle routing problem when demand at nodes is uncertain is considered in [19]. Their model is based on the well-known heuristic "sweeping algorithm" combined with a set of fuzzy rules and approximate reasoning to construct routes. They assume that every demand has to be served and that it is possible to return to the depot and then revisit the customer again. During the bunkering process considered here there is not enough time to travel twice. If the demand of a ship cannot be served completely during the planned trip the exceeding demand is lost.

Therefore a fuzzy multi-criteria mathematical programming model is suggested here to solve the tanker routing problem with uncertain demands. An interactive approach is used to find the compromise solution, which considers the minimal total distance of all routes necessary for all bunkering operations and the maximal potential total sales of fuel. For small problems it can be solved optimally by using standard mixed integer linear programming software. For larger problems an adequate heuristic has to be chosen.

2.1. GENERAL PROBLEM STATEMENT

The port where the bunkering company is located is the only depot for the tankers. Here the orders are announced from the ship owners who know the respective demands of their ships approximately. The ships, which must be served by the tankers, have various capacities and different demands. Usually the captain of each ship orders the final crisp volume of fuel he needs at the moment when tanker and served ship meet in the j-th port, $j = 1, \ldots, N$. The decision maker of the bunkering company has to solve the tanker routing problem before bunkering operations start. At the depot 0 the bunkering company has K tankers for bunkering operations at its disposal. The tankers ($k = 1, \ldots, K$) are identical and they have the same capacity Cap to transport fuel. We suppose that the bunkers contain sufficient fuel for all demands of the N ships in the marine region and that the demand of each single ship is less or equal to the capacity of a tanker. Taking into account that the above-mentioned customers' announcements for all ships to be served are uncertain the demand of each ship can be presented using a fuzzy set. We model the uncertain demand with triangular fuzzy numbers $\widetilde{d}_j = \left(\underline{d}_j, \hat{d}_j, \overline{d}_j \right)$ as shown in fig. 1 [7, 24].

The fuzzy number of each demand can be chosen based on the preliminary customer's orders and the decision maker's experience which deals with his or her intuition and the "a priori" knowledge about the type of ship, type of ship's

cargo, region of voyage, prescribed ports, ship's captain decisions for analogous bunkering operations and others. The real world bunkering process with *K* tankers and N ships is parallel: each tanker serves all ships on its route which is planned by the decision maker at the bunkering company. The time period for the whole bunkering process is much shorter than in the case of an approach where the demands of all the ships must be served sequentially.
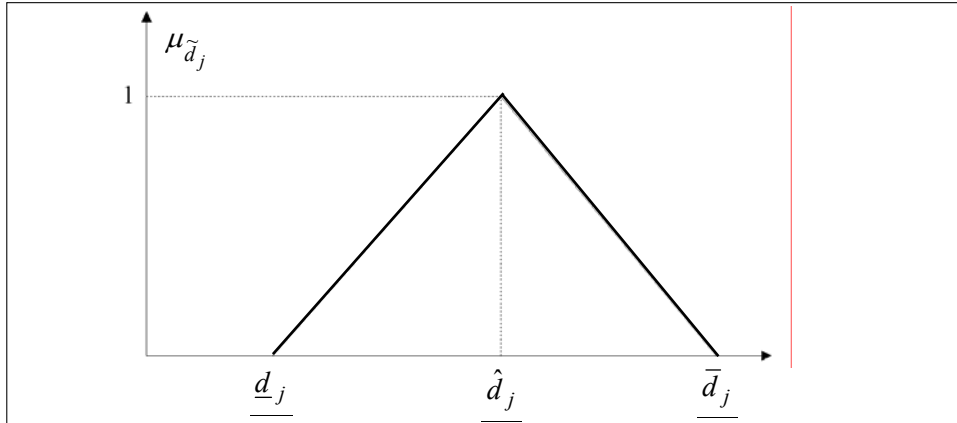

Fig. 1. Triangular membership function of the fuzzy demand $\tilde{d}_j$ of ship *j*

The problem discussed in this paper deals with solving the tanker routing problem for fuzzy demands of served ships. A "compromise solution" for the specific marine operations called "bunkering operations" is determined which takes into account the following goals:

• maximum possible total quantity of unloaded fuel during the entire bunkering process (this value is a main component defining bunkering company's profit);

• minimum possible total distance for tankers which serve ordering ships on all planned routes;

• maximal possibility to serve the demand.

The specific requirements for the bunkering process under consideration are:

• each port *j* ($j = 1, \ldots, N$) is visited only once by only one tanker during the planning period for which the routes are designed and executed;

• each tanker starts and finishes its route at the depot 0;

• each demand $\overline{d}_j$ at port *j* ($j = 1, \ldots, N$) is lower than the capacity *Cap* of a tanker;

• there are enough tankers at the depot 0 to serve any total demand $\sum_{j=1}^{N} d_j \leq \sum_{j=1}^{N} \overline{d}_j$ of all orders of the ships,

Additionally, the locations of the depot and all ports with ordering ships, capacity of the tanker, the distance resp. the traveling costs between the ports must be known.

In the following a fuzzy multi-criteria programming approach is developed to solve the above stated problem. The decision maker interactively to improve the degree of satisfaction with the different goals can modify the compromise model.

2.2. TANKER ROUTING PROBLEM WITH CRISP DEMANDS

The problem is modeled as an extension of the one-depot capacitated vehicle routing problem which can be represented by the following model. Considering several trips to satisfy given demands at locations with tankers having restricted capacity available leads to the capacitated vehicle (here tanker) routing problem. Several models exist for this problem. In accordance with the above formulation the following model for the capacitated vehicle routing problem is chosen.

The capacitated vehicle routing problem with well-known demand can be modeled considering two parts of constraints. One part containing constraints (2) and (3) is the generalized assignment problem. It groups cities to a tour considering the respective demands and the capacity of the vehicles. The other part models a traveling salesperson problem for each of these tours. This part contains constraints (4) to (8).

First we assume that the demand of each customer is exactly known in advance and all other information is strictly given. The distribution starts from a single depot and the vehicle fleet is homogenous that is all vehicles have the same capacity.

Considering different tours (vehicles, tankers) k ($k = 1, \ldots, K$) and cities (ports) i, j ($i, j = 1, \ldots, N$) three types of variables are necessary:

variable x_{ijk} is binary with

$$x_{ijk} = \begin{cases} 1 & \text{for } j \text{ follows } i \text{ on tour } k, \\ 0 & \text{else} \end{cases}$$

variable y_{jk} is binary with

$$y_{jk} = \begin{cases} 1 & \text{for city } j \text{ belongs to tour } k, \\ 0 & \text{else} \end{cases}$$

and variable u_{jk} is an integer ≥ 0 and for the optimal solution it is the sequence number of city j on trip k .

Objective function of the classical vehicle routing problem is to minimize the total distance traveled:

$$\text{Min} \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} c_{ij} \, x_{ijk} \,, \tag{1}$$

c_{ij} for $i, j = 0, \ldots, N$ is the distance from city i to city j .

The capacity Cap of the vehicle is sufficient to meet the crisp demand of all cities on tour k

$$\sum_{j=1}^{N} d_j \, y_{jk} \leq \text{Cap} \quad \text{for } k = 1, \ldots, K \,, \tag{2}$$

d_j is the crisp demand at city j .

Each city belongs to exactly one of the tours except city 0 with the depot

$$\sum_{k=1}^{K} y_{jk} = 1 \quad \text{for } j = 1, \ldots, N \,. \tag{3}$$

Each city j must be entered exactly once on the trip to which tour it belongs

$$\sum_{i=0}^{N} x_{ijk} = y_{jk} \quad \text{for } j = 0,...,N, \ k = 1,...,K . \tag{4}$$

Each city i must be exited exactly once on the trip to which tour it belongs

$$\sum_{j=0}^{N} x_{ijk} = y_{ik} \quad \text{for } i = 0,...,N, \ k = 1,...,K . \tag{5}$$

No city can follow itself on the tour

$$x_{iik} = 0 \quad \text{for } i = 0,...,N, \quad k = 1,...,K . \tag{6}$$

Subtours are forbidden

$$u_{jk} \geq u_{ik} + 1 - \left(1 - x_{ijk}\right) N \quad \text{for } i = 0,...,N, \ j = 1,...,N, \ j \neq i . \tag{7}$$

Each trip starts in the depot

$$u_{ok} = 1 \quad \text{for } k = 1,...,N . \tag{8}$$

This is a mixed integer linear programming model. For small instances it can be solved exactly by using standard software. For larger instances a heuristic must be chosen. Constraint (7) is not the only formulation to avoid cycling and might cause difficulties when solving larger problems [17]. Some of the heuristics suggested in literature first group cities to tours and then solve the traveling salesperson problem to optimize the respective routes. This vehicle routing model can be used to plan the routes for tankers, which are located in a depot and have to travel to different ports for bunkering.

2.3. EXAMPLE: TANKER ROUTING WITH CRISP DEMAND

Let us consider a small example with 5 ships in different ports. The demand d_j, $j = 1,...5$, for fuel is known (table 1). Several tankers are located in depot 0. The capacity of each tanker is 1000. Table 2 contains the distances c_{ij}, $i, j = 0,...,N$, between the ports in miles. In this example $c_{ij} = c_{ji}$ holds for all $i, j = 0,...,N$.

The optimal solution of the capacitated vehicle routing problem is calculated using the model (1) to (8) above and the software package ILOG AMPL CPLEX System vers. 7.0 (2000). The optimal routes $0 - 2 - 4 - 0$ and $0 - 3 - 1 - 5 - 0$ are shown in fig. 2. The total distance traveled is 340.2 miles. Because this graph is symmetric the tours $0 - 4 - 2 - 0$ and $0 - 5 - 1 - 3 - 0$ are optimal too. Exactly two tankers can serve the demands.

T a b l e 1. Demands of the ships

j	1	2	3	4	5
d_j	500	400	300	600	200

T a b l e   2 . Distances between the ports

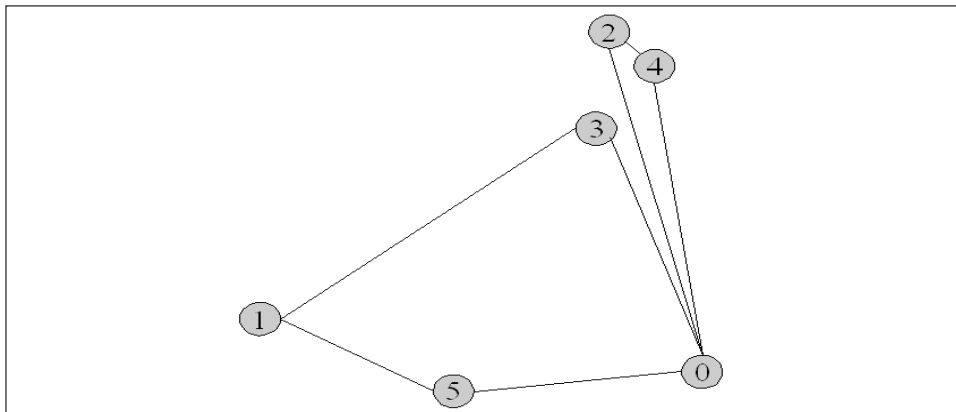|	0	1	2	3	4	5
0	–	70.2	67.5	48.6	59.4	32.4
1	70.2	–	97.2	78.3	86.4	43.2
2	67.5	97.2	–	21.6	10.8	81
3	48.6	78.3	21.6	–	16.2	70.2
4	59.4	86.4	10.8	16.2	–	81
5	32.4	43.2	81	70.2	81	–

Fig. 2. Optimal routing considering crisp demands

3.1. ROUTING PROBLEM WITH FUZZY DEMANDS

If the demand at the different cities is uncertain and imprecise we suggest to model it using a fuzzy set theoretical approach and to use a fuzzy set $\widetilde{d}_j$, $j = 1, \ldots, N$, to represent the demand. The mathematical programming model is the same as above with the exception of capacity constraint (2). Instead of the crisp capacity constraint the following fuzzy constraint for the fuzzy demands on tour k has to be considered:

$$\sum_{j=1}^{N} \widetilde{d}_j y_{jk} \leq \text{Cap} \quad \text{for } k = 1, \ldots, K \tag{9}$$

$$\text{with} \quad \widetilde{D}_k = \sum_{j=1}^{N} \widetilde{d}_j y_{jk} \quad \text{for } k = 1, \ldots, K .$$

Modeling the fuzzy demand we suggest to consider the possibility that the actual demand of all ships on one tour is less or equal to the capacity of the tanker to a certain degree. Because of the following considerations this leads to a crisp equivalent model. This approach to handle these fuzzy constraints is similar to chance constraints programming in stochastic optimization. If the demand is not known exactly we suggest to find a solution for which the possibility to serve the

demand is required at least to a certain degree $\alpha \in [0,1]$. The decision maker has to determine α in advance. Considering a fuzzy number as a method of representing uncertainty in a given quantity by defining a possibility distribution for the quantity is analyzed in [10]. An even stronger condition is to determine a certain degree of necessity β that the demand on the tour can be served. That is

$$\text{Pos}\left(\sum_{j=1}^{N} \tilde{d}_j y_{jk} \leq \text{Cap})\right) \geq \alpha \quad \text{for } k = 1, \ldots, K, \ \alpha \in [0,1]\text{Pos} \qquad (10)$$

or even stronger

$$\text{Nec}\left(\sum_{j=1}^{N} \tilde{d}_j y_{jk} \leq \text{Cap})\right) \geq \beta \quad \text{for } k = 1, \ldots, K, \ \beta \in [0,1]. \qquad (11)$$

If the fuzzy demand of each of the ships can be modeled using triangular fuzzy numbers a crisp equivalent formulation can be developed. For the calculation let us first consider the possibility and necessity of a triangular fuzzy number $\tilde{a}$ to be greater or equal to zero [13].

$$\text{Pos}\left(\tilde{a} \geq 0\right) = \text{Pos}_{\tilde{a}}\left(\{x | x \geq 0\}\right) = \sup_{x \geq 0} \mu_{\tilde{a}}(x), \qquad (12)$$

$$\text{Nec}\left(\tilde{a} \geq 0\right) = \text{Nec}_{\tilde{a}}\left(\{x | x \geq 0\}\right) = 1 - \text{Pos}_{\tilde{a}}\left(\{x | x < 0\}\right). \qquad (13)$$

For a triangular fuzzy number $\tilde{a} = (\underline{a}, \hat{a}, \overline{a})$ with $\hat{a} \neq \overline{a}$ and $\hat{a} \neq \underline{a}$ these possibility (Pos) and necessity (Nec) can be determined using the following formulas

$$\text{Pos}\left(\tilde{a} \geq 0\right) = \begin{cases} 1 & \hat{a} \geq 0, \\ \dfrac{\overline{a}}{\overline{a} - \hat{a}} & \hat{a} < 0 \leq \overline{a}, \\ 0 & \overline{a} < 0. \end{cases} \qquad (14)$$
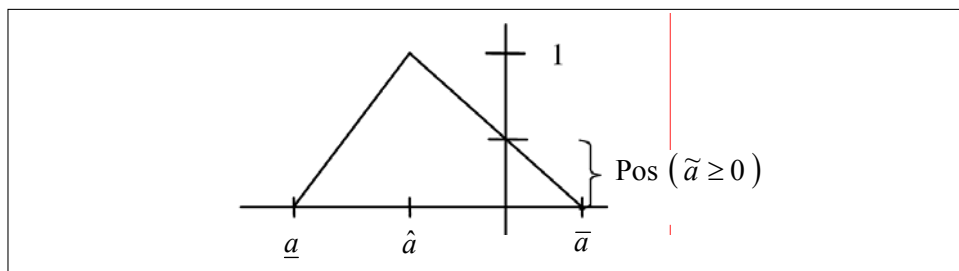

Fig. 3. $\text{Pos}\left(\tilde{a} \geq 0\right)$ for a triangular fuzzy number $\tilde{a}$

$$\text{Nec}\left(\tilde{a} \geq 0\right) = \begin{cases} 1 & \underline{a} \geq 0, \\ \dfrac{\hat{a}}{\hat{a} - \underline{a}} & \underline{a} < 0 \leq \hat{a}, \\ 0 & \hat{a} < 0. \end{cases} \qquad (15)$$
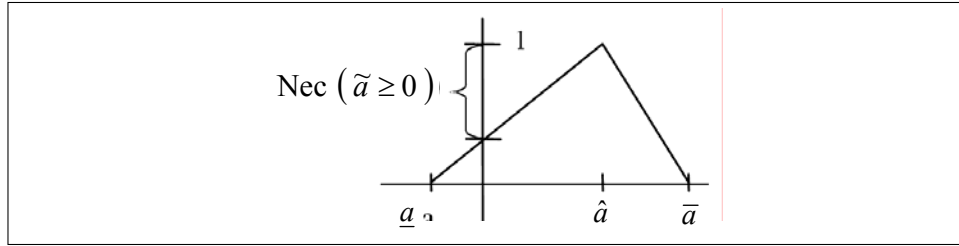
Fig. 4. $\text{Nec}\left(\widetilde{a} \geq 0\right)$ for a triangular fuzzy number $\widetilde{a}$

So the requirement for capacity in the above model can be determined as follows

$$\text{Pos } (\sum_{j=1}^{N} \widetilde{d}_j y_{jk} \leq \text{Cap}) \geq \alpha \qquad \text{for } k = 1, ..., N,$$

$$\Leftrightarrow \text{Pos } (\text{Cap} - \sum_{j=1}^{N} \widetilde{d}_j y_{jk} \geq 0) \geq \alpha \quad \text{for } k = 1, ..., N. \tag{16}$$

If all demands $\widetilde{d}_j = (\underline{d}_j, \hat{d}_j, \overline{d}_j)$, $j = 1, ..., N$ are triangular fuzzy numbers then

$$\text{Cap} - \sum_{j=1}^{N} \widetilde{d}_j y_{jk} = \left(\text{Cap} - \sum_{j=1}^{N} \overline{d}_j y_{jk}, \text{Cap} - \sum_{j=1}^{N} \hat{d}_j y_{jk}, \text{Cap} - \sum_{j=1}^{N} \underline{d}_j y_{jk} \right) \tag{17}$$

is a triangular fuzzy number too.

Thus the possibility that the capacity of the vehicle is sufficient to serve all demands $\widetilde{D}_k$ on tour k is equal to

$$\text{Pos}\left(\text{Serve } \widetilde{D}_k\right) = \begin{cases} 1 & \text{Cap} \geq \sum_{j=1}^{N} \hat{d}_j y_{jk}, \\ \dfrac{\text{Cap} - \sum_{j=1}^{N} \underline{d}_j y_{jk}}{\sum_{j=1}^{N} (\hat{d}_j - \underline{d}_j) y_{jk}} & \sum_{j=1}^{N} \underline{d}_j y_{jk} < \text{Cap} \leq \sum_{j=1}^{N} \hat{d}_j y_{jk}, \\ 0 & \text{Cap} < \sum_{j=1}^{N} \underline{d}_j y_{jk}. \end{cases} \tag{18}$$

The requirements for the necessity that the capacity is sufficient on tour *k* is

$$\text{Nec}\left(\sum_{j=1}^{N} \widetilde{d}_j y_{jk} \leq \text{Cap} \right) \geq \beta \qquad \text{for } k = 1, ..., N,$$

$$\Leftrightarrow \text{Nec } (\text{Cap} - \sum_{j=1}^{N} \widetilde{d}_j y_{jk} \geq 0) \geq \beta \quad \text{for } k = 1, ..., N. \tag{19}$$

It can be calculated similarly to

$$\text{Nec}\,(\text{Serve}\,\widetilde{D}_k) = \begin{cases} 1 & \text{Cap} \geq \sum_{j=1}^{N} \overline{d}_j\, y_{jk}, \\[2em] \dfrac{\text{Cap} - \sum_{j=1}^{N} \hat{d}_j\, y_{jk}}{\sum_{j=1}^{N} (\overline{d}_j - \hat{d}_j)\, y_{jk}} & \sum_{j=1}^{N} \hat{d}_j\, y_{jk} < \text{Cap} \leq \sum_{j=1}^{N} \overline{d}_j\, y_{jk}, \quad (20) \\[2em] 0 & \text{Cap} < \sum_{j=1}^{N} \hat{d}_j\, y_{jk}. \end{cases}$$

For every possibility and associated necessity measure and every set $A \subset X$ the following implication is satisfied [13]

$$\text{Pos}\,(A) < 1 \Rightarrow \text{Nec}\,(A) = 0.$$

The consequence for our application is that it is more demanding to request the necessity to be greater than 0 than to request the possibility to be less or equal to 1. For $\alpha > 0$ we can model the following constraints as crisp equivalents for the fuzzy constraint (9):

$$\text{Pos}\,(\text{Serve}\,\widetilde{D}_k) \leq \alpha \Leftrightarrow$$

$$\sum_{j=1}^{N} (\alpha\, \hat{d}_j + (1-\alpha)\, \underline{d}_j)\, y_{jk} \leq \text{Cap}, \quad k = 1,\ldots,K, \;\; \alpha \in (0,1] \qquad (21)$$

respectively for $\beta > 0$.

$$\text{Nec}\,(\text{Serve}\,\widetilde{D}_k) \leq \beta \Leftrightarrow$$

$$\sum_{j=1}^{N} (\beta\, \overline{d}_j + (1-\beta)\, \hat{d}_j)\, y_{jk} \leq \text{Cap}, \quad k = 1,\ldots,K, \;\; \beta \in (0,1]. \qquad (22)$$

To solve this fuzzy mathematical programming model we suggest to determine the optimal solutions with respect to a given degree of possibility or even stronger a given degree of necessity that the capacity is sufficient to meet the total demand of the customers on each tour.

If the decision maker is not able or does not want to require in advance a certain degree of possibility or necessity for the demand served we suggest to give him or her additional information about the dependencies between those values and the minimal total distance the ships have to travel. On behalf of that first the model (1) and (3) to (8) with constraint (21) is solved with $\alpha \in [0,1]$ parametrically increasing from 0 to 1. Afterwards the second constraint (22) is included into the model instead of (21). It is stronger than (21) for every $\beta > 0$. Here too, $\beta \in [0,1]$ is parametrically increased from 0 to 1. The parametrical modifications for this integer linear programming model exact a lot of calculations. Therefore it might be more efficient to calculate the maximal degree of possibility or necessity each time an optimal solution has been determined for the model. The next optimal solution can then be determined by improving this maximal degree by a small proportion and including this new value into the constraint. The result is an over-

view over all fuzzy efficient solutions with respect to minimal distance and maximal possibility respective necessity meeting the fuzzy demand. A solution is fuzzy efficient if it is not possible to improve one of the values without deteriorating some of other ones [22, 23].

3.2. EXAMPLE: TANKER ROUTING WITH FUZZY DEMANDS

For demonstration purposes we again consider the tanker problem above but now the demands of the different ships are not exactly known. What the decision maker gets is an order d which is not binding and he has some experience what the demand might be. This is modeled using triangular fuzzy numbers $\widetilde{d} = (\underline{d}, \hat{d}, \overline{d})$. For the small example the following data are to be considered with triangular fuzzy numbers for the uncertain demands.

T a b l e 3. Uncertain demands

Ship No.	1	2	3	4	5
$\underline{d}$	400	250	150	500	150
$\hat{d}$	500	400	300	600	200
$\overline{d}$	600	500	350	800	400

The crisp equivalent model for a requested possibility $\alpha = 0.2$ to serve the demand is solved by using CPLEX. The optimal solution of the mixed integer linear programming model are the two tours $0 - 3 - 2 - 4 - 0$ and $0 - 1 - 5 - 0$ with a total travel distance of 286.2.

Parametric optimization is applied first with constraint set (21) and parameter $\alpha \in [0,1]$ and afterwards with constraint set (22) and parameter $\beta \in [0,1]$. This will give a general idea of efficient solutions with respect to total distance traveled and possibility or necessity to serve the demand. Solving the mixed integer linear programming model yields the following results:

For the possibility α of serving the demand with $0 \le \alpha \le 0.25$ the optimal routes are $0 - 3 - 2 - 4 - 0$ and $0 - 1 - 5 - 0$ with a total distance of 286.2.

For $0.25 < \alpha \le 1$ the optimal total distance is 340.2 and the optimal routes are $0 - 2 - 4 - 0$ and $0 - 3 - 1 - 5 - 0$. Now the decision maker can decide whether he wants to accept the very small possibility 0.25 of meeting the demand with a total distance of 286.2 compared to a distance of 340.2 and a possibility of 1 to meet the requirements.

The necessity to meet the demand with the above solution is 0. A higher necessity can be obtained by the optimal routes $0 - 5 - 1 - 0$, $0 - 4 - 0$ and $0 - 3 - 2 - 0$ with a total distance of 402.3 and 3 tankers involved. This solution is — besides symmetrical ones - the only one with necessity greater than 0. Its necessity to serve the demand is 1. Now the decision maker can decide which of these three different solutions fits his or her preferences best.

Instead of parametrically varying degrees of possibility and necessity in the mathematical model it can be more efficient to determine the respective degree of possibility and necessity by calculations outside the mixed integer linear pro-

gramming model (MILP). Starting with possibility 0 we get the optimal solution of the MILP model. This solution contains optimal tours with their fuzzy demands. Using this information the maximal possibility to meet the demand of this solution can be calculated by using (21) resp. (22) to calculate Pos (Serve $\widetilde{D}_k$) resp. Nec (serve $\widetilde{D}_k$) for each k, $k = 1, \ldots, K$, and then calculating $\min\limits_{k=1}^{K} \text{Pos}(\text{Serve}\,\widetilde{D}_k)$.
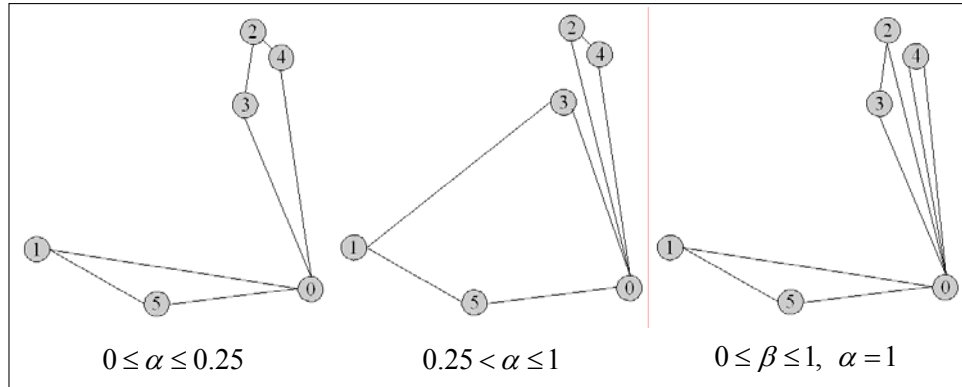

Fig. 5. Optimal routes for different α resp. β

For this example with $\text{Pos}(\text{Serve}\,\widetilde{D}_k) = 0$ the optimal routes are $0 - 3 - 2 - 4 - 0$ and $0 - 1 - 5 - 0$ with fuzzy demands $\widetilde{D}_1 = (900, 1300, 1650)$ and $\widetilde{D}_2 = (550, 700, 1000)$ and $\min\{0.25, 1\} = 0,25$. Then the MILP model is solved again with $\alpha > 0.25$, e.g. $\alpha = 0.3$. The results of both approaches are identical. They are shown in fig. 6.
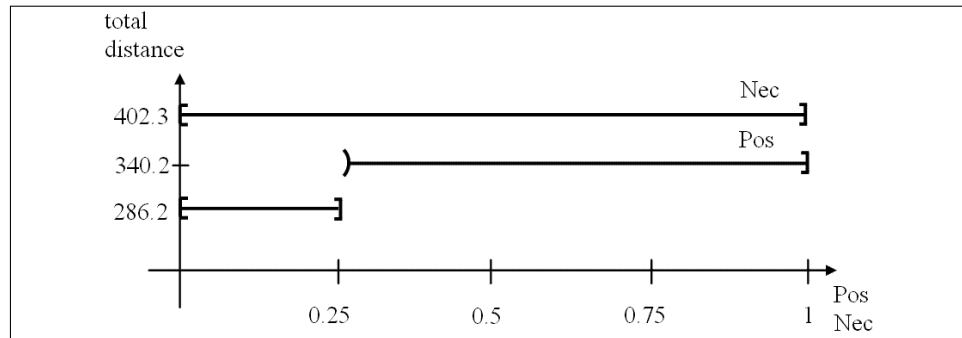

Fig. 6. Dependency of total distance and possibility and necessity to serve the demand

4.1. FUZZY MULTI-CRITERIA APPROACH

In general the bunkering company is not only interested in the possibility or even necessity of serving the demand but the company wants to sell as much fuel as possible. Sales is restricted by the demand and by the capacity of the tanker for a tour. A solution is preferable if the amount of the demand served is high. If the demand is known in advance the capacity constraint (2) ensures that there is

enough capacity for each tour to serve the entire demand and thus maximize sales. In the fuzzy environment considered here it is also possible to serve the entire demand. But here it means that so many tankers must travel so long distances that their capacity is high enough to meet every possible demand even if the membership degree is very low. Therefore these membership degrees have to be taken into account too. To maximize sales in this fuzzy context means to determine and maximize a fuzzy set which results from the fuzzy demand, the tour and the capacity of the tanker.

The fuzzy set sales $\widetilde{S}_k$ on tour k depends o the minimum of the demand $\widetilde{D}_k$ and the capacity of the tanker. The membership function can be calculated

$$\mu_{\widetilde{S}_k}(x) = \begin{cases} \mu_{\widetilde{D}_k}(x) & x < \text{Cap}, \\ \text{Pos}\left(\widetilde{S}_k \geq \text{Cap}\right) & x = \text{Cap}, \\ 0 & x > \text{Cap}. \end{cases}$$

The membership function of the total demand to be served on all routes can be calculated by extended addition of the fuzzy sales. For the extension principle and extended operations see e.g. (Dubois and Prade, 1980; Zimmermann, 1992).

The fuzzy total sales depended on the routes is $\left(\sum\right)_{k=1}^{K} \widetilde{S}_k$ with

$$\mu_{\bigoplus \widetilde{S}_k}(z) = \sup_{\substack{K \\ \sum_{k=1} x_k = z}} \min_{k=1}^{K} \left\{ \mu_{\widetilde{S}_k}(x_k) \right\}. \tag{23}$$

There are rather many calculations to be done to determine this fuzzy set. Therefore we suggest to use an easy to calculate defuzzification method. We suggest to use

$$\frac{1}{3} \min\left\{ \sum_{j=1}^{N} \underline{d}_j y_{jk}, \text{Cap} \right\} + \frac{1}{3} \min\left\{ \sum_{j=1}^{N} \hat{d}_j y_{jk}, \text{Cap} \right\} + \frac{1}{3} \min\left\{ \sum_{j=1}^{N} \overline{d}_j y_{jk}, \text{Cap} \right\} = D\widetilde{S}_k$$

$$\tag{24}$$

to determine a crisp approximation $D\widetilde{S}_k$ for the sales on tour k and $\sum_{k=1}^{K} D\widetilde{S}_k$ as a crisp approximation of total sales.

4.2. EXAMPLE: MULTI-CRITERIA APPROACH

In the example for the routes $0 - 3 - 2 - 4 - 0$ and $0 - 1 - 5 - 0$ the demands on the tours are as above and the capacity of each tanker is 1000. The membership function of the fuzzy demand to occur and to be served on tour 1 that is the membership function of sales $\mu_{\widetilde{S}1}$ is:

$$\mu_{\widetilde{S}1}(x) = \begin{cases} 0 & x \leq 900, \\ \dfrac{1300 - x}{400} & 900 < x < 1000, \\ 1 & x = 1000, \\ 0 & x > 1000. \end{cases}$$
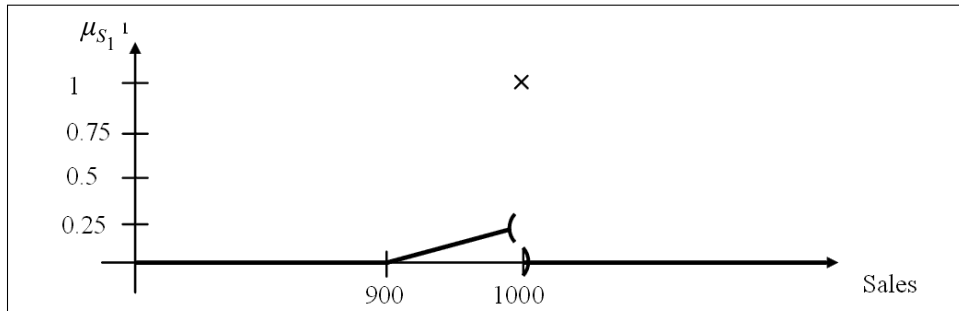
It is shown in fig. 7.

Fig. 7. Membership function of sales $\widetilde{S}_1$

The fuzzy total sales for both routes is a fuzzy set with the membership function depicted in fig. 8.
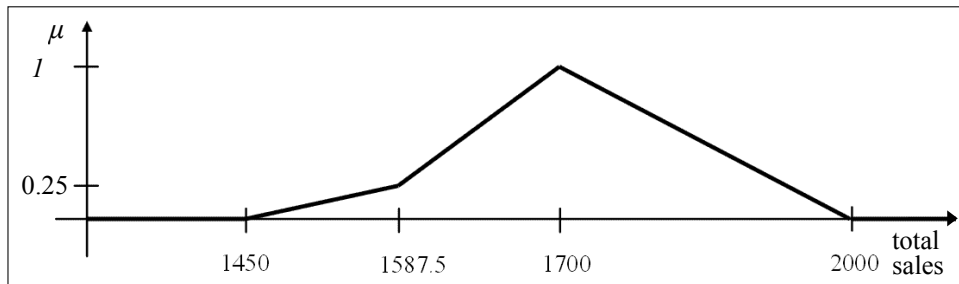

Fig. 8. Membership function of total sales

The crisp approximation of the total sales is

$$\frac{1}{3}(900 + 1{,}000 + 1{,}000) + \frac{1}{3}(550 + 700 + 1{,}000) \approx 1{,}717 .$$

4.3. FUZZY MULTI-CRITERIA MODEL

The fuzzy multi-criteria optimization model to be considered is

$$\min z^1(x, y, u) = \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} c_{ij} x_{ijk} ,$$

$$\max z^2(x, y, u) = \sum_{k=1}^{K} \widetilde{\min} \left\{ \sum_{j=1}^{N} \widetilde{d}_j y_{jk} , \mathrm{Cap} \right\}, \tag{25}$$

$$\text{s.t.} \quad \mathrm{Pos} \left(\mathrm{Cap} - \sum_{j=1}^{N} \widetilde{d}_j y_{jk} \geq 0 \right) \geq \alpha \quad k = 1, \ldots, N ,$$

$$\mathrm{Nec} \left(\mathrm{Cap} - \sum_{j=1}^{N} \widetilde{d}_j y_{jk} \geq 0 \right) \geq \beta \quad k = 1, \ldots, N .$$

and constraints (3) to (8) x, y, u stand for the vectors of variables in this model. $\widetilde{\min}$ means the extended minimum of the two fuzzy sets $\widetilde{D}_k$ and Cap. The crisp set Cap is considered as a fuzzy set with membership function 1_{Cap}.

We suggest to use the interactive approach developed by Werners [21,22,23] for the solution of this tanker problem with some modifications. Due to the fact that objective function (25) and constraints (16) resp. (19) are not independent of each other they should be handled differently and adequately. For (16) and (19) the crisp equivalent models (21) and (22) are used.

The individual optimum of z^1 i.e. the minimal total distance should be determined so that at least the lower bound of the fuzzy demand $\sum_{k=1}^{K} \sum_{j=1}^{N} \underline{d}_j y_{jk}$ is served.

The individual optimum of the objective "minimal total distance" is the solution of the model

$$\min \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} c_{ij} x_{ijk} \ ,$$

$$\text{s.t.} \quad \sum_{j=1}^{N} \underline{d}_j y_{jk} \leq \text{Cap} \qquad k = 1,...,K \tag{26}$$

and constraints (3) to (8).

The optimal solutions are x_{ijk}^1, y_{jk}^1 and u_{jk}^1, $i, j = 0,...,N$, $k = 1,...,K$ with $z^1(x^1, y^1, u^1) = z^{1*}$ given by the solution algorithm and $z^2(x^1, y^1, u^1) = z_*^2$ to be calculated using (24).

To determine the individual optimum of z^2 the crisp equivalent model (24) instead of (25) is used. It is easy to prove that for demands $\widetilde{d}_j$, $j = 1,...,N$ the maximum of (24) cannot exceed $\frac{1}{3} \sum_{j=1}^{N} (\underline{d}_j + \hat{d}_j + \overline{d}_j)$. In case that the number of vehicles is not restricted and each single demand is less than the capacity then a solution can be found which takes this value. So this individual optimum can be calculated without solving a mixed integer linear programming problem. But in general, there are several solutions which are not all fuzzy efficient. To determine a fuzzy efficient solution all solutions $\left(x^2, y^2, u^2\right)$ have to be considered implicitly and that one with $\min z^1\left(x^2, y^2, u^2\right)$ has to be found. Those $\left(x^2, y^2, u^2\right)$ which are optimal with respect to z^2 all satisfy constraint (22) with $\beta = 1$.

Thus, a fuzzy efficient individual optimum z^2 can be ascertained by solving

$$\min \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} c_{ij} x_{ijk} \ ,$$

$$\text{s.t.} \quad \sum_{j=1}^{N} \overline{d}_j y_{jk} \leq \text{Cap} \quad k = 1,...,K \tag{27}$$

and constraints (3) to (8).

The optimal solution is $\left(x^2,\, y^2,\, z^2\right)$ with $z^2\left(x^2,\, y^2,\, z^2\right)=z^{2*}$ and $z^1\left(x^2,\, y^2,\, u^2\right)=z_*^1$.

Now individual optimal and pessimistic solutions can be used to model membership functions for the two goals.

$$\mu_{z^1}(x)=\begin{cases} 1 & z^1(x,y,u)\le z^{1*}, \\ \dfrac{z_*^1-z^1(x,y,u)}{z_*^1-z^{1*}} & z^{1*}<z^1(x,y,u)<z_*^1, \\ 0 & z^1(x,y,u)\ge z_*^1 \end{cases} \tag{28}$$

and

$$\mu_{z^2}(x)=\begin{cases} 1 & z^2(x,y,u)\ge z^{2*}, \\ \dfrac{z^2(x,y,u)-z_*^2}{z^{2*}-z_*^2} & z_*^2<z^2(x,y,u)<z^{2*}, \\ 0 & z^2(x,y,u)\le z_*^2. \end{cases} \tag{29}$$

The first compromise model is

$$\max\ \lambda$$

$$\text{s.t. } (z_*^1-z^{1*})\lambda+\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=1}^{K}c_{ij}x_{ijk}\le z_*^1, \tag{30}$$

$$(z_*^2-z^{2*})\lambda+\sum_{k=1}^{K}\frac{1}{3}\left(\min\left\{\sum_{j=1}^{N}\underset{-j}{d}\,y_{jk},\text{Cap}\right\}+\min\left\{\sum_{j=1}^{N}\hat{d}_j y_{jk},\text{Cap}\right\}+\right.$$

$$\left.+\min\left\{\sum_{j=1}^{N}\overline{d}_j y_{jk},\text{Cap}\right\}\right)\ge z_*^2 \tag{31}$$

and constraints (3) to (8), $0\le\lambda\le1$.

Constraints (16) and (19) can be omitted because they are implicitly contained in (31).

One can get an equivalent linear model by substituting the following constraints for the nonlinear constraint (31).

$$\left(z_*^2-z^{2*}\right)\lambda+\sum_{k=1}^{K}\frac{1}{3}\left(\underset{-k}{t}+\hat{t}_k+\overline{t}_k\right)\ge z_*^2, \tag{32}$$

$$\underset{-k}{t}\le\sum_{j=1}^{N}\underset{-j}{d}\,y_{jk}\quad k=1,\ldots,K, \tag{33}$$

$$\hat{t}_k\le\sum_{j=1}^{N}\hat{d}_j y_{jk}\quad k=1,\ldots,K, \tag{34}$$

$$\overline{t}_k\le\sum_{j=1}^{N}\overline{d}_j y_{jk}\quad k=1,\ldots,K, \tag{35}$$

$$0 \le \underset{-k}{t}, \hat{t}_k, \bar{t}_k \le Cap \quad k = 1,...,K .\tag{36}$$

4.4. EXAMPLE: COMPROMISE SOLUTION

Considering the small tanker routing example the following table shows the individual optimal solutions determined using models (1), (26), (3) to (8) for z^1 and (1), (27), (3) to (8) for z^2. Both are efficient with respect to the two goals.

T a b l e 4. Individual optimal and pessimistic solutions

Individual optimal routes	z^1	z^2
0–3–2–4–0 and 0–1–5–0	$z^{1^*} = 286,2$	$z^2_* = 1,717$
0–1–5–0 and 0–4–0 and 0–3–2–0	$z^1_* = 402,3$	$z^{2^*} = 2,367$

This small example also demonstrates that the approach to calculate the individual optimum of z^2 straight forward is not appropriate. Maximizing the defuzzified version of (25) subject to constraints (3) to (8) without taking into account the total distance leads to the following result. The optimal routes are $0 - 2 - 5 - 0$ and $0 - 3 - 1 - 0$ and $0 - 4 - 0$ with identical possible sales of 2,367 but longer total distance of 497,1.

The crisp compromise model is

$$\max \lambda$$

$$\text{s.t. } 116.1\lambda + \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} c_{ij} x_{ijk} \le 402,3 ,$$

$$-650\lambda + \sum_{k=1}^{K} \frac{1}{3}\left(\underset{-}{t}_k + \hat{t}_k + \bar{t}_k\right) \ge 1,717$$

and constraints (33) to (36) and (3) to (8) and $0 \le \lambda \le 1$.

The optimal solution of the comprise model are the routes $0 - 1 - 5 - 0$, $0 - 3 - 0$ and $0 - 2 - 4 - 0$, the membership degree is $\lambda^0 \approx 0,19$ with $z^1 = 380,7$ and $z^2 \approx 1,933$.
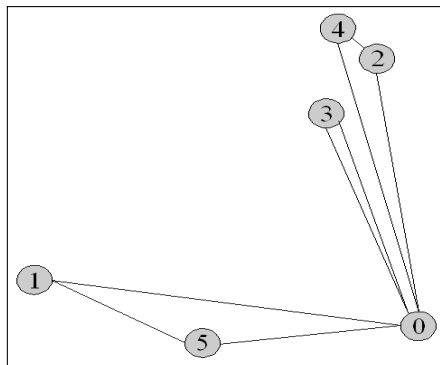

The possibility to serve the demand is 1 and the necessity is 0. The advantage of this solution is the higher possible sales compared with the routes $0 - 3 - 2 - 4 - 0$ and $0 - 1 - 5 - 0$ and even compared with $0 - 2 - 4 - 0$ and $0 - 3 - 1 - 5 - 0$ with possible sales 1,816 and a lower total distance compared with $0 - 1 - 5 - 0$, $0 - 4 - 0$ and $0 - 3 - 2 - 0$. Here the decision maker accepts the first compromise solution, which is shown in fig. 9.

Fig. 9. Compromise solution

5. CONCLUDING REMARKS

In this paper a fuzzy multi-criteria programming model is developed to solve the tanker routing problem with fuzzy demands. Using standard MILP software a compromise solution is determined which takes into account several criteria. One of the main advantages of this model is its generality. Additional aspects can be included into the model appropriately. Exemplarily the model can be extended to consider fuzzy distances between the ports. That can be necessary to model weather conditions. In case that the number of ships to be served is rather large then the model cannot be solved optimally and a heuristic has to be chosen. In principle the well known heuristic approaches can be used. Additional research is necessary to develop modifications which consider multiple criteria and find a compromise solution very efficiently.

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