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DETERMINING THE MAXIMUM LOADABILITY FOR THE PLASTIC CYCLOIDAL GEARS

ABSTRACT

The article presents the analysis of stresses and displacements which occur in the cycloidal gears working in the gerotor machine. The authors considered a possibility of using plastic (polyoxymethylene POM) for manufacturing the gears. The analysis has been conducted by means of the finite element method. Determining the maximum value of pressure which can be applied in the gerotor machine featuring plastic gears is the result of the analysis.

INTRODUCTION

Gerotor pumps belong to the group of internal cycloidal gearing machines of the tooth difference $z_2-z_1=1$. Such machines are also called 'gerotor machines'. Their characteristic features are more compact structure, lower weight, lower displacement (inlet flow rate) pulsation, and lower noise level.

The classic gerotor machine is presented in fig. 1 [4]. In the body consisting in the front plate (1), central plate (2), and back plate (3) there is a gear set (4), (5) featuring the internal cycloidal gearing of the tooth difference $z_2-z_1=1$. The gear set is driven by shaft (7) which is supported by the bearings (6).



Fig. 1. Design and operation principle of the third generation gerotor machine: 1,2,3 – body elements; 4, 5 – cycloidal gears; 6 – bearing; 7 – shaft; I, O – inlet and outlet ports; CH_i, CH_o – inlet and outlet channels; CH – displacement chamber; IZ, OZ – outlet and inlet loss; LB, UB – upper and lower bridge.

Between the teeth of the mating gears displacement chambers (CH) are located. While the gears rotate, the working medium under pressure p_i flows into the chambers (CH) through the inlet port (I) and the inlet channel (CH_i). Further rotation of the chambers causes compression of the working fluid to pressure p_0 and pumping it out through the outlet channel (CH₀) and the outlet port (O).

The authors claim that it is important to look for a way of simplifying the process of the pumps manufacture as well as to consider a possibility of the machines' application in water hydraulic systems. One way of achieving this aim could be making the cycloidal gears of plastics using the moulding method. This method is less work consuming and cheaper than the currently used method of milling steel wheels. In this situation it is necessary to take up theoretical research regarding the plastic cycloidal gears used in hydraulic machines.

2. THE COMPUTABLE MODEL OF THE CYCLOIDAL GEAR SET

By means of the constructional diagram of the gerotor pump shown in fig. 1 a geometric model of the cycloid gear set was prepared and shown in fig. 2. The toothing outline of the outer gear was determined from the equations describing the epicycloid [4]. The toothing of the driven gear was shaped in the form of the arcs of the circle.

The geometric model is influenced by the mechanical load resulting from the transmission of the torque M by the driver shaft of diameter $d_1=25$ mm onto the key situated in the outer tooth gear. The model is also affected by the hydraulic load resulting from the total pressure head p generated in displacement chambers. The gear set keeps its balance because the torque M applied to the shaft is equal to the torque generated by the pressure p.



Fig. 2. Model of the cycloidal gear set: a) loads distribution; b) finite element grid.

The model should be fixed and the fixing method also stems from the operation principle of the gerotor pump. Fig. 1 shows that the inner tooth gear is fixed on the shaft (7) and can only rotate around the axis O_1 . Correspondingly in the model shown in fig. 2a the gear is fixed radially on the shaft of diameter $d_1=25$ mm and can rotate around the axis Z. The inner tooth gear shown in fig. 2a is fixed in the seat of the central body (2) and can rotate around the axis O_2 . Correspondingly in the model shown in fig. 2a the gear is fixed radially on diameter $d_2=75$ mm but can rotate around the axis Z. It stems from fig. 1 that the gear set is put under the pressure load p of the working medium situated in the axial clearance formed between the elements of the body (2) and (3). Because of the fact the gear set is pressed against the body (1). On account of this the end face of the model is put under the load of the pressure p while the opposite face is fixed, which is to see in fig. 2a.

The finite element grid shown in fig. 2b was made by means of HEXA elements being 8-node cubes. The finite element grids for individual working positions of the gear set are made up of ca. 500 000 HEXA elements. The number of nodes indicates that it is the 1st order element. The nodes are located in vertexes of each element. The HEXA element has three degrees of freedom in each node, that means the shift in relation to axes X, Y as well as Z and is a standard element to create spatial models.

In the anticipated loci the grid refinements were made. The loci of refinements are tooth contacts of the mating gears at the suction side (the left side of the model in the fig. 2b) and the area of the splineway in the active gear. The refinements made in the corners of the tooth bearing of the passive gear are smaller.

The grid was made up of 15 layers of HEXA elements along the axis OZ and each of the layers was located at ca. 0,7mm intervals along the dimension b= 0,4mm, namely along the width of the gear.

The numerical model of the cycloid gear set was made by means of the ABAQUS system. The system enabled modelling of the surface contact by means of a special algorithm characteristic of the ABAQUS system. The license to use the ABAQUS system was given to our division by WCSS (Wroclaw University of Technology).

It is especially important because while in operation the gears are in a constant contact, therefore the loads are transmitted among them. The contact was located in the area of the tooth contact at the suction side of the gear set. (the left side of fig.2a and 2b). The friction factor m was selected considering the plastic type and the conditions present in the area of the contact (e.g. high pressure). The value of the factor m=0.4was determined on the research basis [3].

As the plastic for the matched gears pure polyoxymethylene (POM) was chosen. It is characterized by relatively considerable mechanical strength and not a very big strain. Admittedly, the material absorbs water, but to a slight extent, and its processing is well mastered. It must also be added that the authors had already conducted strength tests of the material and specified the limits of its stress plasticity as $\sigma \approx 80$ MPa [2].

3. FINDINGS

Fig. 3,4,5 show the research results of the model of the cycloidal gears put under load with the total state of load i.e. the pressure p=4MPa operating in displacement chambers and the torque M =7,16Nm. The figures show the state of reduced stresses (fig. 3a, 4a and 5a) as well as the states of the gears' deformations (fig. 3b, 4b, 5b) whereas the deformed profiles seen in the gears have been shown on the background of the unstrained profile depicted in a form of the continuous black line.

Correspondingly fig. 3a and 3b examine the state of stresses and strains for the initial position of the external teeth gear($\alpha = -\pi/z_1$). It is to see in fig. 3a that the greatest stress (interteeth forces) is made at the tooth base 1' and equals 33,17MPa. Additionally, between the tooth base 1' and the splineway the stress bridge is formed.

In the next pairs of teeth $2^{2}-2^{2}$ and $3^{2}-3^{2}$ the stresses (interteeth forces) decrease, and in pairs $4^{2}-4^{2}$, $5^{2}-5^{2}$, $6^{2}-6^{2}$ the stresses do not occur.



Fig.3. The state of stresses and strains in the plane XY of the model of cycloidal gears for its different positions. The pressure load of the model p = 4 MPa and the torque load M = 7,16 Nm. a, b) – the initial position of the outer gear $\alpha = -\pi/z_1$.

Fig. 3b shows the biggest deformations in tooth pairs 4'-4" and 5'-5" between which the greatest radial clearances are made. The maximum value of this clearance was registered for the pair of teeth 4'-4", where $h_r = 0.065$ mm.

Successively, fig.4a and 4b show the state of stress and strain for the central position of the outer gear (α =0). Fig. 4a shows that the greatest stress is formed, similarly to the above described situation, on the contact area of tooth 1' and on the side of the splineway. The value of the stresses comes to 26MPa. It the subsequent tooth pairs 2'-2", 3'-3" the stress gradually decreases, and in pairs 4'-4", 5'-5", 6'-6" the stress does not occur.

Fig.4b shows that the biggest deformations are present in tooth pair 4'-4", between which the greatest radial clearance h_r =0,082mm is made. The value of the clearance h_r being produced as a result of the strain comes to $h_r = 0,082mm$ and it is the highest for the entire gear set in this position.



Fig.4. The state of stresses (a) and strains (b) in the plane XY of the model of cycloidal gears for its different positions. The pressure load of the model p = 4 MPa and the torque load M = 7,16 Nm. a,b) - the central position of the outer gear α =0

Fig. 5a and 5b illustrate the state of stresses and strains for the final position of the gears $(\alpha = +\pi/z_1)$.



Fig.5. The state of stresses (a) and strains (b) in the plane XY of the model of cycloidal gears for its different positions. The pressure load of the model p = 4 MPa and the torque load M = 7,16 Nm.

a, b) - the central position of the outer gear $\alpha = +\pi/z_1$.

It is possible to see that the stress values (forces) on the contact points of the teeth are more regular for gear position ($\alpha = +\pi/z_1$), than for the previous positions. According to the authors, it is caused by shifting the splineway with torque M, which results in greater pressure of the teeth in pairs 6'-7", 1'-1" and 2'-2". The highest stress value was recorded on the right side of the splineway, namely 25MPa.

Fig. 5b shows that the biggest deformations can be observed in the teeth in the lower part of a particular set and those are tooth pairs 3'-3" and 4'-4" between which the greatest intertooth clearance h_r is made. The value of the clearance is $h_r=0,037$ mm.

Considering the state of the strains and stresses together it is noticed that the external gearing driving wheel is deformed towards the inlet zone (IZ), whereas the internal gearing driven wheel is deformed in the opposite direction, that is towards the outlet zone (OZ). It causes mutual pressing of teeth 1'-1", 2'-2", 3'-3" and results in creating stresses between them which when the gears are loaded with torque M=7,16NM and pressure p=4MPa do not exceed 33,17MPa. At the same time, teeth 4'-4", 5'-5", 6'-6" are pushed away from each other, and between them intertooth radial clearances $h_r=0,037-0,082mm$ are made.

The mechanism of creating axial clearance h_a has been presented in fig. 6. The front surface of the gears is worked on by pressure which results in compressing the gears and reducing their primary width of b=10,4mm (fig. 2). The value of the clearance is h_a =b'-b'' (fig.6).



Fig. 6. Schematic illustration of axial clearance h_a

Axial clearance h_a occurs in the outlet zone of the gear set and its maximum value was recorded in the area of the upper bridge (UB in fig.1) on the border line of the inlet and outlet zones.

According to the research program, the analysis of strains and stresses was conducted in the range of pressure load p=4, 8, 12, 16 MP and in the range of torque load M=7,16; 14, 32;21,48; 28,64Nm. The research results in a form of radial clearance h_r and axial clearance h_a are presented in table 1.

Analysing table 1 it can be seen that on account of the plastic strains cycloidal gear set consisting of the plastic POM may be put under the working pressure load up to the value of p=10MPa. The maximal value of reduced stresses is approximately equal to σ =80MPa. It can also be noticed that the radial clearance h_r and the axial clearance h_a increases in direct

proportion to the increase of the working medium pressure affecting the gear set. For the terminal load pressure p=10MPa the clearances amount to: $h_r = 0,204$ mm and $h_a = 0,046$ mm.

Table. 1.

| Working pressure p [MPa] | Radial clearance h _r [mm] | Axial clearance h _a [mm] | Maximum stress [MPa] |
|--------------------------------|--|--|-------------------------|
| 4 | 0,037 - 0,082 | 0,018 | 33,17 |
| 8 | 0,074 - 0,164 | 0,036 | 66,34 |
| 10 | 0,092 - 0,204 | 0,046 | 82,96 |
| 12 | 0,11 - 0,246 | 0,055 | 99,51 |
| 16 | 0,148 - 0,328 | 0,073 | 132,68 |

Values of stresses, the radial clearances h_r and the axial clearances h_a depending on the working pressure p

4. CONCLUSION.

The possibility of applying the gears consisting of the plastic POM in the gerotor machines should be considered from the viewpoint of stresses and deformations altogether. It should not be allowed for the stresses in the gears to exceed the yield point resulting in the gear damage. At the same time it should not be allowed for the teeth and gear deformation as well as the radial clearances h_r and the axial clearance resulting from them to exceed the boundary values received for the given class of the hydraulic machine. The condition of deformations is more important than the condition of stresses in this case.

The research done so far proved the POM cycloidal gears to work correctly in the gerotor pump loaded with the working pressure 1MPa [1]. Numerical analysis presented in this article makes it possible to claim that in the gear set made up of the plastic POM the condition of stresses is kept in the range of the pressure load up to ca. 10MPa, where the maximal stress in the gears amounts to ca. 80MPa. The pressure p=10MPa may not yet be reached by the gerotor machine. The radial clearance $h_r=0,204$ mm and the axial clearance $h_a=0,046$ mm produced by the pressure will be so big that they will result in the loss of internal integrity of the machine. In this situation the condition of deformations may not be met. Hence, it is possible to substitute the steel cycloidal gears with the POM gears, but the limit of loading them with pressure should be lowered down to p=4÷6MPa.

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