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# IMAGE INFORMATION DENSITY AND LIDAR OBJECT INTERPRETABILITY

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*The information densities of perfect and real images are defined based upon classical information theory. This definition accounts for image dynamic range, noise, dropouts, false alarm, diffraction effects and image aberrations. The functional form of these theoretical equations for information density is found to be similar to the general form of the empirically derived image quality equations that predicts the interpretability of imagery. This functional similarity suggests that interpretability of images and the lidar detection, recognition and characterization of objects within those images may be directly related to the information density of the image.*
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## Introduction: Image Information Density Definition

The mean information density can be defined as the total information content of the image divided by the total, geospatial area of the image or

$$\overline{K} = \frac{I_{Total}}{A_{Total}}, \qquad (1)$$

where $\overline{K}$ is the mean information density in bits per square meter, $I_{Total}$ is the total information content of the image, and $A_{Total}$ is the total physical area covered by the image in square meters.

If a digitized image is considered in the context as a message to be communicated and each pixel contains the same amount of information, then the total information content of that image can be expressed as

$$I_{Total} = N \times H(X), \qquad (2)$$

where $N$ is the number of pixels in the image, $H(X)$ is the Shannon entropy [1] of the random variable $X$ which can be interpreted at the intensity of an individual pixel. In this context, each pixel may be regarded as a symbol in the message.

The total area can be expressed as

$$A_{Total} = N \times GSD^2, \qquad (3)$$

where $GSD$ is the mean ground sampling distance in meters and all other terms are as previously defined. In this context, the $GSD$ is the distance between the centers of adjacent pixels when projected to object space.

By substituting (2) and (3) into (1), it is obvious that the mean information density

$$\overline{K} = \frac{H(X)}{GSD^2}. \qquad (4)$$

## Information Density of Perfect Imagery

If all values of each symbol are completely independent of the value of any other symbol, the signal does not contain any noise and there are no dropouts, then the Shannon entropy of a continuous signal may be expressed as

$$H_{Source}(x) = -\int_{-\infty}^{\infty} p(x)\log_2 p(x)dx, \qquad (5)$$

where $x$ is the individual values that the random variable can assume, $p(x)$ is the probability that the random variable will take on the value $x$ and all other terms are as previously defined. If the image is digitized so that $x$ can take on only a finite number of discrete values, then the information may be expressed as

$$H(x) = -\sum_{i=1}^{n} p(x)\log_2 p(x), \qquad (6)$$

where $n$ is the number of possible discrete values that each pixel may take on and all other terms are as previously defined.

If the digitized imaging system is perfect, then all of the information will be preserved and the mean information density of the image will be given by

$$\overline{K} = \frac{-\sum_{i=1}^{n} p(x)\log_2 p(x)}{GSD^2}, \qquad (7)$$

where all terms are as previously defined. The information density has units of bits per square meter.

**Information Density of Real Imagery**

Unfortunately, noise, dropouts and false alarms corrupt real images. The intensities and textures of real images are spatially correlated and the imaging system is limited by diffraction, aberrations, scattering and other imperfections. All of these effects must be considered in order to determine the information density of a real image.

First, noise introduces uncertainty in the measured value of the signal. Noise causes the measurement (i.e., equivalent to the reception of a transmitted communications signal) to be different from that of the source (i.e., transmission). Noise results in a reduction of the information received. If the source intensity is known, this reduction in information is known as equivocation. If the measurement is known but the source is not, the information loss is known as uncertainty. The two formulations are equivalent. It fact, it can be shown that [1]

$$H(x) - H_y(x) = H(y) - H_x(y), \qquad (8)$$

where $H(x)$ and $H(y)$ are the entropy of the source and the received signal, respectively, $H_y(x)$ is the equivocation and $H_x(y)$ is the uncertainty.

Second, dropouts also reduce the information content of the image, but their effect is different from noise. While noise corrupts the information in an unknown way, dropouts result in missing pieces of information, but the omission is known. Consequently, dropouts reduce the information of the source and reduce the corrupting effects of noise proportionately. The probability of detection then becomes a coefficient to the source entropy and to the entropy of uncertainty.

Third, false alarms further reduce the information content of the image by introducing totally spurious detection events that are not correlated with any element of the signal generated by the source. “Snow” in a video image is one manifestation of false alarms. Unlike dropouts, these spurious detections cannot be identified *a priori*. They also reduce the information content of the image, but not in a deterministic manner. This reduction in the information is given by [1]

$$H_{fa}(X) = -\sum_{i=1}^{n} p_{fa}(x)\log_2 p_{fa}(x), \tag{9}$$

where $H_{fa}(X)$ is entropy of the false alarms associated with the random variable $X$, $p_{fa}(x)$ is the probability of a false alarm having the value of $x$ and all other terms are as previously defined.

Fourth, imperfections, whether from diffraction, aberrations or scattering, in the optical systems will result in image blur. The intensity associated with an individual pixel is not independent of the intensity of adjacent pixels. This also results in a loss of independence in the actual measurements. When a symbol transmitted by a communications channel depends, in some fashion, on the symbols that preceded it, the communications channel is said to have memory.

Fifth, the intensities in the image are not independent. Real images, just like messages composed in real languages, contain redundancy [1].

As a result, the entropy of a measured image is less, possibly considerably less, than the entropy of the original source. The resulting image entropy is given by

$$H(X) = P_d\left[H_{Source}(X) - H_Y(X)\right] - P_{fa}H_{fa}(X), \tag{10}$$

where $P_d$ is the probability of detection, $P_{fa}$ is the probability that a false alarm will occur and all other terms are as previously defined.

By substituting (10) into (2), it can be seen that the mean information density of a real image may be expressed as

$$\overline{K} = \frac{P_d\left[H_{Source}(X) - H_Y(X)\right] - P_{fa}H_{fa}(X)}{GSD^2}, \tag{11}$$

where all terms are as previously defined.

### Comparison of Information Density and Image Quality Equations

The original General Image Quality Equation (*GIQE*) developed for visible imagery is given by [2]

$$NIIRS = 11.81 + 3.32\log_{10}\left(\frac{RER}{GSD}\right) - 1.48J - \frac{G}{SNR}, \tag{12}$$

where $NIIRS$ is the image quality rating assigned in accordance with the National Image Interpretability Rating Scale, $RER$ is the relative edge response, $J$ is the mean height overshoot caused by edge sharpening, $G$ is the noise gain resulting from edge sharpening and $SNR$ is the signal to noise ratio. The original *GIQE* used the symbol “$H$” for the height overshoot. Shannon also used the same symbol to represent the information entropy and the same symbol is used here in deference to Shannon’s symbology. The symbol “$J$” has been substituted here in the *GIQE* to avoid confusion with the Shannon Entropy.

This form of the *GIQE* was entirely empirically derived. It has been refined [3] and extended [4] to other imaging technologies, but the basic form has remained largely the same.

Studies have shown good, but not perfect, correlation between *NIIRS* predicted by the *GIQE* and trained analysts’ assessments (i.e., correlation coefficients greater than 0.9). Images used in these studies had been optimized for dynamic range and contrast, and had been processed to sharpen edges within the image.

The high correlation between predicted and assessed *NIIRS* values suggests that the terms included in the *GIQE* are important to the interpretability of an image, but it is not conclusive. The strongest functional dependencies are for *SNR* and *RER*, both of which are logarithmic. With respect to the underlying engineering parameters, the correlation is much lower. In fact, these same studies have shown more than an order of magnitude dispersion between the predicted and assessed *NIIRS* values.

Noting that *SNR* is just the peak signal (i.e., measured intensity) divided by the root of the variance of the signal and that $\log_2 x = 3.32\log_{10} x$. Equation 12 can be rewritten as

$$NIIRS = 11.81 + \log_2 RER - \log GSD - 1.48J - \frac{G\sigma}{S_{max}}, \qquad (13)$$

where $S_{max}$ is the peak signal, $\sigma$ is the standard deviation of the signal (i.e., the square root of the variance), and all other terms are as previously defined. The *RER* is actually just a measure of how quickly the intensity in the image can change. It is therefore a measure of the correlation of the signal.

If the contrast has been optimized for a human observer in a real image, all intensities will have nearly equal probability. A human typically has the ability to discern approximately 32 (i.e., $2^5$) shades of gray. If the image contains 32 shades of gray and the intensity of each pixel in the image is independent of all others, then each pixel will contain 5 bits of information. Likewise, false alarms may be expected to take on the same range of values. Each false alarm can be expected to subtract 5 bits of information from the total information content of the image.

Unfortunately, the intensity of each pixel in an image is not independent of the intensity of adjacent pixels. The *RER* is the average (in $x$ and $y$) slope of the response of one pixel to the next. It is measured using a step function input and is, therefore, a measure of the information redundancy introduced by the optical system. Each pixel contains only $RER^2$ new and independent information. Consequently, the information of each pixel or a contrast-enhanced image is given by

$$H(X) = -RER^2 \times \sum_{i=1}^{32} \frac{1}{32}\log_2\frac{1}{32} = RER^2 \times 5\,\text{bits}, \qquad (14)$$

where all terms are as previously defined. This does not account for the fact that the intensity and its mean are locally correlated. This is the phenomenon of texture. Consequently, real images will have less information per pixel that indicated by (14), which should be regarded as an upper bound on the information present in a single pixel.

If the noise is Gaussian, then the uncertainty is given by

$$H_Y(X) = \sigma \ln \sqrt{2\pi e}\,, \tag{15}$$

where all terms are as previously defined. By substituting (14) and (15) into (11), it can be shown that

$$\overline{K} = \frac{P_d\left[RER^2 \times 5 - \sigma \ln \sqrt{2\pi e}\right] - 5P_{fa}}{GSD^2}, \tag{16}$$

where all terms are as previously defined. By taking the logarithm to the base of 2 of both sides of (16), rearranging yields

$$\log_2 \overline{K} = \log_2 \left\{P_d\left[5 \times RER^2 - \sigma \ln \sqrt{2\pi e}\right] - 5P_{fa}\right\} - 2\log_2 GSD, \tag{17}$$

where all terms are as previously defined. The original *GIQE* and its subsequent revisions did not consider either the probability of detection or the probability of false alarm on the interpretability of an image. The similarity between the *GIQE* and the logarithm of the information density is obvious. Although, the functional dependence upon the standard deviation of the intensity is different.

## Conclusions

The similarity between the functional form of the *GIQE* that predicts the *NIIRS* value of an image and the logarithm of the information density suggests that both may be related to image interpretability. However, the information density includes more quantifiable image characteristics that do previous *GIQE*'s and include alternative functional forms for previously included parameters. Incorporation of these new characteristics and functional forms may improve the reliability of interpretability predictions.

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