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ECONOMIC DESIGN OF MULTISTAGE SYSTEMS FOR SCREENING INSPECTION BY ATTRIBUTES

A THESIS

Presented to

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by

Pakorn Adulbhan

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

The general purpose of this study is to develop methods for determination of economically-optimal design parameters for in-process screening inspection of multistage production systems, wherein items must be processed sequentially through a series of production operations and where there is a maximum number of defects allowed for a completed unit. Screening inspection is defined as the type of inspection by which the product is classified into categories by inspection of every item. Defects are assumed to be generated according to stationary Poisson distributions.

There is introduced the concept of tightening inspection specifications--that is, rejecting an in-process item which has number of defects less than the tolerated number. It is then shown how the incorporation of in-process inspection into a production line is related to a sequential decision problem.

Two production environments considered in the study are described. In the first case, a producer, who already owns a fixed stock of raw material, is manufacturing until he exhausts a fixed level of resource availability. In the second situation, a manufacturer is producing to satisfy a fixed production goal, so that any defective item must be replaced by reprocessing a substitute. Economic consequences associated with courses of action for disposition of inspected items for the two environments are discussed.

Economic factors involved in the design of in-process inspection system are presented. The system measure of effectiveness is selected to be the maximum expected gain per item. Gain is defined as the difference between revenue and cost related to a processing item in the system. Revenue is obtained from sale of finished product or scrap. Cost is considered to be the expenditure of monetary resources in processing an item through the system. It is classified into two categories: (a) inspection costs, and (b) production costs. Each group is subdivided into two classes: fixed and variable. It is reasoned that the magnitude of production costs at a given stage would be larger than that of inspection costs, because of the relative nature of production and inspection operations.

The analysis of a production system when producing from a fixed stock is then discussed. The first decision problem considered is the determination of inspection specifications associated with an inspection program. A dynamic programming model is formulated. A numerical example is solved for a three-stage production system. Dynamic programming methods can be used to determine the most economical specifical limits for a given inspection program. The computational procedure is not complex and is practical. In addition, savings in computing effort can be obtained in the case where there are runs of no-inspection stages in the inspection program. Graphical and numerical illustrations are given in such case.

The next class of decision problem presented is that of locating the economically optimal inspection points in a production line--that is, determination of the optimal inspection program. Using the dynamic programming model formulated and the expected value criterion as bases, a procedure is developed to solve this decision problem. A four-stage production system is used to demonstrate the procedure. The same numerical data for the first decision problem also are used to illustrate the procedure. The computational scheme is rather easily implemented. This class of decision problem is interrelated with that of finding specification limits. Using the number of inspection programs completely evaluated as the basis for comparison, the procedure developed would save computing effort up to half of the complete enumeration approach.

Then a production system when producing to a fixed quota is discussed under two variations. Variation 1 employs special processing wherein an item rejected, but not defective, is processed to completion under conditions highly controlled so that no more defects are introduced. This special treatment will result in increased production costs. A replacement for a defective item also uses this special processing. Variation 2 is the one in which a rejected item results in another item being started into regular processing. The decision problems of determining locations of inspection operations and specifications are considered. It is noted that, for a given inspection program, the two production environments result in different specification limits, even though the identical decision rules are used for final inspection. A similar argument holds for the optimal inspection program problem.

The third class of decision problem presented is the determination of the minimum cost inspection sequence to be followed at each inspection operation. The item may acquire multiple types of defects at all production operations of the system. A model of expected unit cost as a function of the inspection sequence is developed. A procedure for finding the minimum cost inspection sequence is given and illustrated with a numerical example. The procedure proves to be useful in reducing computing effort as compared with the complete enumeration. It is reasoned that this decision problem can be treated separately from the problems of determining locations of inspection operations and specifications at each stage.

As a result of this study, it is recommended that there be further study in considering process control for the multistage inspection system, in relaxing the assumption that no defects are removed by subsequent operations, and in developing minimum cost testing sequence where defect types are not of the same degree of severity.

CHAPTER I

INTRODUCTION

One of the most important factors to be considered in the control of a multistage production process is the quality of the resulting product. The conventional approach is to inspect completed product items to discover whether they conform to specified design characteristics. However, such an inspection policy used as the only means of quality control may result in unnecessary expenses. When the manufacturing item becomes defective at some early stage of the production process, the processing of the item through the remaining stages of the process would result in unnecessary expenses both for manufacturing the item in those stages and for inspecting the finished product. Such a defective item will not bring in revenue at the market price and thus, perhaps, should have been removed from the process at that previously mentioned "early stage".

It is therefore logical to reason that there may exist some in-process inspection policies which might prove to be better from the economic point of view, than that of "final inspection" only. Most research work has been concerned with inspection procedures applied at a single inspection station. It is only rather recently that attention has been given to the interrelationships which exist in multistage manufacturing systems.

Based upon these facts, it seems that there is a need for further study in which the inspection program of a multistage production process is determined through treatment as a system of interrelated operations.

Definition of Terms

Throughout this study, unless stated otherwise, the following definitions will be used:

Screening Inspection will mean the type of inspection by which the product is classified into categories by the inspection of every item. Occasionally, the term *Detail Inspection* will be used interchangeably.

Inspection by Attributes will be used to denote the type of inspection whereby the inspector observes not only whether or not the item is defective, but also the type and its corresponding number of defects in the item.

By *In-Process Inspection* is meant inspection carried out between production operations in the same organization.

The *Manufacturing Process* (and also *Manufacturing System, Production Process,* and *Production System*) will refer to a set of interrelated operations for the acquisition, production, and distribution of material. Handling, transportation and storage will be considered, for simplicity, as operations, as well as processes which are designed to change the properties of material. It is not impossible for the item to be damaged in handling, transportation, or to deteriorate in storage. Feasible points for inspection may exist before all production operations.

Scope and Limitations

The research reported herein considers the design of the attribute in-process inspection operations of the production system. Economic criteria are emphasized in considering given design alternatives.

The present study is confined to product control inspection wherein inspection is performed for the purpose of making decisions regarding a product item already in the process, whether or not the item should be allowed to the next operation stage in the system. This implies that no attention is given to process control inspection, even though it should be realized that information obtained from product control inspection may be of some value to the control of the production process.

It is assumed that all manufacturing operations affecting product quality are stable to the extent that defects are generated according to stationary Poisson distributions. These operations are assumed to be mutually independent, so that the number of defects acquired in one operation are independent of those acquired at any other operation. Furthermore, no defects are removed by subsequent operations.

It is assumed that inspection is carried out in a highly efficient manner such that the cumulative number of defects generated by preceding production operations are noted at all inspection points, and that the inspection process itself will neither produce nor remove defects.

The study reported herein is a conceptual analysis of the effect of decision making on the economics of in-process inspection. No attempt was made to investigate the manufacturing process of any particular organization.

Objectives

The general objective of this study is to develop methods for determining economically-optimal design parameters for in-process inspection of multistage production systems, wherein items must be processed sequentially through a series of production operations and where there is a maximum number of defects allowed for a completed unit.

The specific objective of the research is the development of an economic model and associated optimization methods to solve the following decision problems:

1. Determination of where in-process inspection should take place.

2. Determination of what inspection specifications should be employed at the stages where inspection is to take place.

3. Determination of the sequence for inspection of defect types at each stage.

Two production environments are considered. In the first case, a producer is manufacturing until he exhausts a fixed level of resource availability, so that a rejected item results in a loss of revenue. In the second situation, a manufacturer is producing to satisfy a fixed production goal, so that any defective item must be replaced by reprocessing a substitute.

CHAPTER II

A REVIEW OF MULTISTAGE INSPECTION SYSTEMS

Introduction

In this chapter, reported studies concerning the interrelationships of inspection policy at the stages of a production-line system are described briefly. They are presented in order of their time of publication. Comments on these studies conclude the chapter.

Review of Multistage Inspection System Studies

Beightler (1,2) was the first to consider a multistage inspection system of a production line as a sequential decision process. A possible inspection point followed by an actual production operation comprises each stage in the process. Items being manufactured are assumed to arrive at the line in a lot of fixed size. A crucial assumption made is that whenever defective items are discovered, they are immediately replaced with non-defective ones. The input to and output from any stage is thus that fixed size lot. At any stage, the inspection process itself is allowed to produce defectives, as well as to remove those found in the sample. Such effects of inspection process upon the lot quality are expressed by introducing transition matrices at each inspection point of the system. Similar assumption is made for production operation at each stage of the system and is represented by another set of transition matrices. Cost is associated with a transition in number of defectives in the lot between stages of the process. This cost is defined to include the expenses incurred in sampling and those of replacing defective items encountered in the processing operation. The criterion function is then given as the minimization of the sum of expected cost at all stages. Based on these assumptions, a dynamic programming model is formulated. The input state variable is the probability of the lot having a certain number of defective items. The decision variable vector depends upon the sampling plan. An illustrative example is given wherein single sampling plans are used at all inspection points of the system. The decision variables are then sample size and acceptance number.

Lindsay and Bishop (7) considered a problem of determining the inspection levels and locations of inspection points in a single line, multistage production process, with material moving through at some constant rate. The measure of system effectiveness is to minimize the total sum of inspection costs and scrap costs per unit time. The scrap cost at any stage of the system is defined as the cumulative manufacturing costs at all prior stages, plus any costs associated with the disposal of defective items at that stage. The constraint imposed is the requirement of a specified average outgoing quality. The system parameters are the fraction defectives at all stages of the production process, and that of entering raw material. It is assumed that a defective item can be discovered only by inspection of that item at a subsequent stage in the process. The authors approach the problem with dynamic programming method. Each stage consists of a possible inspection point, followed by a manufacturing operation. The decision variables are the inspection levels at the various stages. They first show analytically that inspection between any two production operations should either be applied to all items or be nonexistent. In other words, the total cost of inspection and scrap for a multistage process will, in general, be at a minimum for some allocation of screening inspection effort, provided that the inspection level at each stage is either 0 or 1. Therefore, there would be 2^N possible screening inspection policies for an N-stage production system. It is then shown by means of an example that dynamic programming methods could be used to find the optimal inspection program. The authors also considered the case where the requirement for a specified fraction defective is removed, but instead there is explicit consideration of the costs incurred from those defective items which reach customers. In other words, the measure of effectiveness in this case is the minimization of the total of the costs of finding and removing defective items from the line, and of the costs associated with those defective items which are not removed from the process. It is concluded that the optimal inspection level at all stages will again be at an extreme point, either 0 or 1. The dynamic programming computation procedure is also claimed to be applicable to find the optimal screening inspection program.

Johnson (5,6) structures the problem of finding an optimal in-process inspection plan of an ordered production line, as a multistage decision process with an application-conscious attitude rather than a problem-solving one. His measure of effectiveness is the maximization of net return. The production goal concept of the organization is introduced and is incorporated into the modelling process. There are two types; namely, production to a fixed quota, and production from a fixed stock. Items being manufactured are assumed to arrive at the system with some constant rate. Each stage consists of a possible inspection point, and a production operation, in that order. Based upon Lindsay and Bishop's finding that an optimal inspection program has the property that at every stage either all items are inspected or otherwise no inspection is done at all, Johnson takes a feasible optimal inspection policy where all items are inspected at all stages. It is then reasoned that under most real situations in industry, there is a maximum number of defects allowed for a completed unit. That allowable number is, most of the time, greater than one. Furthermore, he proposes the concept of tightening the inspection specifications--that is, rejecting an in-process item which has a number of defects less than the tolerated number--at the inspection point of earlier stages. Economic consequences associated with the available courses of action for disposition of inspected items are explicitly discussed. Based upon these, a dynamic programming model is formulated for each production objective, along with an illustrated example. It is concluded that artificially severe specification limits should be considered, and that dynamic programming methods can be utilized to determine the most economic limits of number of defects in an item. Suggestions are made in case there is more than one type of defect.

White (11) postulates a multistation inspection model for an ordered production process in a very similar way to that of Beightler (1,2). However, he makes his cost structure more explicit by breaking it up into two components--inspection cost and cost of replacement. At the final stage, there is, in addition to the two components mentioned, a cost associated with each defective item that is not discovered. White assumes that manufacturing cost at all stages is zero, and that the fraction defectives of the production operations are constant. The model of finding an optimal inspection plan is then formulated by the functional equation approach of dynamic programming. White shows that an optimal inspection level at any stage exists at the extreme points. In other words, the optimal inspection plan has the property that at every stage of the manufacturing process either the whole lot (or batch) is inspected or otherwise no inspection should be performed at all. An illustrative example is then given to show how the formulated dynamic programming model can be used to determine an optimal in-process inspection program.

Pruzan and Jackson (9) are the first group outside the United States to study the allocation of in-process inspection effort for a sequential production system. They used Lindsay and Bishop's finding, that the optimal inspection level at all stages is at the extreme point (i.e., detail inspection or none at all), as the basis to develop two dynamic programming models. Their decision problem is thus confined to that of where to make screening inspections. In both models, a production operation, followed by a possible inspection point, comprises each dynamic programming stage. It is assumed that a defect type incurred at any operation is different from types incurred at any other operation and its probability of occurrence is constant over time. Any item possessing any type of defect once inspected will be discarded and has no value. The measure of system effectiveness is to minimize the total sum of inspection costs, cost of unnecessary machining of defective items, and costs of permitting defective items to reach the customers. The raw material moves into the process at some constant rate. Inspection costs, both fixed and variable, at any possible inspection point are structured on the important assumption that they depend on the point at which the most recent actual screening inspection occurred. The farther the inspection point under consideration is from the last inspection point, the higher the unit inspection costs are. In the first dynamic programming model, the number of defective items scrapped at any given inspection point is not recorded and, thus is not available at later points in the process. However, the information about where the most recent inspection took place is available. An example is illustrated in connection with this model. In the second model, both the point of previous inspection and the number of discarded items at that point are known. No illustration is given for this model.

Lindsay (8) extends the second model of his and Bishop's article (i.e., the one in which there is a cost associated with any defective item reaching the customer) to consider the situation wherein there is more than one type of defect generated at all production operations of the system. Again, dynamic programming is used as a method to determine an optimal inspection program. A partial illustration of the model is performed by means of a simple example.

In addition to those already discussed, other studies relating to in-process inspection plans for a multistage manufacturing process are made by Schmidt and Sorber (10), and Heermans (3). However, examination of those articles reveals that they do not seem to be sufficiently general to account for the interdependencies between inspection operations of the production system under study.

Comments and Summary

If the arriving nature of material at the production system is used as a basis of classification, those studies described would fall into two categories; namely, lot (Beightler, White), and continuous at some constant rate per unit time (Lindsay, Johnson, Pruzan and Jackson). What both groups have in common is their finding about the property of the optimal inspection plan of the system. This seems logical; for the two classifications are not significantly different if the lots arrive at the constant interval, which is not impossible under general conditions.

When comparison is based on the measure of inspection program effectiveness, it is apparent that all but Johnson employ the minimum cost as their criterion function. In this regard, most of cost structures do not seem to be realistic.

For the case in which defective items found by in-process inspection are not replaced, all but Johnson assume that such rejected products have no value. Such assumption would seem to be doubted in practice. In addition, for the situation where a defective item, when found, must be immediately replaced with goods ones to keep the lot size fixed, there is no indication as to how this can be carried out in reality. It is argued here that there is no assurance that new processing items will not become defective before reaching that inspection point again.

Though Pruzan and Jackson's work seems to be similar to that of Lindsay and Bishop in the nature of the measure of system effectiveness, its inspection cost structure of the former deviates significantly from the latter. What seems to be questionable is as follows: if the manufactured item having even one type of defect is discarded when inspected, why would it be necessary to inspect for all various types of defects incurred between the previous inspection point and the one under consideration? Such inspection cost structure would seem to be debatable. Furthermore, it might not be feasible to obtain such unit cost data in industry.

In general, it seems reasonable to conclude that most studies do not explicitly discuss the economic consequences associated with the choice of a design alternative of the inspection system.

It may be apparent at this point that the research reported herein is the extension of Johnson's work in the sense that the assumption of having to make screening (100 per cent) inspection at all stages is removed, and that more than one type of defect is considered.

CHAPTER III

ECONOMIC IMPLICATIONS IN THE DESIGN OF MULTISTAGE SCREENING INSPECTION SYSTEM

Introduction

In this chapter, an attempt will first be made to illustrate how the incorporation of screening inspection into a production line is related to sequential decision process. Alternatives for the design of the multistage inspection system in this study, will then be discussed. That will, in turn, be followed by the discussion of economic factors affected by the choice of a design alternative.

Multistage Screening Inspection System Design-- A Sequential Decision Process Problem

In a typical production line where items are processed through an ordered series of operations, there are possibilities for defects[1] to be generated in the manufacturing units at any of the operations. Under general conditions, a final product is considered to have acceptable quality for use, if the total number of defects it contains does not exceed the maximum allowable number. The final inspection operation is naturally concerned with determining whether the completed unit has defects more than that maximum number to be tolerated.

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[1]According to Johnson (5, p. 77), the number of defects created in a unit by a production operation is universally assumed to be a stationary Poisson random variable.

In some situations, it may be desirable to have acceptance inspection operations performed between production operations for the purpose of improving the value of product flow. It has been shown[2] that prior to any given production operation, if inspection is to be carried out, it has to be done to all items, or no inspection should be performed at all. Furthermore, it has become necessary to determine acceptance criteria for in-process items at stages where screening inspection are decided to be made. Johnson [5, pp. 141-199] has demonstrated that: if in-process inspections are conducted at all stages, it is then economically desirable to consider rejecting a manufacturing unit having defects less than the maximum number to be tolerated. As it will be seen, such concept of tightening specifications may still be desirable even if inspections are to take place before only some production operations. Thus, at stage k where in-process inspection is decided to be made, the specification at inspection operation k prior to production operation k would be of the form:

> Reject the incoming unit if $T_{k-1} > D_k$, where T_{k-1} is the cumulative number of defects acquired through production operation k - 1 and D_k is a non-negative integer satisfying $D_k \leq L$, which is the maximum allowable number of defects for a completed unit. Otherwise, accept the item and proceed it to production operation k for manufacturing.

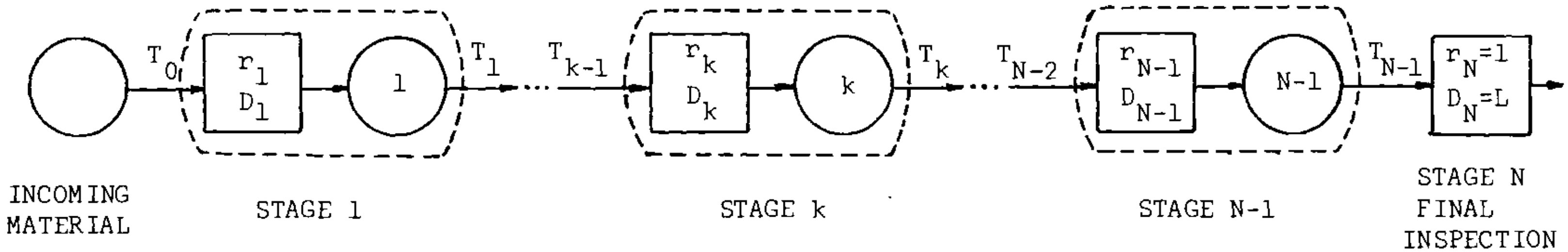
---

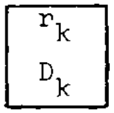
[2]See Lindsay and Bishop [7] and White [11].

It may seem apparent that the incorporation of in-process inspection operations would structure the system design problem as a sequential decision process. Each stage consists of a possible screening inspection point followed by a production operation.

Figure 1 may be considered as a representation of an N-stage inspection system composed of the following: N - 1 possible inspection points; N - 1 production operations; and a final product inspection operation. The circles represent manufacturing activities, whereas the squares denote possible inspection operations. At each stage the system designer has to decide whether to allocate screening inspection effort there or not. Should he decide to do so, a specification limit D_k must be determined. Thus, if r_k is a set of two numbers: 0 and 1; then $r_k = 1$ means that screening inspection is to be located at stage k and D_k is to be set. On the other hand, if no inspection is to be conducted at this stage--that is, $r_k = 0$--then D_k is undefined.

Input state to any stage k is T_{k-1}, the cumulative number of defects from raw material quality through production operation k - 1, inclusive. Decisions to choose r_k and D_k would affect: T_k, the output of stage k and therefore the input to the next stage k + 1; and the return from the item at that stage. By similar reasoning, the input T_{k-1} is influenced by decisions made at all preceding stages. Such interdependency between stages of inspection system indicates the important characteristic of the sequential decision process. The problem is to choose $\{r_k, D_k\}$ optimally such that the expected return per item is maximized.

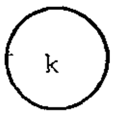




Potential Screening Inspection Operation

$r_k = \begin{cases} 0, & \text{indicates no inspection at stage } k \\ 1, & \text{indicates inspection at stage } k \end{cases}$

D_k = Specification limit at stage k



Production Operation at Stage k

T_{k-1} = Cumulative number of defects through production operation k-1.

Figure 1. Multistage Inspection System

Inspection System Design Alternatives

Characteristics of an optimal inspection system will depend upon the economic consequences associated with the available courses of action for disposition of inspected items. They, in turn, are affected by the environmental circumstances under which the system operates. In this study, two situations are considered: (a) production from a fixed stock and (b) production to a fixed quota.

Production from a Fixed Stock

In this case, a producer already owns a fixed stock of raw material. His primary objective is to continue production of a certain product until he exhausts all raw material available. A unit completing $N - 1$ production operations with defects less than the maximum number to be tolerated L will bring in V monetary units of net revenue--that is, gross revenue less packaging, shipping and selling costs. Any completed unit having more than L defects also can be sold and earns a net revenue of V_N monetary units. It is logical to require that V is greater than V_N.

At any given stage k, $(k < N)$, where inspection operation is carried out, the specification like the one mentioned previously is employed. Any item having defects more than D_k is to be removed from the process and is sold at a reduced price for a net revenue of V_k monetary units. It is required that V_k is less than V.

Production to a Fixed Quota-Variation I

In this situation, a manufacturer has a goal of producing a certain number of certain product. A final product with defects no more than the maximum allowable number, L, is classified as having acceptable quality, and will bring in a net revenue of V monetary units. On the other hand, a completed item with more than L defects is classified as defective and will earn a net revenue of V_N units. It is required that V_N is less than V. Again, the concept of tightening the specification may be applied at any stage where screening inspection is to be conducted.

A rejected, but not defective item--that is, T_{k-1} defects is greater than D_k but less than L--will be set aside and carefully processed under conditions highly controlled[3] so that no more defects are introduced. This special treatment will presumably result in an increase in production expenditure. This unit cost will be hereafter referred to as the cost of rejection of a non-defective item, and be symbolized as S_k.

For a defective item found at any stage k, including final inspection, it must be replaced by processing an additional item for the reason that a fixed production quota goal has already been set and must be attained. This new item for making up production shortages will not be started into regular manufacturing. It will be set aside and carefully manufactured under the above-mentioned controlled conditions[3] so that no defects are created at any production operation. This results in increased production expenditures for acquiring the new unit, and for the use of special processing from the beginning.

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[3]This type of special treatment is motivated by suggestion of certain apparel manufacturer [6, p. 4].

Production to a Fixed Quota--Variation 2

In this variation, a manufacturing item that fails to meet the specification limit at any stage will be removed from the line. A replacement unit is then started into regular processing.

Economic Factors in the Design of In-process Inspection System

System Effectiveness

Like any other design problem, the multistage inspection system must be analyzed with respect to some measure of effectiveness. In this study, the maximization of expected gain per item has been selected as the appropriate one. It will be expressed in monetary terms. This is motivated by what Johnson [5, p. 124] concludes in his study that monetary measures are the best available measure of the utility of a decision.

Gain, as used here, will mean the difference between revenue and cost related to a processing item in the system.

The problem of the system analyst is then to choose optimal value of design parameters r_k and D_k to maximize the expected gain per item.

Revenue

Revenue, as used in multistage inspection design, may probably be defined as the acquisition of monetary resources from sale of product or scrap.

Production from a Fixed Stock. In this environmental situation, revenue may arise from:

(1) Sale of acceptable final product;

(2) Sale of defective final product; and

(3) Sale of rejected item having defects more than specification limit.

Production to a Fixed Quota. In the first variation, revenue may arise from:

(1) Sale of acceptable final product;

(2) Sale of defective item found at any stage.

In the second variation, revenue may be obtained from:

(1) Sale of acceptable final product.

(2) Sale of rejected item having defects more than specification limit.

Cost

Cost may probably be defined as the expenditure of monetary resources in processing an item through the multistage inspection system. It may be classified into two categories: (a) inspection cost, and (b) production cost. Each category may be subdivided into two classes: fixed, and variable. Fixed costs are those incurred that do not vary with volume of output, while those of variable costs do.

Inspection Costs

Those costs result from carrying out in-process screening inspection activities.

Inspection Fixed Costs. One of its components is the cost of setting up inspection equipment at any given stage of the system. The setup cost generally is the same regardless of the number of operations between consecutive setups.

To incorporate in-process inspection activities with a production line will normally require some investment expenditure for inspection equipment and facilities, such as testing instruments, gauges, test racks, inspection exhibits.

Screening inspection activities would naturally affect the flow of materials through the production line, and, as a result, would inevitably create the situation of having in-process inventory at those inspection stations. It would, therefore, generally become necessary to allocate some expenditure in building storage space for such inventory and in carrying such inventory.

Another component of inspection fixed costs is concerned with operating costs, such as the following: plant services in terms of power, fuel, equipment calibration and maintenance.

It may be desirable to transform those fixed inspection costs into monetary units per producing item, for the convenience in the mathematical formulation of an economic model. To compute an average setup cost per item, the system designer, perhaps through his technical knowledge of that particular inspection operation could estimate the average number of items inspected between two consecutive setups and could use this production rate as a basis for the determination of setup cost per item produced.

So far as the investment cost of inspection equipment is concerned, it may be desirable to charge the investment back in terms of the maximum rate of return that could be earned by investment in other activities, which had to be postponed by the manufacturer's course of action to invest in the in-process inspection activity. That rate of return estimate along with the economy life of the equipment would enable the system analyst to determine the amount to be charged annually by the capital recovery method. With the information about raw material arrival rate, the equipment cost per item could be approximately estimated.

In general, the system analyst with good understanding in the nature of both inspection and production operations would make a reasonably close estimation of the fixed costs on the per item basis.

Inspection Variable Costs

As it has been mentioned above, the installation of screening inspection stations would create the existence of in-process inventory waiting to be inspected. This "tie-down" situation would necessitate the increase in working capital for such inventory, whose time in the system is prolonged by the system analyst's decision to inspect.

Labor is another significant component of inspection variable costs. This cost could perhaps be based on the amount of labor time used for testing the item, analyzing inspection results, reporting the result. Based upon the information concerning hourly wage, and arrival and service rate of material at the inspection station, the labor cost per item could be approximately determined.

Production Costs

These are expenditures associated with the processing the item through production operations in the system.

Production Fixed Costs. Their components are similar to those of inspection costs. However, it would seem logical to assume that the monetary magnitude of the former would be larger than that of the latter, because of the relative nature of production and inspection operations. For instance, investment cost for a manufacturing machinery at a given stage typically would be more expensive than that for inspection equipment at that same stage.

Production Variable Costs. The main component will be labor cost. It would, in general, be higher than inspection labor in a given stage because of technical complexity knowledge required. Another important component that must always be included is the costs of material.

It should be noted that two possible situations might arise in connection with production costs. In the event that the production line is not yet in existence and the system analyst is assisting the manufacturer to design multistage inspection system, he would be in the position to select machine capacities such that they are compatible with production flow rates affected by the existence of prior inspection operations, thereby avoiding idle machine and operator time. On the other hand, if the production line with final inspection is already in operation, and the analyst is helping the producer to put in in-process inspection activities, he would face a rather complex problem. There would highly likely be some unused capacities of the machines, and corresponding unproductive labor time. There would be some chance that he could reduce the number of machines to be used in the line and reassign them and their operators to some other section in the same organization for productive use. If this is the case, the expenditures averted by utilization of this freed capacity should be used to help justify inspection operations.

Summary

It has been shown that a multistage inspection system design may be considered as a sequential decision process problem. Two environmental circumstances under which the system is to be established--that is: (a) a producer producing until he exhausts a fixed stock of raw material, and (b) a producer producing to satisfy a fixed production quota--were discussed, along with their economic consequences associated with the disposition of inspected item. Economic factors affected by the design alternative were also identified.

CHAPTER IV

ANALYSIS OF SEQUENTIAL MANUFACTURING SYSTEM WHEN PRODUCING FROM A FIXED STOCK

Introduction

In this chapter an attempt will be made to analyze the potential of economic advantage of in-process inspection activities. The environmental situation under consideration is that the manufacturer, who owns a fixed stock of raw material, is producing a certain type of product until he exhausts all available. The analysis will make use of the property of the optimal inspection policy[1] that at every stage either all items are inspected or otherwise no inspection is performed at all.

The manufacturing system under study is that consisting of N - 1 sequential production operations and a final inspection point. Raw material items are assumed to be fed into the system at constant rate per unit time, and move through the production line continuously. Item may be inspected prior to any production operation. At all inspection operations, inspection by attribute only is performed.

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[1]This has already been discussed in Chapter II. It is based upon the work of Lindsay and Bishop (7), and White (11).

Determination of Specification Parameter for an Inspection Program

Based upon the optimal in-process inspection property, there would be 2^{N-1} possibilities for allocating screening inspection activity in the system. Each possible allocation may be called an inspection program.[2] The decision problem concerned then is to determine specification limits associated with each inspection program.

Symbols for Modelling

Unless otherwise stated, symbolic notations to be used are as follows:

x_k = number of defects introduced in production operation of stage k, $(k = 1,2,3,\cdots,N-1)$. x_k is generated according to stationary Poisson Process.

x_o = number of defects in an item before entering the manufacturing system. It is also a Poisson random variable.

λ_k = parameter of the distribution of x_k, $(k = 0,1,2,\cdots,N-1)$.

T_k = cumulative number of defects in an item through production operation of stage k, $(k = 1,2,\cdots, N-1)$.

Λ_k = parameter of the distribution of T_k, $(k = 0,1,2,\cdots,N-1)$.

$p(x_k;\lambda_k)$ = probability function of x_k, whose parameter is λ_k, $(k = 0,1,2,\cdots,N-1)$.

$p(T_k;\Lambda_k)$ = probability function of T_k, whose parameter is Λ_k, $(k = 0,1,\cdots,N-1)$.

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[2] This term will be symbolically defined in the next subsection.

$P_{x_k}(M)$ = cumulative distribution function of random variable x_k, $(k = 0,1,\cdots,N-1)$, evaluated at the positive integer M.

L = maximum number of defects to be tolerated in an item completing all production operations of the system.

D_k = inspection specification at stage k, $(k = 1,2,\cdots,N-1)$. It is the maximum number of defects permitted on an item prior to production operation at stage k. For the final inspection stage N, D_N is equal to L.

C_k = cost associated with manufacturing a unit at production operation of stage k, $(k = 1,2,\cdots, N-1)$.

C_o = cost of an item before entering the first stage of the system (acquisition cost).

a_k = cost associated with inspecting an item at stage k, $(k = 1,2,\cdots,N-1,N)$.

r_k = inspection level at stage k, $(k = 1,2,\cdots,N)$.

r_k = $\{0,1\}$ for $k = 1,2,\cdots,N-1$. r_k assumes the value 0, if no inspection occurs at the potential inspection point of stage k. It is equal to 1 if inspection does occur.

r_N = 1 for the final inspection stage N.

V = Net revenue of a completed item having no more than L defects.

V_k = Net revenue associated with an item removed from the process after being inspected at stage k, $(k = 1,2,3,\cdots,N)$, for having number of defects more than D_k.

R = inspection program of the production system. This is an N component row vector $(r_1,r_2,r_3,\cdots,r_N)$ in which r_k $(k = 1,2,\cdots,N)$ are defined as mentioned above.

$f_{N-k+1;(r_k,r_{k+1},\cdots,r_N)}(T_{k-1})$ = maximum expected gain per item from the last N-k+1 stages, using partial inspection program $(r_k,r_{k+1},r_{k+2},\cdots,r_{N-1},r_N)$; when the unit possesses T_{k-1} defects after the first k-1 stages.

$W_k(T_{k-1};r_k,D_k)$ = return at stage k when the unit has T_{k-1} defects through production operation of stage k-1, and the inspection level r_k, and specification D_k are used during inspection operation of stage k.

$H(x_k)$ = real function of random variable x_k.

$E_{x_k}[H(x_k)]$ = expected value of $H(x_k)$ with respect to the distribution of random variable x_k.

Model Formulation of the Decision Process

Based upon the design alternative already discussed in Chapter III, the symbolic representation leads to the recurrent relation:

$$f_{N-k+1;(r_k,r_{k+1},\cdots,r_N)}(T_{k-1}) = \underset{D_k}{\text{Max}}\ E_{x_k}[W_k(T_{k-1};r_k,D_k) + f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)] \qquad (4\text{-}1)$$

for $k = 1,2,\cdots,N-1$.

and

$$f_{1;(1)}(T_{N-1}) = \max_{D_N} \begin{cases} V - a_N, \text{ if } T_{N-1} \leq D_N = L \\ \\ V_N - a_N, \text{ if } T_{N-1} > D_N = L \end{cases} \tag{4-2}$$

With the transformation that:

$$T_k = T_{k-1} + x_k \qquad \text{for } k = 1,2,\cdots,N-1 \tag{4-3}$$

and

$$T_o = x_o \tag{4-4}$$

The immediate return at stage k $(k = 1,2,\cdots,N-1)$ is given as:

$$W_k(T_{k-1};r_k,D_k) = \begin{cases} [-(a_k+C_k)(r_k)] + [-C_k(1-r_k)], \text{ if } T_{k-1} \leq D_k \\ \\ (V_k-a_k)(r_k) - C_k(1-r_k) - \\ \quad (f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k))(r_k); \\ \hfill \text{if } T_{k-1} > D_k \end{cases} \tag{4-5}$$

Substituting (4-5) into (4-1) would give:

$$f_{N-k+1;(r_k,r_{k+1},\cdots,r_N)}(T_{k-1}) = \underset{D_k}{\text{Max}} \begin{cases} E_{x_k}[(-(a_k+c_k)(r_k) + (-c_k(1-r_k)) + f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k)], \text{ if } T_{k-1} \le D_k \\ E_{x_k}[(V_k-a_k)(r_k) - c_k(1-r_k) - (f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k))(r_k) \\ \quad + f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)], \text{ if } T_{k-1} > D_k \end{cases} \tag{4-6}$$

$$f_{N-k+1;(r_k,r_{k+1},\cdots,r_N)}(T_{k-1}) = \underset{D_k}{\text{Max}} \begin{cases} E_{x_k}[-(a_k+c_k)(r_k) - c_k(1-r_k) + (f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k))(r_k) \\ \quad + (f_{N-k,(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k))(1-r_k)], \text{ if } t_{k-1} \le D_k \\ E_{x_k}[(V_k-a_k)(r_k) - c_k(1-r_k) + (f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k))(1-r_k)], \text{ if } T_{k-1} > D_k \end{cases} \tag{4-7}$$

Terms common to r_k and $(1-r_k)$ are then collected.

$$f_{N-k+1;(r_k,r_{k+1},\cdots,r_N)}(T_{k-1}) = \underset{D_k}{\text{Max}} \begin{cases} E_{x_k}[\{-(a_k+c_k) + f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)\}(r_k) \\ \quad + \{-c_k + f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)\}\,(1-r_k)], \quad \text{if } T_{k-1} \le D_k \\ E_{x_k}[(V_k-a_k)(r_k) + \{-c_k + f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)\}\,(1-r_k)], \text{ if } T_{k-1} > D_k \end{cases} \tag{4-8}$$

Applying the algebra of expectations to (4-7) gives

$$f_{N-k+1;(r_k,r_{k+1},\cdots,r_N)}(T_{k-1}) = \underset{D_k}{\text{Max}} \begin{cases} (r_k)[E_{x_k}\{-(a_k+c_k)\} + E_{x_k}\{f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)\}] \\ \quad + (1-r_k)[E_{x_k}\{-c_k\} + E_{x_k}\{f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)\}], \text{ if } T_{k-1} \le D_k \\ (V_k-a_k)(r_k) + (1-r_k)[E_{x_k}\{-c_k\} + E_{x_k}\{f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)\}], \text{if } T_{k-1} > D_k \end{cases} \quad (4\text{-}9)$$

It follows that the recursive relation can be expressed as:

$$f_{N-k+1;(r_k,r_{k+1},\cdots,r_N)}(T_{k-1}) = \underset{D_k}{\text{Max}} \begin{cases} [-(a_k+c_k) + E_{x_k}[f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)]](r_k) \\ \quad + [-c_k + E_{x_k}[f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)]](1-r_k), \text{ if } T_{k-1} \le D_k \\ (V_k-a_k)(r_k) + [-c_k + E_{x_k}[f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)]](1-r_k), \text{ if } T_{k-1} > D_k \\ \qquad \text{for } k = 1,2,\cdots,N-1, \end{cases} \quad (4\text{-}10)$$

with

$$f_{1;(1)}(T_{N-1}) = \underset{D_N=L}{\text{Max}} \begin{cases} V - a_N, \text{ if } T_{N-1} \le L \\ V_N - a_N, \text{ if } T_{N-1} > L \end{cases} \quad (4\text{-}11)$$

The solution to this decision problem by dynamic programming method involves the application of (4-10) successively. That is, starting with the one-stage process (consisting of only stage N), the computations are performed backward, proceeding recursively to a two-stage, then on up to finally the N-stage problem. Thus, one can find a set of values $D_k (k = 1,2,\cdots,N-1)$ associated with an inspection program $R = (r_1,r_2,\cdots,r_k,\cdots,r_N)$, which maximize the above-mentioned (4-10).

An Illustrative Example

Consider a three-stage production system. The assumed data are given as follows:

$\lambda_0 = 1$

$\lambda_1 = 3$ $c_1 = 10$ $a_1 = 2$ $V_1 = 45$ $L = 6$

$\lambda_2 = 2$ $c_2 = 20$ $a_2 = 5$ $V_2 = 55$ $V = 100$

$a_3 = 15$ $V_3 = 70$

Based upon the given data, it is obvious that,

$\Lambda_0 = 1$ $D_3 = L = 6$

$\Lambda_1 = 4$

$\Lambda_2 = 6$

For this problem, there would be $2^{3-1} = 4$ possible inspection programs. Their analyses are in order.

<u>Case I</u>. This is the situation in which R is given to be (1,0,1).

For the one-stage process, k = 3,

$$f_{1;(1)}(T_2) = \underset{D_3=6}{\text{Max}} \begin{cases} V - a_3, & \text{if } T_2 \leq 6 \\ V_3 - a_3, & \text{if } T_2 > 6 \end{cases}$$

$$= \underset{D_3=6}{\text{Max}} \begin{cases} 85, & \text{if } T_2 \leq 6 \\ 55, & \text{if } T_2 > 6 \end{cases}$$

For the two-stage process, k = 2,

$$f_{2;(0,1)}(T_1) = -c_2 + E_{x_2}[f_{1;(1)}(T_1+x_2)]$$

$$= -20 + \sum_{x_2} f_{1;(1)}(T_1+x_2) \cdot p(x_2;2)$$

Values of this function $f_{2;(0,1)}(T_1)$ are given in Table 1.

Table 1. Values of the Function $f_{2;(0,1)}(T_1)$

T_1	$f_{2;(0,1)}(T_1) = -20 + \sum_{x_2} f_{1;(1)}(T_1+x_2) \cdot p(x_2;2)$
0	64.85
1	64.49
2	63.41
3	60.71
4	55.31
5	47.18
6	39.05
7	35.00
8	35.00
9	34.95
10	34.73
11	34.07
12	32.09

For the three-stage process, k = 1,

$$f_{3;(1,0,1)}(T_0) = \underset{D_1}{\text{Max}} \begin{cases} -(a_1+c_1) + E_{x_1}[f_{2;(0,1)}(T_0+x_1)], & \text{if } T_0 \le D_1 \\ V_1 - a_1 & \text{, if } T_0 > D_1 \end{cases}$$

$$f_{3;(1,0,1)}(T_0) = \underset{D_1}{\text{Max}} \begin{cases} -(2+10) + E_{x_1}[f_{2;(0,1)}(T_0+x_1)], & \text{if } T_0 \le D_1 \\ 45-2 & \text{, if } T_0 > D_1 \end{cases}$$

$$f_{3;(1,0,1)}(T_0) = \underset{D_1}{\text{Max}} \begin{cases} -12 + E_{x_1}[f_{2;(0,1)}(T_0+x_1)], & \text{if } T_0 \le D_1 \\ 43 & \text{, if } T_0 > D_1 \end{cases}$$

$$= \underset{D_1}{\text{Max}} \begin{cases} -12 + \sum_{x_1} f_{2;(0,1)}(T_0+x_1) \cdot p(x_1;3), & \text{if } T_0 \le D_1 \\ 43 & \text{, if } T_0 > D_1 \end{cases}$$

Values of the function $f_{3;(1,0,1)}(T_0)$ are shown in Table 2. D_1 is found to be 0.

Table 2. Values of the Function $f_{3;(1,0,1)}(T_0)$

T_0	Accept $-12 + \sum_{x_1} f_{2;(0,1)}(T_0+x_1) \cdot p(x_1;3)$	Reject 43	$f_{3;(1,0,1)}(T_0)$
0	45.86	43	45.86
1	41.47	43	43
2	36.20	43	43
3	30.89	43	43
4	26.54	43	43
5	23.65	43	43
6	21.72	43	43

Table 3 summarizes the expected gain per item associated with the inspection program.

Table 3. Maximum Expected Gain for the Inspection Program R = (1,0,1), with $D_1 = 0$, $D_3 = 6$

FIRST STAGE		SECOND STAGE		THIRD STAGE	
T_0	$f_{3;(1,0,1)}(T_0)$	T_1	$f_{2;(0,1)}(T_1)$	T_2	$f_{1;(1)}(T_2)$
0	45.86	0	64.85	0	85
1	43	1	64.49	1	85
2	43	2	63.41	2	85
3	43	3	60.71	3	85
4	43	4	55.31	4	85
5	43	5	47.18	5	85
6	43	6	39.05	6	85
		7	35.00	7	55
		8	35.00	8	55
		9	34.95	9	55
		10	34.73	10	55
		11	34.07	11	55
		12	32.09	12	55
				13	55
				14	55
				15	55
				16	55

Case 2. Inspection policy R is supposed to be (0,1,1).

For k = 3

$$f_{1;(1)}(T_2) = \underset{D_3=6}{\text{Max}} \begin{cases} \bar{V} - a_3 & \text{, if } T_2 \le 6 \\ V_3 - a_3 & \text{, if } T_2 > 6 \end{cases}$$

$$= \underset{D_3=6}{\text{Max}} \begin{cases} 85 & \text{, if } T_2 \le 6 \\ 55 & \text{, if } T_2 > 6 \end{cases}$$

For k = 2

$$f_{2;(1,1)}(T_1) = \underset{D_2}{\text{Max}} \begin{cases} -(a_2+c_2) + E_{x_2}[f_{1;(1)}(T_1+x_2)], & \text{if } T_1 \le D_2 \\ V_2 - c_2 & \text{, if } T_1 > D_2 \end{cases}$$

$$= \underset{D_2}{\text{Max}} \begin{cases} -(5+20) + \sum_{x_2} f_{1;(1)}(T_1+x_2) \cdot p(x_2;2), & \text{if } T_1 \le D_2 \\ 55 - 5 & \text{, if } T_1 > D_2 \end{cases}$$

$$= \underset{D_2}{\text{Max}} \begin{cases} -25 + \sum_{x_2} f_{1;(1)}(T_1+x_2) \cdot p(x_2;2), & \text{if } T_1 \le D_2 \\ 50 & \text{, if } T_1 > D_2 \end{cases}$$

The value of $f_{2;(1,1)}(T_1)$ for each possible value of T_1, is given in Table 4. D_2 is found to be equal to 4

Table 4. Values of the Function $f_{2;(1,1)}(T_1)$

T_1	Accept $-25 + \sum_{x_2} f_{1;(1)}(T_1+x_2) \cdot p(x_2;2)$	Reject 50	$f_{2;(1,1)}(T_1)$
0	59.85	50	59.85
1	59.49	50	59.49
2	58.41	50	58.41
3	55.71	50	55.71
4	50.31	50	50.31
5	42.18	50	50
6	34.05	50	50
7	30.00	50	50
8	30.00	50	50
9	29.95	50	50
10	29.73	50	50
11	29.07	50	50
12	27.09	50	50

For k = 1

$$f_{3;(0,1,1)}(T_0) = -c_1 + E_{x_1}[f_{2;(1,1)}(T_0+x_1)]$$

$$= -10 + \sum_{x_1} f_{2;(1,1)}(T_0+x_1) \cdot p(x_1;3)$$

Values of this function are given in Table 5.

Table 5. Values of the Function $f_{3;(0,1,1)}(T_0)$

T_o	$f_{3;(0,1,1)}(T_o) = -10 + \sum_{x_1} f_{2;(1,1)}(T_0+x_1) \cdot p(x_1;3)$
0	45.12
1	43.08
2	41.34
3	40.28
4	39.82
5	39.40
6	38.30

Summary of the expected gain relating to the inspection program is presented in Table 6.

Table 6. Maximum Expected Gain for the Inspection Program R = (0,1,1), with $D_2 = 4$, $D_3 = 6$

FIRST STAGE		SECOND STAGE		THIRD STAGE	
T_0	$f_{3;(0,1,1)}(T_0)$	T_1	$f_{2;(1,1)}(T_1)$	T_2	$f_{1;(1)}(T_2)$
0	45.12	0	59.85	0	85
1	43.08	1	59.49	1	85
2	41.34	2	58.41	2	85
3	40.28	3	55.71	3	85
4	39.82	4	50.31	4	85
5	39.40	5	50	5	85
6	38.30	6	50	6	85
		7	50	7	55
		8	50	8	55
		9	50	9	55
		10	50	10	55
		11	50	11	55
		12	50	12	55
				13	55
				14	55
				15	55
				16	55

<u>Case 3</u>. This is the case when the inspection program $(1,1,1)^3$ is selected.

For k = 3 and k = 2, the analyses would be the same as those of Case 2, with $D_2 = 4$, and $D_3 = 6$.

For k = 1

$$f_{3;(1,1,1)}(T_0) = \underset{D_1}{\text{Max}} \begin{cases} -(a_1+c_1) + E_{x_1}[f_{2;(1,1)}(T_0+x_1)], & \text{if } T_0 \leq D_1 \\ V_1 - a_1, & \text{if } T_0 > D_1 \end{cases}$$

$$= \underset{D_1}{\text{Max}} \begin{cases} -(2+10) + E_{x_1}[f_{2;(1,1)}(T_0+x_1)], & \text{if } T_0 \leq D_1 \\ 45-2, & \text{if } T_0 > D_1 \end{cases}$$

$$= \underset{D_1}{\text{Max}} \begin{cases} -12 + \sum_{x_1} f_{2;(1,1)}(T_0+x_1) \cdot p(x_1;3), & \text{if } T_0 \leq D_1 \\ 43, & \text{if } T_0 > D_1 \end{cases}$$

Value of $f_{3;(1,1,1)}(T_0)$ for each possible value of T_0 is in Table 7. D_1 is identified to be 0.

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[3]This is what Johnson [4 and 5] considered in his analysis.

Table 7. Values of the Function $f_{3;(1,1,1)}(T_0)$

T_0	Accept $-12 + \sum_{x_1} f_{2;(1,1)}(T_0+x_1) \cdot p(x_1;3)$	Reject 43	$f_{3;(1,1,1)}(T_0)$
0	43.12	43	43.12
1	41.08	43	43
2	39.34	43	43
3	38.28	43	43
4	37.82	43	43
5	37.40	43	43
6	36.30	43	43

Expected gain summary for this inspection policy is shown in Table 8.

Table 8. Maximum Expected Gain for the Inspection Program R = (1,1,1), with $D_1 = 0$, $D_2 = 4$, and $D_3 = 6$

FIRST STAGE		SECOND STAGE		THIRD STAGE	
T_0	$f_{3;(1,1,1)}(T_0)$	T_1	$f_{2;(1,1)}(T_1)$	T_2	$f_{1;(1)}(T_2)$
0	43.12	0	59.85	0	85
1	43	1	59.49	1	85
2	43	2	58.41	2	85
3	43	3	55.71	3	85
4	43	4	50.31	4	85
5	43	5	50	5	85
6	43	6	50	6	85
		7	50	7	55
		8	50	8	55
		9	50	9	55
		10	50	10	55
		11	50	11	55
		12	50	12	55
				13	55
				14	55
				15	55
				16	55

Case 4. In this case, R is considered to be (0,0,1). It is to be noted that this is equivalent to the final inspection policy.

For k = 3, and k = 2, the analyses would be the same as those of Case 1.

For k = 1

$$f_{3;(0,0,1)}(T_0) = -C_1 + E_{x_1}]f_{2;(0,1)}(T_0+x_1)]$$

$$= -10 + \sum_{x_1} f_{2;(0,1)}(T_0+x_1) \cdot p(x_1;3)$$

Values of the function appear in Table 9.

Table 9. Values of the Function $f_{3;(0,0,1)}(T_0)$

T_0	$f_{3;(0,0,1)}(T_0) = -10 + \sum_{x_1} f_{2;(0,1)}(T_0+x_1) \cdot p(x_1;3)$
0	47.86
1	43.47
2	38.20
3	32.89
4	28.54
5	25.65
6	23.72

Table 10. Maximum Expected Gain for the Inspection Program $R = (0,0,1)$, with $D_3 = 6$

FIRST STAGE		SECOND STAGE		THIRD STAGE	
T_0	$f_{3;(0,0,1)}(T_0)$	T_1	$f_{2;(0,1)}(T_1)$	T_2	$f_{1;(1)}(T_2)$
0	47.86	0	64.85	0	85
1	43.47	1	64.49	1	85
2	38.20	2	63.41	2	85
3	32.89	3	60.71	3	85
4	28.54	4	55.31	4	85
5	25.65	5	47.18	5	85
6	23.72	6	39.05	6	85
		7	35.00	7	55
		8	35.00	8	55
		9	34.95	9	55
		10	34.73	10	55
		11	34.07	11	55
		12	32.09	12	55
				13	55
				14	55
				15	55
				16	55

Again, Table 10 shows the return functions associated with this final inspection policy.

Possible Savings in Computational Effort

There are situations in which computational efforts may be reduced. Such circumstances arise when there are some runs of no-inspection stages in the given inspection program R.

By a run is meant a set of at least two consecutive no-inspection stages. Figure 2 illustrates the case in which the given inspection program R has a run of size e, starting from stage b.

The first step one would have to do is to rewrite the system. Figure 3 shows the system after rewriting. Stages 1 through b-1 remain the same. However, stages b through b + e - 1, inclusive, are represented by stage b'. After the run--that is, stages b + e through N - 1--there is no change. Defects generated at stage b' are considered to be the convolution of those generated by stages b, b + 1, ⋯, b + e - 1 of the system before rewriting. In symbolic form,

$$x_{b'} = x_b + x_{b+1} + x_{b+2} + \cdots + x_{b+e-1} \qquad (4\text{-}12)$$

Thus,

$$\lambda_{b'} = \lambda_b + \lambda_{b+1} + \lambda_{b+2} + \cdots + \lambda_{b+e-1} \qquad (4\text{-}13)$$

Equations (4-10) and (4-11) are then used to analyze the rewritten system as before, with the following modifications at stage k = b':

1. In computing the subscript N - k + 1 of f, the magnitude of b' shall be the same as that of b.

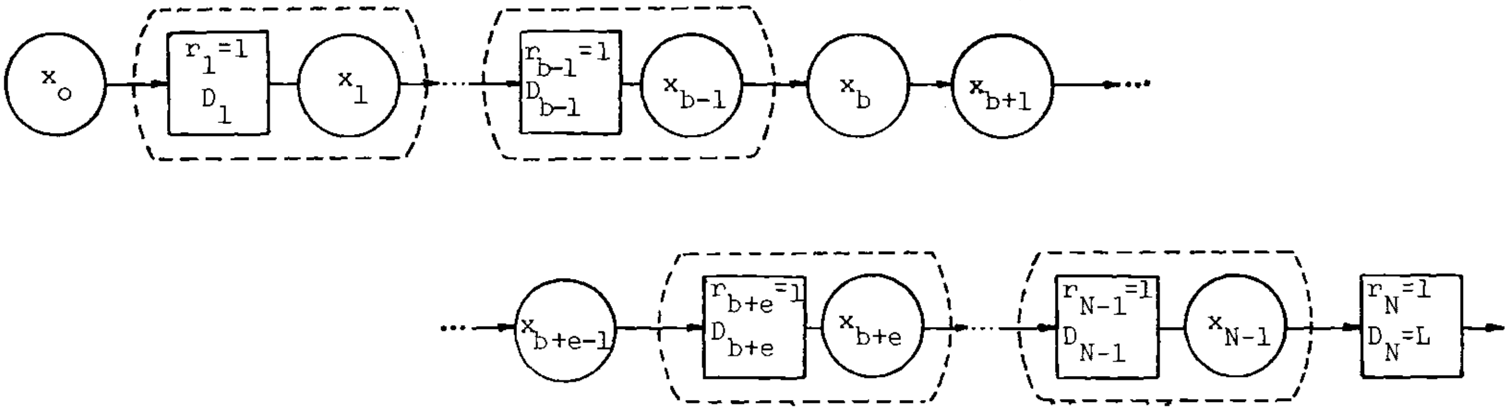

Figure 2. Inspection System with a Run of Size e, Starting at Stage b

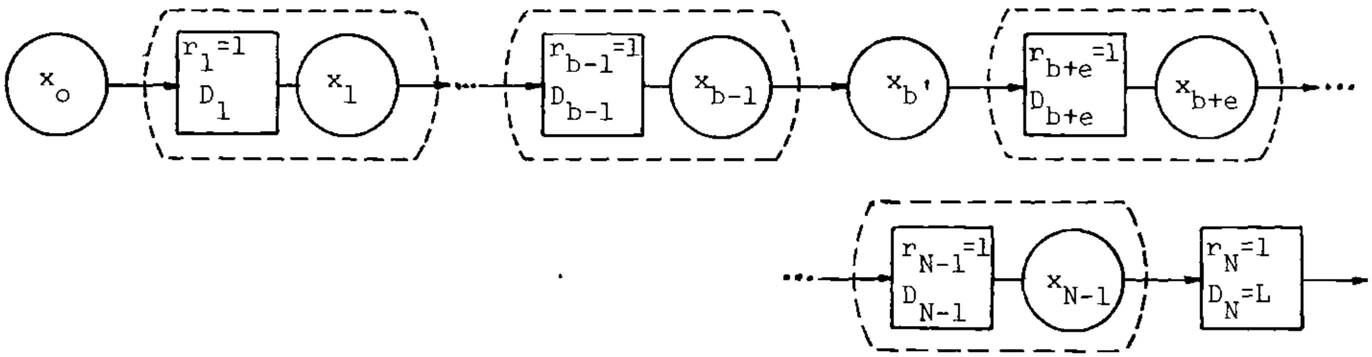

Figure 3. Rewritten System

2. The second subscript of f--the partial inspection program--shall be expressed in the original form.

3. The transformation used is that

$$T_{b+e-1} = T_{b-1} + x_{b'} \tag{4-14}$$

4. The value of $c_{b'}$ will be given as

$$c_{b'} = c_b + c_{b+1} + \cdots + c_{b+e-1} \tag{4-15}$$

Thus, for k = b'

$$f_{N-b+1;(r_b, r_{b+1}, \cdots r_{b+e-1}, r_{b+e}, \cdots, r_N)}(T_{b-1}) = \tag{4-16}$$

$$-c_{b'} + E_{x_{b'}}[f_{N-b-e+1;(r_{b+c}), \cdots, r_N)}(T_{b-1} + x_{b'})]$$

Figure 4 illustrates a five-stage production system with the following assumed data

$$R = (1,0,0,1,1) \qquad L = 16 \qquad V = 100$$

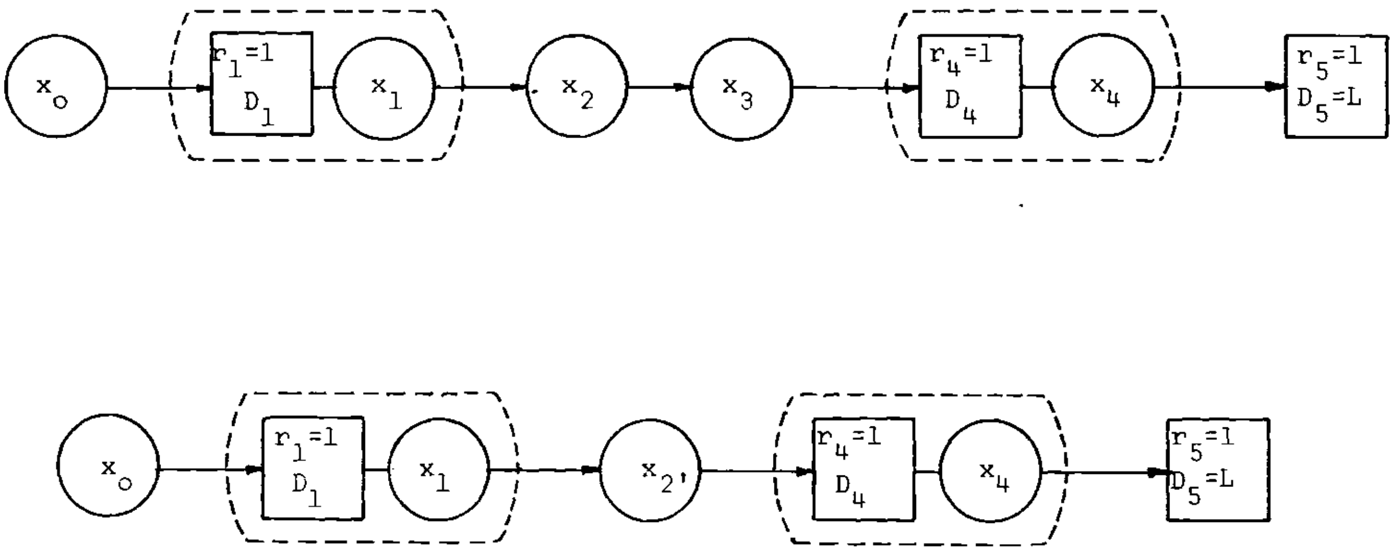

Figure 4. A Five-Stage Production System, Before and After Rewriting

$\lambda_{x_0} = 2$

$\lambda_1 = 3$	$a_1 = 1$	$c_1 = 3$	$v_1 = 30$
$\lambda_2 = 2$	$a_2 = 2$	$c_2 = 5$	$v_2 = 35$
$\lambda_3 = 4$	$a_3 = 3$	$c_3 = 10$	$v_3 = 40$
$\lambda_4 = 3$	$a_4 = 4$	$c_4 = 15$	$v_4 = 45$
	$a_5 = 5$		$v_5 = 50$

In this case there is a run of size 2, starting at stage 2. After rewriting, stage 2' represents stages 2 and 3 of the original one. According to (4-12) and (4-13), $x_{2'}$ is then a Poisson random variable with mean $\lambda_{2'} = 6$. Figure 4 shows the system before and after modification.

Thus, for k = 5

$$f_{1;(1)}(T_4) = \underset{D_5}{\text{Max}} \begin{bmatrix} V - a_5 \text{ , if } T_4 \le D_5 = 16 \\ \\ V_5 - a_5\text{, if } T_4 > D_5 = 16 \end{bmatrix}$$

$$= \underset{D_5}{\text{Max}} \begin{bmatrix} 100 - 5\text{, if } T_4 \le 16 \\ \\ 50 - 5 \text{ , if } T_4 > 16 \end{bmatrix}$$

$$= \underset{D_5}{\text{Max}} \begin{cases} 95, & \text{if } T_4 \le 16 \\ 45, & \text{if } T_4 > 16 \end{cases}$$

For k = 4

$$f_{2;(1,1)}(T_3) = \underset{D_4}{\text{Max}} \begin{cases} -(a_4+c_4) + E_{x_4}[f_{1;(1)}(T_3+x_4)], & \text{if } T_3 \le D_4 \\ V_4 - a_4 & , \text{if } T_3 > D_4 \end{cases}$$

$$= \underset{D_4}{\text{Max}} \begin{cases} -(4+15) + E_{x_4}[f_{1;(1)}(T_3+x_4)], & \text{if } T_3 \le D_4 \\ 45 - 4 & , \text{if } T_3 > D_4 \end{cases}$$

$$f_{2;(1,1)}(T_3) = \begin{cases} -19 + \sum_{x_4} f_{1;(1)}(T_3+x_4) \cdot p(x_4;3), & \text{if } T_3 \le D_4 \\ 41 & , \text{if } T_3 > D_4 \end{cases}$$

Values of this function are given in Table 12. D_4 is found to be equal to 14.

Table 11. Values of the Function $f_{2;(1,1)}(T_3)$

T_3	Accept $-19 + \sum_{x_4} f_{1;(1)}(T_3+x_4) \cdot p(x_4;3)$	Reject 41	$f_{2;(1,1)}(T_3)$
0	76	41	76
1	76	41	76
2	76	41	76
3	76	41	76
4	76	41	76
5	76	41	76
6	76	41	76
7	75.95	41	75.95
8	75.80	41	75.80
9	75.40	41	75.40
10	74.30	41	74.30
11	71.80	41	71.80
12	66.75	41	66.75
13	58.35	41	58.35
14	47.15	41	47.15
15	35.95	41	41
16	28.50	41	41
17	26.00	41	41
18	26.00	41	41
19	25.96	41	41
20	25.82	41	41
21	25.46	41	41
22	24.47	41	41
23	22.22	41	41

For k = 2'

$$f_{4;(0,0,1,1)}(T_1) = -(c_2+c_3) + E_{x_{2'}}[f_{2;(1,1)}(T_1+x_{2'})]$$

$$= -(5+10) + \sum_{x_{2'}} f_{2;(1,1)}(T_1+x_{2'}) \cdot p(x_{2'};6)$$

$$= -15 + \sum_{x_{2'}} f_{2;(1,1)}(T_1+x_{2'}) \cdot p(x_{2'};6)$$

Values of the function $f_{4;(0,0,1,1)}(T_1)$ are given in Table 12.

Table 12. Values of the Function $f_{4;(0,0,1,1)}(T_1)$

T_1	$f_{4;(0,0,1,1)}(T_1)$	T_1	$f_{4;(0,0,1,1)}(T_1)$
0	60.45	8	38.05
1	59.92	9	33.52
2	59.90	10	29.89
3	57.47	11	27.41
4	55.16	12	25.19
5	51.93	13	23.89
6	47.79	14	22.16
7	43.00		

For k = 1

$$f_{5;(1,0,0,1,1)}(T_0) = \underset{D_1}{\text{Max}} \begin{cases} -(a_1+c_1) + E_{x_1}[f_{4;(0,0,1,1)}(T_0+x_1)], & \text{if } T_0 \le D_1 \\ V_1 - a_1 & \text{, if } T_0 > D_1 \end{cases}$$

$$= \underset{D_1}{\text{Max}} \begin{cases} -(1+3) + E_{x_1}[f_{4;(0,0,1,1)}(T_0+x_1)], & \text{if } T_0 \le D_1 \\ 30 - 1 & \text{, if } T_0 > D_1 \end{cases}$$

$$= \underset{D_1}{\text{Max}} \begin{cases} -4 + \sum_{x_1} f_{4;(0,0,1,1)}(T_0+x_1) \cdot p(x_1;3), & \text{if } T_0 \le D_1 \\ 29 & \text{, if } T_0 > D_1 \end{cases}$$

Values of this function are presented in Table 13.

Table 13. Values of the Function $f_{5;(1,0,0,1,1)}(T_0)$

T_0	Accept $-4 + \sum_{x_1} f_{4;(0,0,1,1)}(T_0+x_1) \cdot p(x_1;3)$	Reject 29	$f_{5;(1,0,0,1,1)}(T_0)$
0	52.34	29	52.34
1	49.94	29	49.94
2	46.86	29	46.86
3	43.15	29	43.15
4	39.01	29	39.01
5	34.71	29	34.71
6	30.58	29	30.58
7	26.85	29	29
8	23.49	29	29

D_1 is found to be 6.

Expected gain summary associated with this inspection program (1,0,0,1,1) is shown in Table 14.

For a larger size of run, the savings in computational effort would be even greater. The rewriting method described could also be applied when there is more than one run in the system.

Table 14. Maximum Expected Gain for the Inspection Program (1,0,0,1,1), with $D_1 = 6$, $D_4 = 14$ and $D_5 = 16$

FIRST STAGE		SECOND STAGE		FOURTH STAGE		FIFTH STAGE	
T_0	$f_{5;(1,0,0,1,1)}(T_0)$	T_1	$f_{4;(0,0,1,1)}(T_1)$	T_3	$f_{2;(1,1)}(T_3)$	T_4	$f_{1;(1)}(T_4)$
0	52.34	0	60.45	0	76	0	95
1	49.94	1	59.92	1	76	1	95
2	46.86	2	59.90	2	76	2	95
3	43.15	3	57.47	3	76	3	95
4	39.01	4	55.16	4	76	4	95
5	34.71	5	51.93	5	76	5	95
6	30.58	6	47.79	6	76	6	95
7	29	7	43.00	7	75.95	7	95
8	29	8	38.05	8	75.80	8	95
		9	33.52	9	75.40	9	95
		10	29.89	10	74.30	10	95
		11	27.41	11	71.80	11	95
		12	25.19	12	66.75	12	95
		13	23.89	13	58.35	13	95
		14	22.16	14	47.15	14	95
				15	41	15	95
				16	41	16	95
				17	41	17	55
				18	41	18	55
				19	41	19	55
				20	41	20	55
				21	41	21	55
				22	41	⋮	⋮
				23	41	27	55
						28	55

Determination of Inspection Location For a Production System

In some instances, it may be desirable to locate the optimal screening inspection points in the production system. This class of decision problem is not only concerned with where to make inspection, but simultaneously with what specifications to be employed at such locations. It also implies that the inspection program associated with such optimal location, will be superior to any other one in terms of expected return value. As it shall be seen, the dynamic programming method can still be used to find such optimal inspection program. However, it is necessary to incorporate a procedure, which is described below, into it for the purpose of reducing computational effort. It should be noted that most symbols to be employed have been defined and used in preceding section.

Procedure for the Determination of Optimal Inspection Location

The procedure may be described briefly as follows:

1. At stage N, use (4-11) to determine $f_{1;(1)}(T_{N-1})$.
2. For stage N - 1, use (4-10) to find $f_{2;(r_{N-1},1)}(T_{N-2})$.

Select r^*_{N-1}, which is defined as,

$$E_{T_{N-2}}[f_{2;(r^*_{N-1},1)}(T_{N-2})] = \operatorname*{Max}_{r_{N-1}} \{E_{T_{N-2}}[f_{2;(r_{N-1},1)}(T_{N-2})]\} \qquad (4\text{-}17)$$

This is to find out if it is justified at all to locate screening inspection at stage N - 1, for a two-stage process. Retain the partial inspection program $(r^*_{N-1},1)$.

3. At stage N - 2, determine $f_{3;(0,r^*_{N-1},1)}(T_{N-3})$. Next is to find, by (4-10), $f_{3;(1,r_{N-1},1)}(T_{N-3})$ which consists of $f_{3;(1,1,1)}(T_{N-3})$, and $f_{3;(1,0,1)}(T_{N-3})$. r^*_{N-2} is then selected by

$$E_{T_{N-3}}[f_{3;(r^*_{N-2},r_{N-1},1)}(T_{N-3})] = \underset{r_{N-2}}{\text{Max}} \{E_{T_{N-3}}[f_{3;(0,r^*_{N-1},1)}(T_{N-3})], \quad E_{T_{N-3}}[f_{3;(1,r_{N-1},1)}(T_{N-3})]\} \qquad (4\text{-}18)$$

Retain the partial inspection program $(r^*_{N-2},r_{N-1},1)$. The rest of them are eliminated from further consideration. The retained program will indicate not only if inspection should take place at stage N - 2 for the three-stage process, but also whether it is economical to accompany it with inspection at stage N - 1.

4. In general, at stage k (k = N - 2, N - 3, ···, 3, 2), it is necessary to determine: (a) $f_{N-k+1;(0,r^*_{k+1},r_{k+2},\cdots,r_{N-1},1)}(T_{k-1})$, (b) $f_{N+k+1;(1,r_{k+1},r_{k+2},\cdots,r_{N-1},1)}(T_{k-1})$. Then r^*_k is identified from:

$$E_{T_{k-1}}[f_{N-k+1;(r^*_k,r_{k+1},r_{k+2},\cdots,r_{N-1},1)}(T_{k-1})] = \qquad (4\text{-}19)$$

$$\underset{r_k}{\text{Max}}[E_{T_{k-1}}[f_{N-k+1;(0,r^*_{k+1},\cdots,r_{N-1},1)}(T_{k-1})],$$

$$E_{T_{k-1}}[f_{N-k+1;(1,r_{k+1},\cdots,r_{N-1},1)}(T_{k-1})]]$$

The partial inspection program $(r_k^*, r_{k+1}, r_{k+2}, \cdots, r_{N-1}, 1)$ is retained to be used in the N - k + 2 stage process.

5. At stage 1, things proceed as before. Once r_1^* is selected, the inspection policy associated with it will be the optimal inspection program, symbollically denoted by R^*. Thus,

$$E_{T_0}[f_{N;R^*}(T_0)] = E_{T_0}[f_{N;(r_1^*, r_2, \cdots, r_{N-1}, 1)}(T_0)] \tag{4-20}$$

$$= \underset{r_1}{\text{Max}}\{E_{T_0}[f_{N;(0, r_2^*, r_3, \cdots, r_{N-1}, 1)}(T_0)],$$

$$E_{T_0}[f_{N;(1, r_1, r_2, \cdots, r_{N-1}, 1)}(T_0)]\}$$

It is well to note that one is originally faced with the task of computing 2^{N-1} possible inspection allocation programs for an N-stage production system. Through the utilization of the described procedure, the number of inspection programs completely evaluated are $2^{N-2} + 1$. Saving in computations would become greater as N becomes larger. In general, it would save up to 50 per cent.

In order to demonstrate how the above procedure would work, two examples are given. In the first one, a four-stage production system will be analyzed qualitatively. The second one will make use of the data of the problem discussed in the previous section.

<u>Example 1</u>. One appropriate way to illustrate the structure of screening inspection location problem may probably be done by means of

decision tree. The tree for a four-stage production system is shown in Figure 5. For the two-stage decision process analysis, the tree may be cut at section AA' as shown in Figure 6. Equations (4-11) is then used to find $f_{2;(1,1)}(T_2)$ and $f_{2;(0,1)}(T_2)$. Suppose that,

$$\underset{r_3}{\text{Max}}[E_{T_2}[f_{2;(1,1)}(T_2)],\ E_{T_2}[f_{2;(0,1)}(T_2)]] = E_{T_2}[f_{2;(1,1)}(T_2)]$$

r_3^* is then equal to 1. It implies that it is justified to locate inspection at stage 3 for a two-stage process. The partial program (1,1), corresponding to the upper branch of Figure 6, is retained for further analysis.

Figure 7 illustrates a three-stage decision process, as sectioned at BB' from the main tree. Partial program (1,1) is transformed into that of (0,1,1) by (4-10). $f_{3;(1,1,1)}(T_1)$ and $f_{3;(1,0,1)}(T_1)$ are then determined for the purpose of comparing them with $f_{3;(0,1,1)}(T_1)$. Assuming that

$$\underset{r_2}{\text{Max}}\{E_{T_1}[f_{3;(1,1,1)}(T_1)],\ E_{T_1}[f_{3;(1,0,1)}(T_1)],\ E_{T_1}[f_{3;(0,1,1)}(T_1)]\}$$

$$= E_{T_1}[f_{3;(1,0,1)}(T_1)]$$

It may be stated that, for a three-stage process, if inspection is to take place at stage 2, it is economically justified.

For the four-stage process analysis, as represented by Figure 8, $f_{3;(1,0,1)}(T_1)$ is transformed by (4-10) into $f_{4;(0,1,0,1)}(T_0)$.

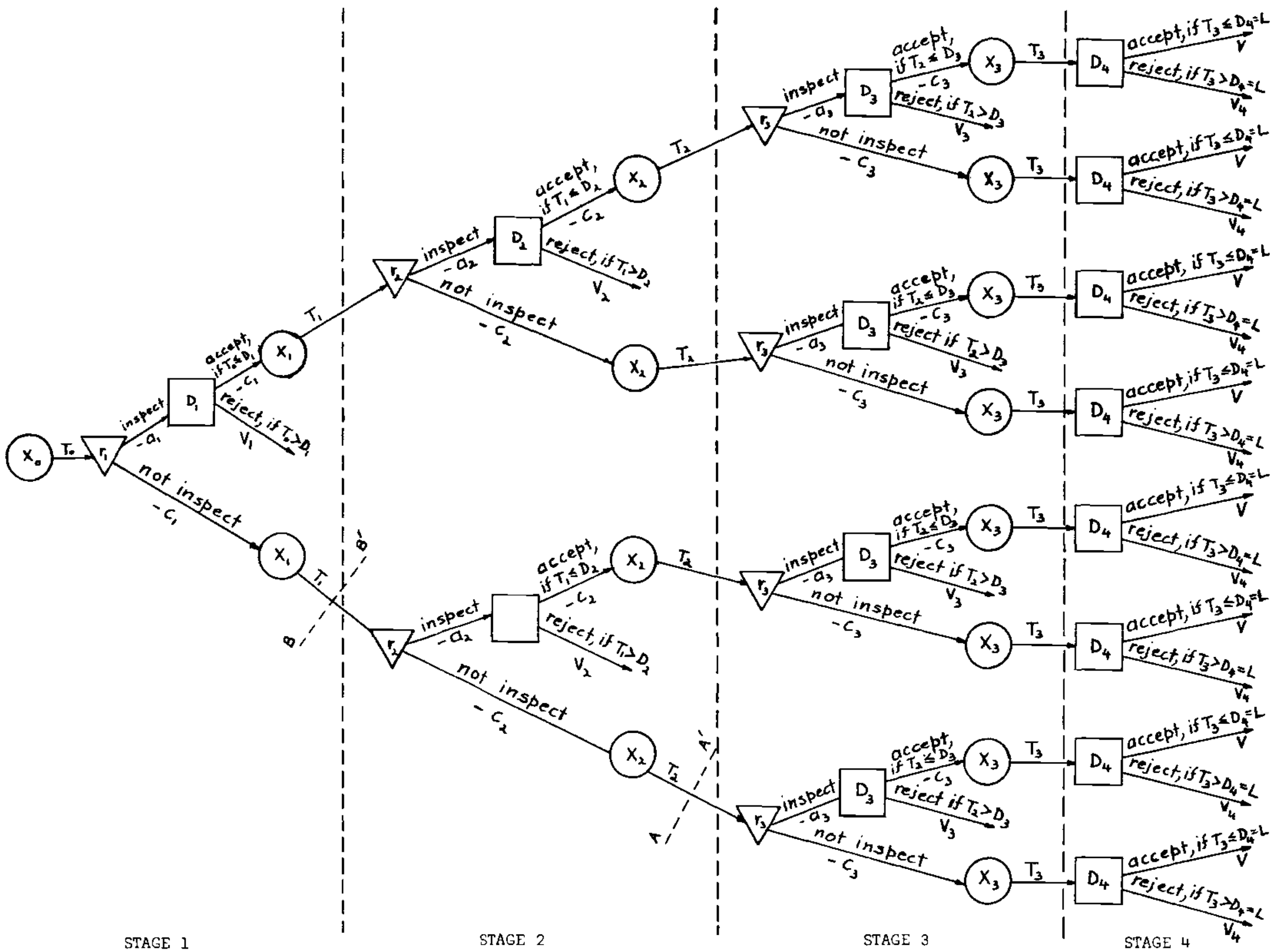

Figure 5. A Decision Tree Diagram of a Four-Stage Production System 1

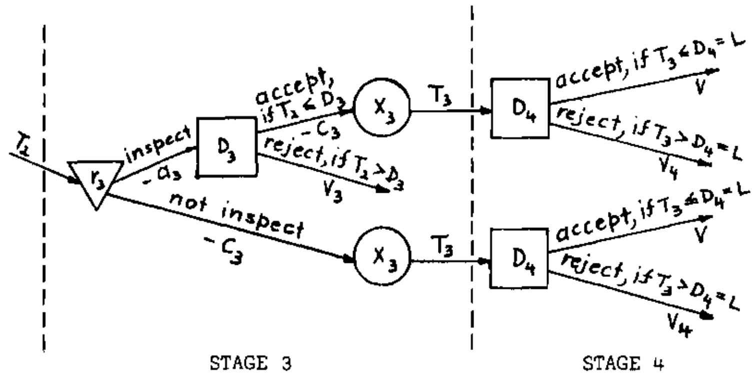

Figure 6. Two-Stage Process

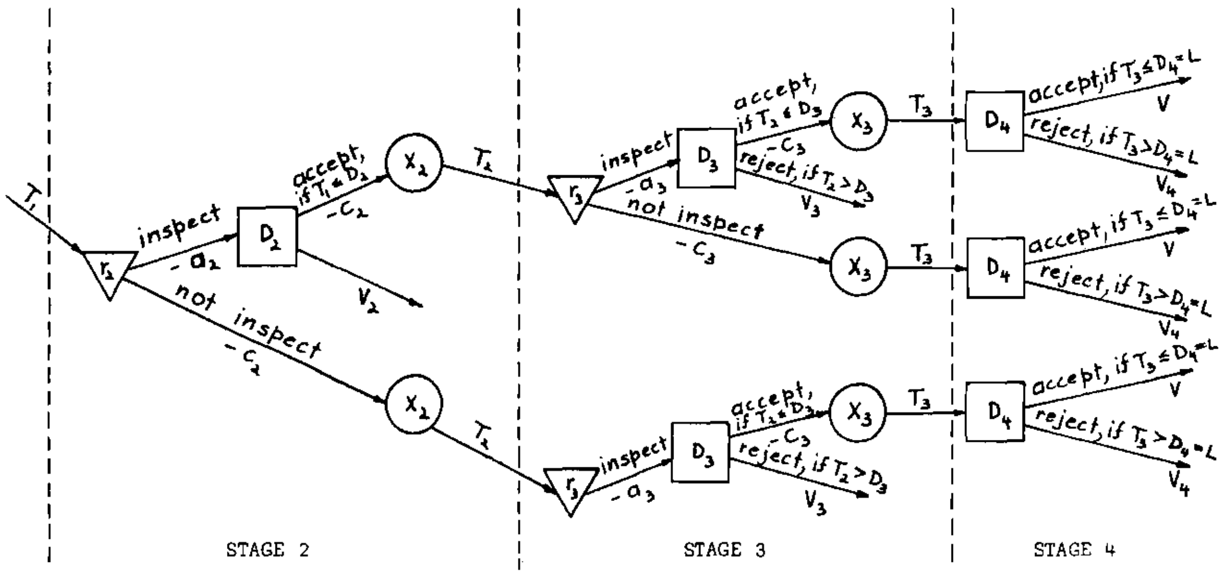

Figure 7. Three-Stage Process

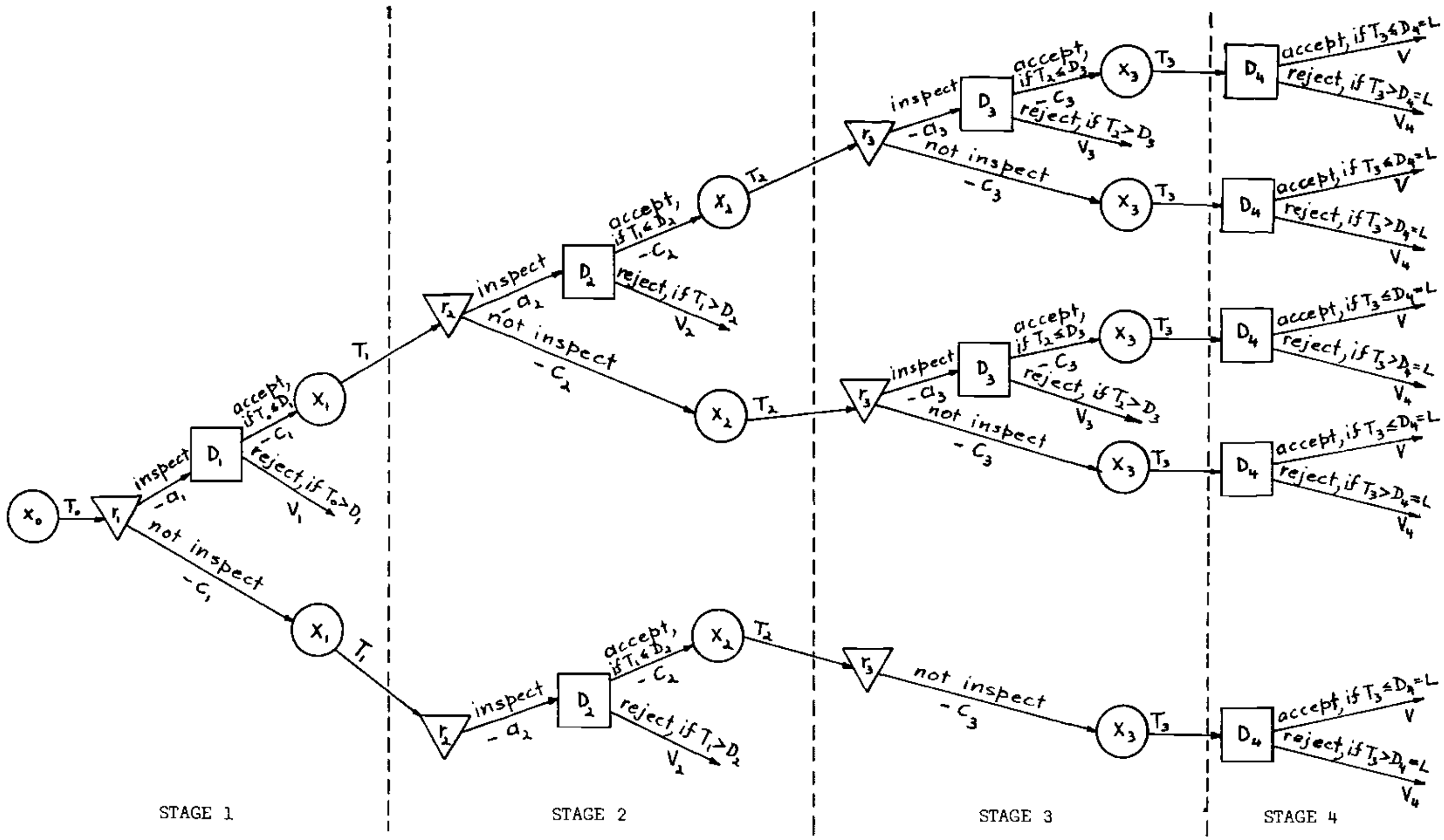

Figure 8. Four-Stage Process

It is then compared with $f_{4;(1,1,1,1)}(T_0)$, $f_{4;(1,1,0,1)}(T_0)$, $f_{4;(1,0,1,1)}(T_0)$ and $f_{4;(1,0,0,1)}(T_0)$. Suppose that

$$\underset{r_1}{\text{Max}}\{E_{T_0}[f_{4;(0,1,0,1)}(T_0)],\ E_{T_0}[f_{4;(1,1,1,1)}(T_0)],\ E_{T_0}[f_{4;(1,1,0,1)}(T_0)],$$

$$E_{T_0}[f_{4;(1,0,1,1)}(T_0)],\ E_{T_0}[f_{4;(1,0,0,1)}(T_0)]\} = E_{T_0}[f_{4;(0,1,0,1)}(T_0)]$$

R^* is then identified to be (0,1,0,1). It is then economical to conduct screening inspection at stage 2 and also final inspection at stage 4.

<u>Example 2</u>. Consider a three-stage production process whose data are already presented on page 32, and whose decision tree is shown in Figure 9. After $f_{2;(0,1)}(T_1)$, and $f_{2;(1,1)}(T_1)$ are determined and presented in Table 1 and Table 4, respectively, it is next to identify r_2^*. It is found that

$$\underset{r_2}{\text{Max}}[E_{T_1}[f_{2;(1,1)}(T_1)],\ E_{T_1}[f_{2;(0,1)}(T_1)]] =$$

$$\underset{r_2}{\text{Max}}[53.281,\ 53.175] = 53.281$$

Thus, $r_2^* = 1$. The partial program (1,1) is retained and $f_{2;(1,1)}(T_1)$ is then transformed into $f_{3;(0,1,1)}(T_0)$, and compared by the expectation criterion with $f_{3;(1,1,1)}(T_0)$ and $f_{3;(1,0,1)}(T_0)$. It is found that

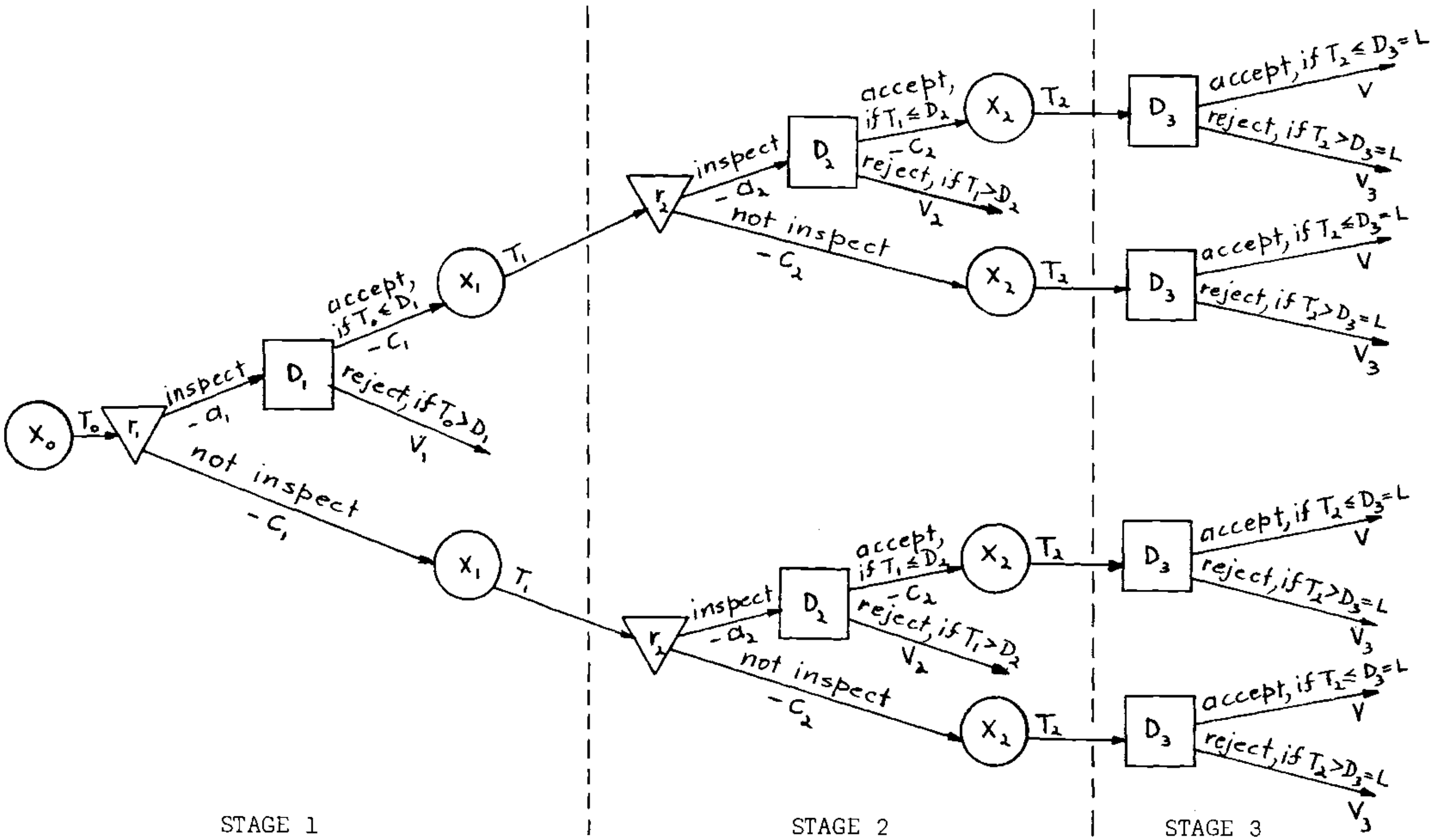

Figure 9. A Decision Tree Diagram of a Three-Stage System

$$\underset{r_1}{\text{Max}}[E_{T_0}[f_{3;(0,1,1)}(T_0)],\ E_{T_0}[f_{3;(1,1,1)}(T_0)],\ E_{T_0}[f_{3;(1,0,1)}(T_0)]]$$

$$= \underset{r_1}{\text{Max}}[43.274,\ 43.044,\ 44.052] = 44.052$$

Thus $r_1^* = 1$ and $R^* = (1,0,1)$. It would be justified economically to locate screening inspection, in addition to the final inspection, at stage 3.

Summary

The production environment considered in this chapter is the case in which a producer is manufacturing until he exhausts a fixed level of resource availability. The first decision problem taken up was the determination of inspection specification limits associated with an inspection program. A dynamic programming model was formulated. A numerical example was solved for a three-stage production system. A method for saving computational effort was presented along with graphical and numerical illustrations.

The second decision problem considered was concerned with locating the optimal screening inspection points in a production line. Using the expected value criterion as the basis, the development of a procedure to solve such decision problem was presented. Two examples were given to demonstrate how to apply the procedure.

It is apparent from the analyses that there is an economic advantage in tightening specifications. It should be noted that the determination of optimal inspection policy has to be carried out simultaneously with that of finding its specification limits.

CHAPTER V

ANALYSIS OF SEQUENTIAL MANUFACTURING SYSTEM WHEN PRODUCING TO A FIXED QUOTA

Introduction

The production environment considered in this chapter is that the producer is manufacturing to satisfy a fixed production goal, so that any defective item must be replaced by reprocessing a substitute. Two variations for processing the new item are discussed. As before, the property of the optimal inspection policy is employed. The structure of the manufacturing system is the same as stated before in Chapter IV.

Variation 1 Analysis

In this variation, an item which fails to meet specification at any stage will be set aside and carefully processed to completion such that no more defects are introduced. For any item that has more than L defects, it will be scrapped and sold at value $V_k (V_k < V)$. A new item will then be acquired and put on special processing to completion.

Most of notation to be used has been defined and used in the preceding chapters, except

S_k = cost of special processing a unit from production operation of stage k to final inspection N, $(k = 1,2,3,\cdots,N-1)$.

Two classes of decision problems are considered; (a) the determination of specifications for a given inspection program; (b) the determination of screening inspection locations in a production line.

<u>Determination of Specification Limits for an Inspection Program</u>

With the use of the transformation that

$$T_k = T_{k-1} + x_k \quad \text{for } k = 1,2,\cdots,N-1 \tag{5-1}$$

and

$$T_0 = x_0, \tag{5-2}$$

The following recursive relationship would be obtained:

$$f_{N-k+1;(r_k,r_{k+1},\cdots,r_N)}(T_{k-1}) = \underset{D_k}{\text{Max}}\ E_{x_k}[W_k(T_{k-1},r_k,D_k) + f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)] \tag{5-3}$$

$$\text{for } k = 1,2,\cdots,N-1$$

with

$$f_{1;(1)}(T_{N-1}) = \underset{D_N=L}{\text{Max}} \begin{bmatrix} V - a_N & , \text{ if } T_{N-1} \le D_N = L \\ \\ (V_N-a_N) + (V-c_0-s_1), & \text{if } T_{N-1} > D_N = L \end{bmatrix}$$

The immediate return at stage k is, according to acceptance rule stated previously, given as

$$W_k(T_{k-1},r_k,D_k) = \begin{cases} [-(a_k+c_k)](r_k) + [-c_k](1-r_k), \text{ if } T_{k-1} \le D_k \\ [V-s_k-a_k-f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)](r_k) + [-c_k](1-r_k), \text{ if } D_k < T_{k-1} \le L \\ [V+V_k-s_1-c_0-a_k-f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)](r_k) + [-c_k](1-r_k), \text{ if } T_{k-1} > L \end{cases} \quad (5\text{-}4)$$

$$\text{for } k = 1,2,\cdots,N-1$$

Thus, Equation (5-3) can be expressed as:

$$f_{N-k+1;(r_k,r_{k+1},\cdots,r_N)}(T_{k-1}) = \underset{D_k}{\text{Max}} \begin{cases} E_{x_k}[[-(a_k+c_k)](r_k) + [-c_k](1-r_k) + f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k)], \text{ if } T_{k-1} \le D_k \\ E_{x_k}[[v-s_k-a_k-f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)](r_k) + [-c_k](1-r_k) \\ \qquad + f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k)], \text{ if } D_k < T_{k-1} \le L \\ E_{x_k}[[V+V_k-s_1-c_o-a_k-f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k)](r_k) + [-c_k](1-r_k) \\ \qquad + f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k)], \text{ if } T_{k-1} > L \end{cases} \quad (5\text{-}5)$$

$$f_{N-k+1;(r_k,\cdots,r_N)}(T_{k-1}) = \underset{D_k}{\text{Max}} \begin{cases} E_{x_k}[[-(a_k+c_k)](r_k) + [-c_k](1-r_k) + f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k) \\ \qquad + [f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)](r_k) \\ \qquad - [f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)](r_k)], \text{ if } T_{k-1} \le D_k \\ E_{x_k}[[V-s_k-a_k](r_k) - [f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k)](r_k) + [-c_k](1-r_k) \\ \qquad + f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k)], \text{ if } D_k < T_{k-1} \le L \\ E_{x_k}[[V+V_k-s_1-c_0-a_k](r_k) - [f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)](r_k) \\ \qquad + [-c_k](1-r_k) + f_{N-k(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)], \text{ if } T_{k-1} > L \end{cases} \quad (5\text{-}6)$$

Since a_k, c_k, r_k, V, s_k, c_0, s_1, V_k are constants. Equation (5-6) is equivalent to:

$$f_{N-k+1;(r_k,r_{k+1},\cdots,r_N)}(T_{k-1}) = \underset{D_k}{\text{Max}} \begin{cases} [-(a_k+c_k)](r_k) + (r_k)[E_{x_k}[f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)]] \\ [-c_k](1-r_k) + (1-r_k)[E_{x_k}[f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)]], \text{ if } T_{k-1} \le D_k \\ [V-s_k-a_k](r_k) + [-c_k](1-r_k) + (1-r_k)[E_{x_k}[f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k)]], \\ \qquad \text{if } D_k < T_{k-1} \le L \\ [V+V_k-s_1-c_0-a_k](r_k) + [-c_k](1-r_k) + (1-r_k)[E_{x_k}[f_{N-k;(r_{k+1},\cdots,r_N}(T_{k-1}+x_k)]], \\ \qquad \text{if } T_{k-1} > L \end{cases} \tag{5-7}$$

By collecting terms common to r_k and $(1-r_k)$, respectively, Equation (5-7) may be rewritten as:

$$f_{N-k+1;(r_k,r_{k+1},\cdots,r_N)}(T_{k-1}) = \underset{D_k}{\text{Max}} \begin{cases} [-(a_k+c_k) + E_{x_k}[f_{N-k;(r_{k+1},r_{k+2},\cdots,r_N)}(T_{k-1}+x_k)]](r_k) \\ \quad + [-c_k + E_{x_k}[f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k)]](1-r_k), \text{ if } T_{k-1} \le D_k \\ [V-s_k-a_k](r_k) + [-c_k+E_{x_k}[f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k)]](1-r_k), \text{ if } D_k < T_{k-1} \le L \\ [V+V_k-s_1-c_0-a_k](r_k) + [-c_k+E_{x_k}[f_{N-k;(r_{k+1},\cdots,r_N)}(T_{k-1}+x_k)]](1-r_k), \text{ if } T_{k-1} > L \end{cases} \tag{5-8}$$

for $k = 1,2,\cdots,N-1$

with

$$f_{1;(r_N)}(T_{N-1}) = f_{1;(1)}(T_{N-1}) = \underset{D_N=L}{\text{Max}} \begin{cases} V - a_N, \quad \text{if } T_{N-1} \le D_N = L \\ V+V_N-S_1-c_0-a_N, \text{ if } T_{N-1} > D_N = L \end{cases} \tag{5-9}$$

The solution to this decision problem by the dynamic programming approach involves the successive application of (5-8).

Illustrative Example. Consider a three-stage manufacturing system. The following data are assumed.

$\lambda_0 = 1$	$c_0 = 45$				
$\lambda_1 = 3$	$c_1 = 10$	$a_1 = 2$	$V_1 = 45$	$s_1 = 60$	$V = 100$
$\lambda_2 = 2$	$c_2 = 20$	$a_2 = 5$	$V_2 = 55$	$s_2 = 40$	$L = 6$
		$a_3 = 15$	$V_3 = 70$		

It is apparent then that:

$$D_3 = L = 6 \qquad \Lambda_0 = 1$$

$$\Lambda_1 = 4$$

$$\Lambda_2 = 6$$

Based upon the property of optimal inspection policy, there would be $2^{3-1} = 4$ possible inspection programs, whose analyses are in order.

Case 1. Suppose that the inspection program R is in the form (1,0,1).

For k = 3,

$$f_{1;(1)}(T_2) = \underset{D_3}{\text{Max}} \begin{cases} V - a_3 & , \text{ if } T_2 \leq 6 \\ V+V_3-s_1-c_0-a_3, & \text{if } T_2 > 6 \end{cases}$$

$$= \max_{D_3} \begin{cases} 100-15 & , \text{ if } T_2 \le 6 \\ 100+70-60-45-15, & \text{if } T_2 > 6 \end{cases}$$

$$= \max_{D_3} \begin{cases} 85 & , \text{ if } T_2 \le 6 \\ 50 & , \text{ if } T_2 > 6 \end{cases}$$

For k = 2,

$$f_{2;(0,1)}(T_1) = -c_2 + E_{x_2}[f_{1;1}(T_1+x_2)]$$

$$= -20 + \sum_{x_2} f_{1;(1)}(T_1+x_2) \cdot p(x_2;2)$$

Values of the function $f_{2;(0,1)}(T_1)$ are presented in Table 15.

Table 15. Values of the Function $f_{2;(0,1)}(T_1)$

T_1	$f_{2;(0,1)}(T_1) = -20 + \sum_{x_2} f_{1;(1)}(T_1+x_2) \cdot p(x_2;2)$
0	64.83
1	64.41
2	63.15
3	60.00
4	53.70
5	44.21
6	34.73
7	30.00
8	30.00
9	29.95
10	29.75
11	29.15
12	27.35

For k = 1,

$$f_{3;(1,0,1)}(T_0) = \underset{D_1}{\text{Max}} \begin{cases} -(a_1+c_1) + E_{x_1}[f_{2;(0,1)}(T_0+x_1)], & \text{if } T_0 \leq D_1 \\ V-S_1-a_1, & \text{if } D_1 < T_0 \leq L \\ V+V_1-s_1-c_0-a_1, & \text{if } T_0 > L \end{cases}$$

$$f_{3;(1,0,1)}(T_0) = \underset{D_1}{\text{Max}} \begin{cases} -(2+10) + E_{x_1}[f_{2;(0,1)}(T_0+x_1)], & \text{if } T_0 \le D_1 \\ 100-60-2 & \text{, if } D_1 < T_0 \le 6 \\ 100+45-60-45-2 & \text{, if } T_0 > 6 \end{cases}$$

$$= \underset{D_1}{\text{Max}} \begin{cases} -12 + \sum_{x_1} f_{2;(0,1)}(T_0+x_1) \cdot p(x_1;3) \\ 38 & \text{, if } D_1 < T_0 \le 6 \\ 38 & \text{, if } T_0 > 6 \end{cases}$$

Values of the function $f_{3;(1,0,1)}(T_0)$ are presented in Table 16. D_1 is found to be 1.

Table 16. Values of the Function $f_{3;(1,0,1)}(T_0)$

T_0	Accept $-12 + \sum_{x_1} f_{2;(0,1)}(T_0+x_1) \cdot p(x_1;3)$	Reject 38	$f_{3;(1,0,1)}(T_0)$
0	44.67	38	44.67
1	39.55	38	39.55
2	33.41	38	38
3	27.22	38	38
4	22.19	38	38
5	18.92	38	38
6	16.95	38	38

Table 17 summarizes the expected gain associated with the policy.

Table 17. Maximum Expected Gain for the Inspection Program $R = (1,0,1)$, with $D_1 = 1$, and $D_3 = 6$

FIRST STAGE		SECOND STAGE		THIRD STAGE	
T_0	$f_{3;(1,0,1)}(T_0)$	T_1	$f_{2;(0,1)}(T_1)$	T_2	$f_{1;(1)}(T_2)$
0	44.67	0	64.83	0	85
1	39.55	1	64.41	1	85
2	38	2	63.15	2	85
3	38	3	60.00	3	85
4	38	4	53.70	4	85
5	38	5	44.21	5	85
6	38	6	34.73	6	85
		7	30.00	7	50
		8	30.00	8	50
		9	29.95	9	50
		10	29.75	10	50
		11	29.15	11	50
		12	27.35	12	50
				13	50
				14	50
				15	50
				16	50

Case 2. This is when R is given to be (0,1,1).

For k = 3

$$f_{1;(1)}(T_2) = \underset{D_3}{\text{Max}} \begin{cases} V-a_3 & , \text{ if } T_2 \leq 6 \\ V+V_3-S_1-c_0-a_3 & , \text{ if } T_2 > 6 \end{cases}$$

$$= \underset{D_3}{\text{Max}} \begin{cases} 85 & , \text{ if } T_2 \leq 6 \\ 50 & , \text{ if } T_2 > 6 \end{cases}$$

For k = 2

$$f_{2;(1,1)}(T_1) = \underset{D_2}{\text{Max}} \begin{cases} -(a_2+c_2) + E_{x_2}[f_{1;(1)}(T_1+x_2)], & \text{ if } T_1 \leq D_2 \\ V-s_2-a_2 & , \text{ if } D_2 < T_1 \leq L \\ V+V_2-s_1-c_0-a_2 & , \text{ if } T_1 > L \end{cases}$$

$$= \underset{D_2}{\text{Max}} \begin{cases} -(5+20) + E_{x_2}[f_{1;(1)}(T_1+x_2)], & \text{ if } T_1 \leq D_2 \\ 100-40-5 & , \text{ if } D_2 < T_1 \leq 6 \\ 100+55-60-45-5 & , \text{ if } T_1 > 6 \end{cases}$$

$$= \underset{D_2}{\text{Max}} \begin{cases} -25 + \sum_{x_2} f_{1;(1)}(T_1+x_2) \cdot p(x_2;2), & \text{if } T_1 \le D_2 \\ 55 & , \text{if } D_2 < T_1 \le 6 \\ 45 & , \text{if } T_1 > 6 \end{cases}$$

For each possible value of T_1, $f_{2;(1,1)}(T_1)$ is determined and tabulated into Table 18. D_2 is found to be 3.

Table 18. Values of the Function $f_{2;(1,1)}(T_1)$

T_1	Accept $-25 + \sum_{x_2} f_{1;(1)}(T_1+x_2) \cdot p(x_2;2)$	Reject 55	$f_{2;(1,1)}(T_1)$
0	59.83	55	59.83
1	59.41	55	59.41
2	58.15	55	58.15
3	55.00	55	55.00
4	48.70	55	55
5	39.21	55	55
6	29.73	55	55
7	25.00	45	45
8	25.00	45	45
9	24.95	45	45
10	24.75	45	45
11	24.15	45	45
12	22.35	45	45

For k = 1

$$f_{3;(0,1,1)}(T_0) = -c_1 + E_{x_1}[f_{2;(1,1)}(T_0+x_1)]$$

$$= -10 + \sum_{x_1} f_{2;(1,1)}(T_0+x_1) \cdot p(x_1;3)$$

Values of this function are given in Table 19.

Table 19. Values of the function $f_{3;(0,1,1)}(T_0)$

T_0	$f_{3;(0,1,1)} = -10 + \sum_{x_1} f_{2;(1,1)}(T_0+x_1) \cdot p(x_1;3)$
0	46.27
1	44.85
2	43.31
3	41.43
4	39.05
5	36.45
6	33.97

Expected gain summary for this inspection program is presented in Table 20.

Table 20. Maximum Expected Gain for the Inspection Program $R = (0,1,1)$, with $D_2 = 3$, and $D_3 = 6$

FIRST STAGE		SECOND STAGE		THIRD STAGE	
T_0	$f_{3;(0,1,1)}(T_0)$	T_1	$f_{2;(1,1)}(T_1)$	T_2	$f_{1;(1)}(T_2)$
0	46.27	0	59.83	0	85
1	44.85	1	59.41	1	85
2	43.31	2	58.15	2	85
3	41.43	3	55.00	3	85
4	39.05	4	55	4	85
5	36.45	5	55	5	85
6	33.97	6	55	6	85
		7	45	7	50
		8	45	8	50
		9	45	9	50
		10	45	10	50
		11	45	11	50
		12	45	12	50
				13	50
				14	50
				15	50
				16	50

Case 3. This is the case in which screening inspection is to be performed at all stages.

For k = 3 and k = 2, the values of $f_{1;(1)}(T_2)$, and $f_{2;(1,1)}(T_1)$, respectively, are the same as those of Case 2.

For k = 1

$$f_{3;(1,1,1)}(T_0) = \underset{D_1}{\mathrm{Max}} \begin{cases} -(a_1+c_1) + E_{x_1}[f_{2;(1,1)}(T_0+x_1)], & \text{if } T_0 \le D_1 \\ V-s_1-a_1 & \text{, if } D_1 < T_0 \le L \\ V+V_1-s_1-c_0-a_k & \text{, if } T_0 > L \end{cases}$$

$$= \underset{D_1}{\mathrm{Max}} \begin{cases} -(2+10) + \sum_{x_1} f_{2;(1,1)}(T_0+x_1) \cdot p(x_1;3), & \text{if } T_0 \le D_1 \\ 100-60-2 & \text{, if } D_1 < T_0 \le 6 \\ 100+45-60-45-2 & \text{, if } T_0 > 6 \end{cases}$$

$$= \underset{D_1}{\mathrm{Max}} \begin{cases} -12 + \sum_{x_1} f_{2;(1,1)}(T_0+x_1) \cdot p(x_1;3), & \text{if } T_0 \le D_1 \\ 38 & , D_1 < T_0 \le 6 \\ 38 & \text{, if } T_0 > 6 \end{cases}$$

Tabulation of $f_{3;(1,1,1)}(T_0)$ is in Table 21. D_1 is identified as 3.

Table 21. Values of the Function $f_{3;(1,1,1)}(T_0)$

T_0	Accept $-12 + \sum f_{2;(1,1)}(T_0+x_1) \cdot p(x_1;3)$	Reject	$f_{3;(1,1,1)}(T_0)$
0	44.27	38	44.27
1	42.85	38	42.85
2	41.31	38	41.31
3	39.43	38	39.43
4	37.05	38	38
5	34.45	38	38
6	31.97	38	38

Summary of expected gain of this policy (1,1,1) are given in Table 22.

Table 22. Expected Gain for the Inspection Program $R = (1,1,1)$, with $D_1 = 3$, $D_2 = 3$, and $D_3 = 6$

FIRST STAGE		SECOND STAGE		THIRD STAGE	
T_0	$f_{3;(1,1,1)}(T_0)$	T_1	$f_{2;(1,1)}(T_0)$	T_2	$f_{1;(1)}(T_2)$
0	44.27	0	59.83	0	85
1	42.85	1	59.41	1	85
2	41.31	2	58.15	2	85
3	39.43	3	55.00	3	85
4	38	4	55	4	85
5	38	5	55	5	85
6	38	6	55	6	85
		7	45	7	50
		8	45	8	50
		9	45	9	50
		10	45	10	50
		11	45	11	50
		12	45	12	50
				13	50
				14	50
				15	50
				16	50

Case 4. This is the case in which R is considered to be (0,0,1). For $k = 3$, and $k = 2$, the values of the function $f_{1;(1)}(T_2)$, and $f_{2;(0,1)}(T_1)$ are the same as those of Case 1.

For k = 1

$$f_{3;(0,0,1)}(T_0) = -c_1 + E_{x_1}[f_{2;(0,1)}(T_0+x_1)]$$

$$= -10 + \sum_{x_1} f_{2;(0,1)}(T_0+x_1) \cdot p(x_1;3)$$

Values of this function are presented in Table 23.

Table 23. Values of the Function $f_{3;(0,0,1)}(T_0)$

T_0	$f_{3;(0,0,1)}(T_0) = -10 + \sum_{x_1} f_{2;(0,1)}(T_0+x_1) \cdot p(x_1;3)$
0	46.67
1	41.55
2	35.41
3	29.22
4	24.19
5	20.92
6	18.95

Table 24 shows the return functions associated with this inspection program.

Table 24. Expected Gain for the Inspection Program $R = (0,0,1)$, with $D_3 = 6$

FIRST STAGE		SECOND STAGE		THIRD STAGE	
T_0	$f_{3;(0,0,1)}(T_0)$	T_1	$f_{2;(0,1)}(T_1)$	T_2	$f_{1;(1)}(T_2)$
0	46.67	0	64.83	0	85
1	41.55	1	64.41	1	85
2	35.41	2	63.15	2	85
3	29.22	3	60.00	3	85
4	24.19	4	53.70	4	85
5	20.92	5	44.21	5	85
6	18.95	6	34.73	6	85
		7	30.00	7	50
		8	30.00	8	50
		9	29.95	9	50
		10	29.75	10	50
		11	29.15	11	50
		12	27.35	12	50
				13	50
				14	50
				15	50
				16	50

Determination of Inspection Location in a Production System

The procedure for the determination of optimal inspection location, developed in the preceding chapter, can also be used in this production environment.

As an illustration, consider the same example. Suppose it is desired to find the optimal inspection program R^*.

After $f_{2;(0,1)}(T_1)$ and $f_{2;(1,1)}(T_1)$ are determined (Table 16 and 19, respectively), the next step is to find r_2^*. It is found that

$$\underset{r_2}{\text{Max}}[E_{T_1}[f_{2;(1,1)}(T_1)],\ E_{T_1}[f_{2;(0,1)}(T_1)]] = \underset{r_2}{\text{Max}}[56.60,\ 51.21] = 56.60$$

Thus $r_2^* = 1$. The partial inspection program (1,1) is retained, and $f_{2;(1,1)}(T_1)$ is transformed into $f_{3;(0,1,1)}(T_0)$. $f_{3;(0,1,1)}(T_0)$ is then compared with $f_{3;(1,1,1)}(T_0)$ and $f_{3;(1,0,1)}(T_0)$. It is determined that

$$\underset{r_1}{\text{Max}}[E_{T_0}[f_{3;(1,0,1)}(T_0)],\ E_{T_0}[f_{3;(1,1,1)}(T_0)],\ E_{T_0}[f_{3;(0,1,1)}(T_0)]]$$

$$= \underset{r_1}{\text{Max}}[41.025,\ 42.788,\ 44.757] = 44.757$$

Thus $r_1^* = 0$ and $R^* = (0,1,1)$. It may be concluded that it is economical to locate screening inspection at stage 2, in addition to that at stage 3.

Variation 2 Analysis

For this variation, a manufacturing item that has defects more than the specification limit, when inspected, will be scrapped and sold for value $V_k'(V_k' < V)$. A new item will be reprocessed into regular processing. The decision problem considered here is the determination of optimal inspection program R^*.

Procedure for Determination of Optimal Inspection Program

1. Given V, $V_k'(k = 1,2,3,\cdots,N-1)$, use the procedure developed in Chapter IV to find R^*. Call this R^*, $R^{[0]}$. Find $E_{T_0}[f_{N;R^{[0]}}(T_0)]$.

2. Use this $E_{T_0}[f_{N;R^{[0]}}(T_0)] + V_k'$ as V_k, find R^*. Call this R^*, $R^{[1]}$. Find $E_{T_0}[f_{N;R^{[1]}}(T_0)]$.

3. Use $E_{T_0}[f_{N;R^{[1]}}(T_0)] + V_k'$ as V_k, find $R^{[2]}$ and $E_{T_0}[f_{N;R^{[2]}}(T_0)]$

4. Continue in this manner until at some iteration, say the ℓth iteration, $R^{[\ell]}$ is found. $E_{T_0}[f_{N;R^{[\ell]}}(T_0)]$ is determined. Suppose there is no change in the value of $E_{T_0}[f_{N;R^{[\ell-1]}}(T_0)]$ and $E_{T_0}[f_{N;R^{[\ell]}}(T_0)]$. Terminate the calculation. The optimal inspection plan for the system is $R^{[\ell]}$.

Summary

The environmental situation of producing to a fixed quota is considered in the chapter. It is subdivided into two variations. Variation 1 uses special processing in reprocessing a substitute. It is shown by an example, that the procedure developed, for the case of production from a fixed stock, can be applied for this variation too in determining the optimal inspection policy. Variation 2 uses regular processing for producing a new item. A procedure using successive iterations is suggested.

CHAPTER VI

DETERMINATION OF THE SEQUENCE FOR INSPECTION OF MULTIPLE DEFECT TYPES

Introduction

The objective of this chapter is to consider the case in which the manufacturing item may acquire multiple types of defects at any production operation of the system. A cost model associated with the testing sequence is developed. A procedure for obtaining the minimum cost inspection sequence is proposed, along with an illustrative example.

Statement of the Problem

Consider the manufacturing system in which M types of defects may be generated on an item at all production operations according to stationary Poisson distributions. The defect types are independently distributed, and are not mutually exclusive. Production operations are assumed to be mutually independent so that the number of defects acquired in one operation are independent of those acquired at any other operation. Furthermore, no defects are removed by subsequent production operations.

At an inspection station of any stage, the item is subjected to a sequence of inspection tests--one for each defect type. The inspection operation is terminated when more than L defects are found; otherwise, it will continue until all M defect types are inspected. It is assumed that the order of testing is insignificant from the technical standpoint, and that the inspection cost per item for each defect type is independent of the position of the test in the sequence. The decision problem is then concerned with determining the testing sequence which will result in the minimum expected inspection cost per item. This expected unit inspection cost associated with optimal testing sequence will then play the role of $a_k (k = 1,2,\cdots,N)$ does in the case of single defect type already discussed in Chapters IV and V.

Symbolic Formulation of the Problem

The following notations are necessarily introduced for the purpose of formulating the symbolic model of this decision problem:

y_{kj} = Number of jth type defects introduced by production operation k $(k = 1,2,\cdots,N-1;\ j = 1,2,3,\cdots,M)$. y_{kj} is generated according to stationary Poisson distribution.

y_{oj} = Number of type j defects coming with raw material, $(j = 1,2,\cdots,M)$. Y_{oj} is also Poisson distributed

w_{kj} = Mean of the distribution of y_{kj}, $(k = 0,1,2,\cdots, N-1;\ j = 1,2,\cdots,M)$.

y_j^{k-1} = Cumulative number of type j defects in an item through production operation k - 1, $(j = 1,2,\cdots,M)$.

Ω_j^{k-1} = Mean of the distribution of y_j^{k-1}, $(j = 1,2,\cdots,M)$.

Thus, the following relations will exist:

$$y_j^{k-1} = y_{0j} + y_{1j} + y_{2j} + y_{3j} + \cdots + y_{k-1,j} \qquad k = 1,2,\cdots,N \quad j = 1,2,\cdots,M \qquad (6\text{-}1)$$

$$\Omega_j^{k-1} = w_{0j} + w_{1j} + w_{2j} + \cdots + w_{k-1,j} \qquad k = 1,2,\cdots,N \qquad j = 1,2,\cdots,M \tag{6-2}$$

x_k, λ_k, Λ_k, T_k are defined as before. The following relationships prevail:

$$x_k = \sum_{j=1}^{M} y_{kj} \qquad \text{for } k = 0,1,2,\cdots,N-1 \tag{6-3}$$

$$\lambda_k = \sum_{j=1}^{M} w_{kj} \qquad \text{for } k = 0,1,2,\cdots,N-1 \tag{6-4}$$

$$T_{k-1} = x_0 + x_1 + x_2 + \cdots + x_{k-1}$$

$$= \sum_{j=1}^{M} y_{0j} + \sum_{j=1}^{M} y_{1j} + \sum_{j=1}^{M} y_{2j} + \cdots + \sum_{j=1}^{M} y_{k-1,j}$$

$$= \sum_{j=1}^{M} (y_{0j} + y_{1j} + y_{2j} + \cdots + y_{k-1,j})$$

Therefore, from (6-1)

$$T_{k-1} = \sum_{j=1}^{M} y_j^{k-1} \tag{6-5}$$

$$\Lambda_{k-1} = \lambda_0 + \lambda_1 + \lambda_2 + \cdots + \lambda_{k-1}$$

$$= \sum_{j=1}^{M} w_{0j} + \sum_{j=1}^{M} w_{1j} + \sum_{j=1}^{M} w_{2j} + \cdots + \sum_{j=1}^{M} w_{k-1,j}$$

$$= \sum_{j=1}^{M} w_{0j} + w_{1j} + w_{2j} + \cdots + w_{k-1,j}$$

Thus, from (6-2)

$$\Lambda_{k-1} = \sum_{j=1}^{M} \Omega_j^{k-1} \tag{6-6}$$

Figure 10 shows the graphical representation of these notations. The circles represent production operations of the system. Above the production line are the defects of M types generated at each stage. The mean of the defects are shown correspondingly below the line.

Let

a_{kj} = cost per item for inspecting type j defect in the kth stage, $(k = 1,2,\cdots,N;\ j = 1,2,\cdots,M)$. Assume $a_{kj} > 0$ for all k's and j's.

Q = testing sequence composing M elements.

= {[1], [2], [3], ···, [j], ···, [M]}, where [j] is the type of defect inspected in the jth position of the testing sequence.

For any inspection sequence, Q, the expected cost per unit would be:

$$a_k(Q) = a_{k,[1]} \cdot 1 + a_{k,[2]} \cdot \Pr[y_{[1]}^{k-1} \le L] + a_{k,[3]} \tag{6-7}$$

$$\cdot \Pr[y_{[1]}^{k-1} + y_{[2]}^{k-1} \le L] + \cdots + a_{k,[j]} \Pr[y_{[1]}^{k-1} + y_{[2]}^{k-1} + \cdots$$

$$+ y_{[j-1]}^{k-1} \le L] + \cdots + a_{k,[M]} \Pr[y_{[1]}^{k-1} + y_{[2]}^{k-1} + \cdots y_{[M-1]}^{k-1} \le L]$$

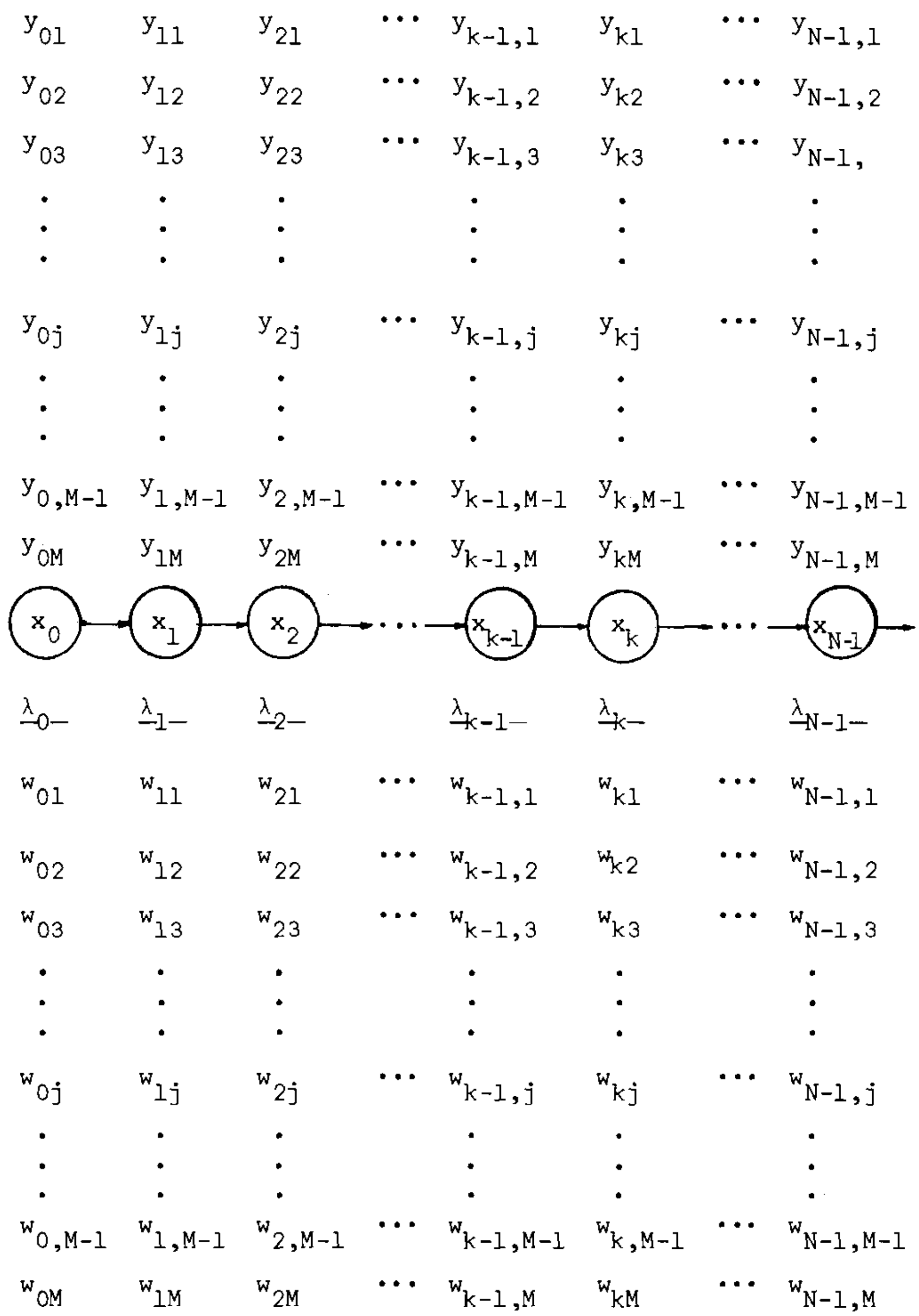

Figure 10. Graphical Representation of Multiple Defect Types

In words, $a_k(Q)$ may be equivalently expressed as

$$a_k(Q) = \sum_{j=1}^{M} a_{k,[j]} \cdot \text{Pr[No rejection through test [j-1]]}$$

or,

$$a_k(Q) = \sum_{j=1}^{M} a_{k,[j]} \cdot \Pr[\sum_{\ell=0}^{j-1} y_{[\ell]}^{k-1} \leq L] \tag{6-8}$$

where,

$$\Pr[y_{[0]}^{k-1} \leq L]^{1} = 1$$

Let Q^* = minimum cost inspection sequence.

In other words,

$$a_k(Q^*) < a_k(Q) \text{ for all } Q$$

One trivial way to find Q^* is through the complete enumeration method. One would have to calculate the cost per item inspected of all M! possible sequences. The sequence giving the least cost is selected as Q^*. For large M, this calculation is to be time-consuming and, for very large M, not feasible. Thus, it is necessary to develop some

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[1]This is due to the fact that for test [1], regardless of the number of defects found, cost $a_{k,[1]}$ is certain to incur.

procedure to find Q^* with less computational efforts.

It should be observed that there are two factors involved in the cost model developed. The first one would be a_{kj}, the unit cost for inspection for each defect type. One would be inclined to select the defect type with the smallest a_{kj} for the first position, the one with the second smallest for the second position, ..., and the defect type with largest a_{jk} for the Mth position. However, one cannot overlook the second factor--the probability of no rejection through test [j-1]. One would be motivated to select defect types successively such that more than L defects will be found as soon as possible. Also implicit in this factor is that, even though the selection of defect type for position [j] is made, its probability property cannot be utilized right away, but is delayed for use in the [j+1]th term. The procedure to select Q^*, therefore, would have to take these two factors into consideration.

Procedure for Selecting Minimum Cost Inspection Sequence

Before the procedure is stated, it is necessary to state and prove the following lemma.

Lemma

In a stationary Poisson process, for a fixed argument, the increase in value of the parameter will result in decreasing the value of the cumulative distribution function.

Proof of the Lemma. Let A and B be independently Poisson distributed with mean λ_1 and λ_2, respectively. Assume than λ_2 is greater than λ_1. L is a positive integer.

$$\Pr[A \le L] = \sum_{A=0}^{L} \frac{e^{-\lambda_1} \lambda_1^A}{A!}$$

According to Haight [4, p. 2],

$$\Pr[A \le L] = \sum_{A=0}^{L} \frac{e^{-\lambda_1} \lambda_1^A}{A!} = \frac{\Gamma(L+1,\lambda_1)}{\Gamma(L+1)}$$

where

$$\Gamma(L+1,\lambda_1) = \int_{\lambda_1}^{\infty} e^{-t} t^L dt$$

$$\Gamma(L+1) = \int_{0}^{\infty} e^{-t} t^L dt$$

Similarly,

$$\Pr[B \le L] = \sum_{B=0}^{L} \frac{e^{-\lambda_2} \lambda_2^B}{B!} = \frac{\Gamma(L+1,\lambda_2)}{\Gamma(L+1)}$$

where

$$\Gamma(L+1,\lambda_2) = \int_{\lambda_2}^{\infty} e^{-t} t^L dt$$

$$\Gamma(L+1) = \int_{0}^{\infty} e^{-t} t^L dt$$

Thus,

$$\Pr[A \le L] - \Pr[B \le L] = \frac{\Gamma(L+1,\lambda_1)}{\Gamma(L+1)} - \frac{\Gamma(L+1,\lambda_2)}{\Gamma(L+1)}$$

$$= \frac{\Gamma(L+1,\lambda_1) - \Gamma(L+1,\lambda_2)}{\Gamma(L+1)}$$

$$= \frac{\int_{\lambda_1}^{\infty} e^{-t}\, t^L dt - \int_{\lambda_2}^{\infty} e^{-t}\, t^L dt}{\Gamma(L+1)}$$

$$= \frac{\int_{\lambda_1}^{\lambda_2} e^{-t}\, t^L dt}{\Gamma(L+1)} > 0$$

Therefore, $\Pr[A \le L]$ is greater than $\Pr[B \le L]$, and the lemma is proved.

The procedure for determining the least cost inspection sequence, Q^*, may be described as follows:

1. For the first position, [1], one will have to determine the quantity:

$$a_{kj}[1-\Pr[y_i^{k-1} \le L]] \qquad \text{for } j \ne i, \quad j = 1,2,3,\cdots,M$$

and then find j that satisfies:

$$a_{kj}[1-\Pr[y_i^{k-1} \le L]] \le a_{ki}[1-\Pr[y_j^{k-1} \le L]] \tag{6-9}$$

Assign such j to [1].

2. For jth position of the inspection sequence, $([j]\varepsilon\{[2],[3],\cdots,[M-1]\})$, evaluate:

$$a_{kj}[\Pr[\sum_{\ell=0}^{j-1} y_{[\ell]}^{k-1} \le L] - \Pr[\sum_{\ell=0}^{j-1} y_{[\ell]}^{k-1} + y_i^{k-1} \le L]]$$

for $j \neq i$ and both j and $i \notin \{[1],[2],\cdots[j-1]\}$.

Find j that satisfies the condition:

$$a_{kj}[\Pr[\sum_{\ell=0}^{j-1} y_{[\ell]}^{k-1} \le L] - \Pr[\sum_{\ell=0}^{j-1} y_{[\ell]}^{k-1} + y_i^{k-1} \le L]] \qquad (6\text{-}10)$$

$$\le a_{ki}[\Pr[\sum_{\ell=0}^{j-1} y_{[\ell]}^{k-1} \le L] - \Pr[\sum_{\ell=0}^{j=1} y_{[\ell]}^{k-1} + y_j^{k-1} \le L]]$$

Designate that j to [j].

3. Use the elimination method to identify the test that does not belong to the set $\{[1],[2],[3],\cdots,[M-2],[M-1]\}$, and assign it to the last position, [M].

It should be noted that, for step 1, if $\Pr[y_i^{k-1} \le L]$ is 1, then use only a_{kj} in comparison.

For step 2, if both $\Pr[\sum_{\ell=0}^{j-1} y_{[\ell]}^{k-1} \le L]$ and $\Pr[\sum_{\ell=0}^{j-1} y_{[\ell]}^{k-1}\, y_i^{k-1} \le L]$ are equal to 1, then use a_{kj} only in comparison.

Whenever $\Pr[\sum_{\ell=0}^{j-1} y_{[\ell]}^{k-1} \leq L]$ of step 2 is zero[2], it implies that L defects are found and the testing sequence is terminated.

Proof

Consider a sequence Q not formed according to the procedure given above. In particular, assume that Q contains tests for inspecting defect types J and J + 1 in positions [i] and [i + 1], respectively[3], such that

$$a_{kJ}[\Pr[\sum_{\ell=0}^{i-1} y_{[\ell]} \leq L] - \Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} + y_{J+1}^{k-1} \leq L]]$$

$$> a_{k,J+1}[\Pr[\sum_{\ell=0}^{i-1} y_{[\ell]} \leq L] - \Pr[\sum_{\ell=0}^{i-1} y_{J}^{k-1} \leq L]].$$

Assume Q and Q^* otherwise identically formed.

By hypothesis,

$$a_{kJ}[\Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} \leq L] - \Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} + y_{J+1}^{k-1} \leq L]] \tag{6-11}$$

$$- a_{k,J+1}[\Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} \leq L] - \Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} + y_{J}^{k-1} \leq L]] > 0.$$

---

[2]Theoretically, $\Pr[\sum_{\ell=0}^{j-1} y_{[\ell]}^{k-1} \leq L]$ is never zero, because of the property of Poisson random variable. However, it may be very close to zero. One way to justify this is to use a standard Poisson Table.

[3]Such tests J and J + 1 must exist or Q and Q^* would be identical.

Contributions to $a_k(Q)$ by terms associated with tests J and J + 1 in positions [i] and [i+1], respectively, are:

$$[i] \qquad a_{kJ} \Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} \leq L]$$

$$[i+1]: \qquad a_{k,J+1} \Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} + y_J^{k-1} \leq L]$$

For Q^*, according to the foregoing procedure, tests J + 1 and J are in positions [i] and [i + 1], respectively. Similarly, contributions to $a_k(Q^*)$ by terms associated with tests J and J + 1 are:

$$[i]: \qquad a_{k,J+1} \Pr[\sum_{\ell=0}^{i=1} y_{[\ell]}^{k-1} \leq L]$$

$$[i + 1]: \quad a_{kJ} \Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} + y_{J+1}^{k-1} \leq L]$$

Therefore,

$$a_k(Q) - a_k(Q^*) = a_{kJ}[\Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} \leq L] - \Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} + y_{J+1}^{k-1} \leq L]]$$

$$+ a_{k,J+1}[\Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} + y_J^{k-1} \leq L] - \Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} \leq L]]$$

or,

$$a_k(Q) - a_k(Q^*) = a_{kJ}[\Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} \leq L] - \Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} + y_{J+1}^{k-1} \leq L]]$$

$$- a_{k,J+1}[\Pr[\sum_{\ell=0}^{i-1} y_{[\ell]} \leq L] - \Pr[\sum_{\ell=0}^{i-1} y_{[\ell]}^{k-1} + y_J^{k-1} \leq L]]$$

According to (6-11), $a_k(Q) - a_k(Q^*) > 0$. Thus, the inspection sequence obtained by the foregoing procedure will yield the minimum expected cost per item.

Illustration

Consider the production system, in which three types of defects (i.e., $M = 3$) are generated at any production operation. At a particular stage k, suppose the data are:

$$\Omega_1^{k-1} = 2 \qquad a_{k1} = 3 \qquad L = 7$$

$$\Omega_2^{k-1} = 4 \qquad a_{k2} = 5$$

$$\Omega_3^{k-1} = 6 \qquad a_{k3} = 7$$

To identify j, ($j = 1,2,3$), for [1], it is determined that

$$a_{k1}[1 - \Pr[y_2^{k-1} \leq 7]] = (3)(0.051) = 0.153$$

$$a_{k1}[1 - \Pr[y_3^{k-1} \leq 7]] = (3)(0.256) = 0.768$$

$$a_{k2}[1 - \Pr[y_1^{k-1} \leq 7]] = (5)(0.001) = 0.005$$

$$a_{k2}[1 - Pr[y_3^{k-1} \le 7]] = (5)(0.256) = 1.280$$

$$a_{k3}[1 - Pr[y_1^{k-1} \le 7]] = (7)(0.001) = 0.007$$

$$a_{k3}[1 - Pr[y_2^{k-1} \le 7]] = (7)(0.051) = 0.357$$

Check if $a_{k1}[1-Pr[y_2^{k-1} \le 7]] \le a_{k2}[1 - Pr[y_1^{k-1} \le 7]]$, but 0.153 is greater than 0.005. Thus, the inequality does not hold; type 1 defect is not considered for position 1.

Next, check if $a_{k2}[1 - Pr[y_3^{k-1} \le 7]] \le a_{k3}[1 - Pr[y_2^{k-1} \le 7]$, but 1.280 is greater than 0.357. Therefore, type 2 defect is not considered for position 1.

This leaves only type 3 defect to be considered for [1]. It is necessary to compare type 1 and type 3. The condition $a_{k3}[1 - Pr[y_1^{k-1} \le 7]] \le a_{k1}[1 - Pr[y_3^{k-1} \le 7]]$ holds, for 0.007 is less than 0.768.

Since type 3 defect satisfies Equation (6-9), it is assigned to position 1. Thus, [1] = 3.

For position 2, it is found that:

$$a_{k1}[Pr[y_3^{k-1} \le 7] - Pr[y_3^{k-1} + y_2^{k-1} \le 7]] = (3)[0.744-0.220] = 1.572$$

$$a_{k2}[Pr[y_3^{k-1} \le 7] - Pr[y_3^{k-1} + y_1^{k-1} \le 7]] = (5)[0.744-0.453] = 1.455$$

According to (6-10), designate type 2 defect into position 2 of the inspection sequence, i.e., [2] = 2.

Using elimination method, type 1 defect is assigned to position 3.

Therefore: $Q^* = \{3,2,1\}$

$$a_k(Q^*) = a_{k3} + a_{k2}\,Pr[y_3^{k-1} \leq 7] + a_{k1} Pr[y_3^{k-1} + y_2^{k-1} \leq 7]$$

$$= 7 + 5(0.744) + 3(0.220)$$

$$= 11.380$$

As a matter of assurance, the $a_k(Q)$ for all possible inspection sequences are calculated and tabulated below:

Q	$a_k(Q)$
{1,2,3}	13.203
{1,3,2}	12.258
{2,3,1}	12.303
{2,1,3}	13.055
{3,1,2}	11.497

As it is seen, no other sequence is better than $Q^* = \{3,2,1\}$

Savings in Computational Effort

It may be of interest to determine the savings in computational effort that may be attained through the use of the procedure. Assume that computing each term of Equation (6-7) represents one unit of computational effort, and so does each term in steps 1 and 2 of the procedure. The complete enumeration of all possible inspection sequences would mean a total computing effort of (M!)(M) units. With the use of the procedure, it will require $\frac{1}{3}M(M^2-1)$ computing units to determine the minimum cost inspection sequence. Comparisons between the two methods, for some selected value of M, are tabulated as follows:

M	(M!)M	$\frac{1}{3}(M)(M^2-1)$
2	4	2
3	18	8
4	72	20
5	360	40
6	2160	70
7	15120	112

It may be apparent that the savings in computational effort would be even greater for larger value of M.

Summary

Initially presented in this chapter was the framework of the production system in which multiple types of defects may be generated at any operation. A model was developed to find the expected cost per item for a sequence of inspection. The development of a procedure to determine the optimal inspection sequence was then presented, along with an illustration. The procedure is efficient in comparison with the complete enumeration method.

It was pointed out that the decision problem of determining optimum inspection sequence can be performed separately from those of locating inspection points, and finding specification limits. All are related in the sense that the expected inspection cost per item associated with an optimal testing sequence, $a_k(Q^*)$, will have to be determined before the other two decision problems could be solved.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

Conclusions evolved from this study are summarized as follows:

1. In general, there is an economic advantage in tightening the specifications for in-process screening inspections.

2. For a given inspection program, a dynamic programming approach can be used to determine the most economical specification limits. The computational procedure is not complex and is practical. Savings in computing effort can be obtained in the case where there are some runs of no-inspection stages in the inspection program. It is observed that, even though identical decision rules are used for final inspection operation, the two production environments considered result in different specification limits.

3. Dynamic programming methods can be utilized as the basis for determining the economically optimal locations of inspection points in a production system--that is, the optimal inspection program. The computational scheme involved is practical, and easily implemented. Decision tree diagrams prove to be an appropriate way to illustrate the structure of the inspection location problem. It is noted that this class of decision problem is interrelated with that of finding specification limits.

4. The procedure for determination of the minimum cost inspection sequence proves to be useful, since significant savings in computational effort are realized in comparison with the complete enumeration approach. It is concluded that this inspection sequence problem can be treated separately from the other two decision problems.

Recommendations

In the course of carrying out this study, some potentially useful areas of research, relating to in-process inspection activity, were found and are listed below:

1. In this study, consideration was given only to product control inspection wherein inspection is performed for the purpose of determining the disposition of the product. However, it should be realized that information obtained from such inspection may be of some value in the control of production process. Further study concerning process control would be of interest.

2. It is assumed in this research that no defects are removed by subsequent production operations. It would be useful to investigate the problem when this assumption is relaxed. It is probable that more information about the process would be required.

3. It would prove valuable to study the case in which multiple types of defects exist and their degrees of severity are not the same.

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VITA

Pakorn Adulbhan, the only son of Mr. Aree and Mrs. Lamai Adulbhan, was born on November 1, 1937 in Bangkok, Thailand. He attended the Bangkok Christian High School through the tenth grade, and completed his high school education at The Pre-University High School in March, 1956.

In June, 1956 he enrolled in the College of Engineering, Chulalongkorn University in Bangkok and graduated in March, 1960 with the degree of Bachelor of Science in Industrial Engineering (BSIE). During the summers as an undergraduate student he worked for the Fang Oil Refinery of Department of Defense, and the Carrier Air Conditioning and Refrigeration Corporation (Thailand) in Bangkok. In order to broaden his background, Mr. Adulbhan decided to pursue another degree in the field of Mechanical Engineering in the same Engineering College. He graduated in March, 1961 with the degree of Bachelor of Science in Mechanical Engineering (BSME).

In January, 1961 he joined the faculty of the Department of Industrial Engineering of Chulalongkorn University as a Junior Instructor. His teaching assignments were in the areas of manufacturing processes and engineering drawing.

In May, 1962, Mr. Adulbhan won the Ford Motor Company International Fellowship for a one-academic-year graduate study in the United States. He enrolled in the Department of Industrial Engineering and

Management Sciences of Northwestern University, Evanston, Illinois in September, 1962. In the summer of 1963, Northwestern University awarded him Walter P. Murphy Fellowship. The degree of Master of Science in Industrial Engineering was conferred on him by Northwestern in June, 1964. A thesis entitled *The Role of Research and Development in the Industrialization of Selected Southeast Asia Countries* was written by him under the direction of Dr. Albert H. Rubenstein, Professor of Industrial Engineering and Management Sciences.

He then enrolled in the Ph.D. program of the School of Industrial Engineering at the Georgia Institute of Technology, and at the same time worked as a graduate teaching assistant. On July 9, 1965, he was married to Darika Mahawan.

In 1966, Mr. Adulbhan joined the faculty of the School of Industrial Engineering of the Georgia Institute of Technology as an Instructor. His teaching assignments were in the areas of engineering statistics, operations research, and engineering economy. He is a member of the American Association for Engineering Education, the Institute of Management Sciences, and the Operations Research Society of America.

Mr. Adulbhan will rejoin the faculty of the Department of Industrial Engineering, Chulalongkorn University in Fall, 1968.