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DEVELOPMENT OF AN INVENTORY MODEL FOR HOSPITAL SUPPLIES

A THESIS

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DEVELOPMENT OF AN INVENTORY MODEL

FOR HOSPITAL SUPPLIES

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SUMMARY

The objective of this study was to provide decision rules for determining optimal purchase quantities and reorder points for hospital supplies. A decision model that considers the following factors was constructed:

1. The distribution of the demand for a single supply item.
2. The distribution of the time to send out and receive orders (lead time distribution).
3. The purchase cost of the item.
4. The costs associated with ordering the item.
5. The costs associated with carrying the item in inventory.

Inventory costs were selected as the measure of effectiveness. To optimize this measure of effectiveness, hospital administrators are herein provided with decision rules which will determine:

1. Economic lot sizes.
2. Reorder points.
3. Protective stock levels.
4. Cost of various inventory policies.

Data was collected at Emory University Hospital, Atlanta, Georgia, on a specific item, surgical rubber gloves, and the operational characteristics of this model were evaluated.

The demand was found to be approximately normally distributed and the lead time was found to vary. Insufficient data on lead time required that an assumption be made as to the nature of this distribution. The Poisson distribution was selected for this purpose.

With information of the two distributions and other relevant factors, it was possible to construct a statistical inventory model to optimize the measure of effectiveness.

Results of this study were as follows:

1. To improve the accuracy of the model all relevant factors should be considered where possible.
2. The inventory records of the hospital frequently lack the necessary information for a complete solution by the model.
3. The decision model can provide an accurate guide to evaluate the various inventory policies.

It is recommended that further study be conducted on this subject in the areas of:

1. Determining methods for accurately estimating ordering costs and inventory carrying costs.
2. Extending the table of reorder points calculated from the joint density function of two Poisson distributions.
3. Calculating tables of reorder points for other typical types of demand and lead time distributions.

CHAPTER I

INTRODUCTION

The objective of this study is to provide decision rules for determining optimal purchase quantities and reorder points for hospital supplies. Inventory costs are used as the measure of effectiveness. Analysis of pertinent literature and discussions with certain hospital administrators indicate a need for improving inventory policies.

The administrators of some hospitals apparently have not taken advantage of even the most elementary inventory tools to help solve their inventory control problems. Many hospitals determine how much to buy on the basis of purchase price, i.e., a cost break motivation. Fair (1)[1] states in his article, "When is Quantity a Good Buy?":

> Price is always the primary factor in determining how large a quantity of any given item we are buying, since the dollar saving economics invariably go along with quantity purchases of any supplies or materials by every hospital. But, many an executive has learned to his sorrow that price can never be the sole determining factor. Where nothing else is considered losses invariably result.

Fair further points out several factors that should be considered in addition to price. These factors are as follows:

1. Rate of use.
2. Danger of obsolescence.
3. Deterioration in storage.
4. Future changes in market price.

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[1]Numbers in parenthesis refer to references listed in bibliography.

5. Earning capacity of capital funds.

6. The storage factor.

7. Added insurance costs.

A second inventory policy, i.e., setting buffer stocks, that has been observed as common practice in some hospitals is that of setting minimum inventory levels on the experience factor of maximum demand per time unit. It seems that these reorder points are rarely adjusted downward. The usual practice is to set a reorder point which is maintained until the demand reaches a new maximum and the reorder point is reset upward.

It is felt that the present trend in type of hospital supplies[2] makes the necessity for correct inventory decisions more acute. The trend seems to be to eliminate the use of reprocessed type supply items and increase the use of disposable items. This change in type of supply items may increase inventories by a factor of from five to twenty, (i.e., the reprocessing factor) depending on the kind of supply item.

This change in type of supply item will create a need for the closer control of inventories because a greater risk of shortage may be encountered using disposable items. At present, if a reprocessable item is in short supply, the reprocessing schedule can often be shortened to provide the necessary item when needed. This is not possible with disposable items because they are discarded after use.

To optimize the measure of effectiveness, hospital administrators

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[2]Smalley, Harold E., Tentative Plans for a Study of Hospital Cost Systems, Bulletin No. 1, Engineering Experiment Station, Georgia Institute of Technology, Atlanta, Georgia, January 1959.

need decision rules that will determine:

1. Economic lot sizes.
2. Reorder points.
3. Protective stock levels.
4. Cost of various policies.

In the development of inventory decision models it is a common procedure to assume a constant lead time. On the other hand, the demand function has been subjected to considerable investigation for both variable and constant conditions. The assumption of a constant lead time, under real conditions of uncertainty, can lead to serious errors in determining the reorder point. Brown (2) has recently proposed a method for handling variable lead time. Since Brown's method is not well defined and utilizes a correlation between order quantity and lead time, it was necessary to develop a new method for this study. The reorder point model developed in this study is based on a statistical evaluation of the distributions of both demand and lead time. A joint density function of the actual demand and lead time is then used to determine the proper reorder points at various levels of probability of a shortage. This reorder point also defines the protective stock level.

To evaluate the operational characteristics of this model a typical supply item, surgical rubber gloves, was selected at Emory University Hospital in Atlanta. While complete operating data was not available to rigorously test this model, it was possible to use certain historical data to approximate costs associated with the use of this model. The model can be tested against the control variable of future inventory costs by collecting operating data concerning this item. The actual testing of the model is not included as a part of this study.

It is assumed that the persons using this model to solve hospital inventory supply problems will be able to evaluate demand and lead time distributions statistically.

To understand how statistical inventory control will help in answering the hospital administrators' questions of, "How much of a supply item to buy?," "When to buy this item?," and "How much of this item to keep on hand?"; one must first understand the general concepts of inventory control. These general concepts will be explained in terms of answers to the above three questions.

Inventory control answers the question of "How much of a supply item to buy?" by finding the order size that gives the lowest total cost. These are the costs associated with placing the order (or ordering costs) and the costs associated with maintaining the inventory. Figure 1 shows how these costs vary in a typical situation.

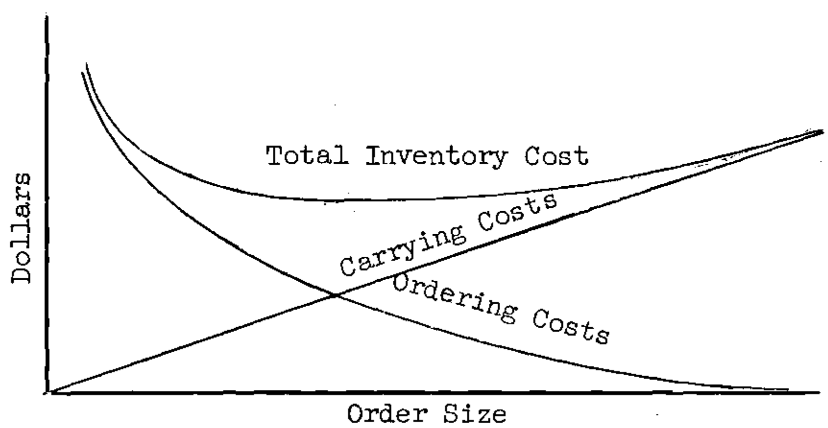

Figure 1 - Typical Inventory Costs

The third curve or total inventory cost curve that appears in Figure 1 shows the sum of these two opposing costs. Inventory control provides a method of finding the order quantity which will give the minimum total cost.

The other two questions that inventory control answers are, "When should this item be bought?" and "How much of this item should be kept on hand?". The answers to these questions require information relative to the demand, to the lead time, and to the level of shortage which will be tolerated. With this information these questions can be answered. Inventory control determines the level of stock (i.e., the reorder point) that must be on hand when an order is placed to insure that the hospital will not have a "stock out" more frequently than indicated by a chosen probability level of a shortage.

Since the early 1950's there has been a remarkable advancement in the development of tools to answer these and other difficult inventory questions. The developments prior to the 1950's and until about 1952 have been summarized and augmented by Whitin (3). Arrow, Karlin and Scarf (4) present a summary of the economic theory involved in inventory control and develop many new mathematical techniques for handling inventory problems with a special emphasis on stochastic inventory processes. Churchman, Ackoff and Arnoff (5) present summaries of the various inventory models developed between the publication of Whitin's book and the recent publication of Arrow, Karlin and Scarf. Some of these models have been designed to accommodate variable costs or price breaks as they are normally called. The majority of these models were developed by people in Operations Research. More Operations Research has been directed toward inventory control than toward any other problem area in business and industry.[3]

Bowman and Fetter (6) present a development of inventory models under two conditions. The first of these conditions is certainty; i.e., situations in which inventory control variables are assumed to be constant. The second condition is uncertainty, i.e., situations in which the inventory control variables are not constant. Morse (7) shows how queueing theory can be utilized to analyze the effects of the variance of supply and demand under certain restricted conditions. Vasonyi (8) presents a summary of statistical inventory control and develops the mathematics necessary to handle the problem of variable demand. He also developes cost equations similar to those in most of the other literature cited. Welch (9) has made an attempt to present many of the concepts of inventory control in the language of elementary mathematics for use by supervisors in business and industry.

This study incorporates many of these principles in the construction of an inventory model appropriate for hospital use. Evaluation of the lead time and demand distributions with subsequent analysis of their joint density function to predict the probability of a shortage at selected reorder points is of importance in the development of this inventory model.

The joint frequency function as shown in Figure 5 is composed of two components of variation--the distribution, $f(x)$, associated with lead time (Poisson) and the weighted conditional demand distribution $f(D_x)$ given a particular lead time (normal).

The summation of the areas for all of these weighted conditional

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[3]Churchman, op. cit., p. 195

demand distributions (normal) over all possible lead times will equal one. The area associated with any lead time is the probability of that lead time occurring. For a particular reorder point the summation of the area from the reorder point over all possible demand in excess of the reorder point for all possible lead times will give the probability of a shortage.

CHAPTER II

EXPERIMENTAL PROCEDURE

In an attempt to determine if any of the inventory models found in the current literature were applicable to the hospitals' inventory problem, it was first necessary to find a hospital in the Atlanta area that would cooperate in providing the necessary information, then to select a typical supply item, gather data concerning this item, and investigate the feasibility of the model selected.

Selection of Supply Item.-- Emory University Hospital agreed to provide the necessary information if it was available in the existing records of the hospital. It was felt that a typical item should be selected for this study. The criteria used to select this typical item were:

1. Intermediate volume of use.
2. Intermediate purchase cost.
3. Necessary information available.

After examining various items in use at Emory University Hospital and determining the availability of the necessary information, surgical rubber gloves were selected as the supply item to be used in testing the inventory models.

Data Collection.-- The demand for gloves was obtained from requisitions at the hospital. This information was collected for a continuous period of 66 weeks and was collected for all sizes of gloves. The data was later enlarged to include a period of 170 weeks. An attempt was made to fit the demand distribution for each size of glove to some theoretical distribution. It was hoped that each size of glove would fit the same theoretical distribution and would differ only in respect to parameters for each size.

An attempt was made to determine the distribution and variability of the lead time. This was not successful since the purchasing records were incomplete with respect to lead time data. However, from the existing information it was found that the lead time does vary. The average of the lead times for five dated orders that were available was found to be one week. This was corroborated by the Purchasing Department personnel as they felt that it took about one week to obtain gloves. The main point is that the assumption of a constant lead time is not appropriate for this inventory model.

Other information that was needed was the inventory carrying cost and the ordering cost. These two costs were estimated with the help of the hospital personnel. Exact costs were impossible to obtain for various reasons. Inventory carrying costs include the cost of storage area and interest on money invested in inventory. These two costs were not known exactly by the hospital administration. However, it is felt that 20 per cent per year of the first cost of the item being studied was in the right range.[1] Emory University Hospital is not an autonomous unit; therefore, the ordering cost of interest to the hospital personnel is only the portion of the cost borne by the hospital directly. If Emory University were considered as a whole, this ordering cost would be higher as the actual work

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[1] Whitin, op. cit., p. 220

of sending out, receiving and accepting bids is performed by the purchasing department of the University. The paper work necessary to pay for the item once it has been received is also performed by this purchasing department. However, it was felt that consideration of these factors would only complicate the problem situation and would detract from the general nature of the study within the time limits of this study. The ordering cost to be used in calculations is arbitrarily set at five dollars. Regardless of the specific values obtained in practice for inventory carrying cost and ordering cost the operational characteristics of this model are valid.

The last factor investigated was the cost of a shortage. In the beginning it was felt that it would be important to know the cost of a shortage, but since this cost was unavailable, it was decided that it would be more practical to determine the cost of added protection. Hospitals have a unique problem in determining this cost. For example, what does it cost to put off an operation or to operate with bare hands if no gloves are available? It could cost the life of the person being operated upon, or it could delay this person's recovery. A determination of these costs on a probability basis is beyond the scope of this study. Therefore, only the real costs incurred at the various levels of risk of a shortage will be considered here.

<u>Model Construction</u>.-- A chi-square test was used to compare the actual demand distribution for each size of glove with the theoretical Poisson distribution having the same mean as the observed data. The hypothesis that the observed demand distributions were Poisson distributed with the same mean as the observed data was accepted for five of the seven sizes of gloves at the five per cent significance level. See Table 18 for actual demand for gloves during 170 weeks.

Since all sizes of gloves did not fit the same type of distribution, a different approach was tried. It was observed that the proportion of each size glove did not vary appreciably, regardless of the total demand for the week. Therefore, it was decided to test the hypothesis that the proportion of each size of glove used each week was independent of the total demand for the week. A chi-square contingency table test was used, e.g. as is presented in Hoel (10). The hypothesis was accepted at the five per cent significance level; therefore, only the total demand for gloves per week would be considered at this point.

With the economic lot size determined for the total gloves needed, the order was prorated by sizes using the proportions to determine what amount of the order would be of each size. This is possible because quantity discounts are based on total gloves ordered rather than on the total for each size. This total demand distribution was compared with the theoretical Poisson distribution having the same mean as the observed distribution. The hypothesis that the observed distribution was from a Poisson distributed population was rejected at the five per cent significance level but could be accepted at the two per cent level. The demand distribution was then compared with the theoretical normal distribution having the same mean and standard deviation as the observed distribution. The hypothesis that the observed distribution was from a normally distributed population could be accepted at the five per cent level of significance. However, it was felt that the sample size of 66 weeks was too small to give conclusive results. The sample size was increased to 170 weeks and the same hypotheses was again tested. The hypothesis that the observed total demand distribution was from a Poisson distributed population was rejected at the five per cent level and up to the one-tenth of one per cent level. The normal hypothesis was again accepted at the five per cent level. With an approximation of the actual demand distribution with a theoretical distribution and the information concerning inventory carrying costs and ordering costs, it was possible to use a "two-bin" inventory model such as the one found in Vazsonyi.[2] A "two-bin" system of inventory control provides an active stock and a lead time stock. These two stocks need not be physically separated into two bins. This inventory model shows the level of inventory at which a new order should be placed. The stock that is on hand when the new order is placed is known as the reorder point. This stock will fill the demand during the lead time with a specified probability of a shortage. This model also provides equations for determining the inventory cost per year.

The model at first appeared to meet all the requirements of the hospital for solving its inventory problem. However, after observing the variability that existed in the lead time, it was felt that the reorder point was incorrect and would produce a much higher level of shortage than would be tolerated under the assumption of a constant lead time. On theoretical grounds the distribution was assumed to be Poisson, although there were insufficient data for testing this hypothesis. On the assumption that these supply item orders have discrete lead times whose mean value is constant, e.g., independent of order size, then the further assumption that the duration of these lead times are random and independent leads to the

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[2]Vazsonyi, op. cit., pp. 330-338

use of the Poisson distribution as the model for this variable. With an assumed theoretical distribution to describe the lead time and another distribution to describe the demand, the joint density function of the two was then evaluated to determine the reorder points for various levels of probability of a shortage.

Controls are provided which will show a significant shift in the proportion of each size of glove used. Controls also are provided which will detect significant shifts in the mean and variability of both the demand distribution and the lead time distribution. With information on the reorder point, the protective stock level could then be calculated, and the related carrying costs evaluated. This completes the information required to use this inventory model.

<u>Calculation of Other Reorder Point Curves</u>.-- In an attempt to provide hospital administrators with another guide to reorder points, curves were calculated from the joint density function of two Poisson distributions. An attempt was made to utilize the IBM 650 computer for compiling this information. A workable program was written using an interpretive system but was found to be no faster than a desk calculator when used in conjunction with Molina's (11) Poisson's Exponential Binomial Limit Tables. Due to this fact the computer was not used for calculations within the range of the tables. However, an attempt was made to utilize the computer for the calculation of curves beyond the range of the tables. This procedure had to be abandoned after the calculation of two curves, as the computer routines immediately accessible do not raise "e" to a sufficiently high power. Therefore, the range of the tables for conditions of variable demand and variable lead time was restricted by both the computer program and time.

CHAPTER III

ANALYSIS OF DATA AND RESULTS

The Demand Distribution.-- The first approach that was used in determining the demand distribution was to examine the requisitions for each size of glove for the period, December 30, 1957 - March 30, 1959. The demand for each size of glove was tabulated by weeks for this 66 week period. This demand data was first plotted by weeks to determine if there were seasonal fluctuations in the demand for each size of glove. The demand fluctuations for each size of glove were comparable to those for the total gloves as shown in Figure 2. No seasonal fluctuations were observed. An $\bar{X}$ control chart was used to check the stability of total demand with respect to time, and it was found to be in control. (See Figure 7).

The actual demand distribution for each size glove was next compared with a theoretical Poisson distribution having the same mean as the observed distribution. Table 2 shows the observed demand and expected demand from Poisson distributed population. A chi-square test was used to test the hypothesis that the observed distributions were from Poisson distributed populations. The results are shown in Table 1. With sizes six and one-half and eight the hypothesis that the actual distribution was from a Poisson distributed population was rejected at the five per cent level of significance. With the other sizes the test gave no reason to reject the hypothesis that the observed distributions were from Poisson distributed populations with the same mean as the observed data at the five per cent level of significance.

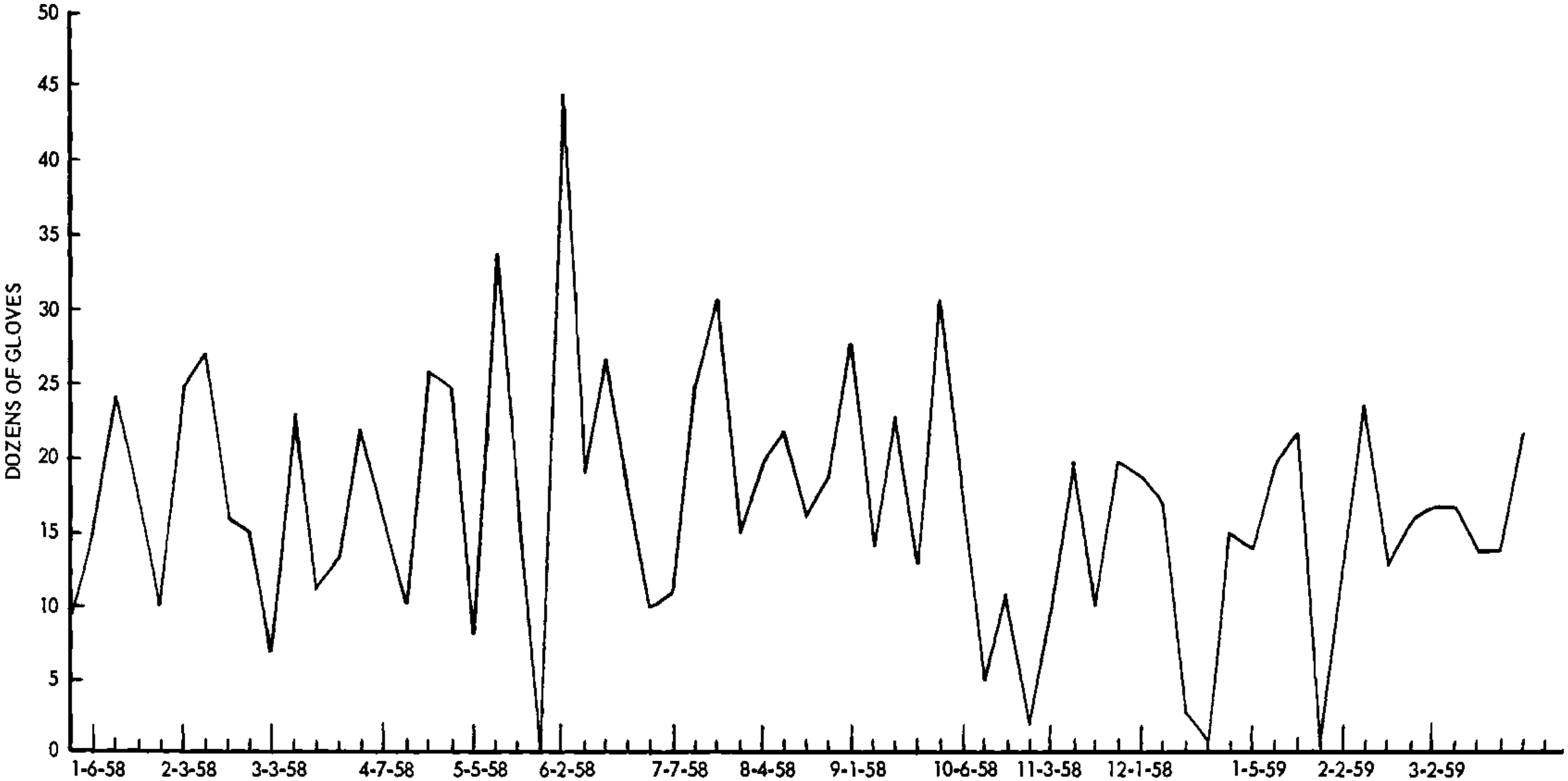

Figure 2. Variation of Total Gloves Requested Per Week from the Store Room.

Table 1. Results of Chi-square Test of Hypothesis that the Demand Distribution for Each Size of Glove is from Poisson Distributed Population

Size	6	$6\frac{1}{2}$	7	$7\frac{1}{2}$	8	$8\frac{1}{2}$	9
χ^2 actual	0.03	24.00	9.91	6.94	12.29	2.55	0.38
χ^2 .05	3.80	11.10	12.59	11.07	11.07	5.99	3.85

Table 2. Observed Demand and Expected Demand from Poisson Distributed Populations for All Sizes of Gloves for 66 Weeks

Dozens of Pairs	Observed	Expected from Poisson Distribution	Dozens of Pairs	Observed	Expected from Poisson Distribution
Size 6			Size 7		
0	43	43	0	8	1
1	18	18	1	5	6
2	5	4	2	8	11
			3	10	13
Size $6\frac{1}{2}$			4	10	13
			5	8	10
0	15	4	6	9	6
1	7	11	7	1	3
2	13	15	8	6	2
3	8	15	9	0	1
4	3	10	10	0	0
5	8	6	11	0	0
6	8	3	12	0	0
7	1	1	13	1	0
8	3	1			

(Cont.)

Table 2 (Cont.). Observed Demand and Expected Demand from Poisson Distributed Populations for All Sizes of Gloves for 66 Weeks

Dozens of Pairs	Observed	Expected from Poisson Distribution
Size 7½		
0	5	0
1	2	1
2	1	3
3	3	6
4	9	9
5	4	11
6	12	11
7	8	9
8	18	7
9	0	4
10	0	3
11	0	1
12	3	1
13	1	0
Size 8		
0	16	3
1	3	9
2	8	14
3	8	15
4	13	12
5	7	7
6	7	4
7	1	2
8	1	1
9	2	0
Size 8½		
0	43	41
1	15	20
2	7	5
3	1	1
Size 9		
0	53	53
1	11	15
2	2	1

Since all seven sizes did not fit the same Poisson distribution a different approach was tried. It was observed that the proportions of each size of glove requisitioned did not vary greatly, regardless of total demand for gloves for the week. These proportions (p_i), where i indicated the i th size, were determined and are shown in Table 3 below.

Table 3. Proportion of Total Demand for Each Size of Glove for 66 Week Period

Size	6	6½	7	7½	8	8½	9
p_i	0.0255	0.1692	0.2284	0.3558	0.1811	0.0291	0.0109

A chi-square contingency table test was used to compare the expected demand for each week of each size with the actual demand for each week. This test is

$$\chi^2 = \sum_{\substack{\text{all} \\ \text{sizes}}} \sum_{\substack{\text{all} \\ \text{weeks}}} \frac{(\text{observed demand} - \text{expected demand})^2}{\text{expected demand}},$$

where the expected demand is the product of the proportion of each size and the total number of gloves ordered for the week. This equation was summed for all seven sizes and for 66 weeks. The chi-square value was found to be

$$\chi^2 = 493.19 \quad .$$

The critical chi-square value must be computed as the degrees of freedom are large. The degrees of freedom are

$$n = (7 \text{ sizes} - 1)(66 \text{ weeks} - 1) = 390 .$$

For large values of degrees of freedom the approximate formula is given by Dixon and Massey (12), page 385, as follows:

$$\chi^2 = n\left(1 - \frac{2}{9n} + Z_\alpha \frac{2}{9n}\right)^3$$

where Z_α is the normal deviate and n is the number of degrees of freedom. The critical value was computed to be

$$\chi^2_{.05} = 572 .$$

As the actual chi-square value is less than the computed value at the five per cent level of significance, there is no reason to reject the hypothesis that the proportions of each size of glove are independent of total demand for the week. Therefore, all sizes of gloves were grouped and only the total demand for gloves was considered.

The total demand distribution was compared with the theoretical normal distribution (Figure 3) having the same mean and standard deviation as the observed data. The mean (μ_d) was found to be 16.89 dozens of pairs of gloves per week and the standard deviation (σ_d) was 8.1 dozens of pairs of gloves per week. Table 4 shows the observed demand and the expected demand from a normally distributed population. The chi-square test was used to test the hypothesis that the observed distribution

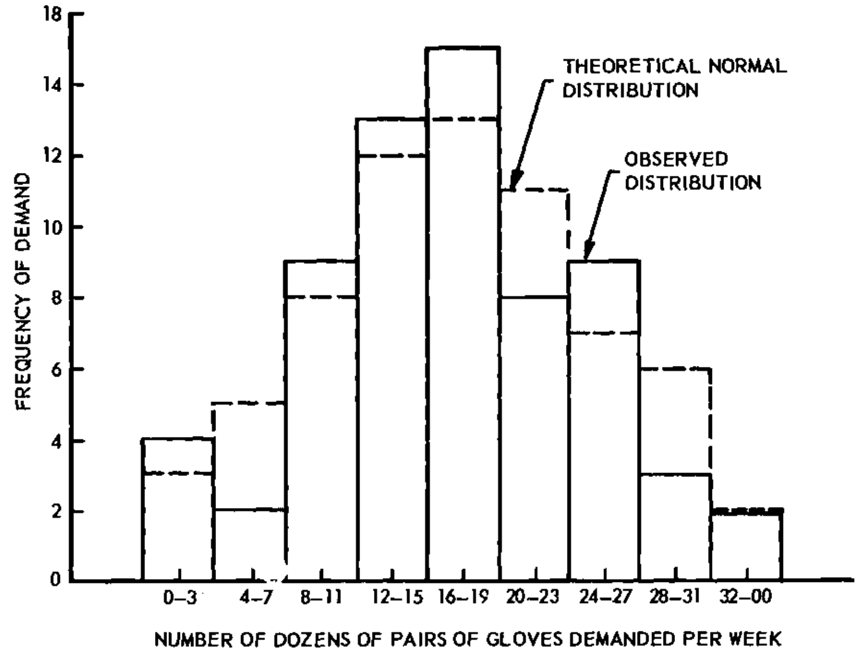

Figure 3. Actual and Theoretical Frequency Distributions of Total Gloves Requisitioned Per Week for 66 Weeks.

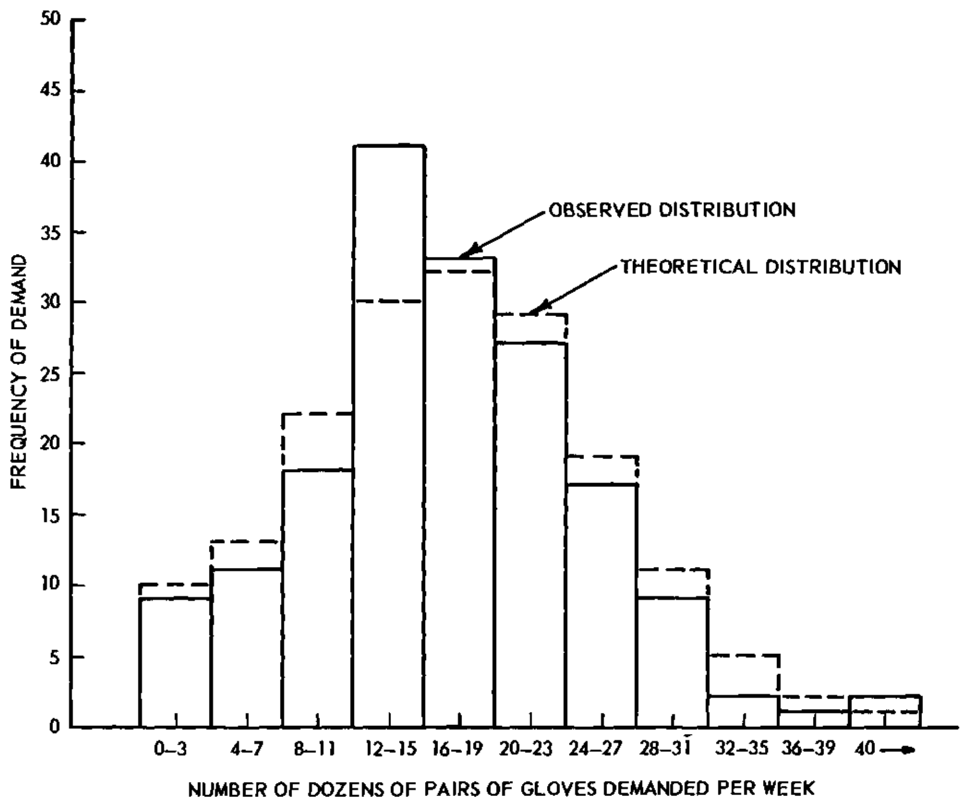

Figure 4. Actual and Theoretical Frequency Distributions of Total Gloves Requisitioned Per Week for 170 Weeks.

Table 4. Observed Demand and Expected Demand from a Normally Distributed Population for Total Gloves for 66 Weeks.

Dozens of Pairs	Observed	Expected from Normal Distribution
0 - 3	4	3
4 - 7	3	5
8 - 11	9	8
12 - 15	13	12
16 - 19	15	13
20 - 23	8	11
24 - 27	9	7
28 - 31	3	4
32	2	2

was from a normally distributed population. The chi-square value was found to be

$$\chi^2 = 2.2 .$$

The critical value at the five per cent level of significance is

$$\chi^2_{.05} = 12.59 .$$

As the actual chi-square value is less than the critical $\chi^2_{.05}$ value there is no reason to reject the hypothesis that the observed distribution is from a normally distributed population.

However, as the distribution is truncated at zero, it was felt that the theoretical Poisson distribution could possibly approximate the actual demand distribution. Table 5 shows the observed demand and the expected demand from a Poisson distributed population. The chi-square

Table 5. Observed Demand and Expected Demand from a Poisson Distributed Population for Total Gloves for 66 Weeks.

Dozens of Pairs	Observed	Expected from Poisson Distribution
0 - 3	4	0
4 - 7	3	0
8 - 11	9	5
12 - 15	13	19
16 - 19	15	24
20 - 23	8	13
24 - 27	9	14
28 - 31	3	1
32	2	0

test was used to test the hypothesis that the observed distribution was from a Poisson distributed population. The actual chi-square value was

$$\chi^2 = 5.48 .$$

The critical chi-square value at the five per cent level of significance is

$$\chi^2_{.05} = 3.84 .$$

As the actual chi-square value is greater than the critical $\chi^2_{.05}$ value the hypothesis that the observed distribution is from a Poisson distributed population was rejected.

Because the sample size was only 66 weeks, it was felt that the results were not conclusive. Therefore, the sample was enlarged to 170 weeks by utilizing data from January 1, 1956, in the records at Emory University Hospital. A new demand distribution was tabulated for this enlarged sample. This distribution was compared with both the normal (Figure 4) and Poisson distributions. Table 6 shows the observed demand and the expected demand from a normally distributed population. Table 7 shows the observed demand and the expected demand from a Poisson distributed population. The results were as follows:

Normal Distribution Comparison

$$\mu_d = 16.89$$
$$\sigma_d = 8.41$$
$$\chi^2 \text{ actual} = 7.42$$
$$\chi^2_{.05} = 12.59 \; .$$

As the actual chi-square value is less than the critical $\chi^2_{.05}$ value the hypothesis that the observed distribution was from a normally distributed population was again accepted.

Poisson Distribution Comparison

$$\mu_d = 16.89$$
$$\chi^2 \text{ actual} = 117.68$$
$$\chi^2_{.05} = 7.82 \; .$$

Since the actual chi-square value is greater than the critical $\chi^2_{.05}$ value, the hypothesis that the observed distribution was from a Poisson distributed population was again rejected.

Table 6. Observed Demand and Expected Demand from a Normally Distributed Population for Total Gloves for 170 Weeks.

Dozens of Pairs	Observed	Expected from Normal Distribution
0 - 3	9	10
4 - 7	11	13
8 - 11	18	22
12 - 15	41	30
16 - 19	33	32
20 - 23	27	29
24 - 27	17	19
28 - 31	9	11
32 - 35	2	5
36 - 39	1	2
40	2	1

Table 7. Observed Demand and Expected Demand from a Poisson Distributed Population for Total Gloves for 170 Weeks.

Dozens of Pairs	Observed	Expected from Poisson Distribution
0 - 3	9	0
4 - 7	11	1
8 - 11	18	14
12 - 15	41	50
16 - 19	33	62
20 - 23	27	33
24 - 27	17	8
28 - 31	9	1
32 - 35	2	0
36 - 39	1	0
40	2	0

Economic Lot Size.-- It is possible to determine the optimum ordering quantity by using conventional economic lot size equations. The economic lot size (q)[1] is found by

$$q = \sqrt{\frac{2YA}{pC}}$$

where Y = yearly demand

A = cost of ordering

p = inventory carrying cost expressed as a per cent of the unit cost

C = purchase cost per unit

The development of this equation is dependent upon the following assumptions:

1. Procurement costs are fixed.
2. There is no interaction between protective stock and economic lot size.
3. The average time between orders, over an extended period of time, is used as a constant.
4. The average demand per fixed unit of time is used as a constant.
5. Interest, risks, depreciation, obsolescence and storage costs may be pooled into one percentage figure.
6. This percentage is constant.
7. The inventory is dispersed in small lots and no back orders allowed.

If it were possible to determine the cost of a shortage, the equation used to determine the economic lot size[2] would become

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[1] Whitin, op. cit., p. 33

[2] Vazsonyi, op. cit., p. 337

$$q = \sqrt{\frac{2Y}{pC}\left\{A + E\left[1 - \Phi(t)\right]\right\}}$$

where E is the cost of the shortage and $[1 - \Phi(t)]$ is the probability of a shortage. However, as this cost cannot be determined at the present time it will be considered implicitly by specifying the tolerated probability of a shortage.

Price Breaks.-- The following values were used in the calculations for the economic order quantity for gloves when price breaks exist:

Y = 878 dozen pairs

A = \$5.00 per order

p = 0.20 per year

The price per unit varies with the order quantity as follows:

Price Break in Dozen Pairs		$b_1 = 13$	$b_2 = 37$	$b_3 = 109$
Quantity	(1-12) dozen	(13-36) dozen	(37-108) dozen	over 108 dozen
Cost	$C_1 = 6.59$	$C_2 = 5.30$	$C_3 = 4.80$	$C_4 = 4.70$

Following the procedure by Churchman[3]

1. Compute q_4. $q_4 = \sqrt{\frac{(2)(878)(5)}{(0.2)(4.70)}}$ = 97 dozen pairs of gloves.
2. Compute q_3. $q_3 = \sqrt{\frac{(2)(878)(5)}{(0.2)(4.80)}}$ = 96 dozen pairs of gloves.

Since $TEK_{b_3} < TEK_{q_3}$, the purchase quantity which will result in a minimum cost is b_3 = 109 dozen pairs. Since q_3 is greater than b_2, the TEK (total expected cost), which includes the variable cost of purchasing the items,

[3]Churchman, *op. cit.*, p. 252

is compared where the order quantity is q_3 and b_3.[4] The quantity which results in the smaller cost will be selected.

$$TEK_{q_3} = \left[C_3Y\right] + \left[A\frac{Y}{q_3} + (\tfrac{1}{2}q_3 + W_\alpha)\ C_3p\right]$$
$$= [\$4214.40] + [\$45.73 + \$59.39]$$
$$= \$4319.52$$
$$TEK_{b_3} = \left[C_4Y\right] + \left[\frac{AY}{b_3} + (\tfrac{1}{2}b_3 + W_\alpha)\ C_4p\right]$$
$$= [\$4126.60] + [\$40.28 + \$64.26]$$
$$= \$4231.14$$

Table 8. Optimum Order Quantity for Each Glove Size

Size	6	6½	7	7½	8	8½	9
No. pairs	33	216	291	454	231	37	14

When the reorder point is reached, the new order will be placed for the number of gloves indicated in Table 8, above. When the controls maintained on the proportions indicate the proportions have changed, a new breakdown of the economic lot size should be made.

Reorder Point.-- In the first calculations for reorder point the assumption will be made that the lead time is constant (one week).

Let f(d) denote: the probability density function of the demand for a fixed time period; and R.P. denote the reorder point. Then the probability of a shortage during the reorder period is given by

$$P\ \{D \geq R.P.\} = 1 - \int_{-\infty}^{R.P.} f(D_x)dD_x\ .$$

Where $f(D_x)$ is the density function of total demand for a constant number (x) of time periods.

[4]The cost of a shortage is excluded from this comparison and W is determined subsequently to be 13.86 dozens of pairs of gloves.

Out of 340 requisitions examined during the 66 week period there were found to be 17 shortages. It will be assumed that this is the shortage level that management will tolerate. Therefore,

$$1 - \int_{-\infty}^{R.P.} f(D_x) dD_x = \frac{17}{340} = 0.05 .$$

The demand distribution f(d) is normal with mean (μ_d) of 16.89 dozen pairs of gloves and standard deviation (σ_d) of 8.4 dozen pairs of gloves. Using the properties of the normal distribution, and recognizing that when x equals 1 that f(d) equals $f(D_1)$. It follows from above

$$\int_{-\infty}^{R.P.} f(d)\, dd = 0.95$$

and therefore R.P. is located at a sigma deviation (t) from the mean μ_d of:

$$+ t\, \sigma_d = + 1.65\, \sigma_d .$$

The reorder point (R.P.) is calculated from the equation,

$$\text{R.P.} = \text{Expected demand during lead time} + \text{Protective stock}$$

$$\begin{aligned} \text{R.P.} &= \mu_d + (t)\, \sigma_d \\ &= 16.89 + 1.65\ (8.4) \\ &= 30.75 \text{ dozens of pairs of gloves} . \end{aligned}$$

It is observed that the insurance against a shortage (W), or protective stock level, is 13.86 dozens of pairs of gloves.

To illustrate how the assumption of constant lead time causes higher probabilities of shortages when the lead time actually varies, the reorder point will again be calculated assuming that the lead time has a Poisson distribution with mean (μ_x) of one week. The same demand distribution will be used as before. The reorder point is calculated from the equation

$$\text{probability of a shortage}^{5} = \sum_{x=0}^{\infty} p(x) \int_{RP}^{\infty} f(D_x|x)dD_x \ .$$

These calculations have been made for the range of probabilities of a shortage from 10 per cent to one-tenth of one per cent. A pictorial representation of this model is shown in Figure 5. An assumption is that the variables of lead time and demand are randomly distributed. These values have been tabulated in Table 9. Figure 5 shows these values plotted.

It will be noticed in comparing the two reorder points that the assumption of Poisson lead time increases the reorder point from approximately 31 dozen pairs of gloves to approximately 53 dozen pairs, and the level of protective stock has been increased from 13.86 dozen pairs to 36.11 dozen pairs. If the lead time is actually Poisson distributed and 31 dozen pairs of gloves were used as the reorder point, the probability of a shortage would be greater than 12 per cent. To show that it is possible to make these calculations using demand and lead time numbers of smaller magnitude, the demand distribution was recalculated in terms of gross per week and the calculations for probabilities of a shortage

[5] See Appendix A for derivation of this equation.

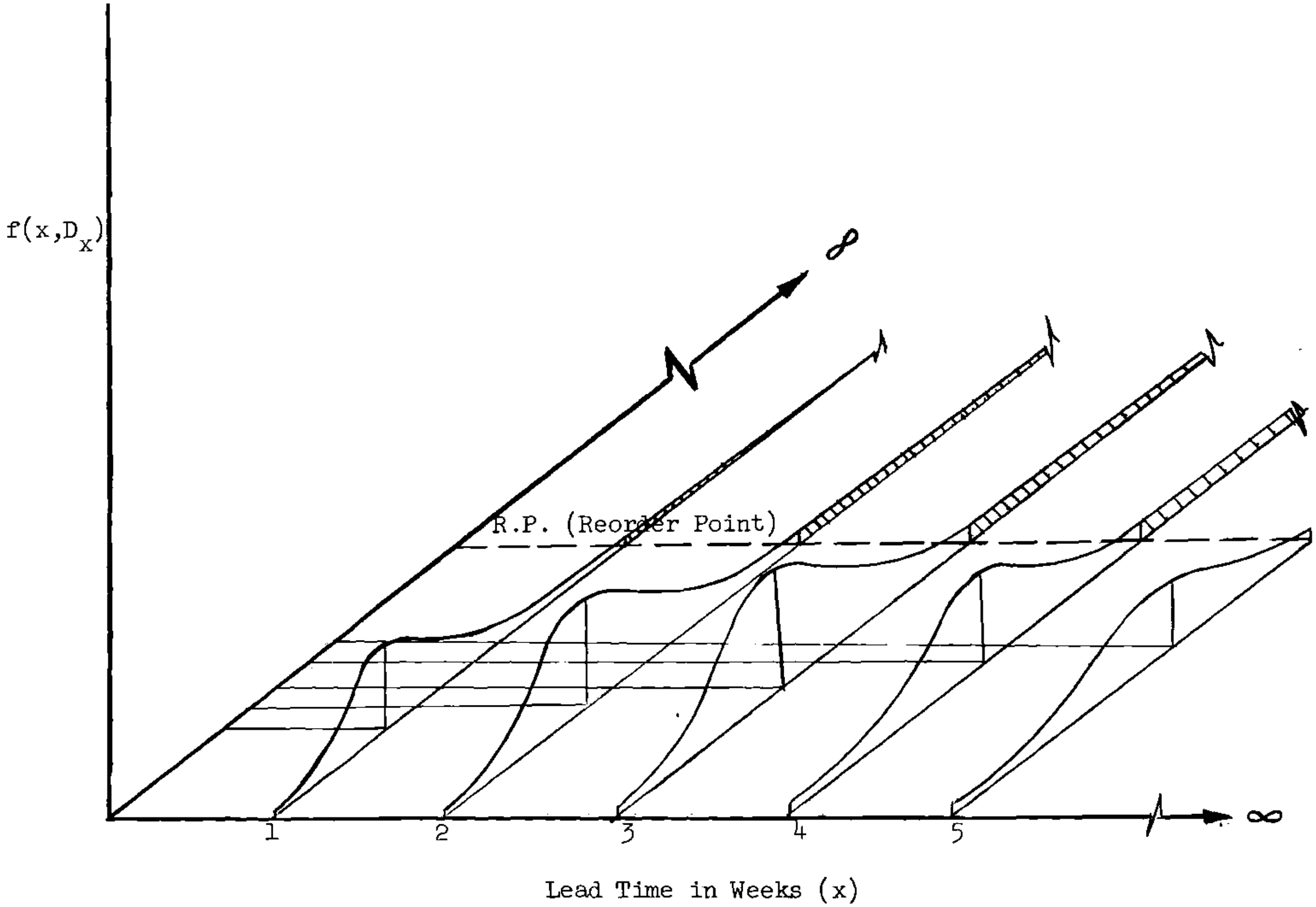

Figure 5. A Pictorial Representation of a Typical Inventory Model for Varying Lead Time and Demand.

Table 9. Tables of Probability of a Shortage at Various Reorder Points Where the Lead Time Has a Poisson Distribution with Mean (μ_x) of 1 Week and the Demand Distribution is Normal With Mean (μ_d) of 16.9 Dozen.

R.P.	Probability of a Shortage	R.P.	Probability of a Shortage
40	0.11844	73	0.01203
41	0.11117	74	0.01113
42	0.10478	75	0.01030
43	0.09815	76	0.00952
44	0.09208	77	0.00879
45	0.08695	78	0.00817
46	0.08122	79	0.00754
47	0.07618	80	0.00694
48	0.07107	81	0.00641
49	0.06656	82	0.00592
50	0.06227	83	0.00545
51	0.05824	84	0.00503
52	0.05445	85	0.00466
53	0.05089	86	0.00431
54	0.04736	87	0.00398
55	0.04423	88	0.00367
56	0.04116	89	0.00337
57	0.03839	90	0.00311
58	0.03601	91	0.02286
59	0.03350	92	0.00264
60	0.03120	93	0.00243
61	0.02902	94	0.00222
62	0.02708	95	0.00206
63	0.02507	96	0.00188
64	0.02273	97	0.00173
65	0.02185	98	0.00159
66	0.02034	99	0.00146
67	0.01888	100	0.00133
68	0.01753	101	0.00123
69	0.01624	102	0.00113
70	0.01506	103	0.00103
71	0.01405	104	0.00095
72	0.01300		

were again performed. These values have been tabulated in Table 10. Figure 6 shows these values plotted. It will be noticed that exactly the

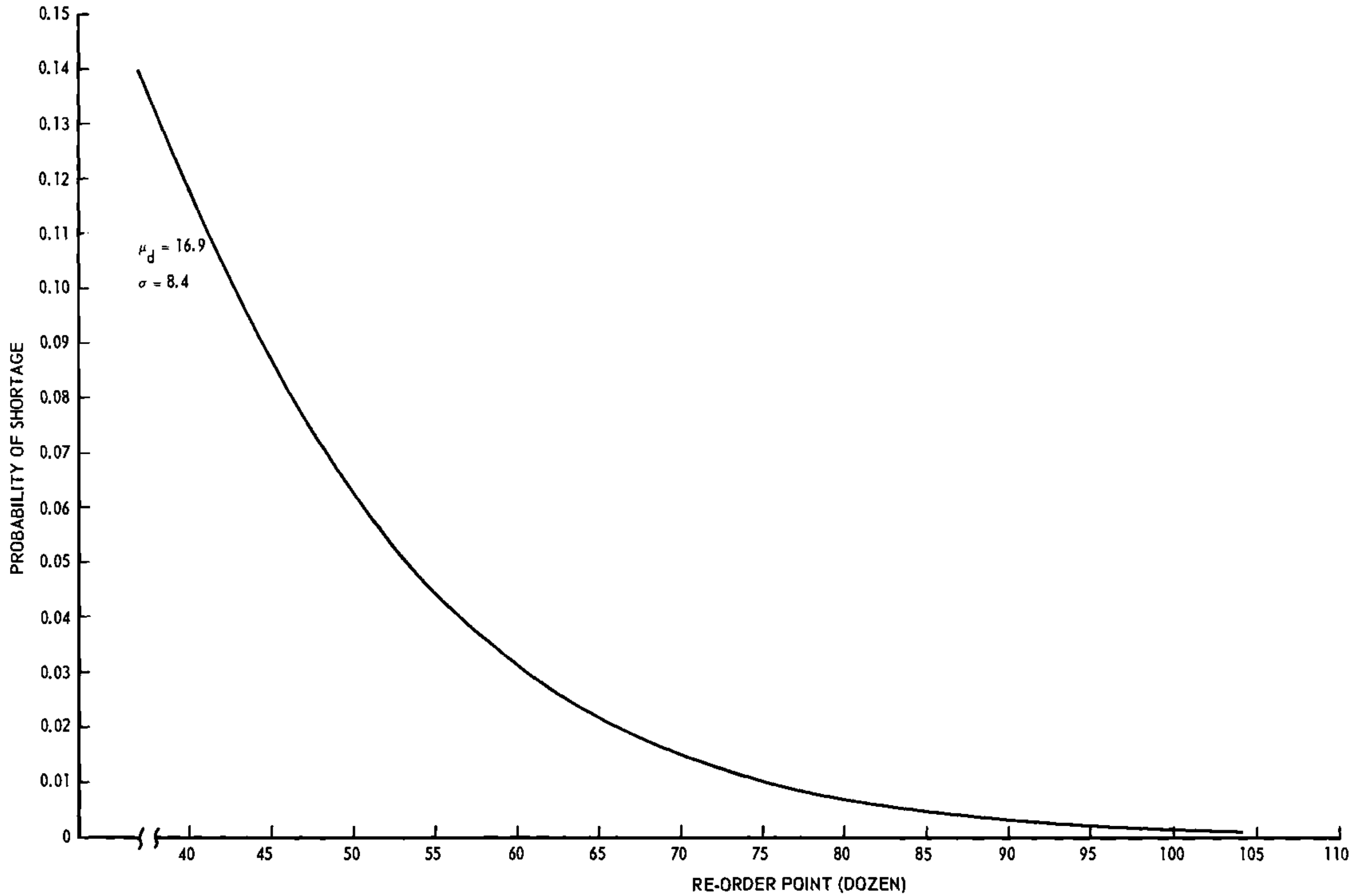

Figure 6. Graph of Two Variables; Probability of Shortage and Reorder Point with the Means of the Poisson Lead Time Function and Normal Demand Function Held Constant.

Table 10. Tables of Probability of a Shortage at Various Reorder Points Where the Lead Time Has a Poisson Distribution With Mean (μ_x) of 1 Week and the Demand Distribution is Normal With Mean (μ_d) of 1.408 Gross.

R.P.	Probability of a Shortage
3	0.14866
4	0.07107
5	0.03120
6	0.01300
7	0.00503
8	0.00188
9	0.00066

same probabilities of shortage were obtained for corresponding reorder points. For example, it will be noted that for a reorder point of 48 dozen pairs of gloves the probability of a shortage is 0.07107 and for a reorder point of 4 gross of pairs of gloves the probability of a shortage is also 0.07107. This indicates that care should be taken in choosing the units for the demand distribution, so that the magnitude of the numbers is as small as is practical.

<u>Total Inventory Costs</u>.-- With the economic lot size and correct protective stock level determined, the total cost (Z) for a year can be calculated from the following equation[6]

$$Z = A\frac{Y}{q} + \left(\tfrac{1}{2}q + W_\alpha\right) cp .$$

This equation is used to determine the cost of various inventory policies, where W_α is the protective stock necessary to insure a probability of α of not having a shortage. The example of gloves shows that for a level of

[6]Vazsonyi, *op. cit.*, p. 333

shortage of five per cent the appropriate level of protective stock is 36.11 dozen pairs of gloves. Therefore, the total annual cost associated with ordering and maintaining the inventory with a probability of a shortage of five per cent is

$$Z = (5.00)\,\frac{878}{109} + \left(\frac{109}{2} + 36.11\right)(4.80)\,(0.20)$$
$$= 40.28 + 85.17$$
$$= \$125.45.$$

However, this figure may not be meaningful. The figure that most hospital administrators would be interested in is the cost per pair of gloves. This cost is

$$\text{cost per pair} = \frac{\$125.45}{878 \text{ dozen x } 12 \text{ pair/dozen}} = \$0.0119 \text{ per pair of gloves.}$$

This figure can be compared with the cost of other inventory policies. For example, the hospital administrator intuitively may feel that five per cent is too high a probability of a shortage to tolerate. The administrator may want to investigate the incremental increase is cost that would be associated with lowering the probability of a shortage to one per cent and to one-tenth of one per cent. From Figure 6 the reorder point is found to be approximately 76 dozen pairs of gloves for a one per cent probability of a shortage and approximately 104 dozen pairs of gloves for a one-tenth of one per cent probability of a shortage. This means that the protective stock level is now 59.11 dozen pairs of gloves for a one per cent probability of a shortage and 87.11 dozen pairs of gloves for a one-tenth of one per cent probability of a shortage. The total cost would now be as follows:

(1) where the probability of a shortage = 1%

$$Z = 5.00 \left(\frac{878}{109}\right) + \left(\frac{109}{2} + 59.11\right)(4.80)(0.20)$$
$$= 40.28 + 106.77$$
$$= \$147.07,$$

and the cost per pair of gloves would be

$$\frac{\$147.07}{878 \times 12} = \$0.0140 \text{ per pair of gloves.}$$

(2) where the probability of a shortage = 0.1%

$$Z = 5.00 \left(\frac{878}{109}\right) + \left(\frac{109}{2} + 87.11\right)(4.80)(0.20)$$
$$= 40.28 + 133.11$$
$$= \$173.39,$$

and the cost per pair of gloves would be $\frac{\$173.39}{878 \times 12} = \$0.0165$.

The hospital administrator is thus able to evaluate the various inventory policies in terms of "out of pocket" costs relative to selected probability levels of a shortage and quantitatively determine what level of protection the hospital can afford.

Detection of Changes in the Model.-- If the proportion of a size of glove used or the total demand or lead time for gloves changes, economic lot size, reorder points and the protective stock level calculations based on these values would not give accurate results. To detect any significant shifts in any of these values control charts have been constructed (Figures 7, 8, 9, and 10) using techniques presented by Duncan (13).

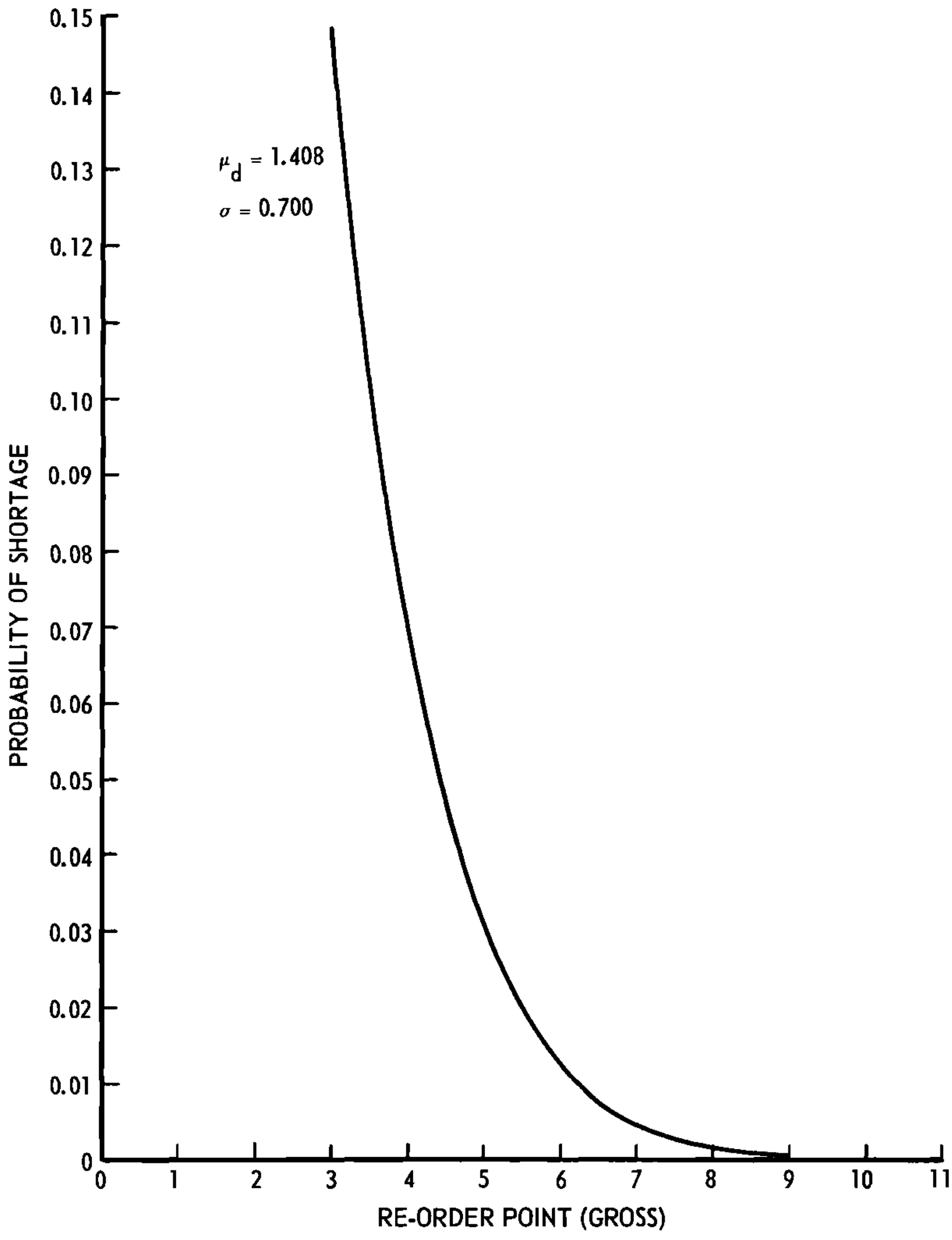

Figure 7. Graph of Two Variables; Probability of Shortage and Reorder Point with the Means of the Poisson Lead Time Function and Normal Demand Function Held Constant.

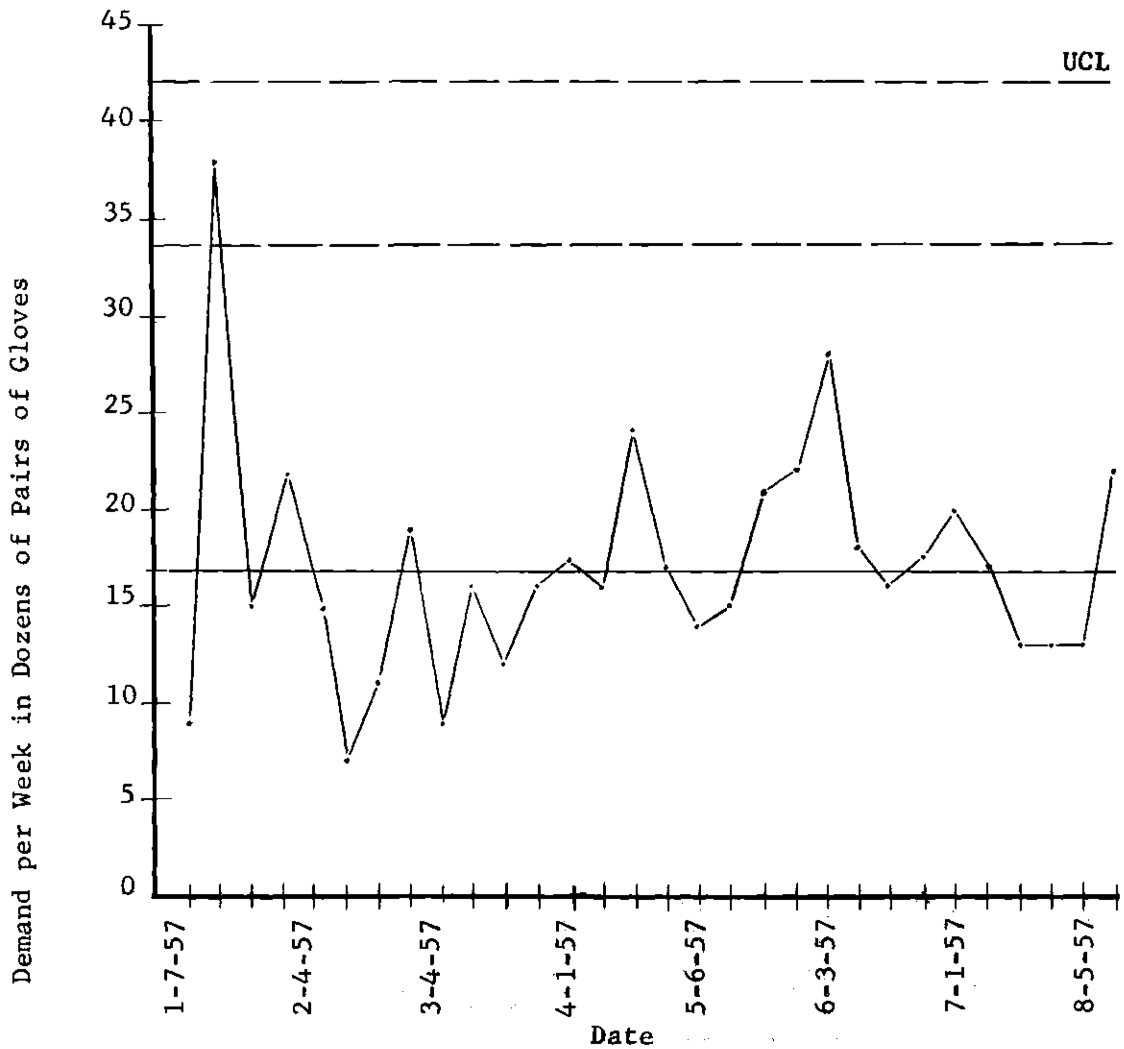

Figure 8. Control Chart for Mean (μ_d)
of Normally Distributed Demand

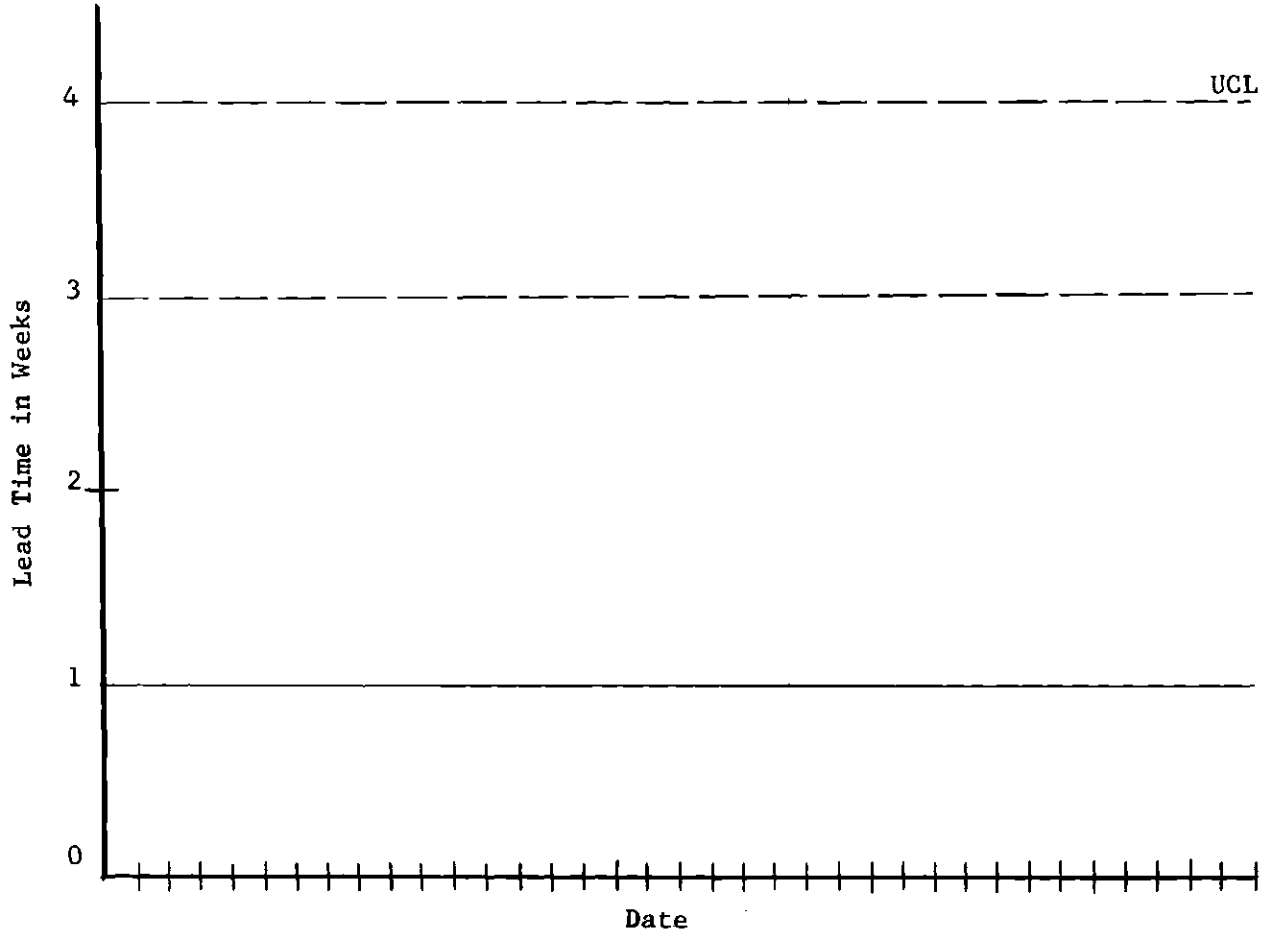

Figure 9. Control Chart for Mean (μ_x) of Poisson Distributed Lead Time

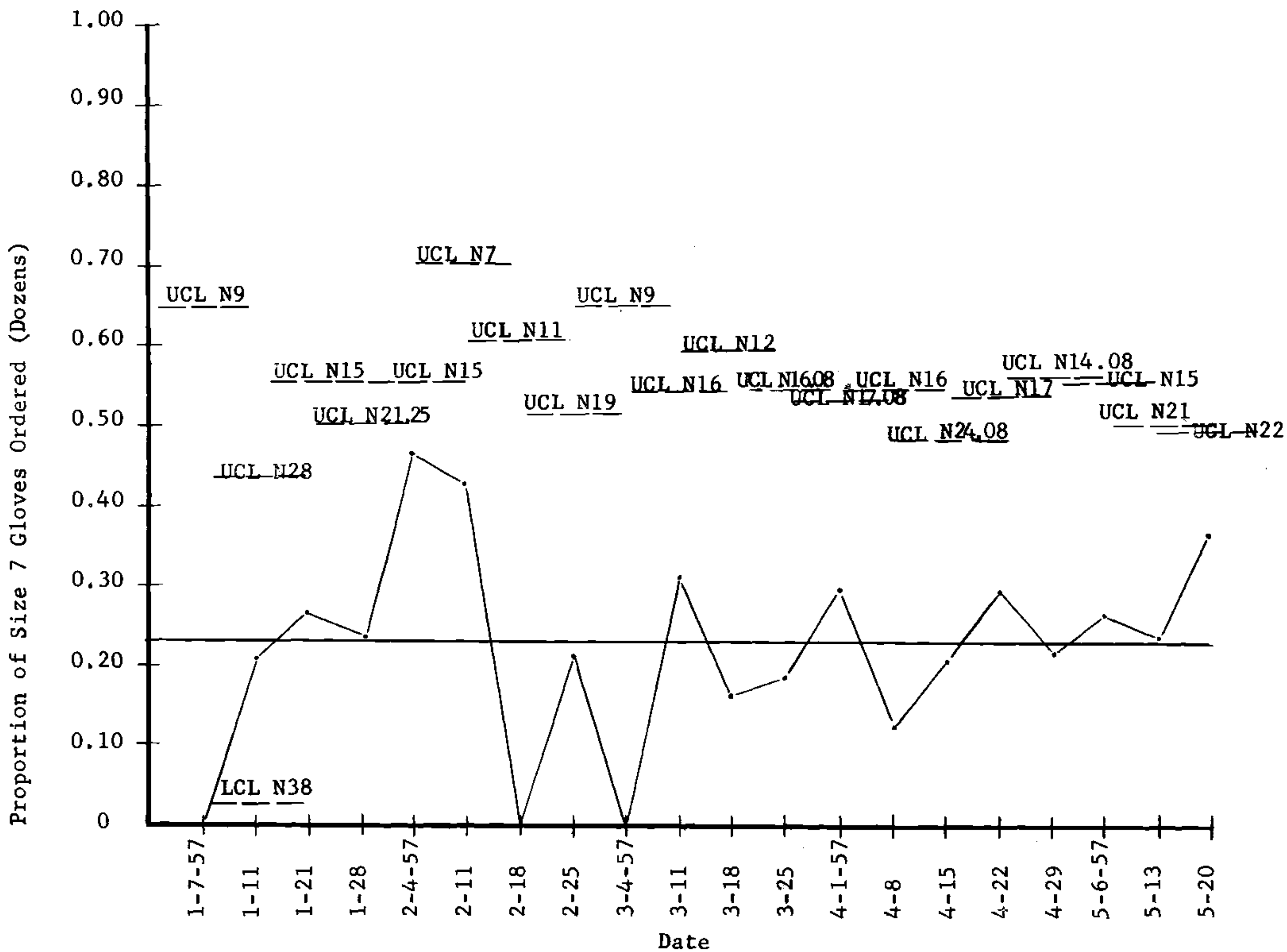

Figure 10. Control Chart for Proportion ($\bar{p}_7$) of Size Seven Gloves Used Showing Various Control Limits.

Figure 7 illustrates an $\bar{x}$ chart[7] that will detect any significant shift in the mean of the total demand distribution for gloves. The demand per week for gloves for the period January 7, 1957 - August 5, 1957 has been plotted to illustrate how the chart is used. To detect any shifts in the variance of the total demand distribution, a σ chart[8] could be constructed.

Figure 8 illustrates a $\bar{c}$ chart[9] that will detect any significant shift in the mean of the lead time distribution.

Figure 9 illustrates a $\bar{p}$ chart[10] that will detect any significant shift in the proportion of the size of glove used. This chart has been constructed for size seven gloves. In actual practice it would be necessary to have six of these charts as the proportions of all sizes would need to be examined for shifts in the proportion used. The upper control limit (UCL) for various total numbers (N) of gloves requisitioned per week are shown. Figure 10 shows the proportion of size 7 gloves requisitioned per week for the period January 7, 1957 - May 20, 1957. This control chart was constructed to illustrate the use of this type of control chart.

To determine if there were significant shifts in any of the values being examined the control charts would be analyzed for runs in addition to points out of control.[11]

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[7] Duncan, _op. cit._, pp. 363-70

[8] _Ibid._, pp. 376-77

[9] _Ibid._, pp. 350-52

[10] _Ibid._, pp. 330-35

[11] _Ibid._, pp. 117-22

Calculation of Other Reorder Point Curves.-- In an attempt to provide hospital administrators with a guide to reorder points when the demand distribution is other than normal, reorder point curves have been calculated from the joint density function of two Poisson distributions. An attempt was made to utilize the IBM 650 computer for these calculations. A workable program was written, using an interpretive system (Figure 11), but this was found to be no faster than the use of a desk calculator, when used in conjunction with Molina's Tables. Because of this fact, the use of the computer was discontinued for calculations within the range of the tables. The following curves were calculated:

$\mu_x = 1$, $\mu_d = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$[12]

$\mu_x = 2$, $\mu_d = 1, 2, 3, 4, 5, 6, 7, 8$

$\mu_x = 3$, $\mu_d = 1, 2, 3, 4, 5, 6$

$\mu_x = 4$, $\mu_d = 1, 2, 3, 4, 5, 6$

$\mu_x = 5$, $\mu_d = 1, 2, 3, 4, 5$

$\mu_x = 6$, $\mu_d = 1, 2, 3, 4, 5$

$\mu_x = 7$, $\mu_d = 1, 2, 3, 4, 5$[12].

Figures 12, 13, 14, 15, 16, 17, and 18 show the results of these calculations and Tables 11, 12, 13, 14, 15, 16, and 17 present these results in tabular form. To utilize these curves it will be necessary to choose the units for the distributions so that they fall within the range of these curves. Appendix B shows a sample calculation for one point on the curve $\mu_x = 2$, $\mu_d = 6$.

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[12]Calculated on IBM 650 computer.

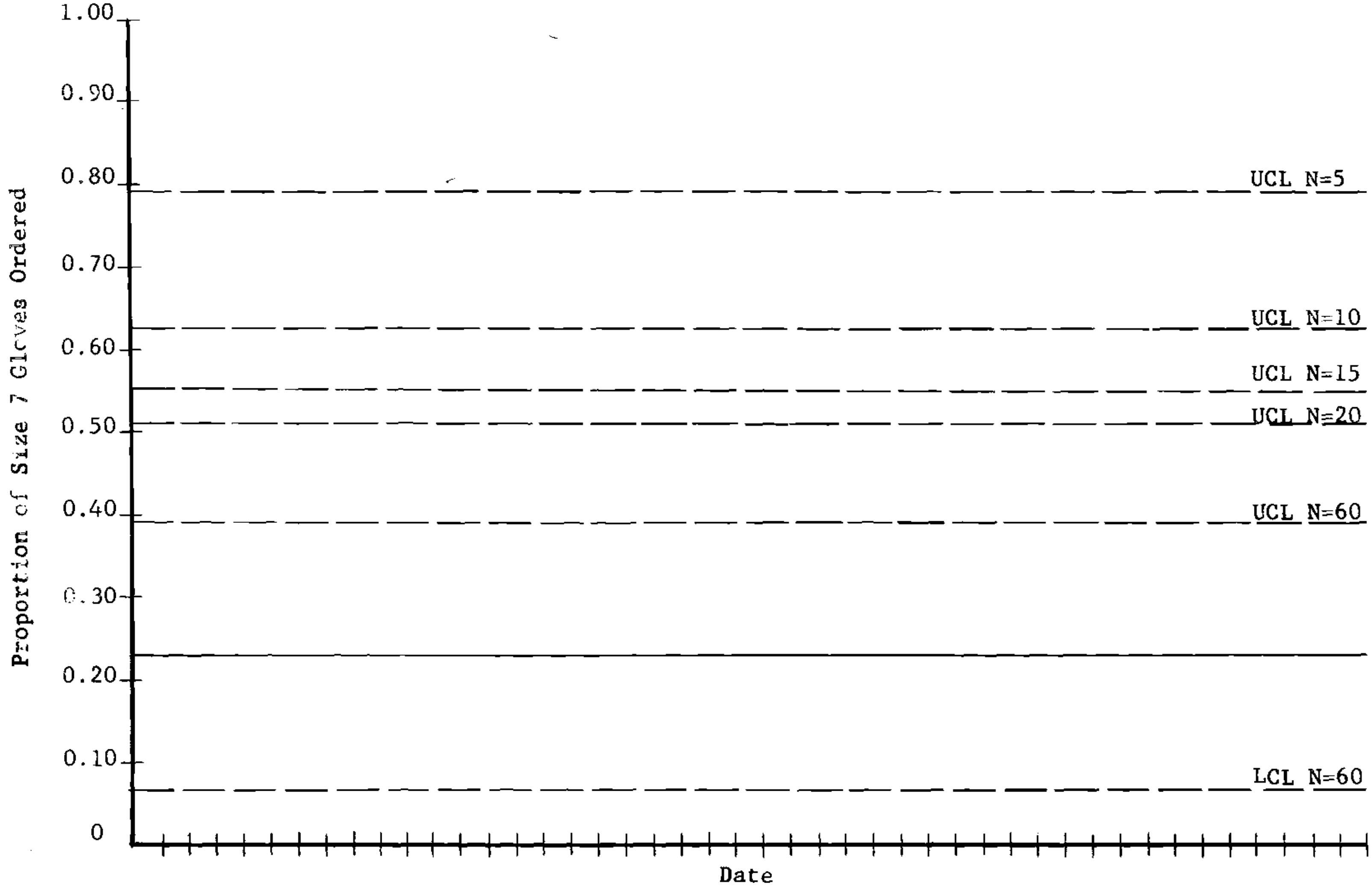

Figure 11. Control Chart for Proportion ($\overline{p}_7$) of Size Seven Gloves Used

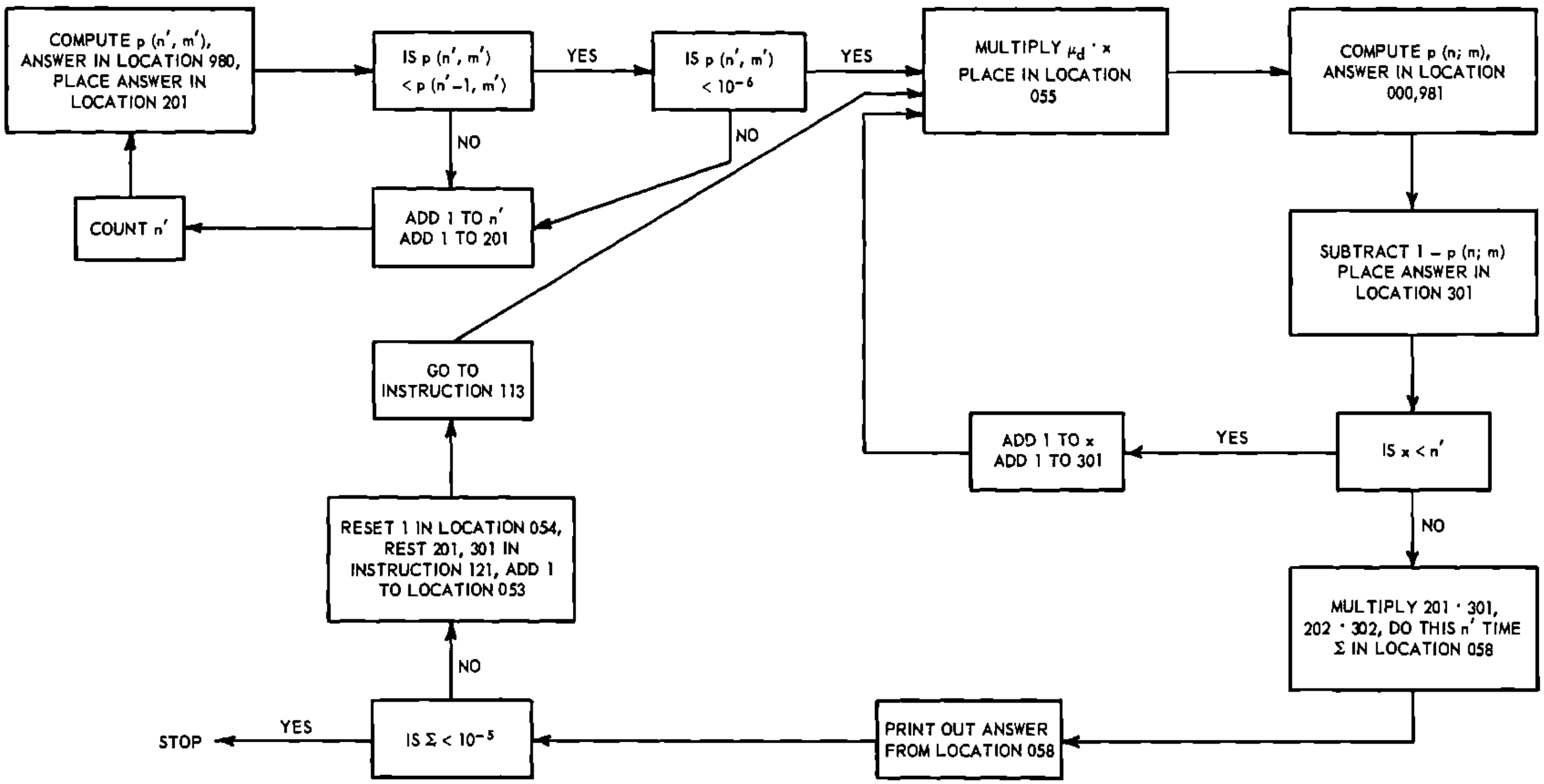

Figure 12. Flow Chart for the IBM 650 Computer Routine.
(Bell Statistical Interpretive System).

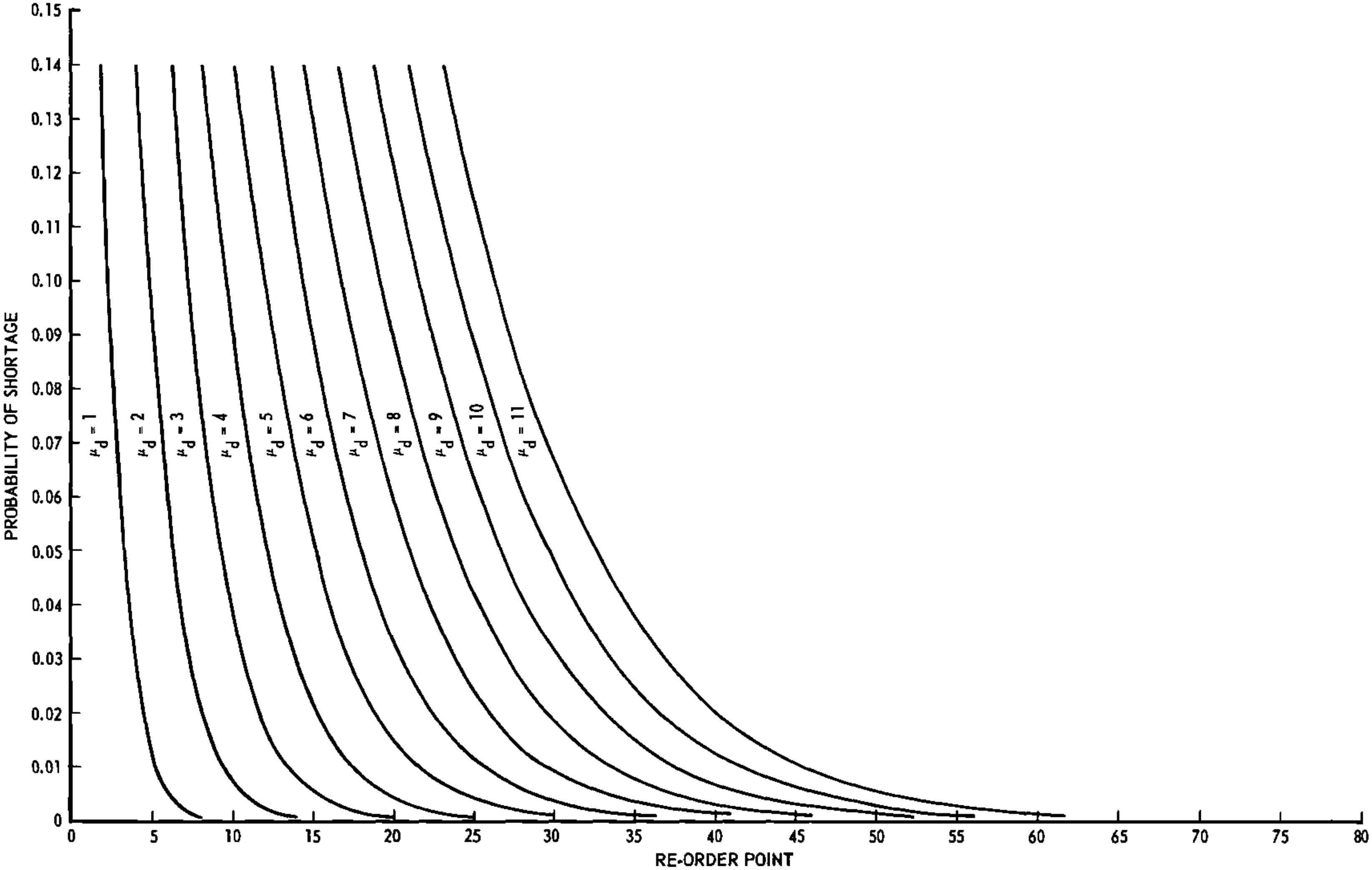

Figure 13. Graph of Three Variables; Probability of a Shortage, Mean (μ_d) of a Poisson Demand Function, and the Reorder Point with the Poisson Lead Time Held Constant, $\mu_x = 1$.

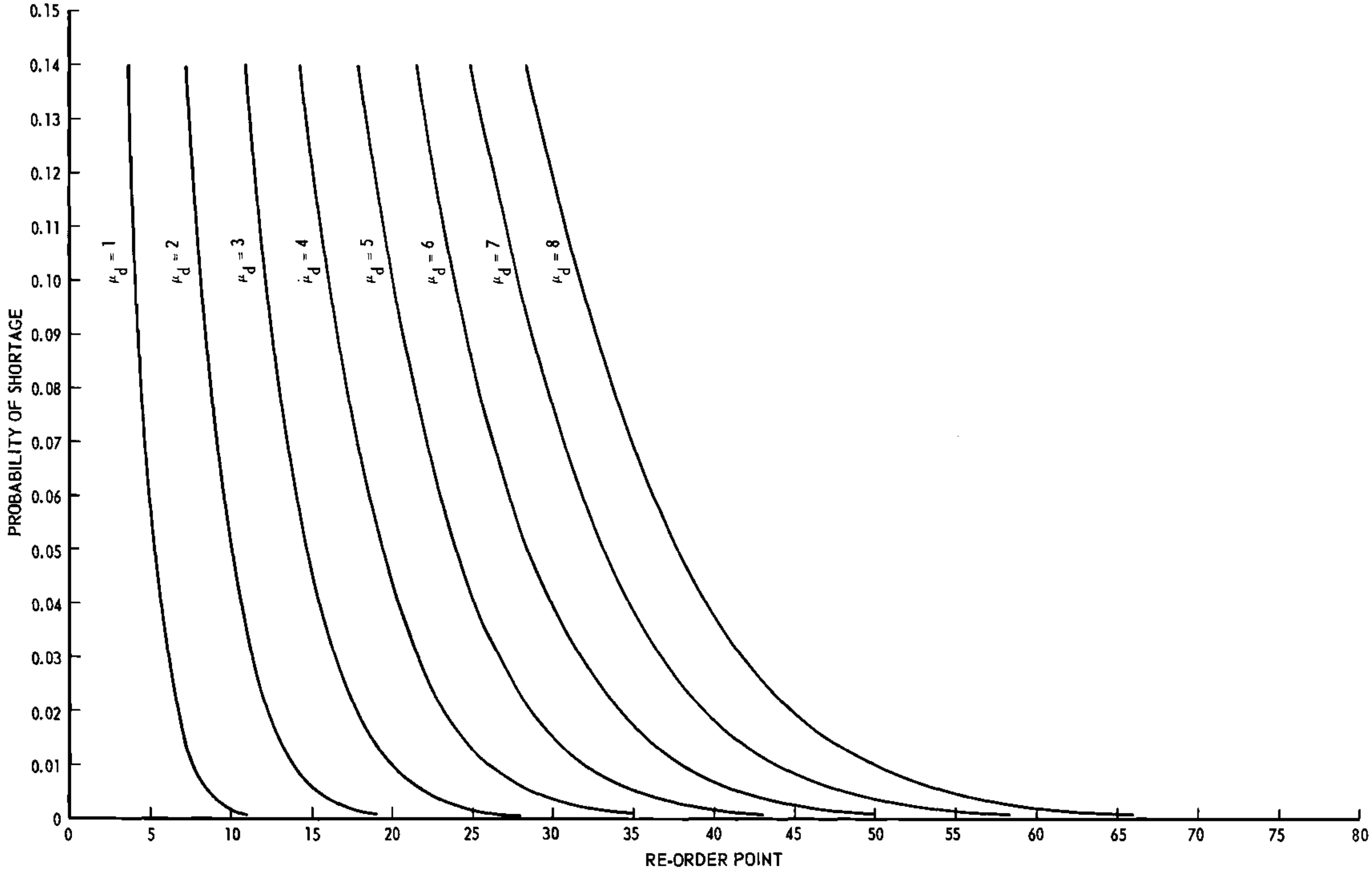

Figure 14. Graph of Three Variables; Probability of a Shortage, Mean (μ_d) of a Poisson Demand Function, and the Reorder Point with the Poisson Lead Time Held Constant, $\mu_x = 2$.

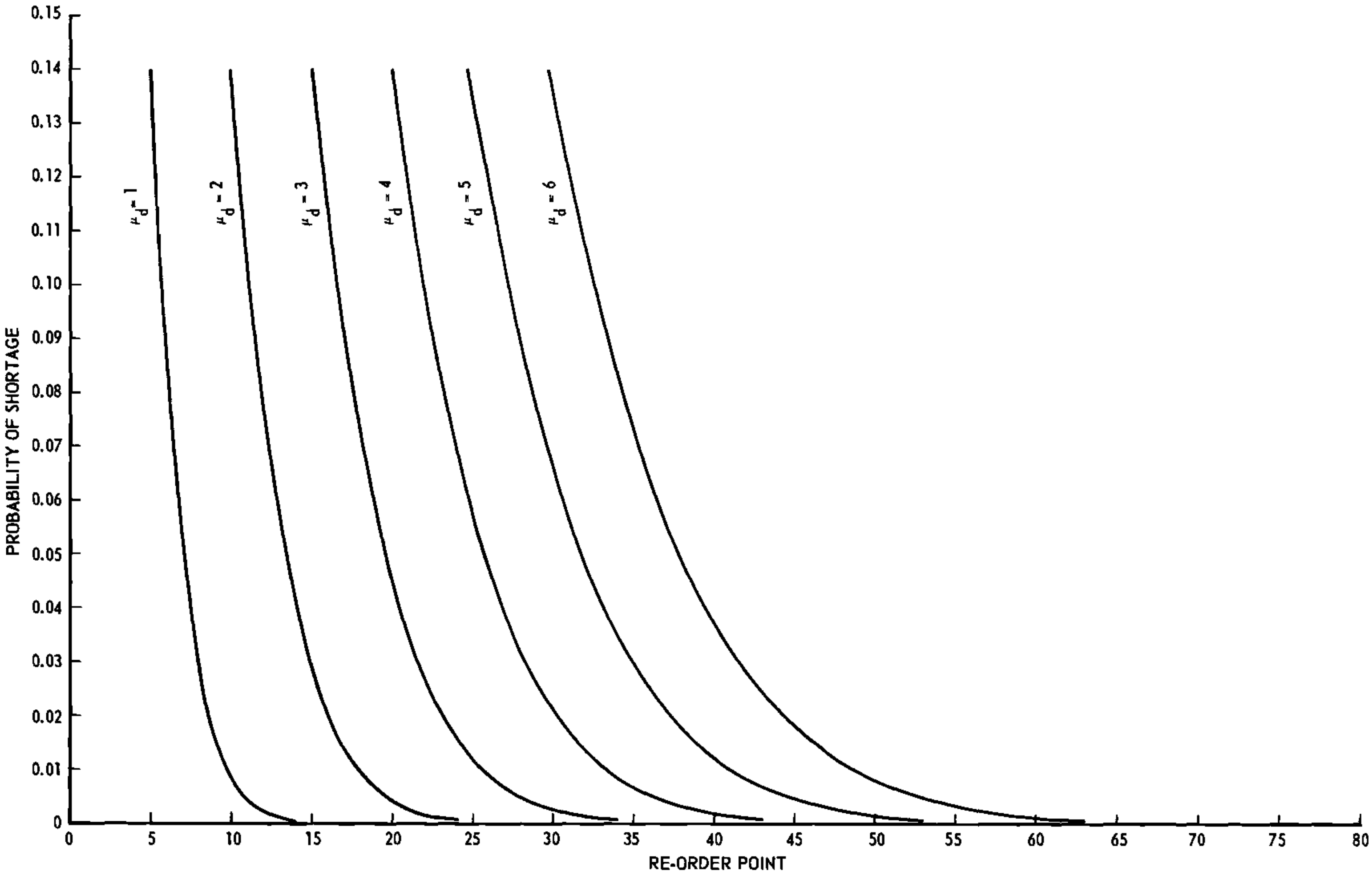

Figure 15. Graph of Three Variables; Probability of a Shortage, Mean (μ_d) of a Poisson Demand Function, and the Reorder Point with the Poisson Lead Time Held Constant, $\mu_x = 3$.

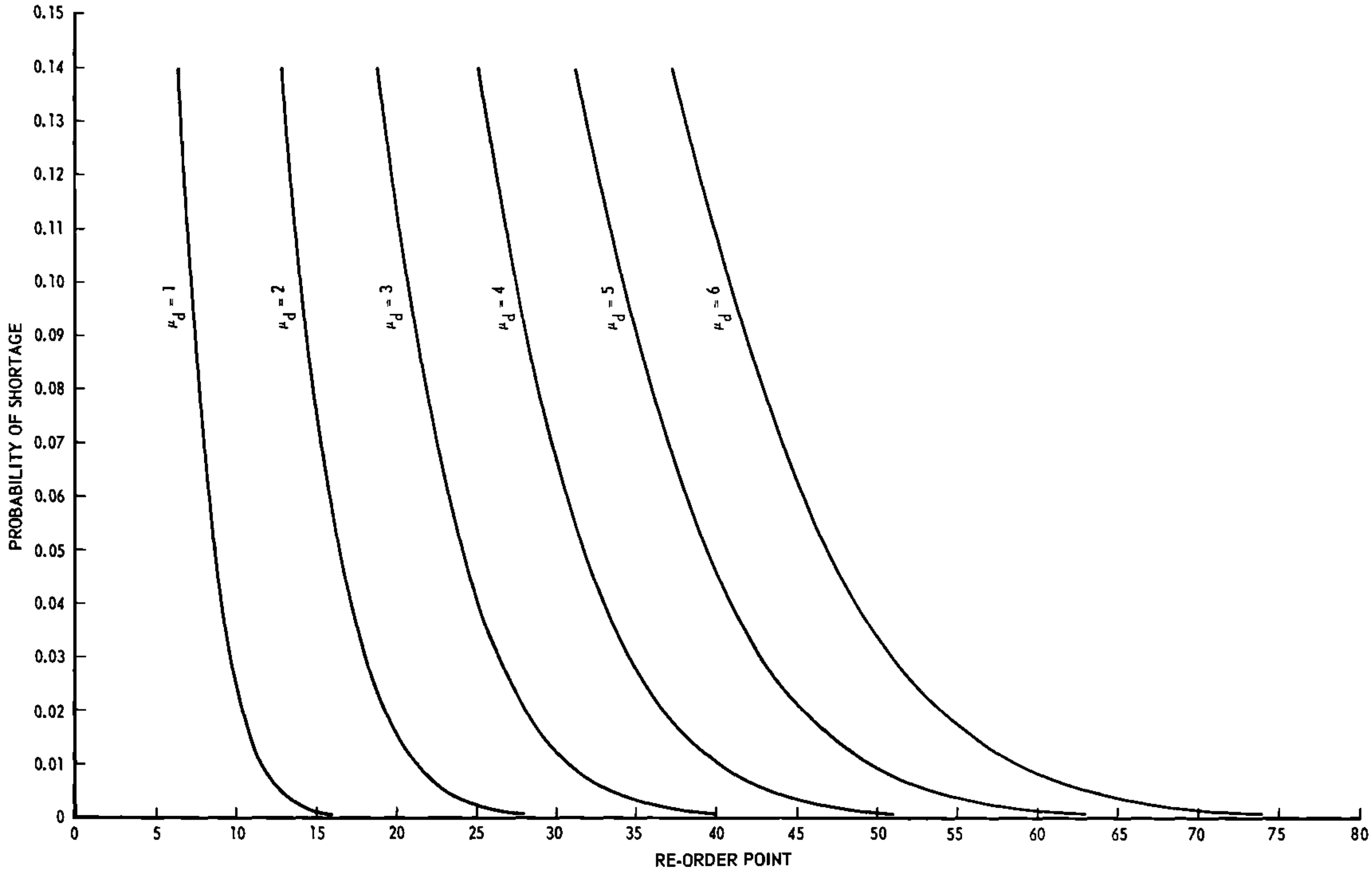

Figure 16. Graph of Three Variables; Probability of a Shortage, Mean (μ_d) of a Poisson Demand Function, and the Reorder Point with the Poisson Lead Time Held Constant, $\mu_x = 4$.

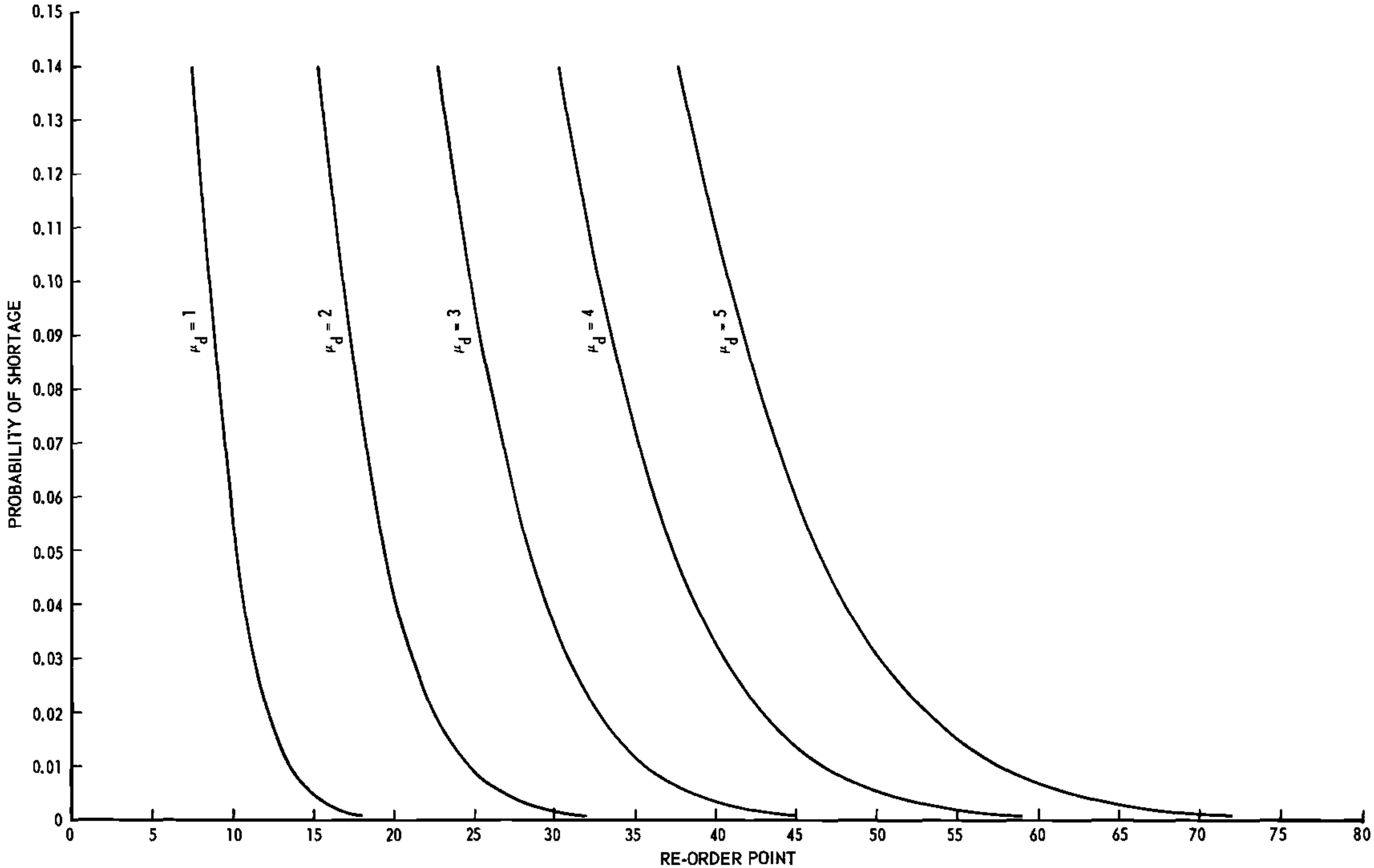

Figure 17. Graph of Three Variables; Probability of a Shortage, Mean (μ_d) of a Poisson Demand Function, and the Reorder Point with the Poisson Lead Time Held Constant, $\mu_x = 5$.

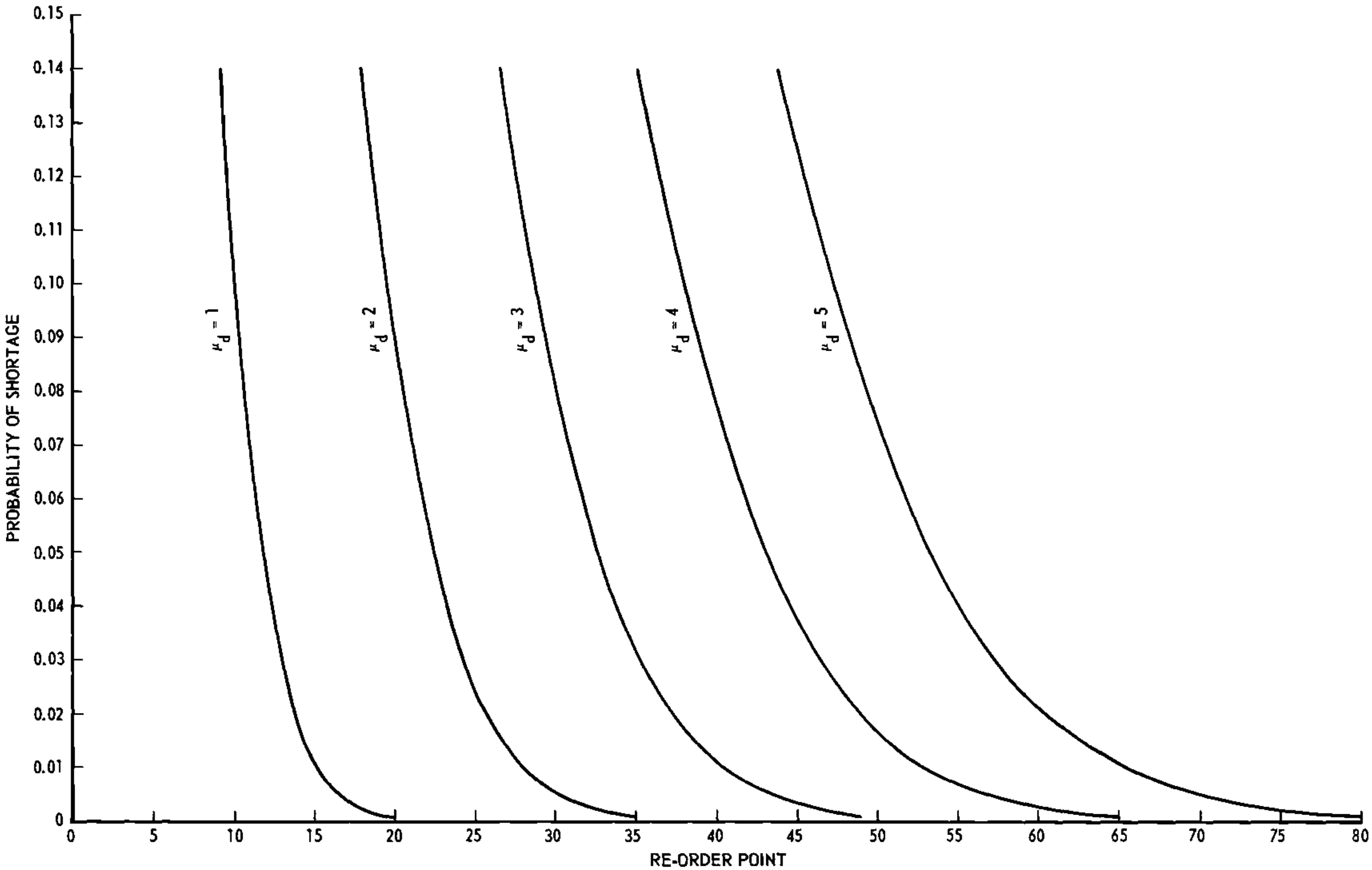

Figure 18. Graph of Three Variables; Probability of a Shortage, Mean (μ_d) of a Poisson Demand Function, and the Reorder Point with the Poisson Lead Time Held Constant, $\mu_x = 6$.

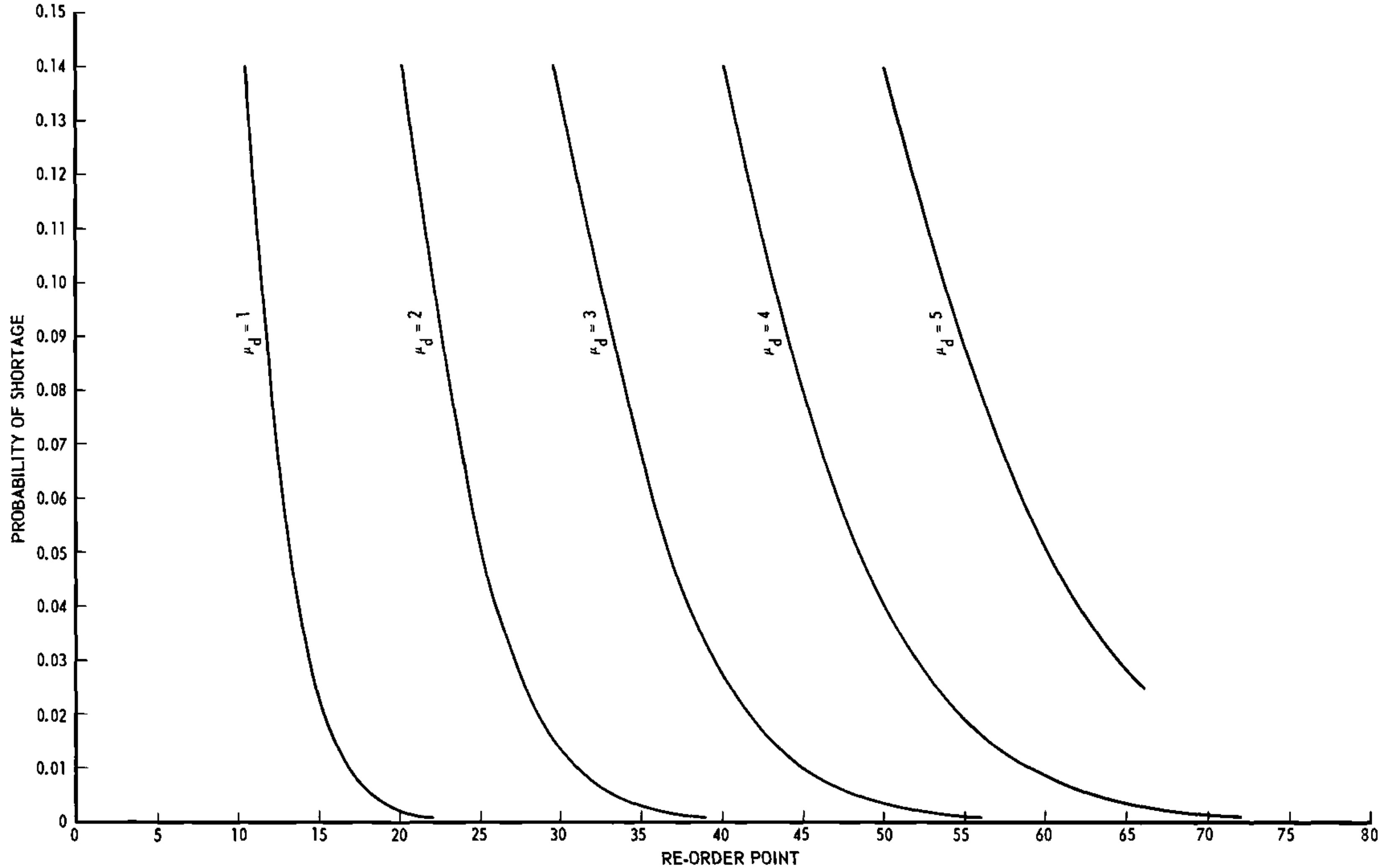

Figure 19. Graph of Three Variables; Probability of a Shortage, Mean (μ_d) of a Poisson Demand Function, and the Reorder Point with the Poisson Lead Time Held Constant, μ_x = 7.

Table 11. Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 1 Week and the Demand Distribution is Poisson with Means (μ_d) of 1 through 11 Dozen.

$\mu_x = 1$

Reorder Point	$\mu_d =$ 1	2	3	4	5	6	7	8	9	10	11
1											
2	0.13930										
3	0.06634										
4	0.03196	0.14884									
5	0.01322*	0.09544									
6	0.00559*	0.05992									
7	0.00229	0.03683	0.11048								
8	0.00092	0.02218	0.07936								
9		0.01312	0.05619	0.11988							
10		0.00764*	0.03926	0.09270							
11		0.00438*	0.02712	0.07091							
12		0.00248	0.01853*	0.05374	0.10237						
13		0.00139	0.01255*	0.04041	0.08355						
14		0.00076	0.00842	0.03182	0.06652	0.10969					
15			0.00559	0.02239	0.05199	0.09100					
16			0.00369	0.01650	0.04106	0.07513	0.11541				
17			0.00241	0.01208	0.03227	0.06181	0.09803				
18			0.00156	0.00879	0.02524	0.05071	0.08298				
19			0.00101	0.00635	0.01965	0.04148	0.07009	0.10373			
20			0.00064	0.00457	0.01521	0.03380	0.05911	0.08946			
21				0.00326	0.01172	0.02744	0.04977	0.07708	0.10844		
22				0.00232	0.00899	0.02218	0.04179	0.06637	0.09488		
23				0.00164	0.00687	0.01785	0.03499	0.05712	0.08300	0.11237	
24				0.00116	0.00522	0.01431	0.02918	0.04907	0.07264	0.09945	
25				0.00081	0.00396	0.01144	0.02425	0.04206	0.06358	0.08805	
26					0.00299	0.00911	0.02008	0.03594	0.00562	0.07805	0.10332

*Computed on IBM 650

(Cont.)

Table 11 (Cont.). Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 1 Week and the Demand Distribution is Poisson with Means (μ_d) of 1 through 11 Dozen.

$\mu_x = 1$

Reorder Point	$\mu_d =$ 1	2	3	4	5	6	7	8	9	10	11
27					0.00225	0.00723	0.01658	0.03061	0.04857	0.06925	0.09238
28					0.00168	0.00573	0.01365	0.02598	0.04231	0.06145	0.08273
29					0.00126	0.00452	0.01210	0.02198	0.03674	0.05548	0.07421
30					0.00094	0.00356	0.00919	0.01855	0.03180	0.04821	0.06663
31						0.00279	0.00751	0.01563	0.02744	0.04269	0.05981
32						0.00219	0.00613	0.01314	0.02361	0.03742	0.05361
33						0.00171	0.00499	0.01102	0.02028	0.03281	0.04795
34						0.00133	0.00405	0.00924	0.01738	0.02869	0.04276
35						0.00103	0.00328	0.00773	0.01488	0.02502	0.03801
36						0.00080	0.00265	0.00645	0.01273	0.02179	0.03368
37							0.00214	0.00538	0.01087	0.01895	0.02976
38							0.00172	0.00447	0.00928	0.01645	0.02613
39							0.00139	0.00371	0.00790	0.01429	0.02305
40							0.00110	0.00307	0.00672	0.01240	0.02030
41							0.00088	0.00254	0.00571	0.01075	0.01787
42								0.00209	0.00483	0.00931	0.01571
43								0.00173	0.00409	0.00805	0.01381
44								0.00142	0.00345	0.00695	0.01213
45								0.00117	0.00291	0.00599	0.01065
46								0.00096	0.00245	0.00515	0.00934
47									0.00206	0.00443	0.00817
48									0.00173	0.00380	0.00715
49									0.00145	0.00325	0.00624
50									0.00121	0.00238	0.00543
51									0.00102	0.00203	0.00473
52									0.00085	0.00173	0.00406

(Cont.)

Table 11 (Cont.). Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 1 Week and the Demand Distribution is Poisson with Means (μ_d) of 1 through 11 Dozen.

$\mu_x = 1$

Reorder Point	$\mu_d =$ 1	2	3	4	5	6	7	8	9	10	11
53										0.00148	0.00353
54										0.00126	0.00307
55										0.00107	0.00267
56										0.00091	0.00231
57											0.00200
58											0.00173
59											0.00150
60											0.00129
61											0.00111
62											0.00096

Table 12. Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 2 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 8 Dozen.

$\mu_x = 2$

Reorder Point	μ_d = 1	2	3	4	5	6	7	8
1								
2								
3								
4	0.11470							
5	0.06311							
6	0.03327*							
7	0.01697							
8	0.00839	0.10872						
9	0.00403	0.07572						
10	0.00189	0.05183*						
11	0.00087	0.03523						
12		0.02342	0.10487*					
13		0.01534	0.08071*					
14		0.00978	0.06151*					
15		0.00624	0.04644*					
16		0.00393	0.03475*	0.10241				
17		0.00245	0.02579*	0.08335				
18		0.00151	0.01898*	0.06741				
19		0.00092	0.01386*	0.05419				
20			0.01005*	0.04344	0.10070			
21			0.00723*	0.03441	0.08497			
22			0.00517*	0.02720	0.07137			
23			0.00367*	0.02138	0.05970	0.11455		
24			0.00259*	0.01672	0.04973	0.09958		
25			0.00182*	0.01301	0.04126	0.08605		
26			0.00127*	0.01008	0.03410	0.07422		
27			0.00088*	0.00777	0.02807	0.06273	0.11142	
28			0.00060	0.00596	0.02302	0.05470	0.09848	

(Cont.)

Table 12 (Cont.). Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 2 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 8 Dozen.

$\mu_x = 2$

Reorder Point	$\mu_d =$ 1	2	3	4	5	6	7	8
29				0.00456	0.01881	0.04674	0.08683	
30				0.00347	0.01532	0.03982	0.07637	
31				0.00263	0.01243	0.03383	0.06700	0.10903
32				0.00198	0.01005	0.02866	0.05864	0.09773
33				0.00149	0.00810	0.02421	0.05120	0.08743
34				0.00112	0.00651	0.02039	0.04460	0.07806
35				0.00083	0.00521	0.01713	0.03876	0.06955
36					0.00416	0.01436	0.03361	0.06227
37					0.00332	0.01200	0.02908	0.05488
38					0.00263	0.01001	0.02511	0.04861
39					0.00208	0.00833	0.02163	0.04265
40					0.00164	0.00691	0.01860	0.03793
41					0.00130	0.00572	0.01596	0.03342
42					0.00102	0.00485	0.01367	0.02939
43					0.00080	0.00389	0.01168	0.02581
44						0.00320	0.00997	0.02262
45						0.00263	0.00849	0.01980
46						0.00215	0.00721	0.01730
47						0.00176	0.00612	0.01509
48						0.00143	0.00518	0.01314
49						0.00117	0.00438	0.01143
50						0.00095	0.00370	0.00992
51							0.00311	0.00860
52							0.00262	0.00745
53							0.00220	0.00644
54							0.00184	0.00556
55							0.00154	0.00479

(Cont.)

Table 12 (Cont.). Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 2 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 8 Dozen.

$\mu_x = 2$

Reorder Point	μ_d = 1	2	3	4	5	6	7	8
56							0.00126	0.00412
57							0.00103	0.00355
58							0.00090	0.00304
59								0.00261
60								0.00223
61								0.00191
62								0.00163
63								0.00139
64								0.00119
65								0.00101
66								0.00086

Table 13. Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 3 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 6 Dozen.

$\mu_x = 3$

Reorder Point	$\mu_d =$ 1	2	3	4	5	6
5	0.15045					
6	0.09152					
7	0.05367*					
8	0.03046*					
9	0.01678					
10	0.00899					
11	0.00470	0.10332				
12	0.00241	0.07907				
13	0.00120	0.05710				
14	0.00059	0.04067				
15		0.02860				
16		0.01987	0.11440			
17		0.01364	0.09172			
18		0.00926	0.07292			
19		0.00623	0.05753			
20		0.00414	0.04505			
21		0.00273	0.03502	0.11786		
22		0.00178	0.02703	0.09926		
23		0.00115	0.02073	0.08315		
24		0.00074	0.01579	0.06931		
25			0.01195	0.05749		
26			0.00899	0.04745		
27			0.00673	0.03899	0.10427	
28			0.00500	0.02880	0.09027	
29			0.00370	0.02596	0.07789	
30			0.00272	0.02105	0.06693	
31			0.00199	0.01700	0.05734	
32			0.00145	0.01367	0.04896	0.10784

(Cont.)

Table 13 (Cont.). Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 3 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 6 Dozen.

$\mu_x = 3$

Reorder Point	μ_d = 1	2	3	4	5	6
33			0.00105	0.01094	0.04167	0.09549
34			0.00076	0.00873	0.03535	0.08427
35				0.00694	0.02990	0.07427
36				0.00549	0.02521	0.06525
37				0.00433	0.02119	0.05719
38				0.00341	0.01776	0.04999
39				0.00267	0.01484	0.04360
40				0.00208	0.01237	0.03794
41				0.00162	0.01028	0.03293
42				0.00126	0.00852	0.02852
43				0.00097	0.00705	0.02465
44					0.00581	0.02125
45					0.00478	0.01829
46					0.00392	0.01570
47					0.00321	0.01345
48					0.00262	0.01150
49					0.00213	0.00981
50					0.00173	0.00836
51					0.00141	0.00710
52					0.00114	0.00603
53					0.00918	0.00510
54						0.00431
55						0.00364
56						0.00306
57						0.00257
58						0.00216
59						0.00181
60						0.00151
61						0.00126
62						0.00105
63						0.00088

Table 14. Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 4 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 6 Dozen.

$\mu_x = 4$

Reorder Point	$\mu_d =$ 1	2	3	4	5	6
7	0.11574					
8	0.07277					
9	0.04438					
10	0.02631					
11	0.01520					
12	0.00857					
13	0.00473					
14	0.00256	0.10280				
15	0.00136	0.07767				
16	0.00071	0.05796				
17		0.04275				
18		0.03117				
19		0.02249				
20		0.01606	0.11723			
21		0.01135	0.09635			
22		0.00795	0.07864			
23		0.00516	0.06375			
24		0.00379	0.05134			
25		0.00259	0.04109			
26		0.00175	0.03269			
27		0.00118	0.02585	0.10809		
28		0.00078	0.02032	0.09253		
29			0.01588	0.07887		
30			0.01235	0.06693		
31			0.00955	0.05657		
32			0.00735	0.04762		
33			0.00562	0.03993	0.11613	
34			0.00428	0.03335	0.10240	

(Cont.)

Table 14 (Cont.). Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 4 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 6 Dozen.

$\mu_x = 4$

Reorder Point	$\mu_d =$ 1	2	3	4	5	6
35			0.00325	0.02774	0.08592	
36			0.00245	0.02300	0.07889	
37			0.00184	0.01899	0.06894	
38			0.00138	0.01563	0.06008	
39			0.00102	0.01282	0.05220	
40			0.00076	0.01048	0.04523	0.10973
41				0.00854	0.03909	0.09850
42				0.00693	0.03369	0.08823
43				0.00561	0.02896	0.07885
44				0.00453	0.02483	0.07033
45				0.00364	0.02123	0.06259
46				0.00292	0.01811	0.05558
47				0.00233	0.01541	0.04926
48				0.00186	0.01308	0.04358
49				0.00148	0.01108	0.03847
50				0.00117	0.00936	0.03389
51				0.00093	0.00789	0.02980
52					0.00664	0.02615
53					0.00557	0.02291
54					0.00466	0.02003
55					0.00390	0.01748
56					0.00325	0.01523
57					0.00270	0.01325
58					0.00224	0.01150
59					0.00186	0.00997
60					0.00154	0.00862
61					0.00127	0.00745
62					0.00105	0.00642

(Cont.)

Table 14 (Cont.). Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 4 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 6 Dozen.

$\mu_x = 4$

Reorder Point	$\mu_d =$ 1	2	3	4	5	6
63					0.00086	0.00553
64						0.00475
65						0.00408
66						0.00349
67						0.00299
68						0.00255
69						0.00218
70						0.00185
71						0.00158
72						0.00134
73						0.00113
74						0.00096

Table 15. Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 5 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 5 Dozen.

$\mu_x = 5$

Reorder Point	$\mu_d =$ 1	2	3	4	5
8	0.12447				
9	0.09022				
10	0.05800				
11	0.03630				
12	0.02218				
13	0.01325				
14	0.00775				
15	0.00444				
16	0.00250	0.12319			
17	0.00138	0.09626			
18	0.00075	0.07427			
19		0.05669			
20		0.04283			
21		0.03203			
22		0.02373			
23		0.01742			
24		0.01267	0.11682		
25		0.00914	0.09766		
26		0.00654	0.08114		
27		0.00464	0.06703		
28		0.00327	0.05505		
29		0.00229	0.04496		
30		0.00159	0.03653		
31		0.00109	0.02951		
32		0.00075	0.02372	0.11293	
33			0.01897	0.09812	
34			0.01510	0.08492	
35			0.01196	0.07323	

(Cont.)

Table 15 (Cont.). Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 5 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 5 Dozen.

$\mu_x = 5$

Reorder Point	$\mu_d =$ 1	2	3	4	5
36			0.00943	0.06292	
37			0.00740	0.05386	
38			0.00578	0.04595	
39			0.00450	0.03906	
40			0.00349	0.03310	0.11037
41			0.00269	0.02795	0.09829
42			0.00207	0.02353	0.08731
43			0.00158	0.01974	0.07735
44			0.00121	0.01651	0.06836
45			0.00092	0.01377	0.06026
46				0.01145	0.05298
47				0.00949	0.04649
48				0.00785	0.04068
49				0.00647	0.03552
50				0.00532	0.03092
51				0.00436	0.02690
52				0.00356	0.02333
53				0.00291	0.02019
54				0.00236	0.01743
55				0.00192	0.01502
56				0.00155	0.01291
57				0.00125	0.01108
58				0.00101	0.00949
59				0.00081	0.00811
60					0.00692
61					0.00589
62					0.00500
63					0.00424

(Cont.)

Table 15 (Cont.). Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 5 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 5 Dozen.

$\mu_x = 5$

Reorder Point	μ_d = 1	2	3	4	5
64					0.00359
65					0.00304
66					0.00256
67					0.00216
68					0.00181
69					0.00152
70					0.00127
71					0.00107
72					0.00089

Table 16. Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 6 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 5 Dozen.

$\mu_x = 6$

Reorder Point	μ_d = 1	2	3	4	5
10	0.10600*				
11	0.07095*				
12	0.04633*				
13	0.02956*				
14	0.01846*				
15	0.01129*				
16	0.00677*				
17	0.00399*				
18	0.00231*				
19	0.00132	0.11290			
20	0.00074	0.08933			
21		0.06999			
22		0.05431			
23		0.04176			
24		0.03182			
25		0.02404			
26		0.01800			
27		0.01338			
28		0.00986	0.11463		
29		0.00721	0.09703		
30		0.00524	0.08169		
31		0.00378	0.06841		
32		0.00270	0.05700		
33		0.00192	0.04726		
34		0.00136	0.03899		
35		0.00095	0.03201		
36			0.02612		
37			0.02127		

(Cont.)

Table 16 (Cont.). Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 6 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 5 Dozen.

$\mu_x = 6$

Reorder Point	$\mu_d =$ 1	2	3	4	5
38			0.01723	0.10125	
39			0.01389	0.08862	
40			0.01115	0.07732	
41			0.00891	0.06724	
42			0.00709	0.05828	
43			0.00562	0.05036	
44			0.00444	0.04338	
45			0.00349	0.03725	
46			0.00274	0.03189	
47			0.00214	0.02722	0.10391
48			0.00129	0.02317	0.09321
49			0.00099	0.01967	0.08342
50				0.01665	0.07449
51				0.01405	0.06637
52				0.01182	0.05901
53				0.00992	0.05234
54				0.00831	0.04634
55				0.00694	0.04093
56				0.00578	0.03608
57				0.00480	0.03174
58				0.00398	0.02786
59				0.00329	0.02441
60				0.00271	0.02134
61				0.00223	0.01862
62				0.00183	0.01622
63				0.00150	0.01410
64				0.00122	0.01223
65				0.00100	0.01059

(Cont.)

Table 16 (Cont.). Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 6 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 5 Dozen.

$\mu_x = 6$

Reorder Point	μ_d = 1	2	3	4	5
66					0.00916
67					0.00790
68					0.00681
69					0.00585
70					0.00502
71					0.00430
72					0.00368
73					0.00314
74					0.00268
75					0.00228
76					0.00194
77					0.00164
78					0.00139
79					0.00118
80					0.00099

Table 17. Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 7 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 5 Dozen.

$\mu_x = 7$

Reorder Point	$\mu_d =$ 1	2	3	4	5
11	0.12029				
12	0.08318				
13	0.05619				
14	0.03714				
15	0.02403				
16	0.01526				
17	0.00951				
18	0.00582				
19	0.00351				
20	0.00208				
21	0.00121				
22	0.00070	0.10321			
23		0.08255			
24		0.06545			
25		0.05144			
26		0.04009			
27		0.03099			
28		0.02377			
29		0.01809			
30		0.01367			
31		0.01025			
32		0.00764	0.11151		
33		0.00565	0.09530		
34		0.00415	0.08106		
35		0.00303	0.06861		
36		0.00220	0.05781		
37		0.00158	0.04849		
38		0.00114	0.04049		

(Cont.)

Table 17 (Cont.). Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 7 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 5 Dozen.

$\mu_x = 7$

Reorder Point	μ_d = 1	2	3	4	5
39		0.00081	0.03366		
40			0.02786		
41			0.02297		
42			0.01885		
43			0.01541	0.10279*	
44			0.01255	0.09077*	
45			0.01018	0.07991*	
46			0.00823	0.07014*	
47			0.00662	0.06139*	
48			0.00531	0.05358*	
49			0.00424	0.04663*	
50			0.00338	0.04047*	
51			0.00269	0.03502*	
52			0.00212	0.03023*	0.11891
53			0.00167	0.02602*	0.10771
54			0.00131	0.02234*	0.09737
55			0.00103	0.01913*	0.08773
56			0.00080	0.01634*	0.07901
57				0.01392*	0.07080
58				0.01183*	0.06367
59				0.01003*	0.05698
60				0.00848*	0.05089
61				0.00716*	0.04526
62				0.00602	0.04030
63				0.00506	0.03580
64				0.00424	0.03174
65				0.00354	0.02809
66				0.00295	0.02481

(Cont.)

Table 17 (Cont.) Tables of Probability of a Shortage at Various Reorder Points where the Lead Time has a Poisson Distribution with Mean (μ_x) of 7 Weeks and the Demand Distribution is Poisson with Means (μ_d) of 1 through 5 Dozen.

$\mu_x = 7$

Reorder Point	μ_d = 1	2	3	4	5
67				0.00246	
68				0.00204	
69				0.00169	
70				0.00140	
71				0.00115	
72				0.00095	

Table 18. Demand for All Sizes of Gloves by Weeks for 170 Weeks, January 2, 1956 - March 30, 1959.

Week Ending	Glove Size							
Date	6	6½	7	7½	8	8½	9	Total
1-2-56	0	4	6	6	4	1	1	22
1-9	0	0	7	8	4	0	1	20
1-16	0	0	0	0	0	0	0	0
1-23	0	0	0	0	0	0	0	0
1-30	2	0	3	3	3	1	1	13
2-6-56	0	6	3	4	0	0	0	13
2-13	1	3	3	2	3	0	1	13
2-20	0	3	3	4	4	0	0	14
2-27	0	3	3	4	0	1	0	11
3-5-56	0	1	4	4 6/12	1	1	1	12 6/12
3-12	0	2	3	6	1	0	0	12
3-19	0	0	0	12	8	0	0	20
3-26	0	6	3	4	4	1	0	18
4-2-56	0	8	6	8	8	0	0	30
4-9	0	0	3	3	3	0	0	9
4-16	1	11	15	22	7	6	1	63
4-23	1	4	6	8	4	4	1	28
4-30	0	0	0	0	0	0	0	0
5-7-56	0	4	0	6	4	2	1	17
5-14	0	0	0	0	0	0	0	0
5-21	0	4	2	5	1	2	3	17
5-28	0	5	0	2	2	0	0	9
6-4-56	1	4	9	7	4	2	0	27
6-11	0	0	0	6	0	0	0	6
6-18	0	10	7	7	3	1	0	28
6-25	2	4	4	3	2	0	0	15

(Cont.)

Table 18 (Cont.). Demand for All Sizes of Gloves by Weeks for 170 Weeks, January 2, 1956 - March 30, 1959.

Week Ending	Glove Size							
Date	6	$6\frac{1}{2}$	7	$7\frac{1}{2}$	8	$8\frac{1}{2}$	9	Total
7-2-56	0	1	1	5	3	0	0	10
7-9	2	4	5 6/12	5	3	2/12	0	19 8/12
7-16	0	3	5	6	2	2	2	20
7-23	0	2	2	8	2	0	1	15
7-30	1	2	3	8 3/12	4	2	0	20 3/12
8-6-56	0	3	7	8 6/12	4	1	1	24 6/12
8-13	0	0	0	4	0	1	0	5
8-20	0	4	1	3	5	0	1	14
8-27	0	7	4	5 1/12	14	2	0	32 1/12
9-3-56	0	2	2	9	5	2	0	20
9-10	2	0	5	7 6/12	1	1/12	0	15 7/12
9-17	0	4	1	6	1	0	0	12
9-24	2	4	5	7	4	0	0	22
10-1-56	0	6	1	6	0	0	0	13
10-8	0	3	4	2	4	0	2	15
10-15	0	1	5	0	4	0	1	11
10-22	0	2	2	2	0	0	0	6
10-29	0	4	4	20	1	0	0	29
11-5-56	2	6	5	7	5	0	0	25
11-12	2	3	0	7	6	2/12	0	18 2/12
11-19	0	1	6	1 4/12	4	1	1	14 4/12
11-26	0	6	6	8	0	1	0	21
12-33-56	0	6	6	8	6	1	2	29
12-10	1	6	5	7	5	2	1	27
12-17	0	4	3	8	5	0	0	20
12-24	0	0	0	0	0	0	0	0
12-31	0	0	6	0	0	0	0	6

(Cont.)

Table 18 (Cont.). Demand for All Sizes of Gloves by Weeks for 170 Weeks, January 2, 1956 - March 30, 1959.

Week Ending Date	6	$6\frac{1}{2}$	7	$7\frac{1}{2}$	8	$8\frac{1}{2}$	9	Total
1-7-57	0	6	0	0	2	0	1	9
1-14	0	4	8	16	6	0	4	38
1-21	0	3	4	8	0	0	0	15
1-28	1/4	5	5	7	4	0	0	21 3/12
2-4-57	1	2	7	5	2	1	1	15
2-11	0	2	3	2	0	0	0	7
2-18	0	2	0	5	2	0	2	11
2-25	2	3	4	4	5	1	0	19
3-4-57	0	0	0	6	3	0	0	9
3-11	0	3	5	8	0	0	0	16
3-18	0	2	2	8	0	0	0	12
3-25	0	3 1/12	3	6	4	0	0	16 1/12
4-1-57	0	5 1/12	5	6	0	0	1	17 1/12
4-8	0	0	2	7	5	2	0	16
4-15	0	6	5	10	3 1/12	0	0	24 1/12
4-22	0	7	5	5	0	0	0	17
4-29	0	0	3	8 1/12	3	0	0	14 1/12
5-6-57	0	6	4	0	3	1	1	15
5-13	0	4	5	8	4	0	0	21
5-20	0	4	8	8	0	1	1	22
5-27	0	7	8	7	6	0	1	28
6-3-57	0	3	2	8	5	0	0	18
6-10	0	2	2	7	5	2/12	0	16 2/12
6-17	0	4	5	7 2/12	2/12	1	0	17 4/12
6-24	0	3	3	8	5	1	0	20

(Cont.)

Table 18 (Cont.). Demand for All Sizes of Gloves by Weeks for 170 Weeks, January 2, 1956 - March 30, 1959.

Week Ending Date	6	6½	7	7½	8	8½	9	Total
7-1-57	1	3	6	6	0	1	0	17
7-8	2	2	2	5	2	0	0	13
7-15	0	0	4	6	3	0	0	13
7-22	1	0	0	8	4	0	0	13
7-29	0	8	3	6	3	0	2	22
8-5-57	0	2	6	4	0	1	0	13
8-12	0	6	5	8	5	0	2	26
8-19	0	4	3	6	6	1	1	21
8-26	0	2	0	13	3	0	0	18
9-2-57	0	0	4	8	9	1	0	22
9-9	0	5 10/12	1	3	2	0	0	11 10/12
9-16	2	0	6	8	3	1	0	20
9-23	0	0	4	0	0	1	0	5
9-30	0	8	4	13	5	0	1	31
10-7-57	1	5	2	3	5	0	0	16
10-14	0	8	3	6	3	0	0	20
10-21	0	5	6	2	6	0	1	20
10-28	2	0	2	8	2	0	0	14
11-4-57	0	0	4	6	3	0	0	13
11-11	0	5	5	2	1	1	1	15
11-18	0	3	4	8	0	0	0	14
11-25	1	1	1	8	5	0	0	16
12-2-57	0	4	2	0	1	0	0	7
12-9	0	2	5	6	1	0	0	14
12-16	0	5	6	8	6	1	0	26
12-23	1	1	1	1	0	0	0	4
12-30	0	1	3	4	0	0	1	9

(Cont.)

Table 18 (Cont.) Demand for All Sizes of Gloves by Weeks for 170 Weeks, January 2, 1956 - March 30, 1959.

Week Ending	Glove Size							
Date	6	6½	7	7½	8	8½	9	Total
1-6-58	1	3	2	7	2	0	0	15
1-13	5	4	6	4	2	2	0	23
1-20	2	5	6	4	1	0	0	18
1-27	0	1	2	7	0	0	0	10
2-3-58	1	6	6	8	4	0	0	25
2-10	0	7	8	8	3	1	0	27
2-17	2	0	0	8	5	1	0	16
2-24	0	0	7	8	0	0	0	15
3-3-58	1	2	0	4	0	0	0	7
3-10	0	3	8	8	2	1	1	23
3-17	0	0	3	6	2	0	0	11
3-24	1	0	1	7	3	1	0	13
3-31	1	5	5	6	5	0	0	22
4-7-58	1	0	4	5	4	2	0	16
4-14	0	0	5	3	0	0	2	10
4-21	1	5	6	8	4	1	1	26
4-28	2	2	5	8	6	1	1	25
5-5-58	0	0	0	8	0	0	0	8
5-12	0	6	8	13	6	1	0	34
5-19	0	3	6	4	3	0	0	16
5-26	0	0	0	0	0	0	0	0
6-2-58	0	8	13	12	9	2	1	45
6-9	0	2	6	6	5	0	0	19
6-16	0	5	6	9	7	0	0	27
6-23	0	2	0	0	0	0	0	2
6-30	0	3	2	4	1	0	0	10

(Cont.)

Table 18 (Cont.). Demand for All Sizes of Gloves by Weeks for 170 Weeks, January 2, 1956 - March 30, 1959.

Week Ending	Glove Size							
Date	6	6½	7	7½	8	8½	9	Total
7-7-58	0	2	3	4	3	0	0	12
7-14	0	2	8	6	6	3	0	25
7-21	2	6	8	12	3	0	0	31
7-28	0	6	1	5	2	1	0	15
8-4-58	0	8	3	7	2	0	0	20
8-11	0	5	4	8	0	0	0	17
8-18	1	1	4	8	2	0	0	16
8-25	0	4	4	6	5	0	0	19
9-1-58	0	5	5	8	8	2	0	28
9-8	1	2	3	6	2	0	0	14
9-15	0	6	5	8	4	0	0	23
9-22	0	3	3	4	3	0	0	13
9-29	0	6	4	12	9	0	0	31
10-6-58	1	3	5	3	6	0	0	18
10-13	0	2	2	1	0	0	0	5
10-20	1	2	2	5	1	0	0	11
10-27	0	0	0	0	0	2	0	2
11-3-58	0	3	2	4	0	1	0	10
11-10	1	5	3	6	4	1	0	20
11-17	1	0	1	6	0	1	1	10
11-24	0	6	3	4	5	1	1	20
12-1-58	1	3	5	7	3	0	0	19
12-8	0	1	4	7	5	0	0	17
12-15	1	1	0	4	0	0	0	6
12-22	0	0	1	0	0	0	0	1
12-29	1	2	3	8	1	0	0	15

(Cont.)

Table 18 (Cont.). Demand for All Sizes of Gloves by Weeks for 170 Weeks, January 2, 1956 - March 30, 1959.

Week Ending	Glove Size							
Date	6	$6\frac{1}{2}$	7	$7\frac{1}{2}$	8	$8\frac{1}{2}$	9	Total
1-5-59	1	5	0	3	4	1	0	14
1-12	0	2	4	8	4	1	1	20
1-19	0	4	6	8	6	0	0	24
1-26	0	0	0	0	0	0	0	0
2-2-59	0	4	1	7	0	0	1	13
2-9	2	6	6	6	4	0	0	24
2-16	0	2	1	5	4	0	1	13
2-23	0	2	4	6	4	0	0	16
3-2-59	0	4	2	7	4	0	0	17
3-9	1	6	2	6	2	0	0	17
3-16	2	2	4	2	2	2	0	14
3-23	0	0	3	5	6	0	0	14
3-30	0	0	8	8	6	0	0	22

The following example illustrates the use of these curves. The lead time distribution has been examined and found to be Poisson distributed with a mean (μ_x) of 2 weeks. The demand distribution has been examined and found to be Poisson distributed with a mean (μ_d) of 6 gross per week. The tolerable level of shortage that management will permit is two per cent. From Figure 13 the reorder point is found to be 34 gross. This means that when the inventory level reaches 34 gross, an order should be placed for the economic lot size.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Conclusions.-- The conclusions that can be drawn from this study are as follows:

1. Factors, in addition to price, should be considered when determining the proper quantity of an item to buy.
2. The lead time distribution should be considered when determining protective stock levels as this improves the accuracy of the inventory model. In particular the assumption of constant lead time leads to underestimate of requisite protective stock.
3. Protective stock levels can be set for hospital use by the statistical evaluation of demand and lead time distributions.
4. The inventory records of the hospital frequently lack the necessary information for a complete solution by the model.
5. The model constructed in this study can provide an accurate guide to evaluate the costs of various inventory policies, when the required information is available.

Recommendations.-- In view of the limitations, results, and conclusions of this study, the following recommendations are made with regard to further methodological and further computational studies:

1. Methodological Studies
   a. The cost of various types of shortages, on a probability basis, should be investigated.

b. Methods for accurately estimating the ordering cost and inventory carrying costs should be investigated.

2. Computational Studies

a. The table of reorder points calculated from the joint density function of two Poisson distributions should be extended.

b. A method of interpolating between calculated values in the table of reorder points should be determined.

c. Tables of reorder points should be calculated for typical types of demand and lead time distributions, other than from combinations of normal and Poisson distributions. Typical distributions that should be considered are:

(1) Log Normal

(2) Chi-square

(3) Erlang.

Hospital administrators are encouraged to utilize this decision model to effect real savings in inventory costs. In addition to the cost savings there are other definite advantages in that data required for this model provides an opportunity for administrators to exercise judgement and control over factors which are hidden at present.

It is apparent that the inventory model developed in this study also can be used in industries other than hospitals. Objective decision models of this type are playing an ever increasing role in the reduction of costs and in the improvement of continuity of operations.

APPENDIX

APPENDIX A

Derivation of Joint Density Function for the Calculation of Reorder Points for Specified Probability of a Shortage

$$\text{Probability of a shortage}^{1} = \sum_{x=0}^{\infty} \sum_{D_x > R.P.}^{\infty} p(x)\ p(D_x|x),$$

where x = random variable lead time and

μ_x = mean lead time.

Under the assumption of Poisson distributed lead time

$$p(x) = \frac{e^{-\mu_x} \mu_x^{x}}{x!}$$

where d = random variable (number of units per time unit),

μ_d = mean demand per time unit,

σ_d = standard deviation of demand,

and the total demand (D_x) during lead time is

$$D_x = d_1 + d_2 + d_3 + \ldots + d_x$$

with mean = μ_d .

For the normal distributed demand

the standard deviation (σ_{D_x}) is

$$\sigma_{D_x} = \sqrt{x}\ \sigma_d$$

[1]Course Notes - Special Problem Course I.E. 705, Dr. J. J. Moder, Professor of Industrial Engineering, Georgia Institute of Technology, 1959. See also Harling and Bramson (14).

and $p(D_x|x) = f(D_x|x)\, dD_x$.

The probability of a shortage, for the conditions stated, becomes

$$\text{probability of a shortage} = \sum_{x=0}^{\infty} p(x) \int_{R.P.}^{\infty} f(D_x|x)\, dD_x$$

$$= \sum_{x=0}^{\infty} \frac{e^{-\mu_x} \mu_x^x}{x!} \int_{R.P.}^{\infty} \frac{1}{\sigma_{D_x} \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{D_x - x\mu_d}{\sigma_{D_x}}\right)} dD_x .$$

However, for the condition of a Poisson distributed demand

$$p(D_x|x) = \frac{e^{-x\mu_d} (x\, \mu_d)^{D_x}}{D_x!}$$

and

$$\text{probability of a shortage} = \sum_{x=0}^{\infty} \sum_{D_x > R.P.}^{\infty} \frac{e^{-\mu_x} \mu_x^x}{x!} \cdot \frac{e^{-x\mu_d} (x\mu_d)^{D_x}}{D_x!} .$$

APPENDIX B

A Sample Calculation for One Point on the Curve
$\mu_x = z$, $\mu_d = 6$, with Demand and Lead Time
Poisson Distributed

$\mu_x = 2$ $\mu_d = 6$ R.P. = 34 $D_x = 35$

x	P(x)		$P(D_x \mid x)$		
1	0.270671	x	0.000000	=	0.00000000
2	0.270671	x	0.000000	=	0.00000000
3	0.180447	x	0.000248	=	0.00004475
4	0.090224	x	0.020570	=	0.00185591
5	0.036089	x	0.202692	=	0.00731495
6	0.012030	x	0.588503	=	0.00707696
7	0.003437	x	0.878581	=	0.00301968
8	0.000859	x	0.978679	=	0.00084069
9	0.000191	x	0.997593	=	0.00019054
10	0.000038	x	0.999812	=	0.00003799
11	0.000003	x	0.999989	=	0.00000700
12	0.000001	x	0.999999	=	0.00000100
				Σ =	0.02039220

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