APPLICATION OF DISCRETE DISTRIBUTIONS IN QUALITY CONTROL

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CHAPTER I

INTRODUCTION

1.1 Background

Statistical quality control is divided into two major areas, acceptance sampling and process control through the application of control charts. Control charts can be broadly classified into charts for use with variable data or attribute data. Three types of control charts are often employed when the data is in attribute form, the p-chart or control chart for fraction defective, the c-chart or control chart for defects, and the u-chart or control chart for defects per unit. The c and u charts are investigated in this thesis.

The Poisson distribution is often automatically assumed to represent the underlying distribution of the occurrence of defects for discrete data. Actually, many quality control references provide control limits for c and u charts with little mention of the fact that they hold for only the Poisson distribution, and that for many situations, these limits would not even be good approximation. Consequently, the full economic savings that should result from the use of control charts, may not be realized. In fact, economic loss could possibly result from the incorrect application of the Poisson distribution.

Situations that may lead to other distributions are:

1) When defects occur in clusters.

2) Where the defects are the result of two or more underlying sources.
3) Where the probability of zero defects is not related to the distribution of counts or the occurrence of zero defects cannot be recorded.

No analytical work has been published to date concerning the extent of the dangers associated with application of an incorrect discrete distribution when modeling the occurrence of defects.

1.2 Problem Statement

This thesis considers the development of practical methods for determining the proper discrete distribution to represent the occurrence of defects, and the determination of appropriate control procedures for several alternative probability models. The economic consequences of model misspecification will also be investigated.

1.3 Objectives and Scope

The overall objective of this thesis is to develop a practical methodology for the utilization and measurement of discrete distributions in process control procedures for defects. The specific objectives entail answering the following questions.

1) What discrete distributions do defects frequently follow? The selection of distributions to study will be based primarily on information gleaned from the literature survey.
2) How can an analyst determine which distribution model adequately represents a distribution of defects?

Graphical methods as well as goodness of fit tests to aid an analyst are presented in Chapter III.

3) What are some appropriate control procedures for the distributions found in Objective 1? Economic models of the control procedures based on these distributions are provided in Chapter 4.

4) What are some of the possible economic consequences if the defect distribution is misspecified? The effect of the shape of the selected distributions are analyzed in this portion of the research. For example, what differences in costs arise with the assumption that the underlying distribution of defects is Poisson when in actuality it is the negative binomial distribution with an approximately equal mean and variance. Chapter 5 presents the results of a numerical analysis involving distribution misspecification.

CHAPTER II

LITERATURE SURVEY

The literature survey is divided into three sections. These sections pertain to:

1) The adequacy of the Poisson model;
2) The determination of a probability model which satisfactorily represents a distribution of defects.
3) The economic design of quality control charts.

The numbers in parenthesis in the following discussion refers to the bibliography.

2.1 The Adequacy of the Poisson Model

Hahn and Shapiro (8) note that the Poisson distribution may be used to represent the occurrence of independent events that take place at a constant rate. The negative binomial model arises when the occurrence rate is not constant.

Jackson (9) presents and discusses three situations that would lead to distributions other than the Poisson. These situations are where defects occur in clusters, where defects are the result of two underlying causes, and where the probability of zero defects is not related to the distribution of counts or the occurrence of zero defects cannot be recorded. The selection of the distributions utilized in this research was largely based on Jackson's article and the references listed in it. A more detailed explanation of Jackson's article is presented in Section 2.4. Montgomery (12) reports that it is important to model the occurrence of assignable causes properly when developing control charts for defects. He states that most of the currently available models assume that assignable causes occur according to a Poisson process. He also contends that "if the occurrence of assignable causes can be thought of as random 'shocks' acting on the system, that is, if the probability of a process shift within any small interval of time is directly proportional to the length of the interval, then this assumption is probably appropriate." However, if assignable causes occur as a result of the cumulative effects of heat, vibration, shock, and other similar phenomena, or as a result of an improper setup or excessive stress during process start-up, then use of the exponential distribution to model the interval during which the process is in-control may not be appropriate. He further contends that serious economic consequences may result from incorrectly using the Poisson process assumption. Alternatives to the Poisson distribution were not provided.

2.2 The Determination of a Distribution Model Which Satisfactorily Represents a Distribution of Defects

Pearson (19) noted that for the hypergeometric distribution the ratio (Pj+1 - Pj+1 + Pj) is of the form

$$\frac{\text{linear function of } j}{\text{quadratic function of } j} \; .$$

where

$$P_j = PR[X = j]$$

Pearson used this as a starting point for obtaining (by a limiting process) the differential equation defining the Pearson system of continuous distribution functions.

Ord (16,17) has developed a system for discrete distributions similar to the Pearson system for the discrete case. This system is fairly general at the current time but quite useful for many applications.

Katz (11) shows those parts of the (α,β) plane occupied by the Poisson, negative binomial (Pascal), and binomial distributions, where

$$\alpha = \frac{\mu^2}{\sigma^2} \quad \text{and} \quad \beta=1 - \frac{\mu}{\sigma^2}$$

An analyst can readily determine which, if either of the Poisson, negative binomial or binomial distribution are adequate to model their data by calculating and then plotting the α and β values on this α,β plane.

Johnson and Katz (10) present a summary of all the above mentioned articles, as well as methods of approximation for the negative binomial and Neyman Type A distributions.

Hahn and Shapiro (8) present the fundamentals of probability plotting and tests for checking distributional assumptions.

Probability plots for the normal, exponential, gamma and other continuous distributions are shown. Tests for distributional assumptions include tests to evaluate specifically the assumption of:

1) Normal or Log-Normal Distribution;
2) Exponential Distribution-Origin Known;
3) Exponential Distribution-Origin Unknown

The Chi-Square Goodness of Fit Test is also presented.

Dubey (6) presents graphical tests for the binomial, Poisson and negative binomial distributions. The tests for the Poisson and negative binomial are discussed in Chapter 3.

Anscombe (1) compared the negative binomial form of distribution with the Neyman Type A and six other two-parameter forms of distributions. He shows that they can be arranged in order of increasing skewness and tail length, and that they vary in the number of modes possible in the frequency function.

White, Schmidt, and Bennett (21) describe and provide examples illustrating the Kolmogorov-Smirnov tests, Chi-Square Goodness of Fit test, and a special Poisson-Process test. The latter test is discussed further in Chapter 3.

2.3 The Economic Design of Quality Control Charts

Montgomery, Heikes, and Fuller (14) presents discrete time models for the optimum economic design of both c-charts and u-charts. A grid search procedure is used to select the control chart parameters that minimize the objective function. The sensitivity analysis in this paper indicates that the model reaction to changes in cost coefficients and other model parameters is appropriate. They also state that the cost function is convex and relatively flat in the neighborhood of the optimum.

Montgomery (12) reviews and analyzes several different process models that have been developed and applied to most of the major types of control charts. He also provides some discussion on the practical implementation of economic design procedures for control charts. The remaining articles analyzed in this survey are also discussed in Montgomery (12).

Girshick and Rubin (7) considered a process model in which a machine produces items with a quality characteristic x. This machine can be in one of four states. States 1 and 2 are production states, while states 3 and 4 are repair states. They treat both 100 percent inspection and periodic inspection rules. The economic criterion is to maximize the expected income from the process. The optimal control rules are difficult to derive and consequently, the model's use in practice has been limited. The paper, however, is of significant theoretical value. Girshick and Rubin were the first to propose the expected cost (or income) per unit time criterion and rigorously show its usefulness for this problem. Numerous authors have investigated single assignable cause economic models for the fraction defective control chart. Chui (2,3) has formulated a cost model of the fraction defective chart. Chui uses a variation of Fibonacci search to find the economically optimal design. He also proposes an approximately optimal design procedure.

Chui (4) presents a brief sensitivity analysis of this model. He notes that the model is relatively insensitive to errors in estimating the cost coefficients, but requires more precise estimates of the fraction defective when in the in-control and out-of-control states.

Montgomery, Heikes, and Mance (15) have developed a multiple assignable cause economic model for the fraction defective control chart. They use both the grid search methods and pattern search to minimize the cost function. The article contains solutions to approximately 100 numerical examples. Results of a sensitivity analysis are also reported. The model is not extremely sensitive to the number of out-of-control states utilized, and they note that a properly chosen single cause model would often be a good approximation for a complex multiple cause process. The cost response surface is convex and relatively flat in the vicinity of the optimum, therefore moderate error in estimating the model parameters has only a slight effect.

Montgomery and Heikes (13) investigated the use of the geometric, Poisson and logarithmic series distributions to model the duration that a process is in-control. They note that the choice of process failure mechanism is an important aspect of optimum control chart design, and that misspecification of this property can result in significant economic penalties.

2.4 Distribution Selection

Patil and Joshi (18) contains over 3000 cross-indexed references and information relating to the probability function, generating function, and other descriptive measures for over 150 discrete distributions.

Four distributions were chosen for study in this thesis. They are the Poisson, negative binomial, weighted sums of two Poissons and combination Poisson and negative binomial. The four distribution models chosen for this research should have wide industrial application. For example, Jackson (9) specifies several situations where the four distributions apply. Hahn and Shapiro (8) also provide applications for some of these distributions.

2.5 Parameter Estimation

Parameter estimation is not considered in this thesis; however, the parameters of the distributions must be known or estimated in order to utilize the techniques and models presented in this thesis. The following references are provided for the convenience of the reader in case parameter estimation is found to be necessary.

Johnson and Katz (10) provide methods for estimating the parameters of the Poisson, negative binomial, and Neyman Type A. Cohen (5) provides methods for estimating the parameters of the distribution of the weighted sum of two Poissons and the distribution that results from the mixture of a Poisson and negative binomial.

CHAPTER III

MODELING DISTRIBUTION OF DEFECTS

3.1 Introduction

The procedure most commonly employed (and the one used here) when modeling discrete data is to experimentally investigate several of the standard distributions. Other techniques are discussed in the literature survey. Generally, one of the standard distributions will approximate the actual distribution well enough so that it may be used for significance testing and estimation procedures. The selection of candidate distributions may be aided by investigation of the underlying mechanism of the process that generated the data. This should be done first if possible. The results of such an investigation may suggest a particular distribution or group of distributions. Conversely, if the underlying mechanism is unknown, the distribution having the best fit may provide a clue to the nature of the mechanism.

Techniques to assess the adequacy of a selected model are also provided in this chapter. Two different techniques are discussed: probability plotting and statistical (goodness of fit) tests. Specifically, the method is discussed by which these techniques can be utilized to determine if it is reasonable to assume a Poisson or negative binomial or Neyman Type A model on the basis of the given data. These three distributions are frequently encountered in practice (9). The flow diagram near the end of the chapter presents a logical course of action when modeling distribution of defects. A reference for other candidate distributions is listed on the flow diagram in case the Poisson, negative binomial and Neyman Type A are all found to be inappropriate.

3.2 Probability Plotting

Probability plotting consists of constructing a graphical or pictorial display of the data. The analyst visually examines this pictorial representation in an attempt to determine whether or not the data contradicts the assumed model. Probability plotting is generally very simple, which makes it a very appealing technique. However, it must be remembered that it is a subjective method and may not provide clear-cut answers to the appropriateness of a particular model.

The following graphical tests are proposed by Dubey (6) to determine if the data can be satisfactorily described by the Poisson distribution or negative binomial (Pascal) distribution.

The probability function p(x) of the Poisson distribution is given by

$$p(x) = \text{Prob. } (X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad , \; x = 0,1,2.... \qquad (3\text{-}1)$$

where λ is a positive number. From expression (3-1) we write:

$$p(x+1) = \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!} \qquad (3\text{-}2)$$

Next we see that

$$\frac{p(x)}{p(x+1)} = \frac{x+1}{\lambda} = \frac{1}{\lambda} + \frac{1}{\lambda} x \qquad (3\text{-}3)$$

which is a straight line with $y = p(x)/p(x+1)$ as the dependent variable and x as the independent variable. This straight line has the same intercept and slope $(1/\lambda)$. Thus if we plot experimental data $p(x)/p(x+1)$ against x and the points appear to be a straight line, we can feel confident in modeling the data with the Poisson distribution.

Expression (3-4) is a recurrence relationship:

$$p(x+1) = \frac{\lambda}{x+1} \, p(x). \qquad (3\text{-}4)$$

This relationship may be helpful for computing the theoretical frequency $p(x+1)$ from $p(x)$. Note that by utilizing expressions (3-3) and (3-4), theoretical frequencies may be computed after obtaining a satisfactory estimate of the parameter λ.

Table 3.1 shows discrete data in relative frequency form and the theoretical frequencies that would be obtained from a Poisson distribution with the same mean as that of the sample, $\hat{\lambda} = 3.74$. Figures 3.1 and 3.2 display the results from using these two sets of data and the Poisson Graphical Test.

It is evident by viewing Figure 3.1 that the observed data

Table 3.1 Sample Data

Number of Scratches	Observed Relative Frequencies	Theoretical Poisson Relative Frequencies
0	.1660	.0238
1	.1020	.0888
2	.1240	.1662
3	.1260	.2072
4	.1220	.1936
5	.1120	.1448
6	.0800	.0902
7	.0540	.0482
8	.0400	.0226
9	.0260	.0094
10	.0120	.0035
11	.0160	.0012
12	.0100	-
Over 12	.0100	-
	1.000	1.000

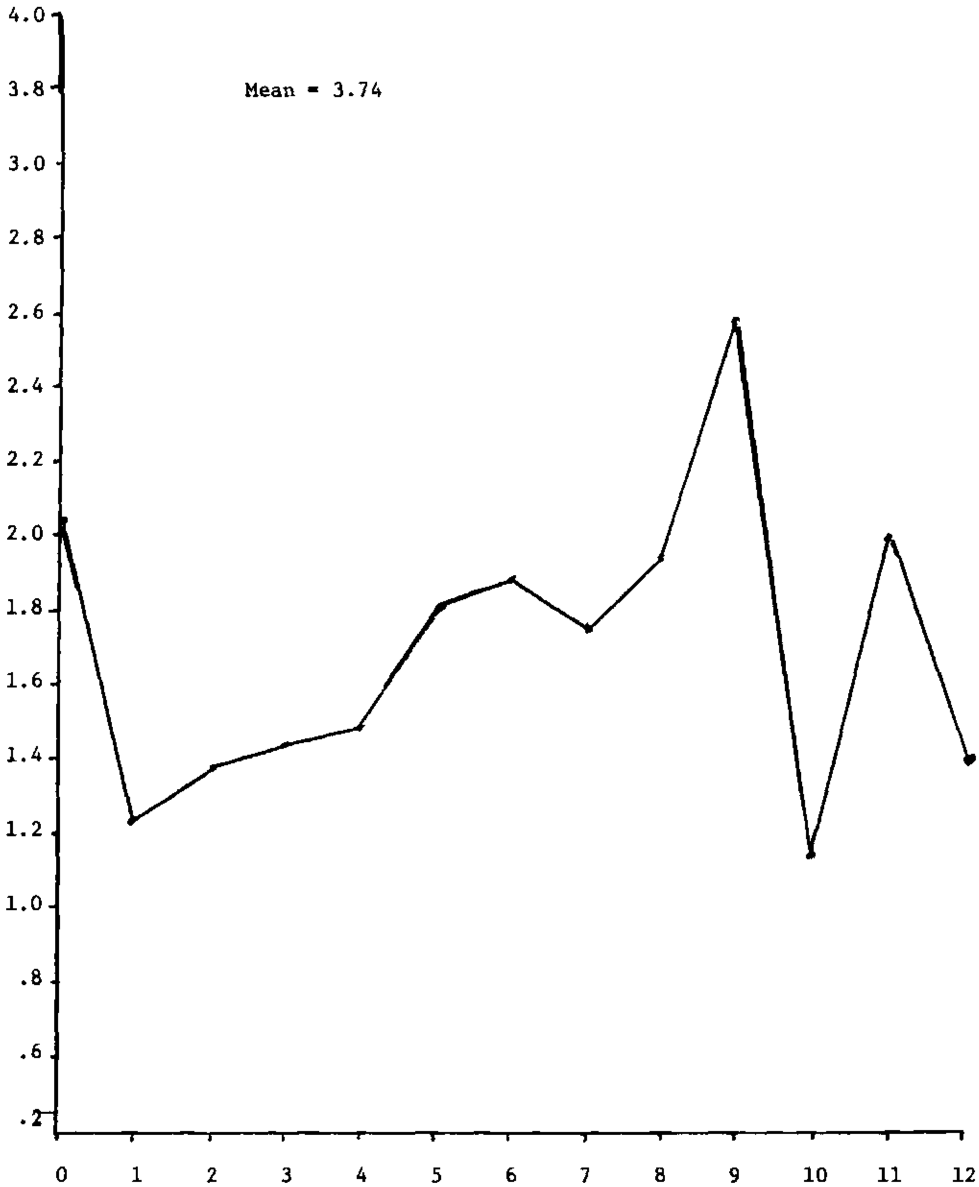

Figure 3.1. Non Poisson Data - Poisson Graphical Test

does not follow the Poisson distribution. Jackson (9) compares the theoretical data in Table 3.1 with the frequencies generated by a Thomas distribution with parameter $\hat{\lambda}_1 = 1.91$ and $\hat{\lambda}_2 = .96$. Jackson contends on the basis of a Chi-Square Goodness of Fit test that the data forms a distribution whose deviation from the Thomas distribution could be attributable to chance alone.

Figure 3.2 displays results when the data follow a Poisson distribution.

The probability function p(x) of the negative binomial distribution is given by:

$$p(x) = \text{Prob }(X{=}x) = \begin{bmatrix} x-1 \\ k-1 \end{bmatrix} p^k q^{x-k} \qquad (3\text{-}5)$$

where:

$\underline{x}$ equals the total number of trials required to encounter $\underline{k}$ successes. See Appendix B for futher explanation of the negative binomial distribution.
$\begin{bmatrix} x-1 \\ k-1 \end{bmatrix}$ is a binomial coefficient, p is a positive number in the open interval (0,1) and q = 1-p.

From expression (3-5) we obtain:

$$p(x+1) = \begin{bmatrix} \frac{x}{k-1} \end{bmatrix} p^k \quad q^{x+1-k} \qquad (3\text{-}6)$$

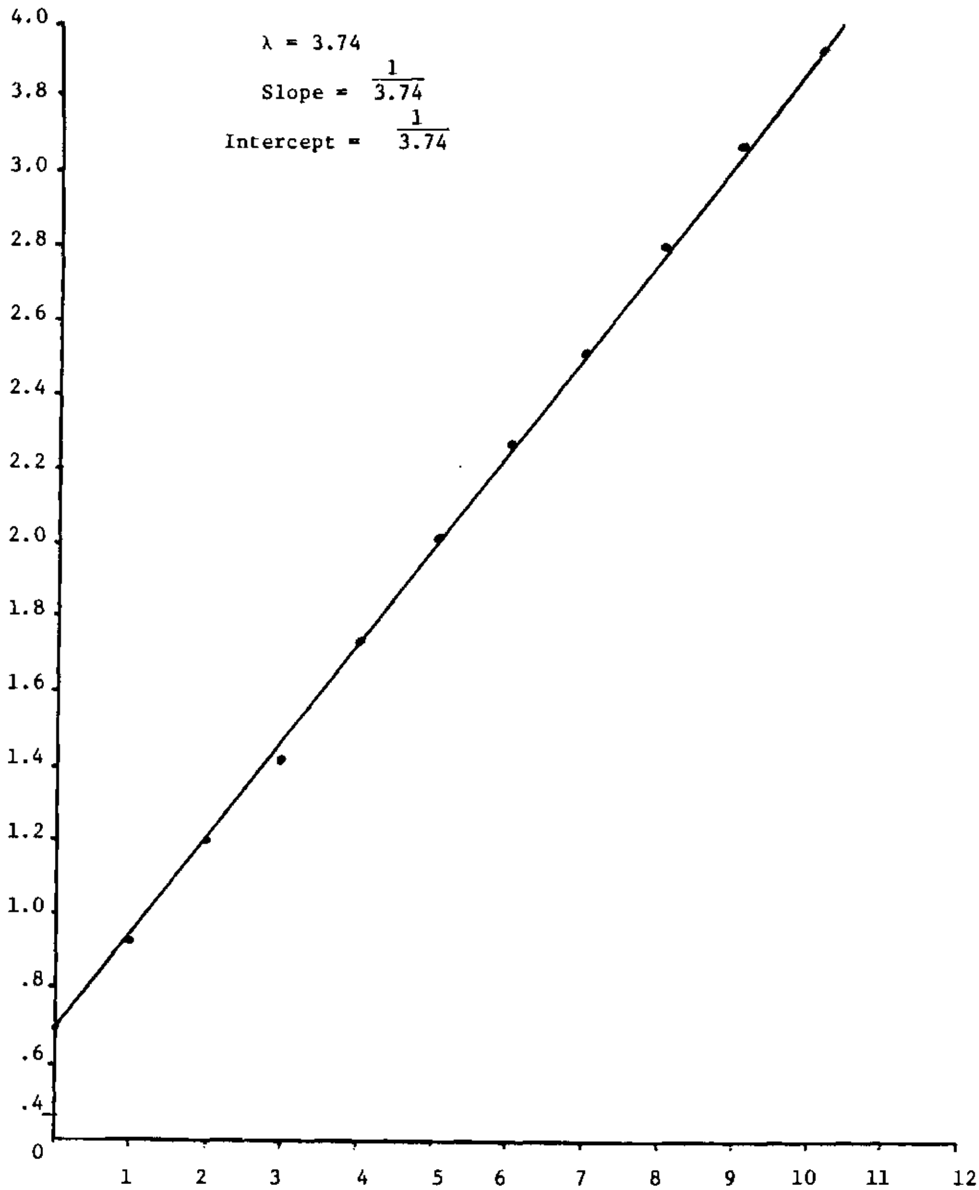

Figure 3.2. Poisson Data - Poisson Graphical Test

and furthermore:

$$\frac{p(x)}{p(x+1)} = \frac{1}{q} - \frac{k-1}{q}\frac{1}{x} \qquad (3\text{-}7)$$

which generates a straight line with y = p(x)/p(x+1) as a dependent variable and z = 1/x as an independent variable. This line has an intercept of (1/q) and a slope of -(k-1)/q. If after plotting the experimental data p(x)/p(x+1) against $\frac{1}{x}$, the points appear to be a straight line, we can feel confident in modeling the data with the negative binomial distribution.

The recurrence relationship below is helpful for computing the theoretical frequencies p(x+1) from p(x) if a good set of negative binomial tables is not easily acceptable. However, notice that a satisfactory estimate of the parameter p must be obtained before the theoretical frequencies can be calculated.

$$p(x+1) = qp(x) \qquad (3\text{-}8)$$

Figure 3.3 shows the results when the negative binomial graphical test is applied to data generated by a negative binomial distribution with parameters: p = .95, k = 70, mean = 3.69 and variance = .95. For additional probability plots, consult Hahn and Shapiro (8).

A decision concerning the appropriateness of a model is not always obvious from visual inspection of a probability plot. Often more objective techniques are necessary. Such techniques

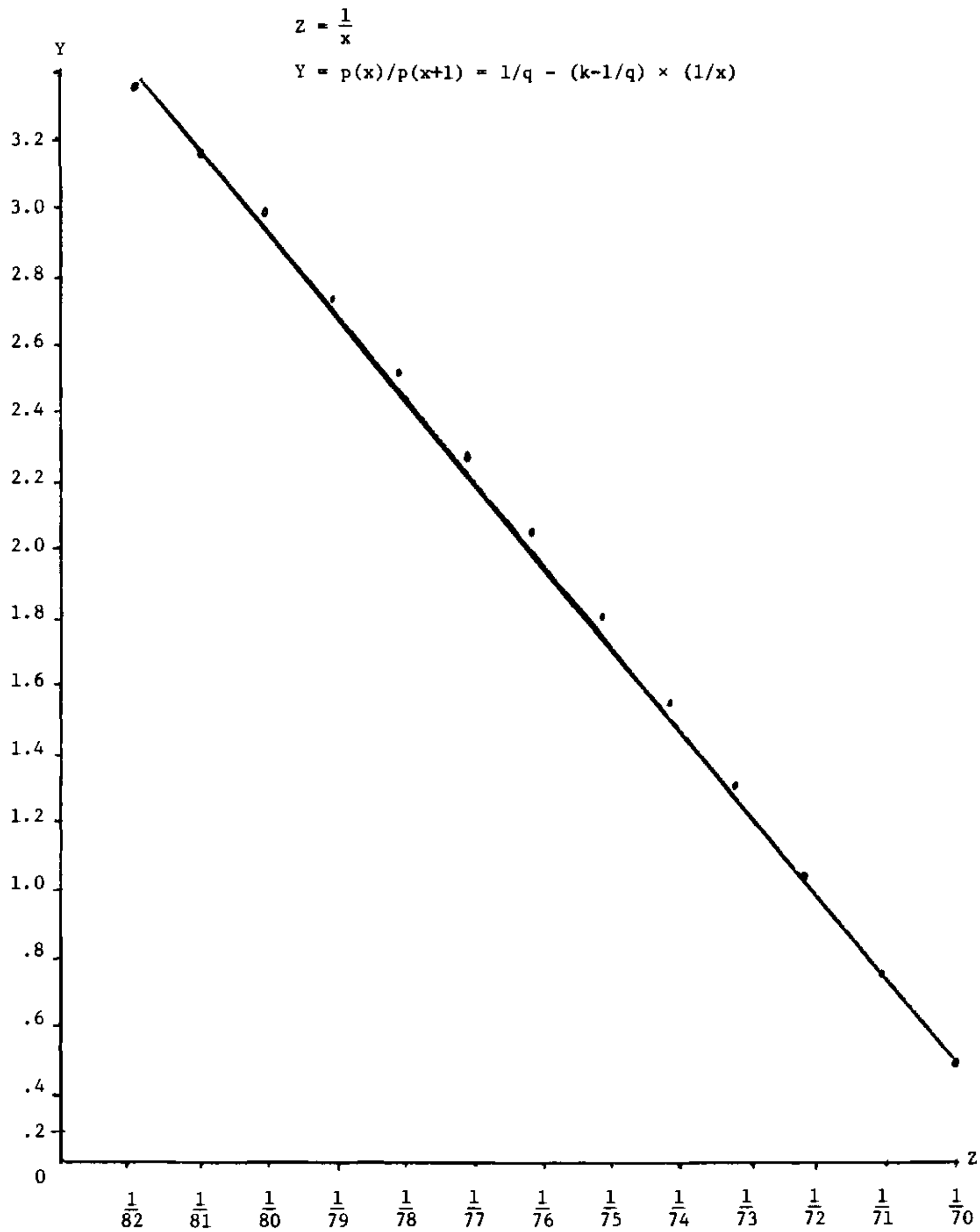

Figure 3.3. Negative Binomial Data - Negative Binomial Graphic Test

are discussed in the next section.

3.3 Statistical Tests

Statistical tests are more objective than probability plots and provide a probabilistic framework in which to evaluate the adequacy of the model. They may be used by themselves or as a supplement to probability plots when the plots fail to provide a clear-cut decision.

Statistical tests allow us to reject a model, they never allow us to prove that the assumed model is correct. The outcome of a statistical test depends greatly on the amount of available data. The chances of rejecting an inappropriate model increases as the amount of data increases.

As mentioned in the introduction, the Poisson distribution is virtually always assumed when distribution of defects is modeled. Therefore, it is of special interest to identify situations where a Poisson process is not present. The absence of a Poisson process can be determined by verifying that the number of defects over a fixed time interval does not have a Poisson distribution.

Several statistical tests have evolved to evaluate distributional assumptions. The Poisson Process test is one often utilized to test for the absence of a Poisson process. Because of the importance of identifying non-Poisson processes, the Poisson Process test is outlined. This test was taken from J. White, Schmidt and Bennett (21).

Poisson Process Test:

Let t_1, t_2 ... t_n denote the times at which each of n defects occurred during a time interval of length T. If these defects are from a Poisson process, then the times are independent and uniformly distributed over the interval 0 to T with mean T/2 and variance $\frac{T^2}{12}$. If the sum below is formed,

$$S_n = \sum_{i=1}^{n} \frac{t_i}{n} \tag{3-9}$$

then by the central limit theorem, for large n the test statistic will be normally distributed with mean

$$E(S_n) = T/2 \tag{3-10}$$

and variance

$$\mathrm{Var}\ (S_n) = \frac{T^2}{12n}. \tag{3-11}$$

Thus, to test the hypothesis that the defects are generated by a Poisson process, first compute the normal test statistic

$$\frac{S_n - T/2}{(T^2/12n)^{1/2}} \tag{3-12}$$

Second choose a level of significance α and locate the critical values $Z_{1-\alpha/2}$ and $Z_{\alpha/2}$ in the cumulative normal table. If $Z < Z_{\alpha/2}$ or $Z > Z_{1-\alpha/2}$, we reject the null hypothesis that the defects were generated by a Poisson process. The steps of the Poisson Process Tests are summarized below.

1) Compute the sum S_n given by (3-9).
2) Compute the normal test statistic Z given by (3-12).
3) Select a level of significance α.
4) Locate the critical values $Z_{\alpha/2}$ and $Z_{1-\alpha/2}$ in a Cumulative Normal Table.
5) If $Z < Z_{\alpha/2}$ or $Z > Z_{\alpha/2}$ then reject the hypothesis that the defects are generated by a Poisson process.

The reader may wonder about the validity of the Poisson Process Tests for finite sample size. White, Schmidt and Bennett (21) state that since this test is based on the central limit theorem, as a rule of thumb we can safely apply the test whenever $n \geq 30$. They further report that tests based on the central limit theorem are more powerful than non-parametric tests such as the Chi-Square Goodness of Fit test.

The times at which defects occur cannot always be conveniently measured in practice. A more convenient method may be to record the number of defects that occur over time intervals of fixed length. When data have been collected in this manner an estimate for the mean defect rate λ is given by

$$\lambda^{-1} = \bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \tag{3-13}$$

where n is the total number of observations. This result is the maximum likelihood estimator for the parameter λ of a Poisson mass function. Data collected by this latter method will not be in the proper format for the Poisson Process test and therefore a non-parametric goodness of fit test, such as the Chi-Square test should be utilized for distribution identification. Explanation of how to use the Chi-Square test is available in virtually every undergraduate level statistic book.

3.4 Flow Diagram for Determination of Discrete Distribution

The flow diagram for determination of discrete distributions was developed so analysts with limited statistical backgrounds could in an organized manner determine an adequate mathematical model to represent a distribution of defects. The flow diagram is presented in Exhibit 3-4.

The flow diagram is a result of the investigation of probability plotting, statistical tests and the knowledge that the Poisson and the negative binomial are unimodal distributions and that the Neyman Type A distribution often exhibits more than one mode.

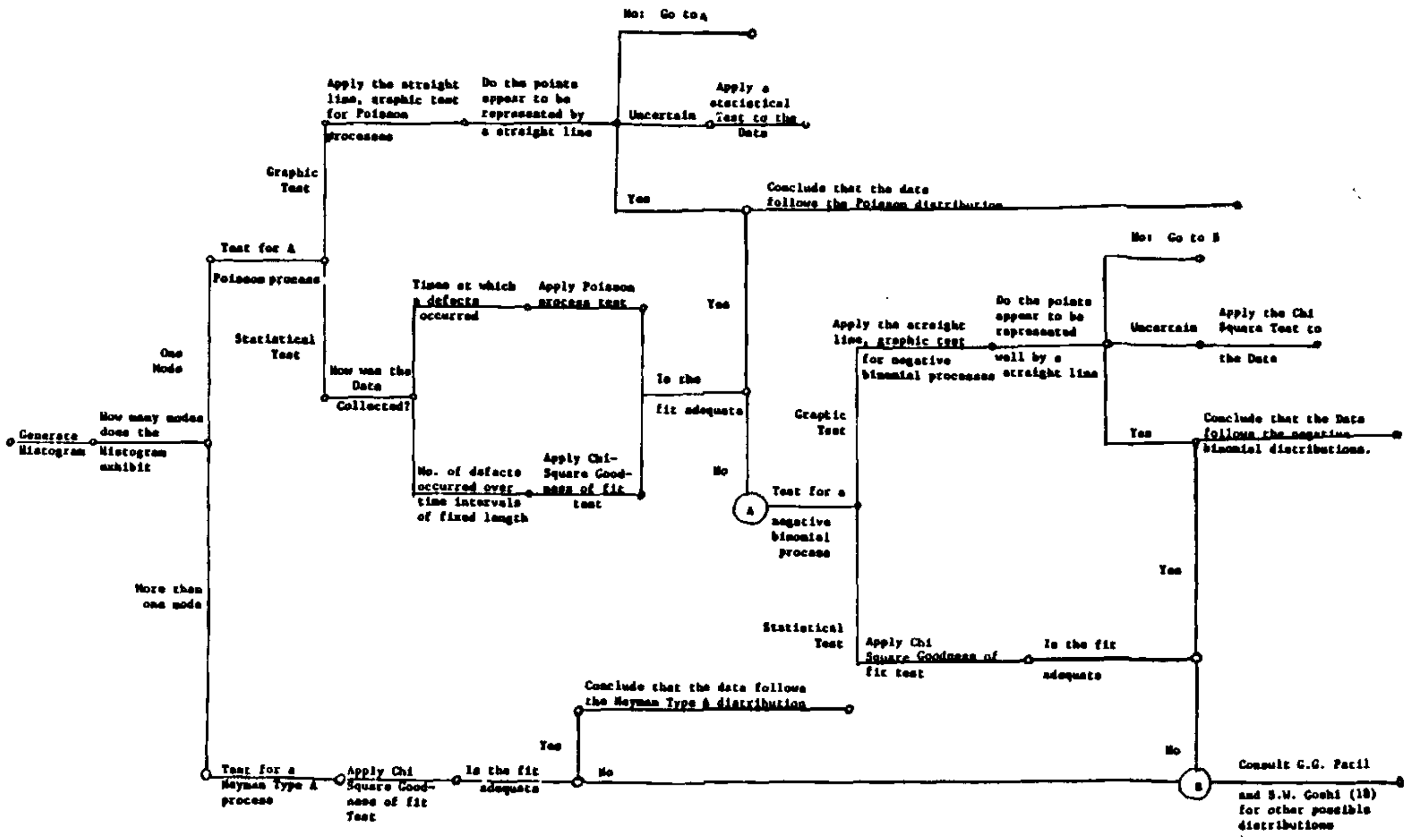

Figure 3.4. Flow Diagram for "Testing" Model Adequacy

CHAPTER IV

MODELS

4.1 Introduction

The third objective of this thesis was to develop appropriate control procedures when the underlying distribution of occurrence of defects was either Poisson, negative binomial, the weighted sum of two Poisson or a combination of Poisson and negative binomial. A computer model capable of giving the economic control limits for c and u charts was developed for each distribution. Optimal inspection plans can be obtained through application of these computer models and an appropriate grid search technique.

The general model structure is the same for each computer model and is described in Section 4.2. A description of each computer model and its application are given in Section 4.3

4.2 General Model Structure

Montgomery (12) reports that it seems that multiple assignable cause processes can usually be well approximated by an appropriately chosen single assignable cause model. The models developed for this thesis are characterized as single assignable cause models and they assume that once the process shifts out-of-control, it remains in the out-of-control state until detected and corrective action is taken.

The models assume that a sample of n units is taken after the production of (N-n) units, where N is the interval between decisions. This interval is called the sample interval in the following discussion. There are three sets of conditions which can arise:

1) $N = N = 1$; 2) $N = n \neq 1$; 3) $N > n$ and $n > 0$

A c-chart results from the first set of conditions. The second set of conditions also results in a c-chart, however the sample unit has been redefined. For example, if $n = 2$, and the original sample unit was one automobile, under the second set of conditions the sampling unit becomes two automobiles. A u-chart results if the third set of conditions are true. The third set of conditions are assumed to prevail in the remainder of the discussion on model structure. The equation can be simply modified to fit the definition of a c-chart by setting $N = n$.

4.2.1 Control Chart Parameters

The control procedure is as follows. Let an item be the basic unit of production. The models assume that the last "n" items of each N items are inspected, where n equals the inspection size and N equals the sampling interval. For example if $N = 50$ and $n = 5$, items 46 through 50 are inspected in every 50 items produced.

The number of defects per item when the process is in the "in-control" state follow a given distribution with mean λ_1. The number of defects per item when in the "out-of-control" state follows the same distribution with mean λ_2.

The control procedure is as follows. The total number of defects in the "n" inspection units, say n_0, is observed. The value of n_c is then plotted on a control chart with centerline $n\lambda_1$ and control limits given by:

$$\text{LCL} = n\lambda_1 - k\ (\text{STD}) \tag{4-1}$$

$$\text{UCL} = n\lambda_1 - k\ (\text{STD}) \tag{4-2}$$

where $n\lambda_1$ is the mean of the distribution under investigation for sample size n, k is the distance from the centerline ($n\lambda_1$) expressed in standard deviations, and STD is the standard deviation of the distribution of interest with mean $n\lambda_1$. Equations for calculating the standard deviations for all the distributions are given in Appendix B. If n_c falls inside the control limits, the process is assumed to be in control.

4.2.2 Cost Structure[1]

Economic schemes are based on the cost that occur because of the application of a control procedure. All the models developed contain four cost components:

A_0 = Variable cost of sampling or testing one item for the presence of a defect.

A_1 = Fixed cost of sampling inspection

A_2 = Cost of a "false" alarm

---

[1] The general cost model structure is due to Girshick and Rubin (7).

A_3 = Cost per unit of operating out-of-control

The variable cost of sampling, A_0, includes all direct sampling costs attributed to an item of production. The fixed cost, A_1 contains all direct and indirect costs that result from the existence of the sampling procedure that are independent of the sample size, n.

The cost of a false alarm, A_2, arises because of the probability of a Type I error; that is, the probability of concluding the process is out-of-control, when in fact it is in control.

The cost per unit of operating out-of-control, A_3, includes all the additional unit costs incurred as a result of the increased (average) number of defects per unit. This cost can take many forms. For one, the additional repair cost per unit that results when the average number of defects per unit increases to that of the out-of-control state is included under this cost heading. The inspection cost per sampling interval is equal to:

$$c = (A_0 \times n) + A_1 + (A_2 \times \alpha) + (A_3 \times B_1) \qquad (4\text{-}3)$$

where n is the sample size, α is the probability of a false alarm and B_1 is the expected number of units produced per sampling interval N, while the process is in the out-of-control state.

The inspection cost per unit equals:

$$c_N = c/N \qquad (4\text{-}4)$$

How to calculate α, β and B_1 is presented in the following sections. The probability of a Type II error, β, must be known before B_1 can be calculated.

4.2.3 Type I and Type II Error

Let n_1 be the number of items in the sampling interval N from the "in-control" state and n_2 equal the number of items produced while in the "out-of-control" state. The total sample interval N equals $n_1 + n_2$.

If d equals the total number of defects in the n items inspected, then the probability of a Type I error, α, can be expressed as:

$$\alpha = PR(d > UCL|n_2 = 0) + PR(d < LCL|n_2 = 0) \qquad (4\text{-}5)$$

The calculation of the probability of a Type II error is much more complicated. Let p = probability of a shift between production of single units, γ_1 = probability of starting a period in-control, γ_2 = the probability of starting any period out-of-control, where a period is equal to the sample interval. The above statements are based on the assumption that the process is not self correcting. The probability of starting a period out-of-control γ_2, is equal to the probability of a shift between items dividing periods plus the probability that a shift occurred undetected (Type II error) in the previous period.

$$\gamma_2 = p + \beta \qquad (4\text{-}6)$$

where β is the probability of a Type II error.

A shift to an out-of-control state can occur during sampling or before samples are taken. The possible number of units in the sample produced while the process is out-of-control is between 1 to N. The probability of a Type II error, β, then equals

$$\begin{aligned}\beta = {} & PR(LCL \leq d \leq UCL|n_2 = 1) \times PR(n_2 = 1) + \\ & PR(LCL \leq d \leq UCL\ n_2 = 2) \times PR(n_2 = 2) + \ldots \\ & PR(LCL \leq d \leq UCL\ n_2 = n - 1) \times PR(n_2 = n - 1 + \\ & PR(LCL \leq d \leq UCL\ n_2 = n) \times PR(n_2 = n) + \ldots \\ & PR(LCL \leq d \leq UCL\ n) \times PR(n_2 = N - 1) + \\ & PR(LCL \leq d \leq UCL\ n) \times PR(n_2 = N)\end{aligned} \tag{4-7}$$

The calculations of β was divided into three segments. The first segment consisted of calculating the probability of a Type II error given the shift to the out-of-control state occurred during inspection of the n units. This is defined as, a.

$$a = \sum_{n_2=1}^{n-1} PR(LCL < d < UCL|n_2) \times PR(n_2) \tag{4-8}$$

The shift to the out-of-control state in the second segment is assumed to occur before inspection and after production of the first unit in the current period. Call this probability b, then

$$b = \sum_{n_2=n}^{N-1} PR(LCL < d < UCL|n_2) \times PR(n_2) \tag{4-9}$$

The shift to the out-of-control state in the third segment is assumed to have occurred before the current period. Denoting this c,

$$c = PR(LCL < d < UCL|n) \times PR(n_2 = N) \tag{4-10}$$

Now equation (4-7) can be rewritten as:

$$\beta = \gamma_1\ a + \gamma_1\ b + \gamma_2\ c \tag{4-11}$$

Equation 4-12 results from substituting p + β for γ_2 (4-6) and noting that $\gamma_1 = 1 - \gamma_2$

$$\beta = \frac{a + b + p[c - b - a]}{1 - c + b + a} \tag{4-12}$$

Setting f = a+b, the above equation can be simplified to

$$\beta = \frac{f + p\ (c - f)}{1 - c + f} \tag{4-13}$$

4.2.4 Expected Number of Units Produced While in the Out-of-Control State, (B_1)

$$B_1 = (\gamma_2)\ (N) + \sum_{n_2=1}^{N-1} (n_2)\ (p)\ (1-p)^{N-n_2} \tag{4-14}$$

B_1 equals the product of the probability of starting out-of-control, γ_2, and the number of units in the sample interval N, plus, the summation of the product n_2 and the probability that the shift occurred after the production of the $(N-n_2)$ item of the period, where n_2 can range from 1 to N-1.

4.2.5 Expanded Cost Equation and an Example

Equation (4-3) can be expanded by substituting the appropriate terms for α and B_1. The expanded cost equation (4-15) is presented on the following page. The most difficult part of this equation is the calculation of the Type I α, and Type II error, β. The following example illustrates how to calculate α and β for the case where the defect distribution is Poisson. It is assumed that the control limits have been converted to integer values in the following equations. Since the data is in discrete form, integer control limits are more practical. As mentioned before the process is considered in control only if the points plotted on the chart lie inside the control limits.

The probability of a Type I error equals the probability that the total number of defects, d in the sample is greater than or equal to the upper limit plus the probability that d is less than or equal to the lower control chart limit. For the case where the defects are generated by a Poisson process:

$$\alpha = \sum_{x=UCL}^{\infty} \frac{\exp(-n\lambda_1)(n\lambda_1)^x}{x!} + \sum_{x=0}^{LCL} \frac{\exp(-n\lambda_1)(n\lambda_1)^x}{x!} \tag{4-5a}$$

(4-15)

$$(A_0 n) + A_1 + A_2 0 \left[\sum_{UCL}^{\infty} p(x) + \sum_{0}^{LCL} p(x) \right] + A_3 \left[\left[pt \quad \frac{a+b+p(c-b-a)}{1-c+b+a} \right] \right.$$

$$\left. (N) + \sum_{n_2=1}^{N-1} (n_2)(p)(1-p)^{N-n_2} \right]$$

where

$$a = \sum_{n_2=1}^{n-1} \sum_{x=LCL+1}^{UCL-1} p(x)(1-p)^k p \quad ;$$

$$b = \sum_{x=LCL+1}^{UCL-1} p(x) \sum_{k=1}^{N-n} (1-p)^k p \quad ;$$

$$c = \sum_{x=LCL+1}^{UCL-1} p(x) \quad ;$$

$P(x)$ = probability function for the given distribution

The calculation of β was divided into three segments. The probability of a, b and c was calculated in the three segment respectively.

$$a = \sum_{n_2=1}^{n-1} \sum_{x=LCL+1}^{UCL-1} \frac{\exp\,[-(n-n_2)\ \lambda_1 - n_2\lambda_2]\ [(n-n_2)\lambda_1 + n_2\lambda_2]^x\ [1-p]^{n}p^{n_2}}{x!} \tag{4-9a}$$

The mean number of defects per unit depends on when the shift occurs. An example will best illustrate the point in question. Assume that the following conditions prevail:

$$\lambda_1 = 5;\ \lambda_2 = 10;\ n = 5$$

If the shift to out-of-control occurs after the second unit is inspected; that is, $n_2 = 3$, then on an average the first two units will have means of 5 defects and the remaining three sample units will have a means of ten defects. Thus, the mean number of defects for the sample is (2)(5) + (3)(10) = 40. In the general case the mean number of defects in the sampled units is:

$$(n - n_2)\ \lambda_1 + n_2\ (\lambda_2)$$

Applying the above equation to the probability function for the Poisson distribution we arrive at the first portion of equation (4-8a), that is

$$\frac{\exp\left[-(n-n_2)\,\lambda_1 - n_2\lambda_2\right]\left[(n-n_2)\lambda_1 + n_2\lambda_2\right]^x}{x!}$$

The remaining portion of the equation is easy. For instance assume $n_2 = 3$ and $N = 50$. The first 47 units in the sampling interval must have been produced while the process was in control. The probability of this occurrence is equal to the probability of a shift not occurring (1-p) raised to the forty seventh power. The probability that a shift occurs after the production of the forty seventh unit and before the forty eighth is simply p. The product of these probabilities makes up the last terms in the equation. The logic behind the summation signs was discussed in connection with equation (4-8).

$$b = \sum_{x=LCL+1}^{UCL-1} \frac{\exp\,(-n\lambda_2)(n\lambda_2)^x}{x!} \sum_{k=1}^{N=n} (1-p)^k p \qquad (4\text{-}9a)$$

The mean number of defects per unit is equal to $n\lambda_2$, since the shift to out-of-control must take place before sampling begins for equation (4-9) and (4-9a) to be significant. The last term in the equation

$$\sum_{k=1}^{N-n} (1-p)^k p$$

represents the cumulative probability of a shift occurring after the k^{th} item is produced in the current period.

$$c = \sum_{x=LCL+1}^{UCL-1} \frac{\exp\ (-n\lambda_2)(n\lambda_2)^{\lambda}}{x!} \quad PR(n_2 = N) \qquad (4\text{-}10a)$$

The terms in equation (4-10a) have already been explained in the previous discussion.

The last step to the development of an equation to calculate the probability of a Type II error when the underlying distribution of defects in a Poisson is to substitute equations (4-8a), (4-9a), and (4-10a) into equation (4-12).

Equations (4-8a), (4-9a) and (4-10a) can easily be transformed for cases where the underlying distribution of defects is not Poisson. The Poisson probability function is simply replaced by the distribution of interest in order to accomplish this feat.

4.3 Computer Models

Seven computer models were developed to aid in the analysis of the economic impact of distribution misspecification. The models can be classified into two categories, single and double distribution models. Four of the models are single distribution models. They are the Poisson model, negative binomial model, weighted sum of two poissons model and the combination Poisson and negative binomial model. The underlying distribution of the occurrence of defects dictates the most appropriate model. For example, if the underlying distribution is negative binomial, then obviously the best model to choose would be the negative binomial model. How much better the negative binomial model is than the Poisson model when the underlying distribution is negative binomial is analyzed for several cases in the next chapter.

All the single distribution models feature three option features discussed at the end of this section. This makes them very versatile. Their applications range from finding the cost of a single inspection plan to the obtainment of optimal inspection plans and sensitivity analysis.

Double distribution models are hybrid of two single distribution models. Three hybrid models were developed, with the combination of:

1) Poisson + Negative Binomial
2) Poisson + Weighted Sum of Two Poissons
3) Poisson + Combination Poisson and Negative Binomial

The application of these models are two fold. First, they enable the analyst to compare results between the use of the Poisson distribution and the other model under similar conditions. Secondly, these models provide data on the effect of incorrectly assuming that the Poisson represents the underlying defect distribution.

The double distribution models feature only the "ALL and Printer Plot" output options. They do not include the "Min Cost" output option. Because of this their use for optimization purposes is extremely limited. These output features are discussed in the next section.

4.3.1 Output

The first two lines of output report input parameters. The first line consists of the range and incremental value of k, N and n that will be analyzed in the current run. Distribution and cost parameters are listed on the second line. The remainder of output depends upon the output options selected by the analyst.

Most of the computer models feature three output options, the "All", "Min Cost" and "Printer Plot" option. For these models the analyst has the choice of utilizing one, two or all three of the output options. Some models, however, only have the "All" and "Printer Plot" options.

The "All" option will produce the values of

$$C_I,\ c_N,\ c,\ B_1,\ N,\ n,\ n\lambda_1,\ \mathrm{LCL},\ \mathrm{UCL},\ k,\ \mathrm{STD},\ \alpha,\ \beta$$

for every inspection plan analyzed during the run. C_I is the number assigned by the program to the output line. The other symbols have been defined in previous discussion and are also defined in the Glossary of Terms.

The next output option is called the "Min Cost" feature. The initial value of the Min Cost is set by the analyst. An initial value of 2.0 was found to be satisfactory for the cases analyzed by the author.

This feature operates as follows. The cost per unit of each plan is compared against the Min Cost value. If this cost is equal to or less than the Min Cost value it replaces the present value of Min Cost and becomes the standard by which the following inspection plans are compared. Each time the Min Cost value is replaced the following data on the new "Min Cost" plan is printed out.

MIN COST, C_I, c_N, c, B_1, N, n, $n\lambda_1$, LCL, UCL, k, STD, α, β

The "Min Cost" at the beginning of the line indicates utilization of the Min Cost option. This option is very useful when conducting a grid search in order to find the minimum cost inspection plan and was exstensively utilized for that purpose throughout the project.

The Printer Plot is the last ouput option. This option requires the use of subroutine USPLX from the IMS Library of programs provided through Computer Services at the Georgia Institute of Technology.

The cost per unit for every inspection plan is plotted with the usage of this feature. The integer values on the ordinate axis corresponds to the number "C_I" assigned to each inspection plan by the internal counter of the models. In other words, if the Printer Plot and "ALL" features are utilized together the number assigned to each inspection plan by use of the "ALL" features corresponds to the integer value on the ordinate of the Printer Plot.

A limit of 100 inspection plans plotted per graph is suggested. The Printer Plot can handle more points, however, every graph has the same width and it becomes increasingly difficult to determine which point corresponds to the appropriate inspection plan beyond 100 points. Even with this limitation, the Printer Plot is very useful for the detection of trends and/or patterns in data.

CHAPTER V

NUMERICAL ANALYSIS

The optimal design of c-charts and u-charts will be investigated in this chapter. Situations where the Poisson distribution is incorrectly chosen to represent the distribution of defects is also analyzed. It was first assumed that the mean and variance of the defect distribution were approximately equal to the mean and variance of the assumed Poisson distribution. The occurrence of defects was assumed to fit the negative binomial distribution in the first situation and the weighted sum of two Poisson distribution and a combination of Poisson and negative binomial in the second and third situation, respectively.

Results from this analysis are presented in the next five sections of this chapter. The last section of the chapter presents results from an analysis where the mean of the defect distribution is equal to that of the assumed Poisson, but its variance was significantly greater than that of the Poisson.

5.1 Parameter Selection

Five sets of parameters, each composed of values for λ_1, λ_2, A_0, A_1, A_2, A_3, and p were utilized to analyze cases where the mean and variance of the defect distribution were approximately equal to the mean and variance of the assumed Poisson[1]. A one

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1 The choice to analyze five sets of parameters was based on the cost and availability of necessary resources.

quarter 2^7 designed experiment was utilized in order to gain insight into how these different parameters influence the cost of quality control. A low and high value were initially selected for each parameter. The selected values are displayed in Table 5.1. It is believed that the values selected are representative of cases frequently encounted in practice.

Table 5.1. Parameter Values

Parameter	λ_1	λ_2	A_0	A_1	A_2	A_3	p
Low Value	5.0	10.0	.10	2.0	150.0	1.0	.01
High Value	7.0	15.0	.50	6.0	200.0	3.0	.03

The generators utilized in the construction of the effect and aliases structure for the 2^{7-2} designed experiment were:

$$P = ABCD \qquad Q=AEFG$$

$$Q=BCDEFG$$

The effect and aliases structure is summarized in Table 5.2. The experiment was run under twelve different inspection plans, where an inspection plan specifies the sample interval, N, sample size n and the half width (or width) of the control chart, k. The cost that resulted from the given set of parameters and inspection plans are displayed graphically in Figures 5.1 through 5.4. The inspection plan that was utilized in the given experimental run is listed at the top of the graphs. The values along the ordinate

Table 5.2. Effect and Alias Structure

EFFECT		ALIASES		
1	1	ABCD	AEFG	BCDEFG
2	A	BCD	EFG	ABCDEFG
3	B	ACD	ABEFG	CDEFG
4	AB	CD	BEFG	ACDEFG
5	C	ABD	ACEFG	BDEFG
6	AC	BD	CEFG	ABDEFG
7	D	ABC	ADEFG	BCEFG
8	AD	BC	DEFG	ABDEFG
9	E	ABCDE	AFG	BCDFG
10	AE	BCDE	FG	ABCDFG
11	BE	ACDE	ABFG	CDFG
12	ABE	CDE	BEF	ACDEF
13	CE	ABDE	ACFG	BDFG
14	ACE	BDE	CFG	ABDEG
15	BCE	ADE	ABCFG	DFG
16	DE	ABCE	ADFG	BCFG
17	F	ABCDF	AEG	BCDEG
18	AF	BCDF	EG	ABCDEG
19	BF	ACDF	ABEG	CDEG
20	ABF	CDF	BEG	ACDEG
21	CF	ABDF	ACEG	BDEG
22	BEF	ACDEF	ABG	CDG
23	DF	ABCF	ADEG	BCEG
24	BDF	ACF	ABDEG	CEG
25	BEF	ACDEF	ABG	CDG
26	G	ABCDG	AEF	BCDEF
27	EF	ABCDEF	AG	BCDG
28	BG	ACDG	ABEF	CDEF
29	CG	ABDG	ACEF	BDEF
30	DG	ABCG	ADEF	BCEF
31	ACG	BDG	CEF	ABDEF
32	ADG	BCF	DEG	ABCEF

SYMBOL	=	PARAMETER
A	=	λ_1
B	=	λ_2
C	=	A_0
D	=	A_1
E	=	A_2
F		A_3
G	=	p

corresponds to the number assigned to each effect and its aliases listed in Table 5.2; for instance, point number 40 corresponds to effect number 8 (40-32=8).

The five sets of parameter values chosen for analysis are presented in Table 5.3 and are circled in Figures 5.1 through 5.4.

Table 5.3. Cost Parameter Sets

Parameter Set #	Value of Parameters λ_1	λ_2	A_0	A_1	A_2	A_3	p
1	5.0	10.0	.10	2.0	150.0	1.0	.01
2	7.0	10.0	.10	2.0	150.0	1.0	.01
3	5.0	10.0	.10	2.0	150.0	1.0	.03
4	5.0	10.0	.10	2.0	150.0	3.0	.01
5	7.0	10.0	.10	2.0	150.0	3.0	.01

The parameters are all at their low levels in the first set. One parameter is at its upper level in the second, third, and fourth parameter sets.

The mean number of defects for the "in-control state", λ_1, is at its upper level in the second set. Inspection cost is not always increased significantly by setting λ_1 at its upper level. The majority of cases in Figure 5.1 and 5.3 indicate that an increase in λ_1 by itself results in significant increases in cost. The opposite is true, however, in the majority of cases in Figures 5.2

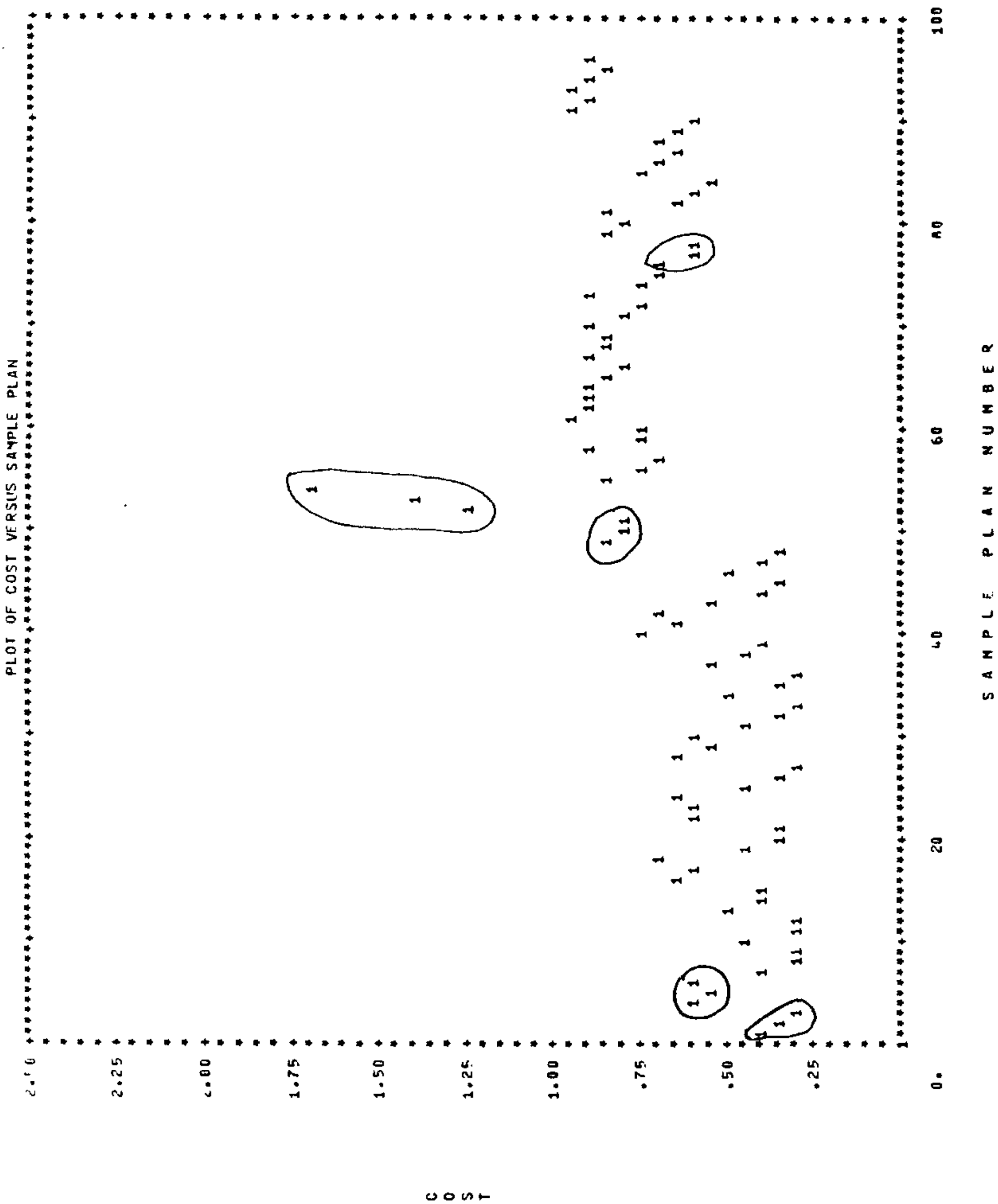


Figure 5.1. Cost Versus Sample Plan Plot #1

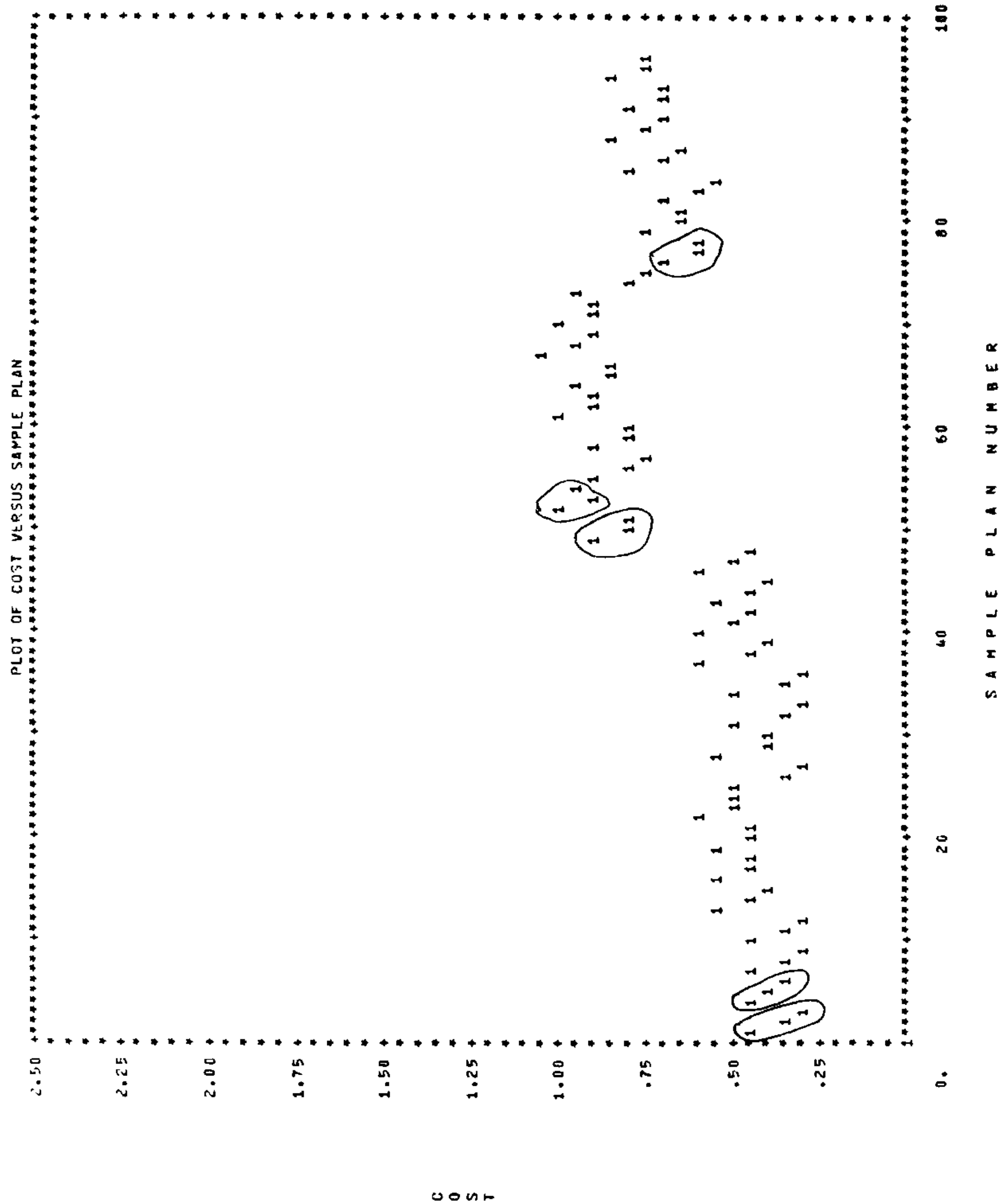


Figure 5.2. Cost Versus Sample Plan Plot #2

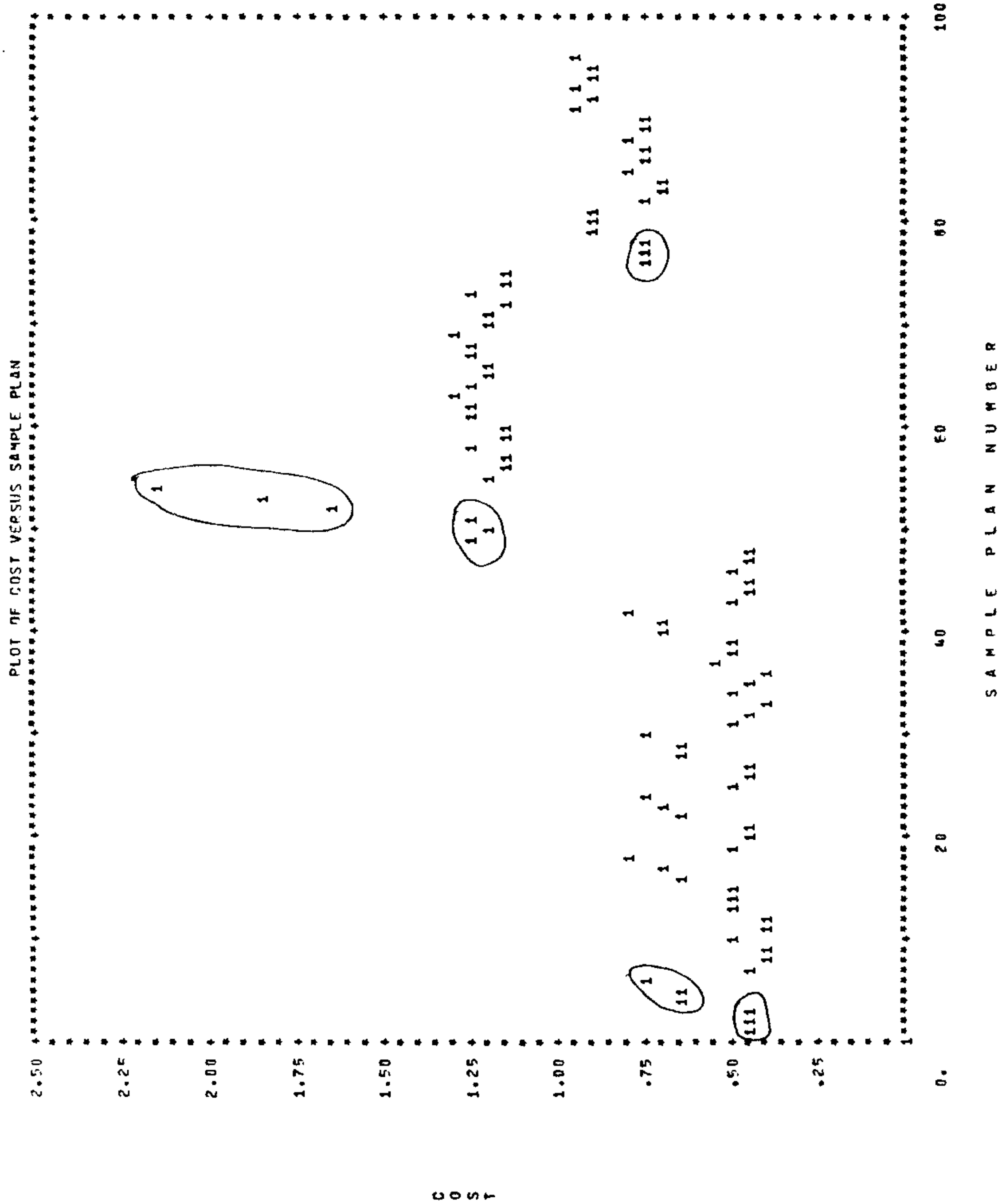


Figure 5.3. Cost Versus Sample Plan Plot #3

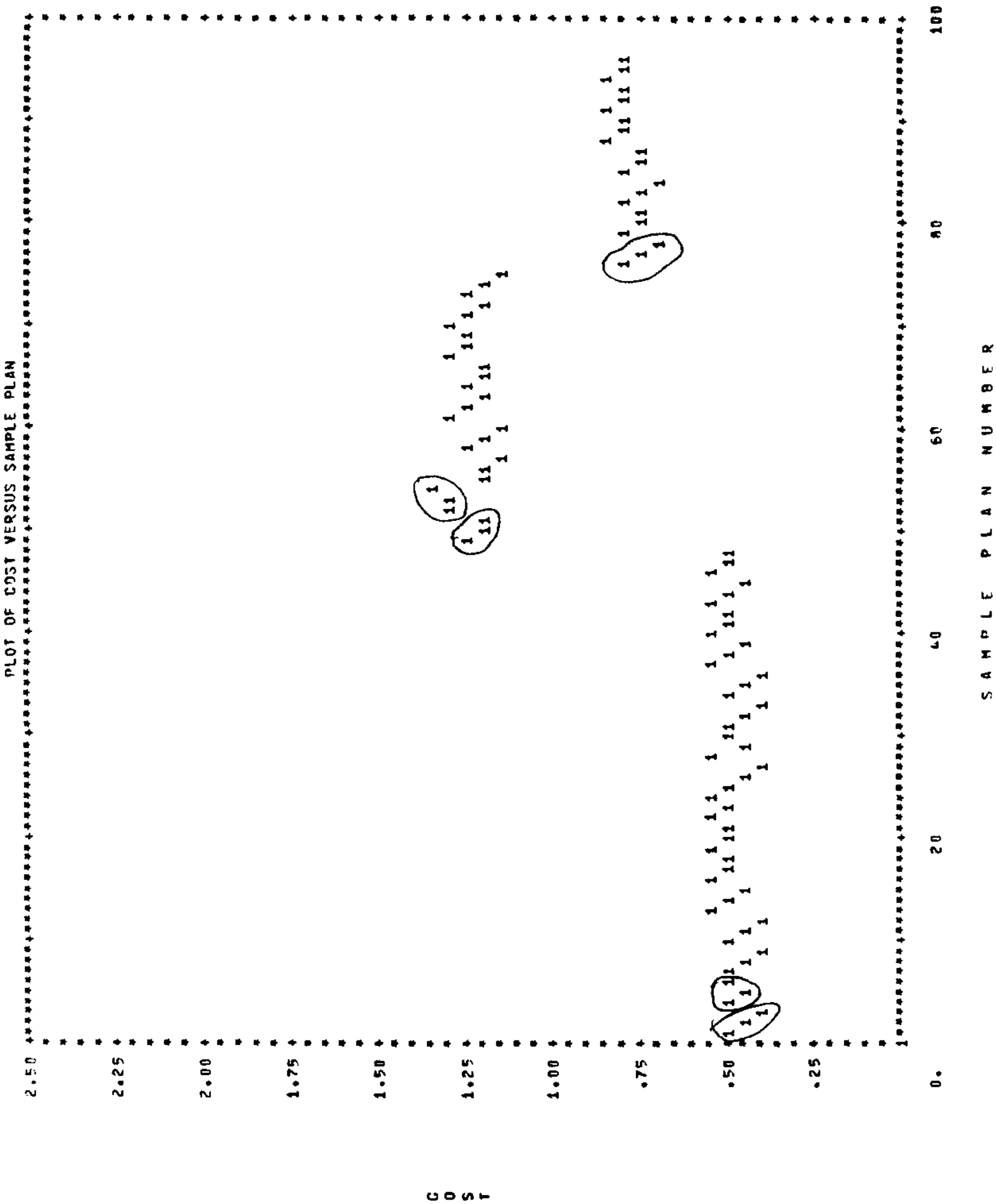


Figure 5.4. Cost Versus Sample Plan Plot #4

and 5.4. The impact of increasing λ_1 to its upper level appears to depend mainly on the sample size of the inspection plan. Sample size is equal to 5 for the cases in Figures 5.1 and 5.3 and equal to 15 for the cases in Figures 5.2 and 5.4.

The probability of a shift from the in-control state to the out-of-control state, p, and the cost of operating out-of-control, A_3, are the parameters at their upper level in the third and fourth parameter sets, respectively. The plots conclusively illustrate that a change in parameter A_3 or p from their low value to their high value has a significant impact on the cost of inspection in the cases analyzed. The cost of operating out of control, A_3, is set at \$3,00 per unit in the parameter sets 49 through 75, and at \$1.00 per unit in all the other parameter sets. The probability of a shift, p, is set at .03 in the last 21 parameter sets and is set at .01 in the first 75 sets.

The set of parameters that generally resulted in the greatest inspection cost had λ_1 and A_3 at their high levels. This fact is illustrated especially well in Figures 5.1 and 5.3. In these figures the three highest points correspond to the fifth parameter set where λ_1 and A_3 are at their high levels.

In summary, the parameter selection procedure was not intended to theoretically justify the selected sets of parameter values. The purpose was to enable the author to select five sets of parameter values in a logical manner: It did accomplish this objective.

Table 5.4. Distribution Parameter Set #1

Distribution	Parameters and Values	Mean	Var.
Poisson	$\lambda = 5.0000$	5.0000	5.0000
	$\lambda = 7.0000$	7.0000	7.0000
	$\lambda = 10.0000$	10.0000	10.0000
Negative	$m = 95.000$; $p = .950$	5.0000	5.2632
Binomial	$m = 133.000$; $p = .950$	7.0000	7.3684
	$m = 190.000$; $p = .950$	10.0000	10.5263
Weight Sum of Two Poissons	$\lambda_1 = 5.040$; $\lambda_2 = 1.000$; $\phi=.990$	4.9996	5.1612
	$\lambda_1 = 7.056$; $\lambda_2 = 1.400$; $\phi=.990$	6.9994	7.3161
	$\lambda_1 = 10.080$; $\lambda_2 = 2.000$; $\phi=.990$	9.9992	10.6455
Combination Poisson Negative Binomial	$\lambda= 5.000$; $m = 95.000$; $p=.950$; $\phi=.50$	5.0000	5.1316
	$\lambda= 7.000$; $m = 133.000$; $p=.950$; $\phi=.50$	7.0000	7.1842
	$\lambda= 10.000$; $m = 190.000$; $p=.950$; $\phi=.50$	10.0000	10.2632

5.2 Distribution Parameter Selection

Three sets of parameters had to be selected for each distribution to generate the three mean values of defects required in the cost parameter sets. The necessary mean values were 5.00, 7.00 and 10.00. The parameters were selected such that the mean and variance of the distributions would be approximately equal. The properties of all the distributions studied, except the Poisson, prevent the means and variances of these distributions from being exactly equal. A more detailed description of the distribution models is provided in Appendix B. The parameters and their assigned values for each distribution, along with the resulting mean and variance are given in Table 5.4.

5.3 Optimal Inspection Plans

The optimal inspection plans were found through the application of a grid search technique. Grid search techniques are often employed to select the control chart parameters that minimize the objective function. Construction of a more efficient optimalization algorithm may be justified if regular application of the algorithm is necessary.

The optimal inspection plans are presented in Tables 5.5 through 5.9. The standard deviation of the assumed distribution and the probability of Type I and II errors associated with the inspection plans are also provided in the tables. A range of control chart widths exist in which the costs remain optimal. This results because the test statistic is a discrete random variable. For a more detailed explanation of this phenomena see (13). The range of these values is provided in column 8. The control chart limits, column 6 and 7, were made into integers, since the data is in discrete form. If the number of defects is equal or outside the control limits the process is considered out of statistical control.

The optimal inspection plans are categorized according to the set of cost parameters for which they produce the optimal cost. For example, inspection plan set #1, Table 5.5 is optimal if cost parameter set #1, Table 5.3 is present.

Table 5.5. Optimal Inspection Plan Set #1

Model	Inspection Cost Per Unit	Sampling Parameters (N) Sample Interval	(N) Sample Size	Control Chart Parameters Center Line	Lower Control Limit Range	Upper Control Limit Range	1/2 Width of Control Charts in Std. Dev.	Standard Deviation of Test Statistic	Probability of a Type I Error	Type II Error
Poisson	.2629	27	7	35.0000	14	56	3.39 3.54	5.9161	.0008	.0355
Negative Binomial	.2652	27	7	35.0000	14	56	3.30 3.45	6.0698	.0011	.0363
Weight Sum of Two Poissons	.3118	35	7	34.9972	15	55	2.91 3.05	6.5509	.0115	.0305
Combination Poisson and Negative Binomial	.2641	27	7	35.0000	14	56	3.34 3.50	5.9934	.0009	.0359

The control chart limits were integerized since the data is in discrete form. If the number of defects in the sample is equal to or outside the control limits the process is considered out-of-control.

Table 5.6 Optimal Inspection Plan Set #2

Model	Inspection Cost Per Unit	Sampling Parameters (N) Sample Interval	(N) Sample Size	Control Chart Parameters Center Line	Lower Control Limit Range	Upper Control Limit Range	1/2 Width of Control Charts in Std. Dev.	Standard Deviation of Test Statistic	Probability of A Type I Error	Type II Error
Poisson	.3540	34	16	112.0000	80	144	2.93 3.02	10.5830	.0034	.0909
Negative Binomial	.3591	34	16	112.0000	80	144	2.86 2.94	10.8579	.0043	.0922
Weight Sum of Two Poissons	.3886	40	15	111.9910	74	135	2.26 2.26	13.2759	.0145	.0815
Combination Poisson and Negative Binomial	.3565	34	16	112.0000	80	144	2.90 2.98	10.7214	.0038	.0916

Table 5.7. Optimal Inspection Plan Set #3

Model	Inspection Cost Per Unit	Sampling Parameters (N) Sample Interval	(N) Sample Size	Control Chart Parameters Center Line	Lower Control Limit Range	Upper Control Limit Range	1/2 Width of Control Charts in Std. Dev.	Standard Deviation of Test Statistic	Probability of A Type I Error	Type II Error
Poisson	.4499	17	6	30.0000	11	49	3.29 / 3.46	5.4772	.0011	.0777
Negative Binomial	.4542	18	7	35.0000	14	56	3.30 / 3.45	6.0698	.0011	.0760
Weight Sum of Two Poissons	.5207	24	7	34.9972	15	55	2.91 / 3.05	6.5509	.0115	.0595
Combination Poisson and Negative Binomial	.4522	18	6	30.0000	11	49	3.25 / 3.42	5.5488	.0012	.0786

Table 5.8. Optimal Inspection Plan Set #4

Model	Inspection Cost Per Unit	Sampling Procedure (N) Sample Interval	(N) Sample Size	Control Chart Parameters Center Line	Lower Control Limit Range	Upper Control Limit Range	1/2 Width of Control Charts in Std. Dev.	Standard Deviation of Test Statistic	Probability of A Type I Error	Type II Error
Poisson	.5083	15	6	30.0000	12	48	$\frac{3.11}{3.28}$	5.4772	.0019	.0317
Negative Binomial	.5148	14	6	30.0000	11	49	$\frac{3.21}{3.38}$	5.6195	.0014	.0356
Weight Sum of Two Poissons	.5973	18	6	29.9976	12	48	$\frac{2.85}{3.00}$	5.9814	.0120	.0310
Combination Poisson and Negative Binomial	.5119	15	6	30.0000	12	48	$\frac{3.08}{3.24}$	5.5488	/0022	.0320

Table 5.9. Optimal Inspection Plan Set #5

Model	Inspection Cost Per Unit	Sampling Parameters (N) Sample Interval	(N) Sample Size	Control Chart Parameters Center Line	Lower Control Limit Range	Upper Control Limit Range	1/2 Width of Control Charts in Std. Dev.	Standard Deviation of Test Statistic	Probability of a Type I Error	Type II Error
Poisson	.7338	17	13	91.0000	63	119	2.84 2.93	9.5394	.0046	.0885
Negative Binomial	.7461	17	13	91.0000	63	119	2.76 2.86	9.7872	.0057	.0892
Weight Sum of Two Poissons	.7979	22	14	97.9922	70	126	2.14 2.21	12.6517	.0167	.0777
Combination Poisson and Negative Binomial	.7400	17	13	91.0000	63	119	2.84 2.89	9.6641	.0052	.0888

The influence of the cost parameters on the cost of inspection for the optimal plans is similar to their impact on the non-optimal plans exhibited in Figures 5.1 through 5.4. The lowest inspection cost naturally occurs when all the cost parameters are at their low levels as was the case for Inspection Plan Set #1. The highest cost occurs for the inspection plans of set #5; where λ, and A_3 are at their high levels. The impact of increasing λ_1, and A_3 both to their upper level is well illustrated by Figures 5.1 and 5.3 and also by the fact that the inspection costs exhibited in Table 5.9 are more than 33% higher than their counterparts in Table 5.8.

Inspection cost for Inspection Plan Set #4, Table 5.8 exhibit the second highest cost of the five sets analyzed. The cost of operating out-of-control, A_3, was the only cost parameter at its upper level when optimal inspection plans were determined for Inspection Plan Set #4. The increase of parameter A_3 to its upper level also significantly increased inspection cost for the plans illustrated in Figures 5.1 through 5.4. Inspection Plan Set #3 and #2 ranked third and fourth, respectively, for highest inspection cost.

The following list of observations pertain to only the inspection plans presented in Tables 5.5 through 5.9.

1) The rank of the models in relationship to cost is independent of the cost parameters utilized. The inspection cost is always lowest for the Poisson model

for any given set of cost parameters. The combination Poisson and negative binomial model results in the second lowest cost followed by the negative binomial and weighted sum of two Poissons models respectively.

2) The rank of the models in relationship to lowest variation parallels the cost rankings. The Poisson model ranks first for it exhibits the lowest variance, the combination Poisson and negative binomial and the weight sum of two Poissons rank second, third, and fourth respectively.

3) Setting λ_1, at its upper level increases the sample interval and sample size by approximately the same number of units for all the models.

4) Setting p at its upper level reduces the sample interval and sample size. The sample size is reduced from 7 to 6 units for all models. The reduction in the sample interval ranges from 9 units for the Poisson model to 17 units for the weighted sum of two Poissons model.

5) The sample interval and sample size are both reduced when A_3 is increased to its upper level. The sample size is reduced by one unit for all models. The magnitude of reduction in the sample interval varied from model to model.

6) Setting both λ_1, and and A_3 at their upper levels had the combined effect of reducing the sampling interval, while

increasing the sample size.

7) The optimal sampling interval, for each set of cost parameters varied by one or less between the Poisson, negative binomial and combination Poisson and negative binomial models. The sampling interval for the weighted sum of two Poissons model was always greater than for the other models.

8) The optimal sample sizes for each set of cost paramaters varied by one or less between all the models.

5.4 Sensitivity Analysis

The cost of inspection is dependent on the value of the four cost parameters and the chosen inspection plan. Inspection plans consist of a sample size, n, sample interval, N, and the half width (or full width) of the control chart, k. The sensitivity of inspection cost was first analyzed for each model under the five sets of cost parameters. Secondly, the difference in sensitivity between the models was analyzed for each set of cost parameters.

Results from the investigation on the sensitivity of inspection cost given the Poisson model are displayed in Tables 5.10 through 5.15. The sensitivity of the expected inspection cost due to deviation from the optimal inspection plan varies for each set of cost parameters.

The first line of each Table gives the optimal inspection plan and corresponding cost for the assumed set of cost parameters. The remainder of the lines in Table 5.10 through 5.14 give the cost

Table 5.10. Sensitivity Analysis: Poisson Model and Cost Parameter Set #1

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
3.44	27	7	.2629	-	-
3.44	27	4	.2955	.0326	12.40
3.44	27	5	.2744	.0115	4.37
3.44	27	6	.2637	.0008	.30
3.44	27	8	.2649	.0020	..76
3.44	27	9	.2684	.0055	2.09
3.44	27	10	.2730	.0101	3.84
3.44	24	7	.2638	.0009	.34
3.44	25	7	.2632	.0003	.11
3.44	26	7	.2629	.0000	0
3.44	28	7	.2632	.0003	.11
3.44	29	7	.2637	.0008	.30
3.44	30	7	.2644	.0015	.57
3.32	27	7	.2631	.0002	.08
3.36	27	7	.2631	.0002	.08
3.40	27	7	.2629	.0000	0
3.48	27	7	.2629	.0000	0
3.52	27	7	.2629	.0000	0
3.56	27	7	.2646	.0017	.65

Table 5.11. Sensitivity Analysis: Poisson Model and Cost Parameter Set #2

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
3.00	34	16	.3540	-	-
3.00	34	13	.3598	.0058	1.64
3.00	34	14	.3560	.0020	.56
3.00	34	15	.3543	.0003	.08
3.00	34	17	.3547	.0007	.20
3.00	34	18	.3563	.0023	.65
3.00	34	19	.3585	.0045	1.27
3.00	31	16	.3547	.0007	.20
3.00	32	16	.3542	.0002	.06
3.00	33	16	.3540	.0000	0.00
3.00	35	16	.3541	.0001	.03
3.00	36	16	.3544	.0004	.11
3.00	37	16	.3549	.0009	.25
2.88	34	16	.3546	.0006	.17
2.92	34	16	.3546	.0006	.17
2.96	34	16	.3540	.0000	0.00
3.04	34	16	.3549	.0009	.25
3.08	34	16	.3549	.0009	.25
3.12	34	16	.3573	.0033	.93

Table 5.12. Sensitivity Analysis: Poisson Model and Cost Parameter Set #3

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
3.40	17	6	.4499	-	-
3.40	17	3	.5513	.1014	22.54
3.40	17	4	.4878	.0380	8.45
3.40	17	5	.4544	.0045	1.00
3.40	17	7	.4514	.0015	.33
3.40	17	8	.4563	.0064	1.42
3.40	17	9	.4640	.0141	3.13
3.40	14	6	.4563	.0064	1.42
3.40	15	6	.4528	.0029	.64
3.40	16	6	.4508	.0009	.20
3.40	18	6	.4500	.0001	.02
3.40	19	6	.4509	.0010	.22
3.40	20	6	.4524	.0025	.56
3.28	17	6	.4505	.0006	.13
3.32	17	6	.4499	.0000	0.00
3.36	17	6	.4499	.0000	0.00
3.44	17	6	.4499	.0000	0.00
3.48	17	6	.4536	.0037	.82
3.52	17	6	.4536	.0037	.82

Table 5.13. Sensitivity Analysis: Poisson Model and Cost Parameter Set #4

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
3.20	15	6	.5083	-	-
3.20	15	3	.6126	.1043	20.52
3.20	15	4	.5412	.0329	6.48
3.20	15	5	.5158	.0075	1.48
3.20	15	7	.5145	.0062	1.22
3.20	15	8	.5204	.0121	2.38
3.20	15	9	.5318	.0235	4.62
3.20	12	6	.5154	.0071	1.40
3.20	13	6	.5107	.0024	4.47
3.20	14	6	.5085	.0002	.04
3.20	16	6	.5097	.0014	.28
3.20	17	6	.5124	.0041	.81
3.20	18	6	.5163	.0080	1.58
3.08	15	6	.5142	.0059	1.16
3.12	15	6	.5083	.0000	0.00
3.16	15	6	.5083	.0000	0.00
3.24	15	6	.5083	.0000	0.00
3.28	15	6	.5083	.0000	0.00
3.32	15	6	.5099	.0016	.31

Table 5.14. Sensitivity Analysis: Poisson Model and Cost Parameter Set #5

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
2.88	17	13	.7338	-	-
2.88	17	10	.7621	.0283	3.86
2.88	17	11	.7449	.0111	1.51
2.88	17	12	.7364	.0026	.35
2.88	17	14	.7353	.0015	.20
2.88	17	15	.7395	.0057	.78
2.88	17	16	.7451	.0113	1.54
2.88	14	13	.7413	.0075	1.02
2.88	15	13	.7367	.0029	.40
2.88	16	13	.7343	.0005	.07
2.88	18	13	.7348	.0010	.14
2.88	19	13	.7371	.0033	.45
2.88	20	13	.7403	.0065	.89
2.76	17	13	.7345	.0007	.10
2.80	17	13	.7345	.0007	.10
2.84	17	13	.7338	.0000	0.00
2.92	17	13	.7338	.0000	0.00
2.96	17	13	.7383	.0045	.61
3.00	17	13	.7383	.0045	.61

Table 5.15. Sensitivity Analysis: Poisson Model and Cost Parameter Set #2

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
3.00	34	16	.3540	-	-
3.00	31	13	.3579	.0039	1.10
3.00	31	19	.3610	.0070	1.98
3.00	37	13	.3631	.0091	2.57
3.00	37	19	.3578	.0038	1.07
2.88	31	16	.3560	.0020	.56
3.12	31	16	.3570	.0030	.85
2.88	37	16	.3550	.0010	.28
3.12	37	16	.3591	.0051	1.44
2.88	34	13	.3580	.0040	1.12
2.88	34	49	.3598	.0058	1.63
3.12	34	13	.3638	.0098	2.77
3.12	34	19	.3584	.0044	1.24
2.88	31	13	.3570	.0030	.85
2.88	31	19	.3628	.0088	2.49
2.88	37	13	.3605	.0065	1.84
2.88	37	19	.3588	.0038	1.07
3.12	31	13	.3612	.0072	2.03
3.12	31	19	.3605	.0065	1.84
3.12	37	13	.3678	.0138	3.90
3.12	37	19	.3581	.0041	1.16

of inspection when one of the inspection parameters are non-optimal and the other two parameters are set at their optimal values. Table 5.15 gives results for when two and three parameters are non-optimal at the same time.

Tables 5.10 through 5.15 indicate that the cost surfaces are most sensitive to changes in sample size. Thus, the shape of the cost surfaces are greatly dependent on the sample size. The cost surfaces become flatter in the vicinity of the optimum as sample size increases. Figure 5.5 displays the impact that deviating from the optimal sample size has on inspection cost. Departure of K from its optimal value has a step function effect on inspection cost. That is, there are ranges of K that have the same impact on inspection cost. Table 5.15 illustrates that the worst set of conditions is to underestimate sample size, n, and overestimate K and N.

The same analysis just described for the Poisson model was also conducted for the negative binomial, weighted sum of two Poissons and combination Poisson, negative binomial models. Results of these analysis' are presented in Appendix C. The conclusions drawn from these analysis were similar to those just described for the Poisson model.

Figures 5.6 through 5.10 illustrate the impact that the choice of the defect distribution had on the expected cost. No consistent pattern concerning the impact of the distribution model on the sensitivity of expected cost could be determined through

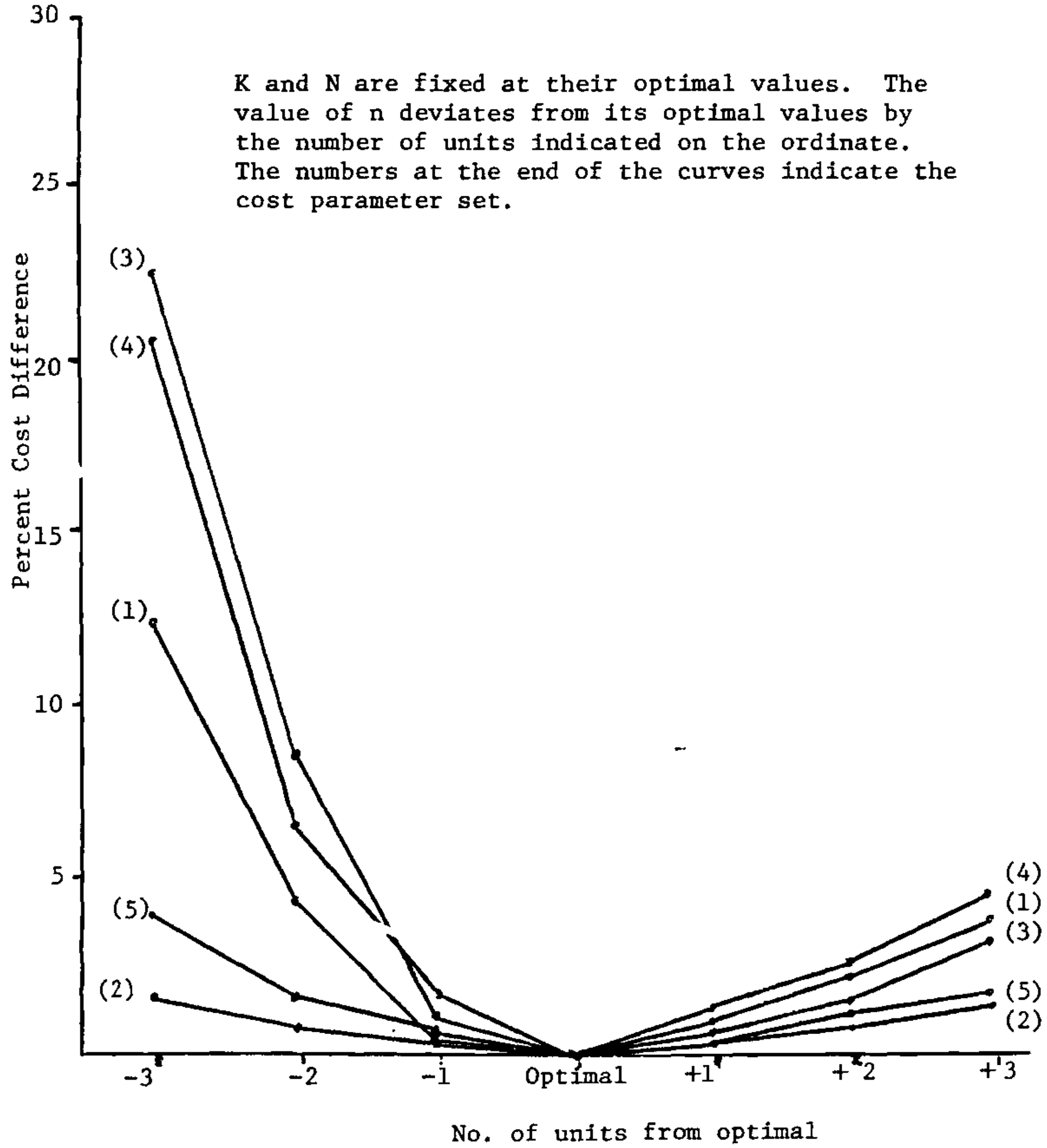


Figure 5.5 Sensitivity Curves for Poisson Model

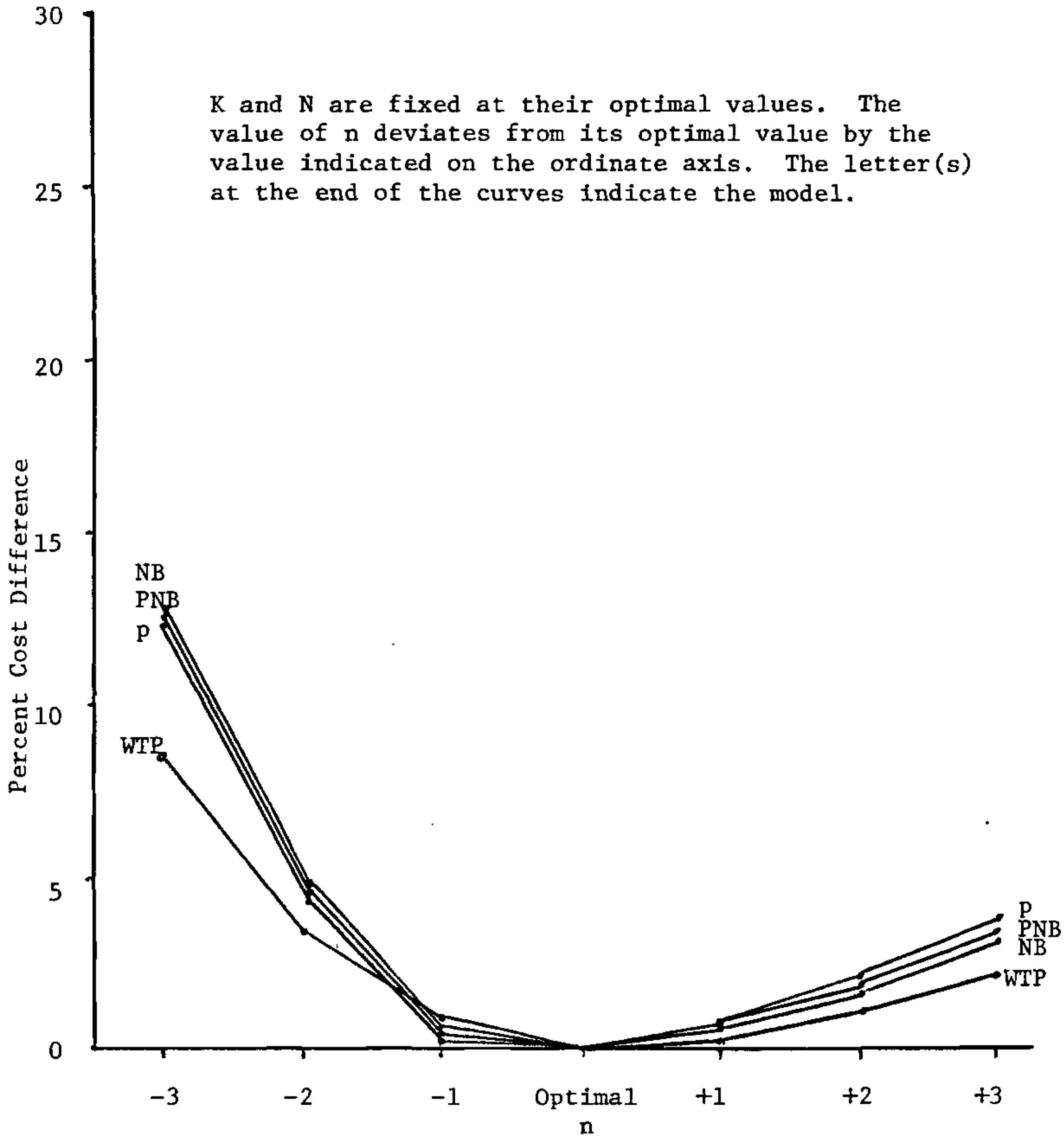


Figure 5.6. Model Sensitivity Using Cost Parameter Set #1

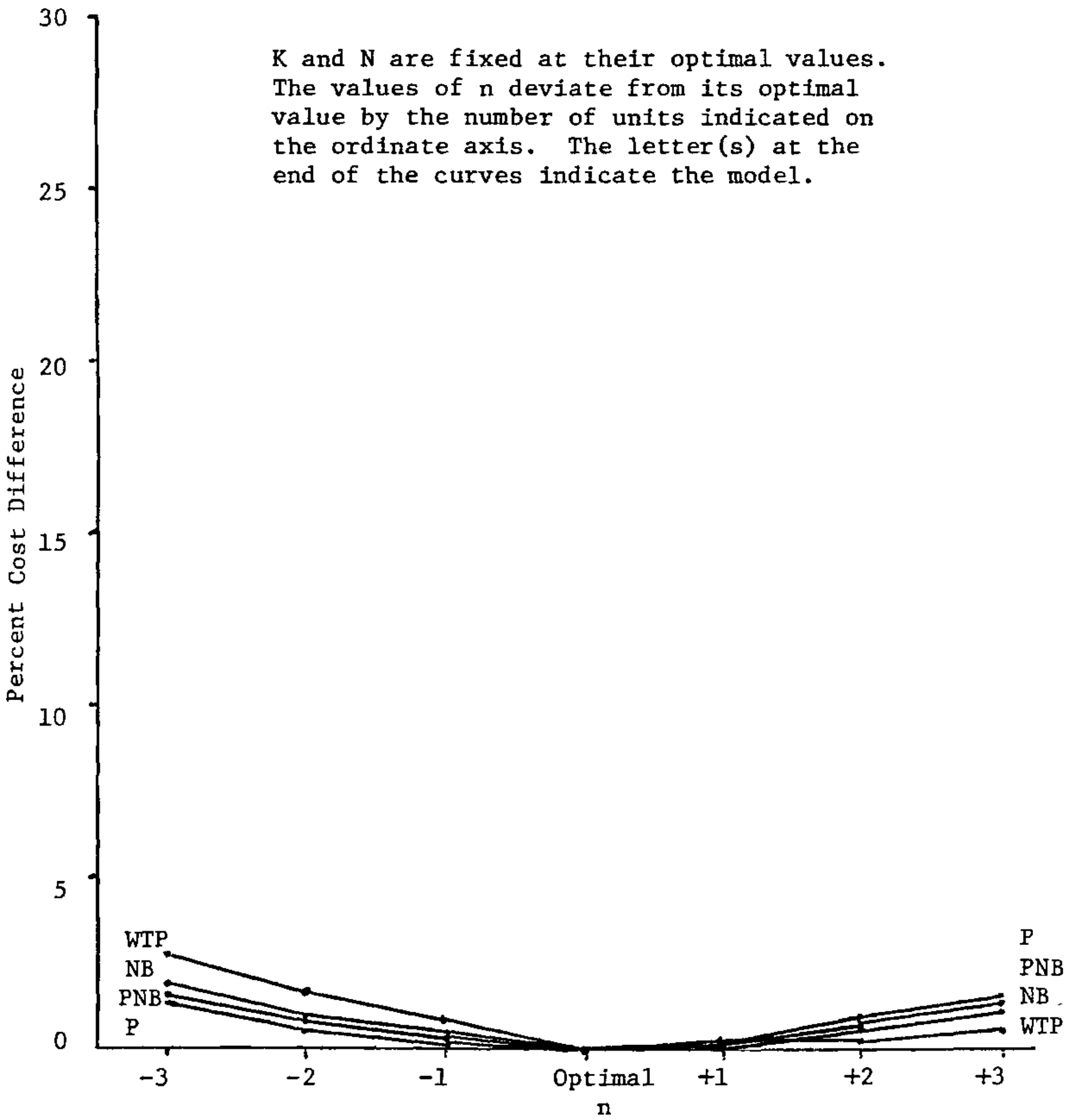


Figure 5.7. Model Sensitivity Using Cost Parameter Set #2

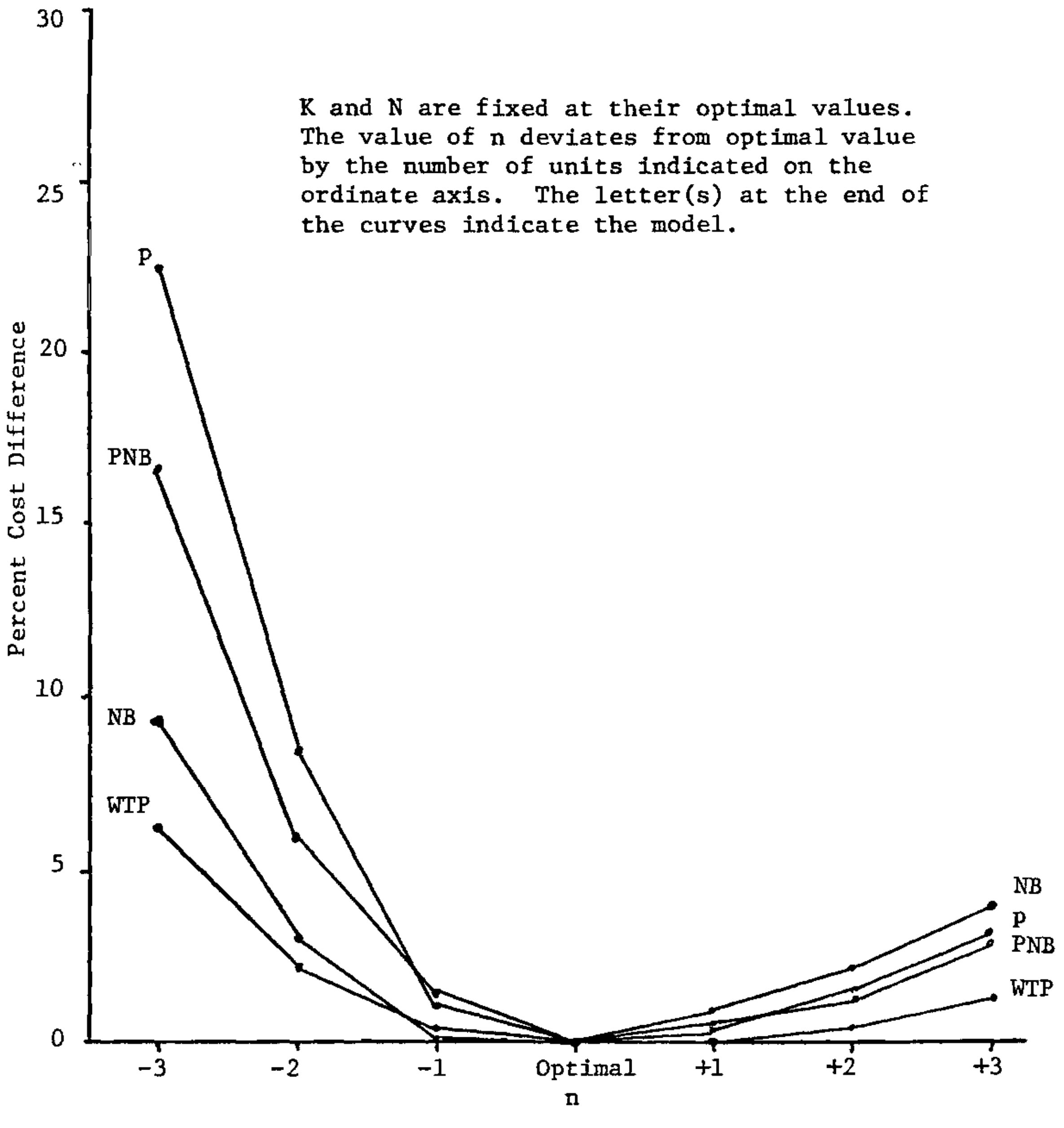


Figure 5.8. Model Sensitivity Using Cost Parameter Set #3

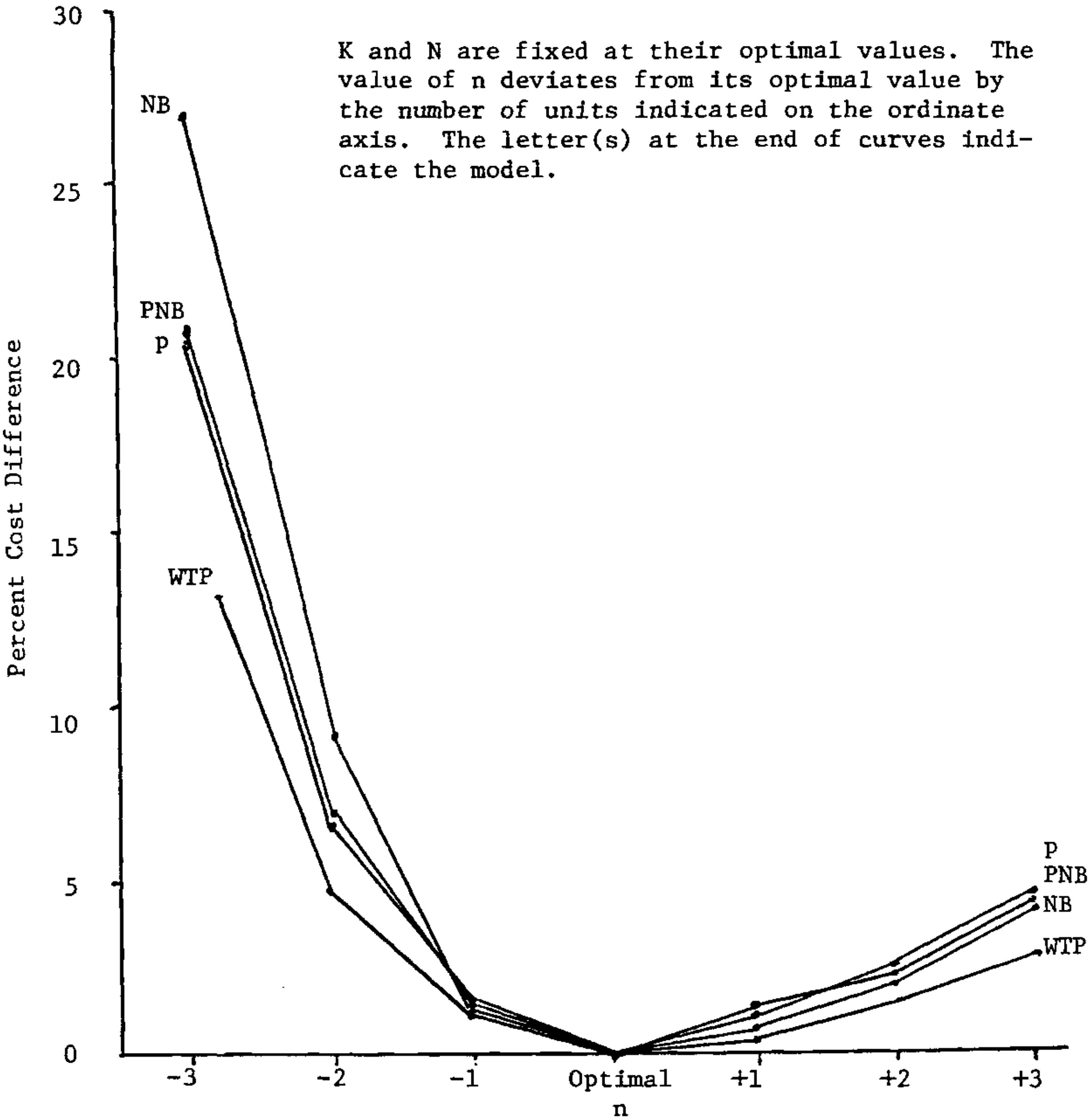


Figure 5.9. Model Sensitivity Using Cost Parameter Set #4

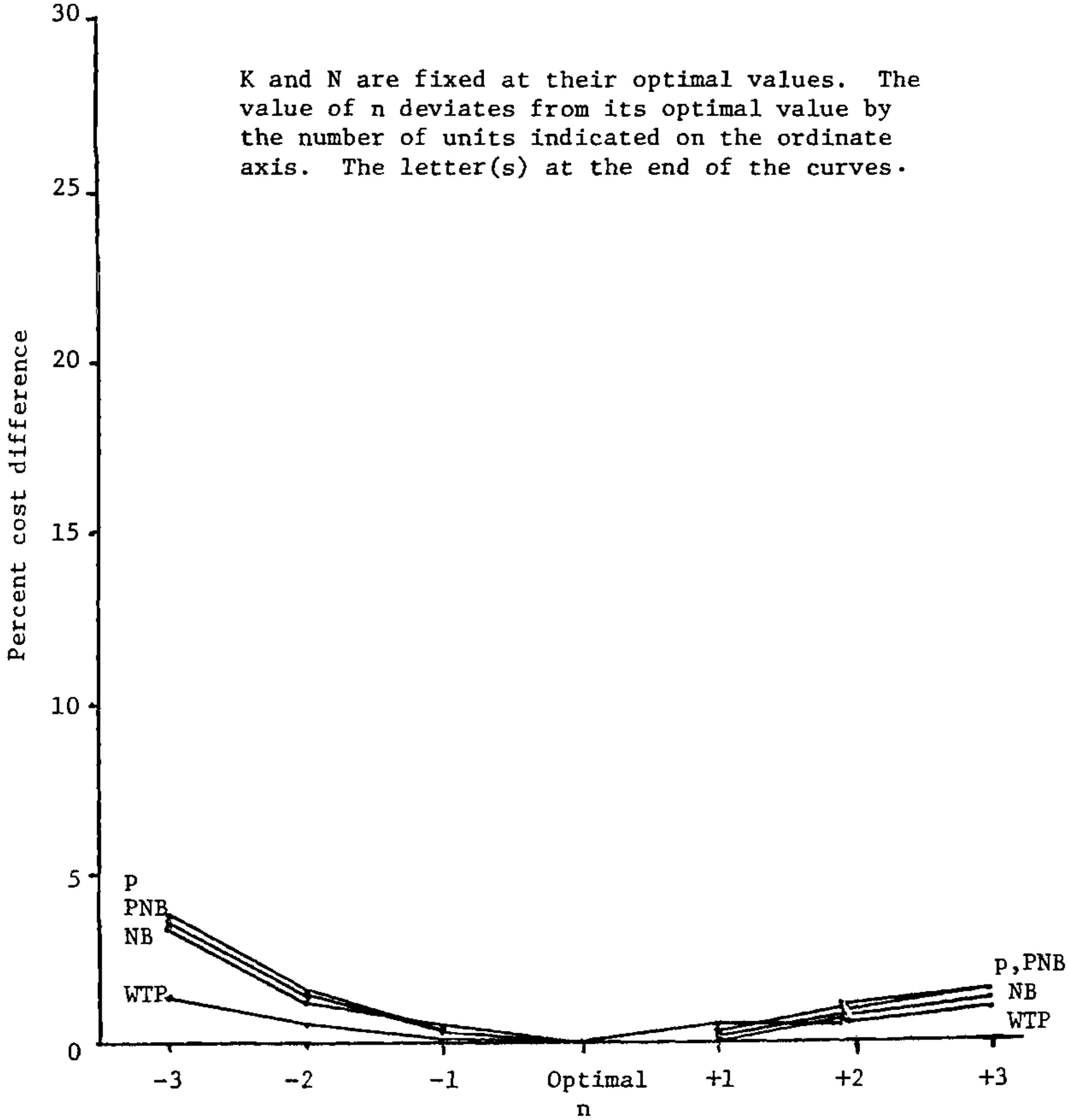


Figure 5.10. Model Sensitivity Using Cost Parameter Set #5

analysis of these five figures. However, analysis of these figures reveals the cost is least sensitive when cost parameter sets #2 and #5 are assumed and that it is generally better to use a larger than optimal sample size than to use a smaller than optimal sample size, no matter which model is utilized. The optimal inspection plans that result when cost parameter sets #2 and #5 are assumed have the highest sample size. This indicated that the sensitivity of the expected inspection cost is strongly influenced by sample size, at least in the cases analyzed in this project.

5.5 The Effect of Misspecifying the Distribution of the Occurrence of Data

The effects of incorrectly assuming that the defects were generated by a Poisson process were analyzed in this section of the thesis. Three cases were analyzed:

1) The Poisson was assumed when in actuality the defects were generated by a negative binomial process;
2) The Poisson was assumed while in fact the defects were generated by the weighted sum of two Poisson processes.
3) The Poisson was assumed when in actuality the defects were generated by two processes, one being Poisson, the other being negative binomial (combination Poisson and negative binomial).

The double-distribution models were utilized to analyze these cases. The parameters utilized in the previous analysis were also used in this investigation.

The results of this investigation are presented in Tables 5.16 through 5.20 and are summarized below. The cost of the optimal inspection plan when the correct defect distribution is assumed is presented in the second column of the tables. The cost when the Poisson is incorrectly assumed to represent the occurrence of defects is presented in the third column. The increase in cost and the percent increase in cost that results from incorrectly assuming the Poisson distribution are presented in the forth and fifth columns, respectively.

1) No change in cost occurred when the underlying distribution of defects was the combination of Poisson and negative binomial model and it was incorrectly assumed to be Poisson.
2) Only slight changes occurred when the underlying defect distribution was the negative binomial model and the Poisson model was incorrectly assumed. The changes ranged from 0.000% to .155%.
3) Potentially significant increases in cost occurred when the Poisson model was incorrectly assumed to model the weighted sum of two poisson distribution of defects. The increases ranged from 1.260% to 2.977%.

Table 5.16. Model Misspecification Analysis Using Cost Set #1

Actual Defect Distribution	Cost when the Correct Defect Distribution was Assumed	Cost when the Poisson was Incorrectly Assumed	Increase in Cost	Percent Increase in Cost
Poisson	.2629	-	-	-
Negative Binomial	.2652	.2653	.0001	.0377
Weight Sum of Two Poissons	.3118	.3119	.0081	2.5978
Combination Poisson and Negative Binomial	.2641	.2641	.0000	0.0000

Table 5.17. Model Misspecification Analysis Using Cost Set #2

Actual Defect Distribution	Cost when the Correct Defect Distribution was Assumed	Cost when the Poisson was Incorrectly Assumed	Increase in Cost	Percent Increase in Cost
Poisson	,3540	-	-	-
Negative Binomial	.3591	.3591	0.0000	0.0000
Weight Sum of Two Poissons	.3886	.3935	.0049	1.2609
Combination Poisson and Negative Binomial	.3565	.3565	0.0000	0.0000

Table 5.18. Model Misspecification Analysis Using Cost Set #3

Actual Defect Distribution	Cost when the Correct Defect Distribution was Assumed	Cost when the Poisson was Incorrectly Assumed	Increase in Cost	Percent Increase in Cost
Poisson	.4499	-	-	-
Negative Binomial	.4542	.4545	.0003	.06605
Weight Sum of Two Poissons	.5201	.5362	.0155	2.9768
Combination Poisson and Negative Binomial	.4522	.4522	0.0000	0.0000

Table 5.19. Model Misspecification Analysis Using Cost Set #4

Actual Defect Distribution	Cost when the Correct Defect Distribution was Assumed	Cost when the Poisson was Incorrectly Assumed	Increase in Cost	Percent Increase in Cost
Poisson	.5083	-	-	-
Negative Binomial	.5148	.5156	.0008	.1554
Weight Sum of Two Poissons	.5973	.6063	.0090	1.5068
Combination Poisson and Negative Binomial	.5119	.5119	0.0000	0.0000

Table 5.20. Model Misspecification Analysis Using Cost Set #5

Actual Defect Distribution	Cost when the Correct Defect Distribution was Assumed	Cost when the Poisson was Incorrectly Assumed	Increase in Cost	Percent Increase in Cost
Poisson	.7338	-	-	-
Negative Binomial	.7461	.7461	0.0000	0.0000
Weight Sum of Two Poissons	.7979	.8083	.0104	1.3034
Combination Poisson and Negative Binomial	.7400	.7400	0.0000	.0000

4) The percent increase in cost appears to be strongly influenced by sample size.

5) The rank of the models in relation to percent increase in cost is independent of the cost parameters utilized. The percentage of cost increase is always lowest for the combination Poisson, negative binomial model. The negative binomial model sometimes ties the combination Poisson, negative binomial model for the lowest percentage increase. The highest percentage increase in cost always occurs when the weighted sum of two Poissons is the underlying defect model.

6) The rank of the models in relationship to lowest variance parallels the percentage increase cost rankings. This fact indicates that at least for the cases analyzed in this project, that the closer the variance is to equalling the mean, the less the impact of incorrectly assuming a Poisson process.

5.6 Mean and Variance - Not Equal

The analysis in the previous section illustrates that the closer the variance is to equalling the mean, the less is the impact on cost because of incorrectly assuming a Poisson process. In this section we analyzed cases where the mean of the defect distribution is equal to the mean of the assumed Poisson, and the variance of the defect distribution is significantly greater than the variance of the assumed Poisson. Tables 5.21 display the parameter values utilized in this analysis. The parameter values assigned to λ_1, λ_2, A_0, A_1, A_2, A_3, and p, (Table 5.21) corresponds to parameter set #2 utilized in the previous section of this chapter.

Table 5.21. Parameter Value Set #2

Parameter	λ_1	λ_2	A_0	A_1	A_2	A_3	p
Value	7.00	10.00	.10	2.00	150.00	1.00	.01

The results of this investigation are provided in Tables 5.22 through 5.28. The values of the distribution parameters utilized to obtain the results in each table are listed near the bottom of the page. The format of these tables is the same as the tables used in section 5.3. The reader will notice that three cost figures are shown in tables 5.23 and 5.27 for the case where the Poisson distribution was incorrectly assumed. For these particular situations small changes in the width of the control chart changes the cumulative probability of the negative binomial probability function which in turn alters the probability of a Type I and Type II errors (see column 11) which results in a different cost. Two major conclusions were drawn from this study.

1) The greater the difference between the variance and mean of the underlying distribution of the occurrence of defects, the greater is the percent increase in cost that results by incorrectly assuming that the under-

Table 5.22. Poisson

Model	Inspection Cost Per Unit	Percent of Increase	Sampling Parameters (N) Sample Interval	Sampling Parameters (N) Sample Size	Control Chart Parameters Center Line	Control Chart Parameters Lower Control Limit Range	Control Chart Parameters Upper Control Limit Range	1/2 Width of Control Charts in Std. Dev.	Std. Dev. of Test Stat.	Probability of A Type I Error	Probability of A Type II Error
Optimal	.3540		34	16	112.0000	80	144	2.93	10.5830	.0034	.0909
Poisson								3.02			

Values of the distribution parameters for the:

"in-control state" - $\lambda = 7.0$; $\mu = 7.0$; $\sigma^2 = 7.0$;

"out-of-control state" - $\lambda = 10.0$; $\mu = 10.0$; $\sigma^2 = 10.0$

Table 5.23. Negative Binomial #1

Model	Inspection Cost Per Unit	Percent of Increase	Sampling Parameters (N) Sample Interval	(N) Sample Size	Control Chart Parameters Center Line	Lower Control Limit Range	Upper Control Limit Range	1/2 Width of Control Charts in Std. Dev.	Standard Deviation of Test Statistic	Probability of A Type I Error	Type II Error
Assumed	2.9300							2.93		.0147	.1008
Poisson	.4122	6.073							12.5219		
Actually	2.94-2.77		34	16	111.992	80	144			.0132	.1008
Negative	.4056	4.375						3.02			
Binomial	2.98-3.02									.0132	.1018
	.4065	4.606									
Optimum								2.90			
Negative	.3886		35	19	133.000	93	173		13.6454	.0037	.1197
Binomial											
								2.92			

Values of the distribution parameters for the:

"in-control state" $k = ,7.50$; $p = .7143$; $\mu = 6.9995$; $\sigma^2 = 9.7998$

"out-of-control state"[1] - $k = 25.00$; $p = .7143$; $\mu = 9.9993$; $\sigma^2 = 13.9997$

1 The increase in cost due to distribution misspecification depends on the width of the control chart.

Table 5.24. Negative Binomial #2

Model	Inspection Cost Ber Unit	% of Inc.	Sampling Parameters (N) Sample Interval	(N) Sample Size	Control Chart Parameters Center Line	Lower Control Limit Range	Upper Control Limit Range	1/2 Width of Control Charts in Std. Dev.	Standard Deviation of Test Statistic	Probability of A Type I Error	Type II Error
Assumed Poisson Negative Binomial	.5235	20.7	34	16	112.0000	80	14	2.93 3.02	14.9666	.0377	.1120
Optimum Negative Binomial	.4336		41	24	168.0000	118	218	2.68 2.72	18.3303	.0076	.1400

Values of the distribution parameters for the:

"in-control state" - $k = 7.0$; $p = .50$; $\mu = 7.0$; $\sigma^2 = 14.0$

"out-of-control state" - $k = 10.0$; $p = .50$; $\mu = 10.0$; $\sigma^2 = 25.67$

Table 5.25. Weight Sum of Two Poissons #1

Model	Inspection Cost Per Unit	Percent of Increase	Sampling Parameters (N) Sample Interval	(N) Sample Size	Control Chart Parameters Center Line	Lower Control Limit Range	Upper Control Limit Range	1/2 Width of Control Charts in Std. Dev.	Standard Deviation of Test Statistic	Probability of A Type I Error	Type II Error
Assumed Poisson; Actually Weight Sum of Two Poissons	.8104	39.412	34	16	112.0000	80	144	$\frac{2.93}{3.02}$	26.2298	.0416	.0436
Optimum Weight Sum of Two Poissons	.5813		87	17	119.0000	80	158	$\frac{1.38}{1.44}$	27.7354	.1035	.0398

Values of the distribution parameters for the:

"in-control state" - $\lambda_1 = 7.50$; $\lambda_2 = 2.50$; $\alpha = .90$; $\mu = 7.00$; $\sigma^2 = 9.25$

"out-of-control state" - $\lambda_1 = 11.00$; $\lambda_2 = 1.00$; $\alpha = .90$; $\mu = 10.00$; $\sigma^2 = 19.00$

Table 5.26. Weight Sum of Two Poissons #2

Model	Inspection Cost Per Unit	Percent of Increase	Sampling Parameters (N) Sample Interval	(N) Sample Size	Control Chart Parameters Center Line	Lower Control Limit Range	Upper Control Limit Range	1/2 Width of Control Charts in Std. Dev.	Standard Deviation of Test Statistic	Probability of A Type I Error	Type II Error
Assumed Poisson; Weight Sum of Two Poissons	1.3439	80.535	34	16	111.9968	80	144	$\frac{2.93}{3.02}$	37.3122	.2288	.3027
Optimum Weight Sum of Two Poissons	.7444		18	7	48.9986	7	91	$\frac{2.40}{2.44}$	17.1475	.0104	.4555

Values of the distribution parameter for the:

"in-control state" – $\lambda_1 = 8.00$; $\lambda_2 = 2.00$; $\alpha = .8333$; $\mu = 7.0$; $\sigma^2 = 12.00$

"out-of-control state" – $\lambda_1 = 11.00$; $\lambda_2 = 1.00$; $\alpha = .8333$; $\mu = 10.0$; $\sigma^2 = 25.67$

Table 5.27. Combination Poisson and Negative Binomial #1

Model	Inspection Cost Per Unit	Percent of Increase	Sampling Parameters (N) Sample Interval	(N) Sample Size	Control Chart Parameters Center Line	Lower Control Limit Range	Upper Control Limit Range	1/2 Width of Control Charts in Std. Dev.	Standard Deviation of Test Statistic	Probability of A Type I Error	Type II Error
Poisson;	2.9300									.0090	.0957
Combination	.3830	2.189						2.93	11.5930		
Poisson and	2.94-2.97		34	16	111.9968	80	144				
Negative	.3797	1.606						3.02		.0083	.0957
Binomial	3.0200										
	.3802	1.739								.0083	.0962
Combination								2.89			
Poisson and	.3737		35	17	119.0000	84	153		11.9498	.0046	.1044
Negative											
Binomial								2.92			

Values of the distribution parameters for the:

"in-control state" - $\lambda = 7.0$; $k = 17.50$; $p = .7143$; $\alpha = .50$; $\mu = 6.9998$; $\sigma^2 = 8.3998$

"out-of-control state" - $\lambda = 10.0$; $k = 25.00$; $p = .7143$; $\alpha = .50$; $\mu = 9.9997$; $\sigma^2 = 11.9927$

Table 5.28. Combination Poisson and Negative Binomial #2

Model	Inspection Cost Per Unit	Percent of Increase	Sampling Parameters (N) Sample Interval	Sampling Parameters (N) Sample Size	Control Chart Parameters Center Line	Control Chart Parameters Lower Control Limit Range	Control Chart Parameters Upper Control Limit Range	1/2 Width of Control Charts in Std. Dev.	Standard Deviation of Test Statistic	Probability of A Type I Error	Probability of A Type II Error
Poisson; Combination Poisson and Negative Binomial	.4384	7.662%	34	16	112.0000	80	144	2.93 / 3.02	12.9615	.0205	.1011
Combination Poisson and Negative Binomial	.4072		38	21	147.0000	103	191	2.90 / 2.96	14.8492	.0063	.1232

Values of the distribution parameters for the:

"in-control state" - $\lambda = 7.0$; $k = 7.0$; $p = .50$; $\alpha = .50$; $\mu = 7.0$; $\sigma^2 = 10.50$

"out-of-control state" - $\lambda = 10.0$; $k = 10.0$; $p = .50$; $\alpha = .50$; $\mu = 10.0$; $\sigma^2 = 15.00$

lying distribution is Poisson. The percent cost increase that resulted from incorrectly assuming the Poisson distribution to represent the defect distribution ranges from 0 to 7.662% for cases analyzed where the variance of the defect distribution was not more than 50% greater than its mean. Table 5.23 illustrates an example where the variance of the defect distribution (negative binomial) was approximately 40% greater than its mean. The percent cost increase ranges from 4.375% to 6.073% depending on the width of the control chart. The range of 2.93 to 3.02 is the optimal 1/2 width control chart width that results when the defect distribution is assumed to be Poisson. The percent increase in cost due to incorrectly assuming the Poisson distribution increased to 20.733% for the example illustrated in Table 5.24. The variance of the defect distribution was twice the value of its mean in this example.

2) The cost of operating the quality control procedure may be significantly underestimated when the Poisson distribution is incorrectly assumed to model the defect distribution. Table 5.22 shows the results of an example where the optimal cost was found to be .3540 when the defect distribution was assumed to be Poisson. However,

if this assumption is incorrect and the defect distribution was negative binomial with the distribution parameters displayed in Table 5.24 the actual cost would be .5235 which is 47.88% greater than the cost found by assuming the Poisson distribution.

The amount by which the actual cost of operating the quality control procedure is underestimated due to incorrectly assuming the Poisson distribution depends on the values of the cost parameters, as well as the defect distribution.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

Based on the research conducted, several conclusions may be drawn. The chapter number shown parenthetically after each conclusion indicates the location of supportive material.

1) A practical methodology was developed that enables analysts to determine an appropriate mathematical model to represent the occurrence of defects (Chapter III).

2) Four single distribution cost models for defects were developed and employed in this thesis. The models are as follows: (1) Poisson model, (2) Negative binomial model, (3) Weighted sum of two Poissons model, and (4) Combination Poisson, negative binomial model. Optimal inspection plans for these models can be obtained through use of a grid search procedure (Chapter IV).

3) Three double-distribution models were also developed. These models have two applications. First they make it possible to quickly and easily compare results between the use of the Poisson model and the other single distribution models. Secondly, these models provide data on the effect of incorrectly assuming that the Poisson distribution represents the underlying

defect distribution (Chapter IV).

4) The closer the variance and mean of the distribution of occurrence of defects are to being equal, the lower is the cost increase that results by incorrectly assuming a Poisson process. The increase in percent cost ranged from 0 to 7.662% for the cases analyzed where the difference between the mean and variance of the defect distribution was less than 50%[1]. The percent increase in cost raises rapidly as the difference between the mean and variance of the distribution of the defects increases beyond 50%.

5) The cost of operating the quality control procedure is underestimated when the Poisson distribution is incorrectly assumed to model a distribution of defects. The discrepancy between the actual cost of operation and that calculated when the Poisson distribution is incorrectly assumed increases rapidly as the difference between the mean and variance of the defect distribution increases.

6.2 Recommendations

1) Additional cases utilizing the models developed in this thesis should be investigated to test the tentative conclusions that resulted from this thesis. One approach would be to vary the cost parameters in order to determine

[1] Percent cost increase was found by calculating the cost when the Poisson distribution was incorrectly assumed and comparing this cost with the cost that results when we knew the defect distribution.

their impact on the percent cost increase. Another approach would be to hold the cost parameters constant and vary the inspection plan parameters. Different sets of distribution parameters could also be used. In any event there are an infinite number of cases in addition to those in this thesis that could be analyzed.

2) A more efficient search technique would be a logical extension of the present models.

3) Many well known discrete distributions were not analyzed in this thesis. Models patterned after those in this thesis could be easily developed to employ other discrete distributions. Work with these models could add to the field of knowledge concerning the impact of model misspecification.

4) Another logical extension would be to analyze cases where distributions other than the Poisson were incorrectly assumed to represent the distribution of defects.

5) The impact of model misspecification was only analyzed for optimal inspection plans in this thesis. The last recommended extension of the thesis is to analyze the impact of model misspecification on non-optimal inspection plans. A logical procedure would be to fix the control chart width at either 4 or 6 standard deviations. These widths are commonly employed in practice. Next determine

the optimal values for N and n given the fixed control chart width. At this point the investigation could be patterned after the investigation presented in this thesis.

APPENDICES

APPENDIX A

GLOSSARY OF TERMS

1) A_0 - Variable cost of sampling or testing one item for the presence of a defect.

2) A_1 - Fixed cost of sampling.

3) A_2 - Cost of a false alarm

4) A_3 - Cost per unit of operating out-of-control.

5) B_1 - Expected number of units produced per period while in the out-of-control state.

6) c - Total cost per period.

7) c-Chart - Control chart for defects.

8) C_I - number assigned by the internal counter of the computer models

9) C_N - Cost per unit.

10) d - Total number of defects in the n items inspected.

11) k - 1/2 width of the control chart in standard deviations.

12) LCL - Lower control limit of the control chart

13) n - Sample size.

14) N - Sample interval.

15) $n\lambda_1$ - Centerline of control chart

16) p - Probability of a shift from the in-control state to the out-of-control state.

17) p-Chart - Control chart for freaction defective.

18) period - Equals sampling interval, N.

19) STD - Standard deviation.

20) u-Chart - Control chart for defects per unit.

21) UCL - Upper control limit of control chart.

22) α - Probability of a Type I error

23) β - Probability of a Type II error.

24) λ_1 - Mean number of defects per unit while in the in-control state.

25) λ_2 - Mean number of defects per unit while in the out-of-control state

26) γ_1 - Probability of starting the period in the in-control state

27) γ_2 - Probability of starting the period in the out-of-control state.

APPENDIX B

DISTRIBUTIONS

The probability function, generating function, and equations for calculating the mean and variance for the five distributions utilized in this thesis are summarized in Table B1. A brief description follows for each distribution.

The Poisson distribution may be used to represent the occurrence of independent events that take place at a constant rate. Most of the current cost models for defects assume that assignable causes occur according to a Poisson process.

The negative binomial gives the probability that the m^{th} success occurs on the $(m + x)^{th}$ trial where the probability of success in a single trial is P. This distribution is very often the first alternative when it is felt that a Poisson distribution is inadequate. While the negative binomial does not have the same flexibility as certain contagious distributions (with more than two assignable parameters) it often gives an adequate representation when the strict randomness requirements for the Poisson distribution are not approximated sufficiently close (18).

The weighted sum of two Poisson and the combination Poisson and negative binomial distributions can arise when mixture problems are present. For additional information on these distributions consult (5), (8), (9), (10) and (18).

Table B1. Summary of Distributions

Random Variable Name	Probability Function	Generating Function	Mean (μ)	Variance (σ^2)	Range of Variables	Parameter Values
Poisson	$\frac{\lambda^x}{x!} e^{-\lambda}$	$e^{-\lambda(1-z)}$	λ	λ	$x=0,1,2,...$	$\lambda > 0$
Negative Binomial	$\binom{x+m-1}{x} p^m (1-p)^x$	$\left[\frac{pz}{1-z(1-p)}\right]^m$	$\frac{m(1-p)}{p}$	$\frac{m(1-p)}{p^2}$ or μ/p	$x=m,m+1...$	$0 < p < 1$ $m=1,2,...$
Weighted Sum of Two Poissons	$\alpha(e^{-\lambda_1}\lambda_1{}^x/x!) +$ $(1-\alpha)(e^{-\lambda_2}\lambda_2{}^x/x!)$	$[e^{-\lambda_1(1-z)}] \times$ $[e^{-\lambda_2(1-z)}]$	$\phi\lambda_1+(1-\phi)\lambda_2$	$\mu[1+(1-\phi)\lambda_1+\phi\lambda_2]$ $-\lambda_1\lambda_2$	$x=0,1,2$	$\lambda > 0$ $0 < \phi < 1$
Combination Poisson and Negative Binomial	$(\alpha)[e^{-\lambda}\lambda^x/x!] +$ $(1-\alpha)\binom{S+m-1}{S} p^m(1-p)^S$	$[e^{-\lambda}(1-z)] \times$ $\left[\frac{pz}{1-z(1-p)}\right]^m$	$\phi\lambda +$ $\frac{(1-\phi)(1-p)^m}{p}$	$\phi\lambda^2 +$ $\frac{(1-\ )(k+1)+(1-p)^2 m}{p^2}$ $+\mu(1-\mu)$	$x=1,2,...$ $S=m,m+1,...$	$\lambda > 0$ $0 < p < 1$ $0 < \phi \leq 1$ $m=1,2,...$

APPENDIX C

Sensitivity Analysis: Negative Binomial Model and Cost Parameter Set #1

Inspection Plan			Inspection Cost		
K	N	n		Difference	Percent Difference
3.36	27	7	.2652	-	-
3.36	27	4	.2991	.0339	12.78
3.36	27	5	.2775	.0123	4.64
3.36	27	6	.2667	.0015	.57
3.36	27	8	.2672	.0020	.75
3.36	27	9	.2701	.0049	1.85
3.36	27	10	.2746	.0094	3.54
3.36	24	7	.2663	.0011	.41
3.36	25	7	.2656	.0004	.15
3.36	26	7	.2653	.0001	.04
3.36	28	7	.2655	.0003	.11
3.36	29	7	.2660	.0008	.30
3.36	30	7	.2666	.0014	.53
3.24	27	7	.2661	.0009	.34
3.28	27	7	.2661	.0009	.34
3.32	27	7	.2652	.0000	0.00
3.40	27	7	.2652	.0000	.00
3.44	27	7	.2652	.0000	.00
3.48	27	7	.2665	.0013	.49

Sensitivity Analysis: Negative Binomial Model and Cost Parameter Set #2

Inspection Plan			Inspection Cost		Percent
K	N	n		Difference	Difference
2.92	34	16	.3591	-	-
2.92	34	13	.3653	.0062	1.73
2.92	34	14	.3615	.0024	.67
2.92	34	15	.3596	.0005	.14
2.92	34	17	.3597	.0006	.17
2.92	34	18	.3611	.0020	.56
2.92	34	19	.3632	.0041	1.14
2.92	31	16	.3601	.0010	.28
2.92	32	16	.3596	.0005	.14
2.92	33	16	.3592	.0001	.03
2.92	35	16	.3592	.0001	.03
2.92	36	16	.3594	.0003	.08
2.92	37	16	.3598	.0007	.19
2.80	34	16	.3607	.0016	.45
2.84	34	16	.3607	.0016	.45
2.88	34	16	.3591	.0000	0.00
2.96	34	16	.3593	.0002	.06
3.00	34	16	.3593	.0002	.06
3.04	34	16	.3610	.0019	.53

Sensitivity Analysis: Negative Binomial Model and Cost Parameter Set #3

Inspection Plan K	N	n	Inspection Cost	Difference	Percent Difference
3.36	18	7	.4542	-	-
3.36	18	4	.4960	.0418	9.20
3.36	18	5	.4680	.0138	3.04
3.36	18	6	.4545	.0003	.07
3.36	18	8	.4583	.0041	.90
3.36	18	9	.4642	.0100	2.20
3.36	18	10	.4719	.0177	3.90
3.36	15	7	.4596	.0054	1.19
3.36	16	7	.4566	.0024	.53
3.36	17	7	.4549	.0007	.15
3.36	19	7	.4543	.0001	.02
3.36	20	7	.4551	.0009	.20
3.36	21	7	.4566	.0024	.53
3.24	18	7	.4551	.0009	.20
3.28	18	7	.4551	.0009	.20
3.32	18	7	.4542	.0000	0.00
3.40	18	7	.4542	.0000	0.00
3.44	18	7	.4542	.0000	0.00
3.48	18	7	.4563	.0021	.46

Sensitivity Analysis: Negative Binomial Model and Cost Parameter Set # 4

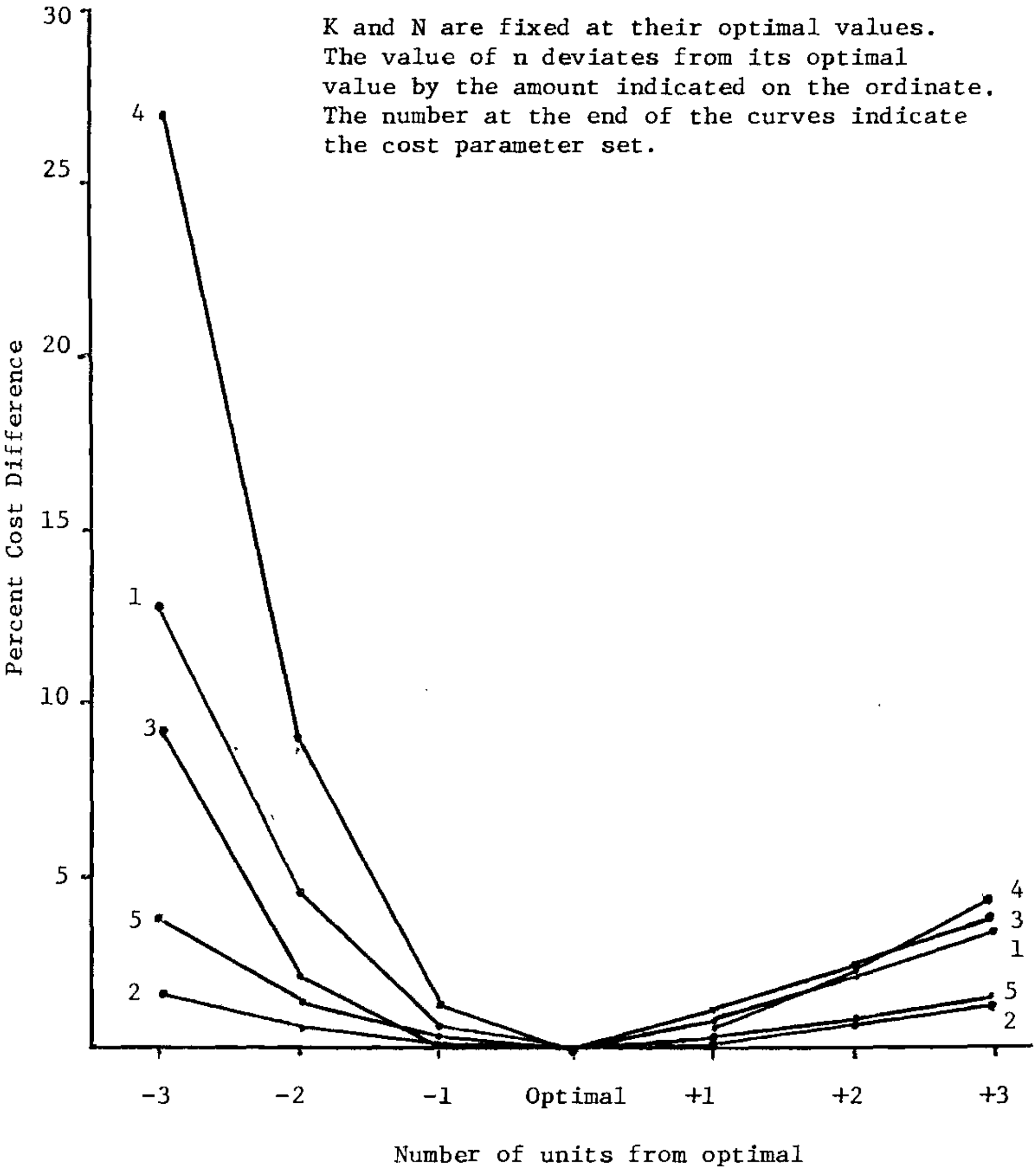
Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
3.28	14	6	.5148	-	-
3.28	14	3	.6542	.1394	27.08
3.28	14	5	.5621	.0473	9.19
3.28	14	5	.5208	.0060	1.17
3.28	14	7	.5183	.0035	.68
3.28	14	8	.5256	.0108	2.10
3.28	14	9	.5366	.0218	4.23
3.28	11	6	.5270	.0122	2.37
3.28	12	6	.5200	.0052	1.01
3.28	13	6	.5161	.0013	.25
3.28	15	6	.5153	.0005	.10
3.28	16	6	.5175	.0027	.52
3.28	17	6	.5209	.0061	1.18
3.16	14	6	.5161	.0013	.25
3.20	14	6	.5161	.0013	.25
3.24	14	6	.5148	.0000	0.00
3.32	14	6	.5148	.0000	0.00
3.36	14	6	.5148	.0000	0.00
3.40	14	6	.5196	.0048	.93

Sensitivity Analysis: Negative Binomial Model and Cost Parameter Set #5

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
2.80	17	13	.7461	-	-
2.80	17	10	.7739	.0278	3.73
2.80	17	11	.7571	.0110	1.47
2.80	17	12	.7488	.0027	.36
2.80	17	14	.7473	.0012	.16
2.80	17	15	.7512	.0051	.68
2.80	17	16	.7571	.0110	1.47
2.80	14	13	.7552	.0091	1.22
2.80	15	13	.7499	.0038	.51
2.80	16	13	.7471	.0010	.13
2.80	18	13	.7467	.0006	.08
2.80	19	13	.7486	.0025	.34
2.80	20	13	.7516	.0055	.74
2.68	17	13	.7494	.0033	.44
2.72	17	13	.7494	.0033	.44
2.76	17	13	.7461	.0000	0.00
2.84	17	13	.7461	.0000	0.00
2.88	17	13	.7484	.0023	.31
2.92	17	13	.7484	.0023	.31

Sensitivity Analysis: Negative Binomial Model and Cost Parameter Set #2

Inspection Plan K	N	n	Inspection Cost	Difference	Percent Difference
2.92	34	16	.3591	-	-
2.92	31	13	.3637	.0046	1.28
2.92	31	19	.3659	.0068	1.89
2.92	37	13	.3683	.0092	2.56
2.92	37	19	.3622	.0031	.86
2.80	31	16	.3625	.0034	.95
3.04	31	16	.3609	.0018	.50
2.80	37	16	.3608	.0017	.47
3.04	37	16	.3628	.0037	1.03
2.80	34	13	.3646	.0055	1.53
2.80	34	19	.3652	.0061	1.70
3.04	34	13	.3684	.0093	2.59
3.04	34	19	.3623	.0032	.89
2.80	31	13	.3639	.0048	1.34
2.80	31	19	.3686	.0095	2.65
2.80	37	13	.3668	.0077	2.14
2.80	37	19	.3639	.0048	1.34
3.04	31	13	.3660	.0069	1.92
3.04	31	19	.3646	.0055	1.53
3.04	37	13	.3722	.0131	3.65
3.04	37	19	.3618	.0027	.75



Sensitivity Curves for Negative Binomial Model

Sensitivity Analysis: Weight Sum of Two Poissons Model and Cost Parameter Set #1

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
3.00	35	7	.3118	-	-
3.00	35	4	.3383	.0265	8.50
3.00	35	5	.3219	.0101	3.24
3.00	35	6	.3138	.0020	.64
3.00	35	8	.3126	.0008	.26
3.00	35	9	.3152	.0034	1.09
3.00	35	10	.3182	.0064	2.05
3.00	32	7	.3122	.0004	.13
3.00	33	7	.3119	.0001	.03
3.00	34	7	.3118	.0000	.00
3.00	36	7	.3121	.0003	.09
3.00	37	7	.3125	.0007	.22
3.00	38	7			
2.88	35	7	.3135	.0017	.55
2.92	35	7	.3118	.0000	.00
2.96	35	7	.3118	.0000	.00
3.04	35	7	.3118	.0000	.00
3.08	35	7	.3124	.0006	.19
3.12	35	7	.3124	.0006	.19

Sensitivity Analysis: Weight Sum of Two Poissons Model and Cost Parameter Set #2

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
2.26	40	15	.3886	-	-
2.26	40	12	.3995	.0109	2.80
2.26	40	13	.3950	.0064	1.65
2.26	40	14	.3918	.0032	.82
2.26	40	16	.3895	.0009	.23
2.26	40	17	.3897	.0011	.28
2.26	40	18	.3910	.0024	.62
2.26	37	15	.3895	.0009	.23
2.26	38	15	.3890	.0004	.10
2.26	39	15	.3887	.0001	.03
2.26	41	15	.3887	.0001	.03
2.26	42	15	.3888	.0002	.05
2.26	43	15	.3891	.0005	.13
2.14	40	15	.3919	.0033	.85
2.18	40	15	.3919	.0033	.85
2.22	40	15	.3902	.0016	.41
2.30	40	15	.3904	.0018	.46
2.34	40	15	.3922	.0036	.93
2.38	40	15	.3922	.0036	.93

Sensitivity Analysis: Weight Sum of Two Poissons Model and Cost Parameter Set #3

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
3.00	24	7	.5207	-	-
3.00	24	4	.5529	.0322	6.19
3.00	24	5	.5324	.0117	2.25
3.00	24	6	.5225	.0018	.35
3.00	24	8	.5207	0	0
3.00	24	9	.5228	.0021	.40
3.00	24	10	.5273	.0066	1.27
3.00	21	7	.5231	.0024	.46
3.00	22	7	.5216	.0009	.17
3.00	23	7	.5209	.0002	.04
3.00	25	7	.5211	.0004	.08
3.00	26	7	.5219	.0012	.23
3.00	27	7	.5231	.0024	.46
2.88	24	7	.5230	.0023	.44
2.92	24	7	.5207	.0000	0
2.96	24	7	.5207	.0000	0
3.04	24	7	.5207	.0000	0
3.08	24	7	.5216	.0009	.17
3.12	24	7	.5216	.0009	.17

Sensitivity Analysis: Weight Sum of Two Poissons Model and Cost Parameter Set #4

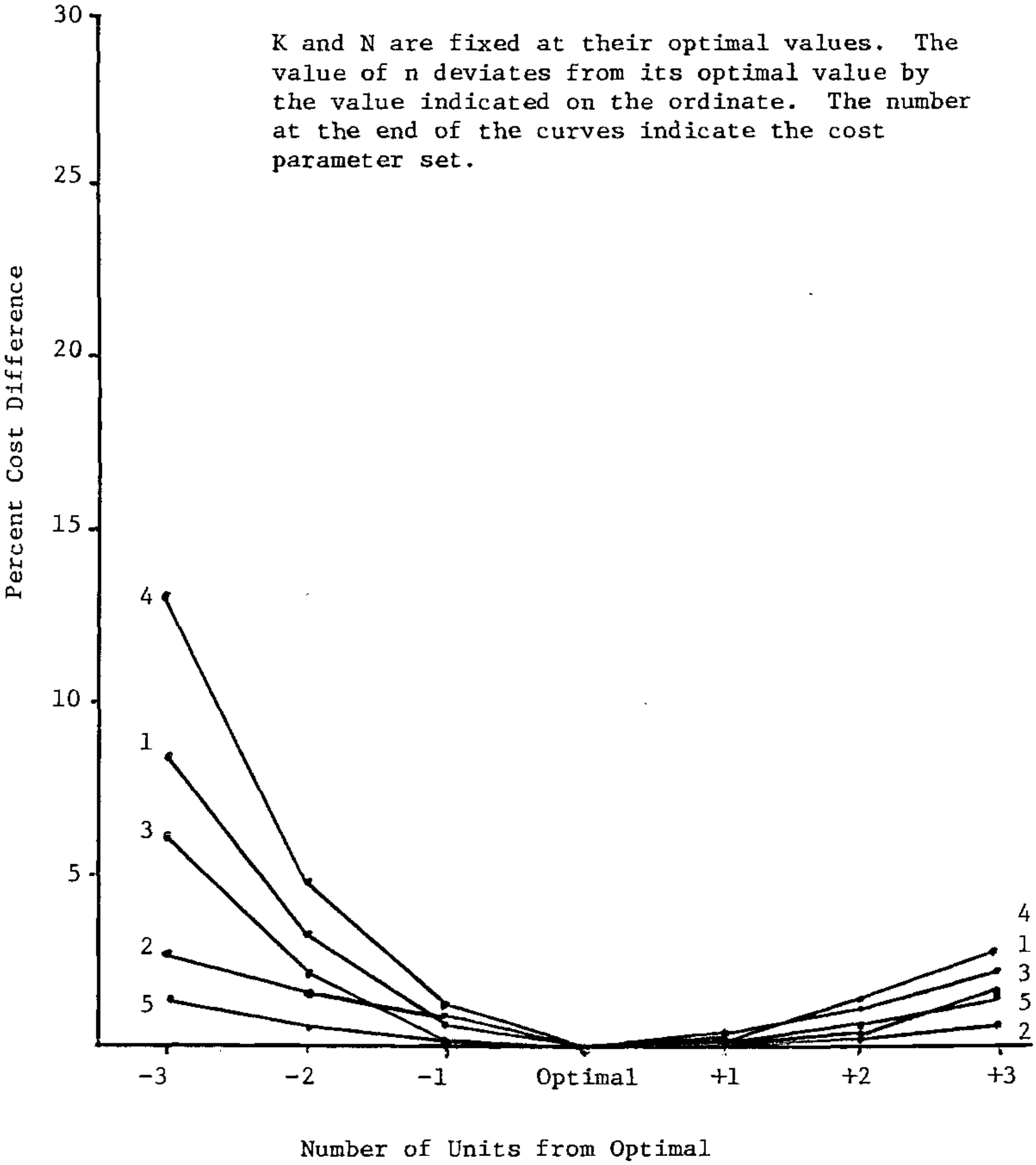
Inspection Plan K	N	n	Inspection Cost	Difference	Percent Difference
2.92	18	6	.5973	-	-
2.92	18	3	.6767	.0794	13.29
2.92	18	4	.6256	.0283	4.74
2.92	18	5	.6041	.0068	1.14
2.92	18	7	.5989	.0016	.27
2.92	18	8	.6052	.0079	1.32
2.92	18	9	.6133	.0160	2.68
2.92	15	6	.6063	.0090	1.51
2.92	16	6	.6014	.0041	.69
2.92	17	6	.5985	.0012	.20
2.92	19	6	.5976	.0003	.05
2.92	20	6	.5990	.0017	.28
2.92	21	6	.6015	.0042	.70
2.80	18	6	.6011	.0038	.64
2.84	18	6	.6011	.0038	.64
2.88	18	6	.5973	.0000	.00
2.96	18	6	.5973	.0000	.00
3.00	18	6	.5973	.0000	.00
3.04	18	6	.6001	.0028	.47

Sensitivity Analysis: Weight Sum of Two Poisson Models and Cost Parameter Set #5

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
2.20	22	14	.7979	-	-
2.20	22	11	.8102	.0123	1.54
2.20	22	12	.8030	.0051	.64
2.20	22	13	.7986	.0007	.09
2.20	22	15	.7998	.0019	.24
2.20	22	16	.8030	.0051	.64
2.20	22	17	.8079	.0100	1.25
2.20	19	14	.8033	.0054	.68
2.20	20	14	.8002	.0023	.29
2.20	21	14	.7984	.0005	.06
2.20	23	14	.7984	.0005	.06
2.20	24	14	.7997	.0018	.23
2.20	25	14	.8018	.0039	.49
2.08	22	14	.8013	.0034	.43
2.12	22	14	.8013	.0034	.43
2.16	22	14	.7979	0.0000	0.00
2.24	22	14	.7990	.0011	.14
2.28	22	14	.7990	.0011	.14
2.32	22	14	.8039	.0060	.75

Sensitivity Analysis: Weight Sum of Two Poisson Models and Cost Parameter Set #2

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
2.26	40	15	.3886	-	-
2.26	37	12	.3992	.0106	2.73
2.26	37	18	.3926	.0040	1.03
2.26	43	12	.4011	.0125	3.22
2.26	43	18	.3907	.0021	.54
2.14	37	15	.3935	.0049	1.26
2.38	37	15	.3923	.0037	.95
2.14	43	15	.3918	.0032	.82
2.38	43	15	.3939	.0053	1.36
2.14	40	12	.4015	.0129	3.32
2.14	40	18	.3917	.0031	.80
2.38	40	12	.4043	.0157	4.04
2.38	40	18	.3922	.0036	.93
2.14	37	12	.4022	.0136	3.50
2.14	37	18	.3941	.0055	1.41
2.14	43	12	.4022	.0136	3.50
2.14	43	18	.3909	.0023	.59
2.38	37	12	.4024	.0138	3.55
2.38	37	18	.3936	.0050	1.29
2.38	43	12	.4072	.0186	4.79
2.38	43	18	.3922	.0036	.93



Sensitivity Curves for Weight Sum of Two Poissons

Sensitivity Analysis: Combination Poisson and Negative Binomial Model and Cost Parameter Set #2

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
2.96	34	16	.3565	-	-
2.96	34	13	.3625	.0060	1.68
2.96	34	14	.3588	.0023	.65
2.96	34	15	.3569	.0004	.11
2.96	34	17	.3572	.0007	.20
2.96	34	18	.3587	.0022	.62
2.96	34	19	.3608	.0043	1.21
2.96	31	16	.3574	.0009	.25
2.96	32	16	.3569	.0004	.11
2.96	33	16	.3566	.0001	.03
2.96	35	16	.3566	.0001	.03
2.96	36	16	.3569	.0004	.11
2.96	37	16	.3574	.0009	.25
2.84	34	16	.3577	.0012	.34
2.88	34	16	.3577	.0012	.34
2.92	34	16	.3565	0	0
3.00	34	16	.3571	.0006	.17
3.04	34	16	.3571	.0006	.17
3.08	34	16	.3592	.0027	.76

Sensitivity Analysis: Combination Poisson and Negative Binomial Model and Cost Parameter Set #1

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
3.40	27	7	.2641	-	-
3.40	27	4	.2973	.0332	12.57
3.40	27	5	.2759	.0118	4.47
3.40	27	6	.2652	.0011	.42
3.40	27	8	.2661	.0020	.76
3.40	27	9	.2693	.0052	1.97
3.40	27	10	.2736	.0096	3.60
3.40	24	7	.2661	.0002	.76
3.40	25	7	.2644	.0003	.11
3.40	26	7	.2641	.0000	0
3.40	28	7	.2643	.0002	.08
3.40	29	7	.2648	.0007	.27
3.40	30	7	.2655	.0014	.53
3.28	27	7	.2646	.0005	1.90
3.32	27	7	.2646	.0005	1.90
3.36	27	7	.2641	0	0
3.44	27	7	.2641	0	0
3.48	27	7	.2641	0	0
3.52	27	7	.2656	.0015	.57

Sensitivity Analysis: Combination Poisson and Negative Binomial Model and Cost Parameter Set #3

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
3.32	17	6	.4522	-	-
3.32	17	3	.5278	.0756	16.72
3.32	17	4	.4789	.0267	5.90
3.32	17	5	.4586	.0064	1.42
3.32	17	7	.4538	.0016	.35
3.32	17	8	.4581	.0059	1.30
3.32	17	9	.4658	.0136	3.01
3.32	14	6	.4589	.0067	1.48
3.32	15	6	.4553	.0031	.69
3.32	16	6	.4531	.0009	.20
3.32	18	6	.4522	0	0
3.32	19	6	.4531	.0009	.20
3.32	20	6	.4545	.0023	.51
3.20	17	6	.4536	.0014	.31
3.24	17	6	.4536	.0014	.31
3.28	17	6	.4522	0	0
3.36	17	6	.4522	0	0
3.40	17	6	.4522	0	0
3.44	17	6	.4554	.0032	.71

Sensitivity Analysis: Combination Poisson and Negative Binomial Model and Cost Parameter Set #4

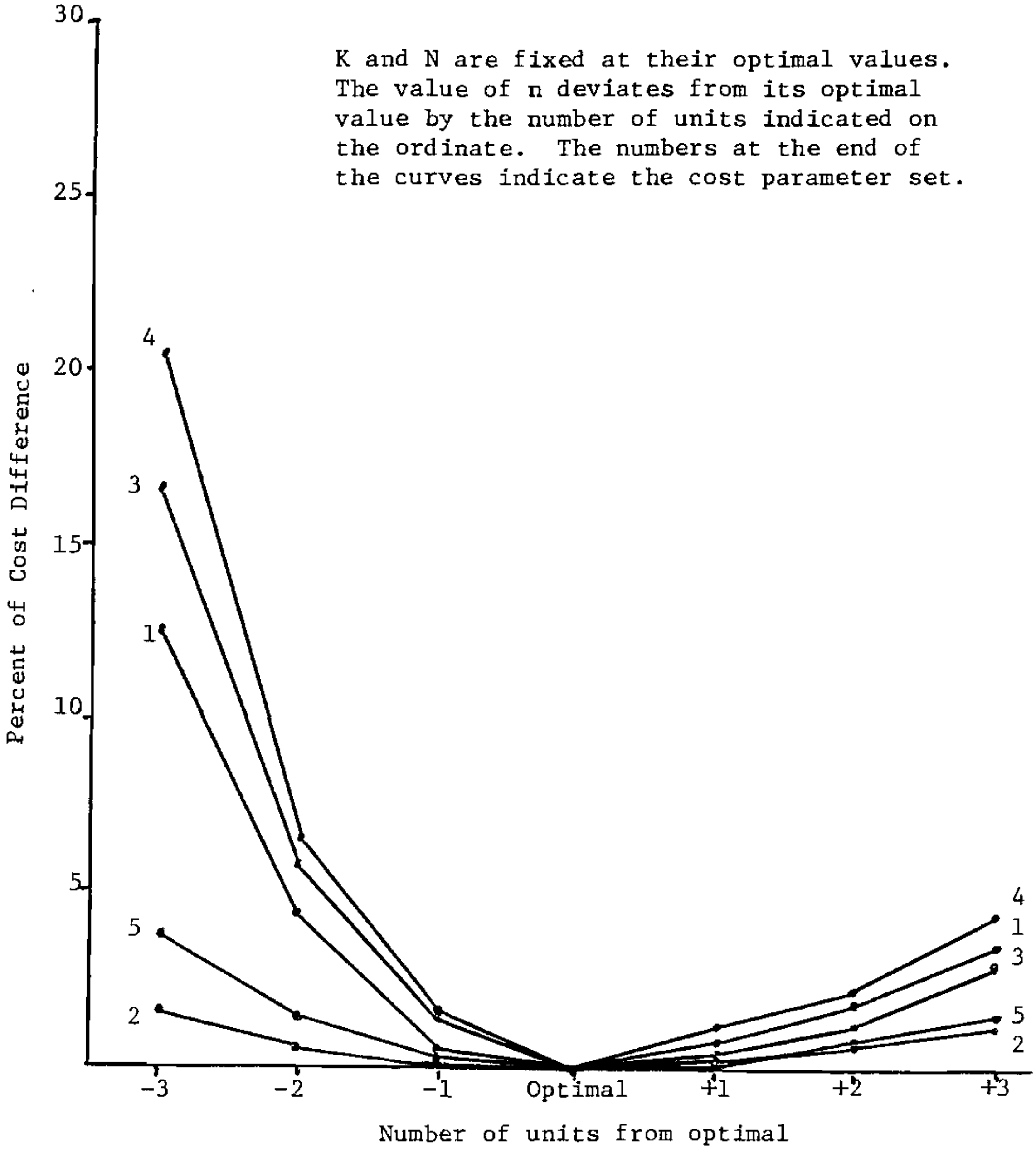
Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
3.16	15	6	.5119		
3.16	15	3	.6173	.1054	20.59
3.16	15	4	.5456	.0337	6.58
3.16	15	5	.5194	.0075	1.47
3.16	15	7	.5183	.0064	1.25
3.16	15	8	.5232	.0113	2.21
3.16	15	9	.5345	.0226	4.41
3.16	12	6	.5197	.0078	1.52
3.16	13	6	.5147	.0028	.55
3.16	14	6	.5123	.0004	.08
3.16	16	6	.5132	.0013	.25
3.16	17	6	.5159	,0040	.78
3.16	18	6	.5196	.0077	1.50
3.04	15	6			
3.08	15	6	.5119	0	0
3.12	15	6	.5119	0	0
3.20	15	6	.5119	0	0
3.24	15	6	.5119	0	0
3.28	15	6	.5126	.0007	.14

Sensitiviy Analysis: Combination Poisson and Negative Binomial Model and Cost Parameter Set #5

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
2.84	17	13	.7400	–	–
2.84	17	10	.7680	.0280	3.78
2.84	17	11	.7510	.0110	1.49
2.84	17	12	.7426	.0026	.35
2.84	17	14	.7413	.0013	.18
2.84	17	15	.7453	.0053	.72
2.84	17	16	.7514	.0114	1.54
2.84	14	13	.7482	.0082	1.11
2.84	15	13	.7433	.0033	.45
2.84	16	13	.7407	.0007	.09
2.84	18	13	.7408	.0008	.11
2.84	19	13	.7428	.0028	.38
2.84	20	13	.7460	.0060	.81
2.72	17	13	.7420	.0020	.27
2.76	17	13	.7420	.0020	.27
2.80	17	13	.7400	0	0
2.88	17	13	.7400	0	0
2.92	17	13	.7434	.0034	.46
2.96	17	13	.7434	.0034	.46

Sensitivity Analysis: Combination Poisson and Negative Binomial Model and Cost Parameter Set #2

Inspection Plan			Inspection Cost	Difference	Percent Difference
K	N	n			
2.96	34	16	.3565		
2.96	31	13	.3608	.0043	1.21
2.96	31	19	.3634	.0069	1.94
2.96	37	13	.3657	.0092	2.58
2.96	37	19	.3600	.0035	.98
2.84	31	16	.3592	.0027	.76
3.08	31	16	.3589	.0024	.67
2.84	37	16	.3579	.0014	.39
3.08	37	16	.3609	.0044	1.23
2.84	34	13	.3613	.0048	1.35
2.84	34	19	.3625	.0060	1.68
3.08	34	13	.3661	.0096	2.69
3.08	34	19	.3603	.0038	1.07
2.84	31	13	.3605	.0040	1.12
2.84	31	19	.3657	.0092	2.88
2.84	37	13	.3636	.0071	1.99
2.84	37	19	.3613	.0048	1.35
3.08	31	13	.3636	.0071	1.99
3.08	31	19	.3625	.0060	1.68
3.08	37	13	.3700	.0135	3.79
3.08	37	19	.3599	.0034	.95



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