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DEVELOPMENT OF EFFECTIVENESS MEASURES FOR WAREHOUSES

A THESIS

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

SUMMARY

Selection of handling systems for warehouses is a complex task, as there are many factors and their interactions which must be considered. The objective of this study is to establish means of evaluating the operation of both proposed and existing warehousing systems.

Relationships are developed for space requirements and operating times for stacker crane, overhead crane, and fork truck systems. These relationships consider the dimensions of each unit load, stacking heights, aisle requirements, and the speed and maneuverability of each system.

The study is theoretical and suffers from the lack of established standard times in maneuvering for stacker and overhead cranes.

Since external contingencies weigh heavily in construction costs, so relationships between square footage or usable height and the cost of the structure are extremely complex. However, the study can provide an experienced practitioner a well defined approach to evaluating different alternatives.

CHAPTER I

INTRODUCTION

The concept of warehousing was born when some prehistoric creature, appetite sated, hid or buried an unfinished meal for later use. Thus the basic premise in warehousing was quite old when man appeared. No doubt caves were used for storage as well as habitation in human antiquity, and as the principles of farming were discovered, the principles of warehousing necessarily followed.

Today's storage center owes its existence to its predecessors. A warehouse is often the best, sometimes the only method of solving fundamental problems created by irregular production or demand rates. The former is exemplified by farming, where the entire product is harvested in a relatively short time span once a year. The latter is represented by seasonal sporting goods, such as skiis and types of clothing. Warehouses are also useful for storing material near the area where it will be used.

Background

Although warehouses and inventories have been extant for a long time, very little was done in scientific inventory management until 30 years ago when Wilson developed the first inventory control model. It is based on the assumptions that demand is constant over time and that the only costs associated with the inventory are ordering and holding costs.

In the time since Wilson's model appeared, there has been a proliferation of modes of analysis. Queueing theory, linear programming, and dynamic programming have been applied to the problems of inventory (14). The advent of the computer has encouraged use of more sophisticated and complex techniques to give far more realistic control of goods in storage, but all these models consider only the inputs and outputs of the system, and ignore completely methods of handling and storage. The only information these models develop for warehouse design is the quantity of a particular product which could be expected to be on hand at a given time.

Definitions and Classifications

There has been a rapid increase in the development of handling methods within the warehouse. In order to categorize these methods, the following definitions will be used:

1. The automated warehouse is one in which goods are stored and/or retrieved by automatic remote controlled devices (8). An example of this type of warehouse is found in the Sara Lee installation in Deerfield, Illinois (15).

2. If the warehouse has manual storage but automatic selection and retrieval, it will be called a semi-automatic warehouse (Type I). This type is exemplified by the Colgate-Palmolive warehouses in Jersey City, Jeffersonville, Indiana, and Kansas City.

3. A semi-automatic warehouse (Type II) is one in which they are to be stored by a programmed device such as a conveyor with automatic switching. Storing and retrieving is done with manually operated

equipment. The Consolidated Cigar Company's warehouse in Port Newark, New Jersey, is an example of this type.

4. Other warehouses are operated by manual or manually controlled mechanized means. The former will be defined as manual warehouses and the latter as mechanized ones.

Warehouses in which more than one method of operation is in use will not be considered.

A unit load, hereafter called a unit, is a number of items or bulk material so arranged or restrained that the mass can be picked up or moved as a single object. An odd lot is a unit load which has been subdivided in some manner.

The marshalling area is defined as the location in which unit loads may be broken down for odd lot orders, and is the area in which groups of unit loads are assembled for storage or shipment.

There are three categories into which automated warehouses may be classified:

1. *Rack/Conveyor.* Material is loaded onto racks, generally of the gravity feed type, and some selection device, such as a remotely operated solenoid, allows one unit at a time to feed onto a conveyor which runs to the marshalling area. Colgate-Palmolive's warehouses are of this type.

2. *Pallet/Rack Stacker Retriever.* This system handles one unit load or pallet load at a time. A stacker crane or some other type of retriever stores and collects the loads. The load can then be placed on a conveyor or automatic train for transportation to the marshalling

area. Sara Lee, the Aldmeda Naval Air Station, and the Peter A. Frasse warehouse in Philadelphia exemplify this type.

A stacker crane (type 1) refers to an overhead crane with a stacker crane attachment. It can cross aisles whenever there is sufficient clearance to do so.

A stacker crane (type 2) is supported by the storage racks or ground rails. It requires a transfer crane at the end of the aisles to change to another aisle.

3. *Overhead Crane.* Items are selected by layers or unit load, depending on the type of holding attachment on the crane. In an automatic warehouse of this type, a vacuum system is used and it picks up one layer of boxes each trip. This warehouse is owned by a food distributor in the midwest, and was designed by Wiretyer's Engineered Handling Systems Division.

In reference to handling equipment, manually controlled shall mean that the device is operated by a button, lever, switch, or pedal which causes the device to move or lift until such time as the button, lever, switch, or pedal is deactivated.

Automatically controlled shall mean that the device is activated by a switch, lever, or computer which causes the device to move to a particular location and stop.

Objective

Valid comparison of the wide variety of equipment available for warehouses is extremely difficult. Although the choice can be simplified by eliminating those types of equipment with characteristics unsuitable for a particular application, measures of effectiveness for evaluating warehouse handling systems are needed.

The objective of this study is to establish means of evaluating the operation of proposed and existing warehousing systems. Relationships for the total operating times, space requirements, and capacities of both existing and proposed warehousing systems will be developed.

Method of Attack

The following procedures will be used:

1. Assuming that the maximum amounts of the various materials to be stored are known, a mathematical model will be structured to develop space requirements. This model will consider the individual items as assembled into units, and their dimensional characteristics. Weight will be considered as having a maximum allowable value per unit volume. The model will include the access required to approach and select each item.

2. A second model will deal with the characteristics of the different operating systems to determine the capacity of goods they are capable of handling. It will be used to predict the turnover rate and the limits within which each system is capable of operating.

3. The interrelation of these models will be investigated. Spatial requirements are dependent on the type of handling system involved, and the speed of the system is dependent on the dimensions of the building.

4. Comparisons of warehouses utilizing stacker crane, overhead crane, and fork truck systems will be made.

Importance

Integration of these models with existing inventory management techniques will give management quantitative principles of choice, enabling it to select the optimal system for its particular requirements. They would enable equipment manufacturers to evaluate their system(s) before completing the design. This permits businesses to obtain and maintain clear perspective on actual costs of operating a total inventory system.

In the face of steadily increasing competition both in this country and from abroad, manufacturers need more sophisticated controls to minimize operating costs. It is hoped that these models will become a useful tool in assisting them in their quest.

Scope

This study will examine the stacker crane, overhead crane, and fork truck systems on a theoretical basis.

Since there are no automatic warehouses in the vicinity, an existing warehousing system will not be evaluated. The prime value of these models will be with proposed systems.

This study is limited to the warehouse area proper. It will not consider the marshalling area where unit loads may be broken down. For this reason, the rack/conveyor system, which selects individual boxes, thus combining some of the aspects of the marshalling area with the storage area, will not be considered.

Assumptions

This study begins where traditional inventory models end. It assumes that when and in what amount an item is needed is given. It is further assumed that the maximum quantities of items to be stored at any particular time are given.

In order to develop mathematical models of the internal functioning of a warehouse, several simplifying assumptions are made to conform to realistic warehousing situations.

In the storage area, only unit loads will be selected or deposited, and they will not be more than one deep. This is the case in the fully automatic warehouses such as Sara Lee's and the food distributor's which was designed by Wiretyer.

Unit loads will be fairly similar in bulk. No restrictions as to size and shape are made, but similar things should be grouped together to make effective use of the equipment. Sheet metal would be stored in one area, eggs in another.

Fragile materials or other materials which require special handling are assumed to be packed in such a manner as to permit normal handling.

All items are assumed to have uniformity of dimension in the vertical plane. For example, a cone on a pallet is assumed to have the dimensions of a cube. If it was not on the pallet it would have the dimensions of a cylinder.

The warehouse management is assumed to have no control over packing the product, which arrives assembled into unit loads. Quite significant savings can be realized if the product is a box and it can be folded or nested in some manner (9). Considerations of this type are extremely important in saving space. However, the models shall take the unit load, whatever its dimensions, as essentially unalterable.

CHAPTER II

LITERATURE SURVEY

Although much has been written about inventory management, and many magazines deal almost exclusively with the types of equipment used in materials handling, very little has been written about quantitative methods for comparing different handling systems.

G. L. Almond's doctoral dissertation at Ohio State University (1) provides historical perspective in warehouse development and the services they perform. It discusses the changes that have improved the warehousing function since World War II. It also discusses conditions necessary for automation: steady volume, a relatively small variety of packages, and a market which permits amortization of the system in four or five years. Unfortunately, Almond ceased gathering information for his dissertation in 1959, and thus he did not cover later developments, such as automatic stacker crane systems.

A few authors have concerned themselves with space utilization within the warehouse. In 1962, Joseph E. Wiltrakis published his V-Spatial formula which establishes, in 20-odd factors, a method for "unlocking space economics." It is primarily concerned with reducing the effective size of the unit load, by such techniques as nesting. However, the model fails to provide an adequate measure of comparison between different equipment types.

Herbert Thornton attempted to optimize the space of a warehouse which uses fork trucks. His primary concerns were aisle width, clearance between pallets, and the angle of the pallet to the aisle center line (13).

J. B. Hemmi's thesis at the Georgia Institute of Technology has provided some useful information in evaluating fork truck performance in terms of space requirements. It proves that there is a highly significant difference between many different types of pallet arrangements if there is a large number of pallets in the warehouse (6).

Two articles dealt with the time involved of a handling system in a warehouse. Roy Lave and Hamdy Taha developed a program for simulating overhead crane operations in material handling situations. They divided the crane cycle into six events and used event oriented simulation, showing an approach to structuring a material handling operation (7).

Bazaraa's thesis at the Georgia Institute of Technology presents an interesting approach to establishing the degree of mechanization of handling equipment. It develops factors in the material, such as quantity and weight, and in the move, such as distance and frequency. He uses these factors to relate the different levels of mechanization for material handling equipment with a graphic technique (2).

CHAPTER III

SPACE REQUIREMENTS

Development of a model to predict spatial requirements is the first step toward an economic model capable of predicting costs. The spatial model is quite useful in its own right for determining the size of a proposed warehouse.

This analysis is based on a known maximum number of units of each type of product item which will be on hand at any time.

Let $n_i(t)$ be the number of units of product i on hand at time t, where t is some future time. Then the total number of units expected to be on hand is $\sum_i n_i(t)$.

Height Establishment

Let H_i denote the required (used) height in the building. It is a function of the heights of the various types of unit loads, h_i, a pallet or rack allowance, p, the number of units in the stack, k_i, and possibly an allowance, θ_i, for overhead handling equipment. Thus,

$$H_i = (h_i+p)k_i + \theta_1 \qquad (1)$$

is the height required by a stack of k_i unit loads of product type i.

Let H^* be the maximum clear height, i.e.

$$H^* = \max_i(H_i)$$

This would then determine the required height throughout the building. H^* must be selected to minimize the amount of unutilized space subject to the limitations of the equipment considered. In other words, H^* cannot have a value exceeding the capabilities of the equipment type. Determination of the optimal H^* depends on the choice of the number of units k_i in a stack of product i.

Different types of unit loads must be considered. Although there may be a great many different items to be stored, there will be far fewer variations in size once the items have been packed and assembled into unit loads. For instance, an appliance manufacturer might make 20 different models of washing machines, but most are approximately the same size, and once they are packed there would probably be no more than two or three different sizes to contend with. He might make dozens of different kinds of small appliances, but if they are packed and assembled on a standard pallet to make up a unit load, height will be the only variable, and its variation will be small. Therefore, it is felt that this problem will not be difficult to solve in practice.

A practitioner might find a safety factor in H^* desirable if it was felt that the height of the units might increase during the expected life of the building. H^* could be determined by considering the anticipated height of the product at some future time.

Floor Area Determination

Floor area requirements are dependent on many factors. These shall be introduced on a progressive basis, beginning with the number of stacks.

Number of Stacks

At any time t, the number of stacks required for product i will be

$$\left\lceil \frac{n_i(t)}{k_i} \right\rceil .$$

This term represents the smallest integer larger than $n_i(t)/k_i$, and hereafter is denoted by $\alpha_i(t)$. Fractional remnants are considered as one stack because stacking different kinds of unit loads on top of one another is not permissible unless racks are used. The total number of stacks in the building at time t is $\sum \alpha_i(t)$.

Cross Sectional Areas of Units

Let a_i represent the cross sectional area of a unit of type i. A rectangular shape permits maximum utilization of storage. All other shapes waste some space and a factor, s_i, must be applied to increase the required amount of space to a rectangle. Let l_i and w_i represent the effective length and width, respectively (Figure 1), which create the smallest possible rectangle which bounds a_i. Length is always along the aisle and width is cross aisle.

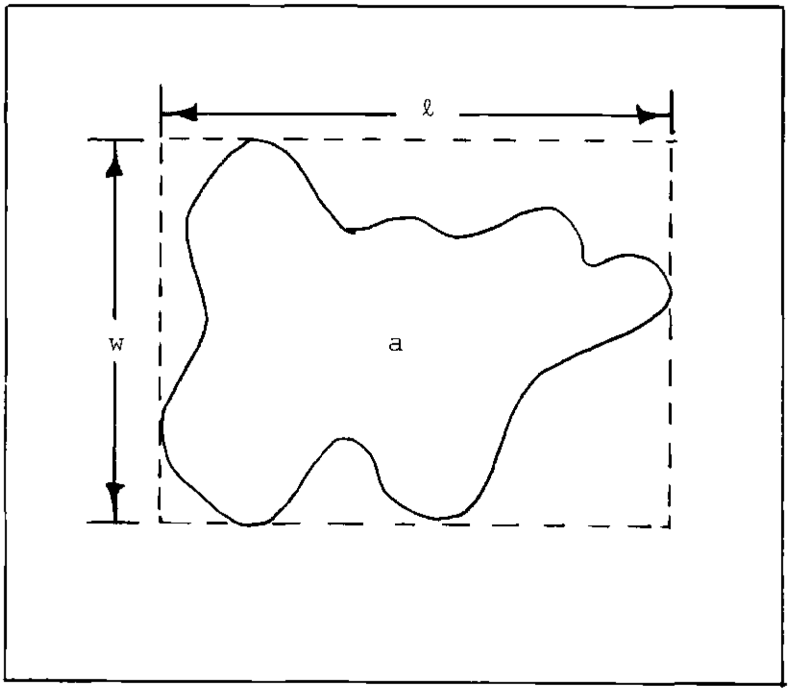

Figure 1. Illustration of Shape Parameters

The shape factor is simple to derive in common geometric shapes. For a circle, s_i is equal to .786 ($4/\pi$). For a triangle, s is equal to .500.

<u>Nesting of Units</u>

Odd shapes might permit nesting. Another factor, e_i, is necessary to encompass this contingency. The nesting factor is applied to reduce the total area required, A_1 (Figure 2). At this point, estimated area requirements are:

$$\sum_i \frac{a_i \alpha_i(t)}{e_i s_i} \quad \text{or} \quad \sum_i \frac{l_i w_i \alpha_i(t)}{e_i} \tag{2}$$

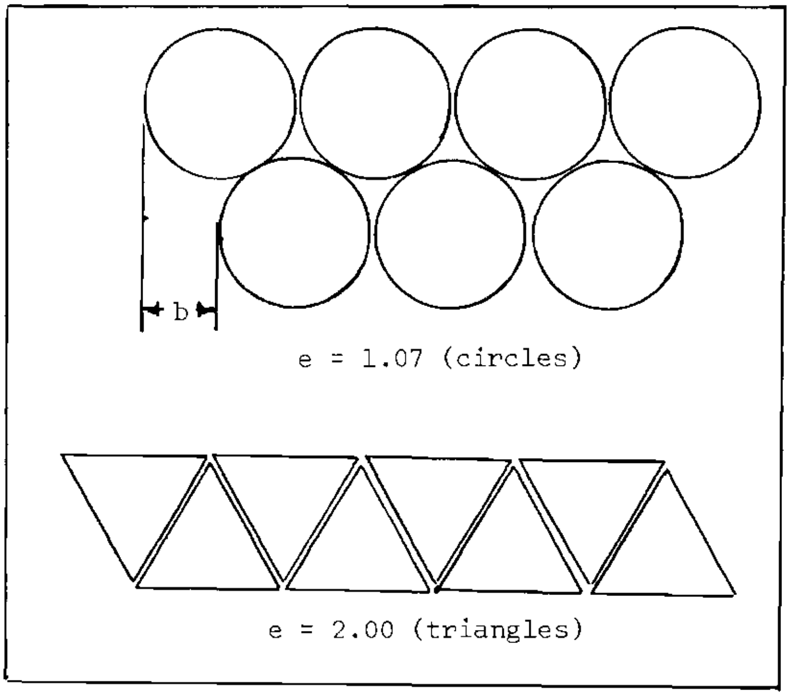

Figure 2. Illustration of Nesting

Row Width

If unit loads have different widths they still must be aligned along an aisle, and actual required width, w_i^* of any row, r, is equal to width of the widest unit load in the row. The relationship is:

$$w_i^* = \max_i(w_i \text{ product type i stored in row r}). \quad (3)$$

If random storage is desired or composition of rows is subject to change over time, it is necessary to use:

$$w^* = \max_i(w_i). \quad (4)$$

Clearance Between Stacks

To this point no provision has been made for clearance between stacks of unit loads. Clearance is necessary for getting unit loads in and out of storage locations. The clearance factor, θ_2, is dependent upon the type of handling equipment. Now the area required is:

$$\sum_i \frac{\theta_2 w^* \alpha_i(t)}{e_i} + \sum_i \frac{l_i w^* \alpha_i(t)}{e_i} \quad \text{or} \quad \sum_i \frac{(l_i+\theta_2) w^* \alpha_i(t)}{e_i} \tag{5}$$

It is possible for this formula to be slightly inaccurate: it assumes a clearance for each stack, and thus there would be an extra one if racks were not used or if one row was not against a wall. This inaccuracy is regarded as negligible.

Aisle Width

Aisle width, W_a, depends on the type of equipment used. Aisles are necessary for access in every case with the exception of the overhead crane. In order to keep the expression consistent, W_a will be used for overhead cranes, but it will represent the clearance, θ_3, along the line the aisle would have been on. Since unit loads cannot be back to back, there is in effect an "aisle" of width θ_3 for every row, and hence

$$W_a(\text{for overhead cranes}) = 2\theta_3 \tag{6}$$

See Figure 4 for illustration.

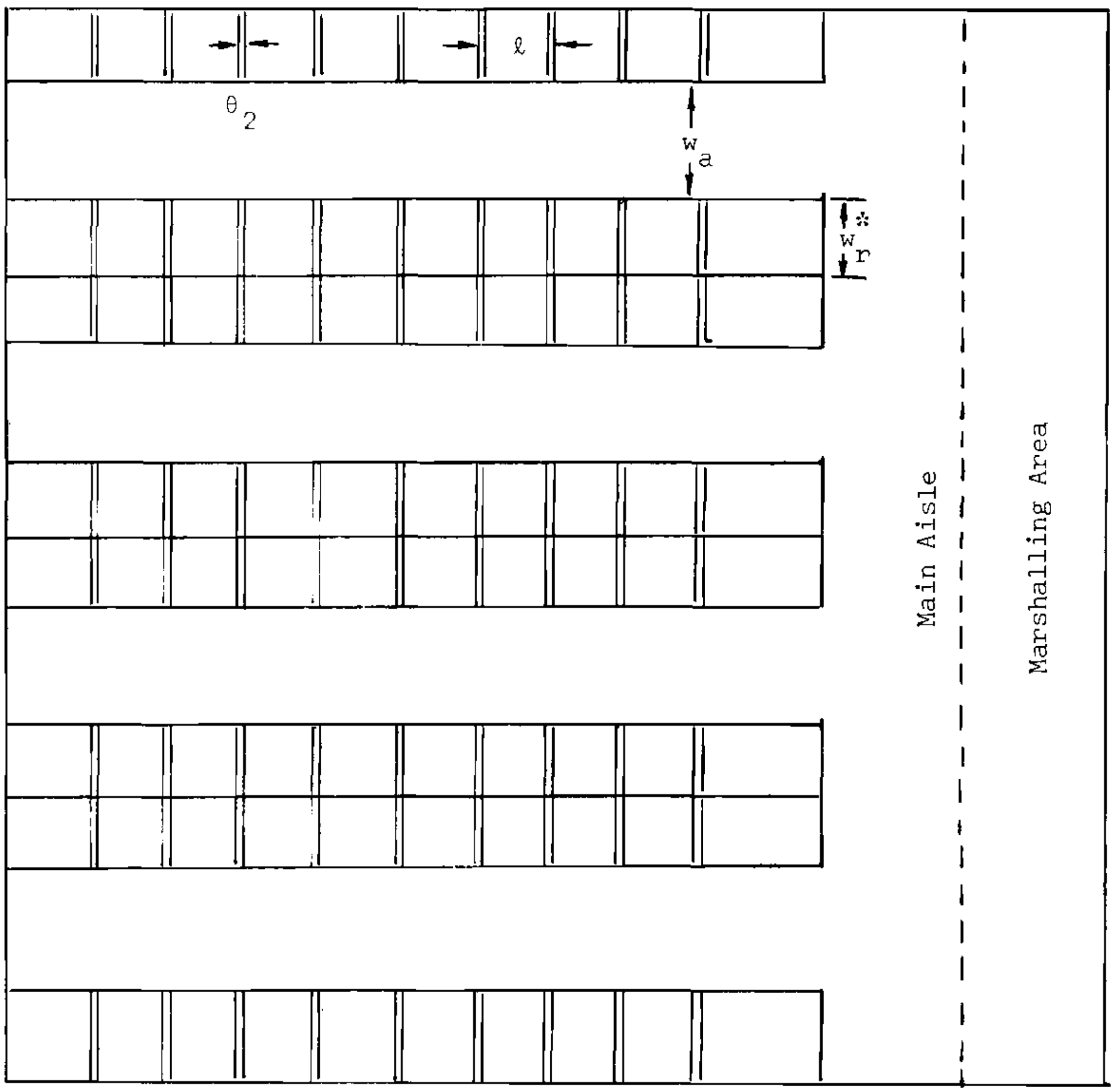

Figure 3. Warehouse Layout, Fixed Path Equipment

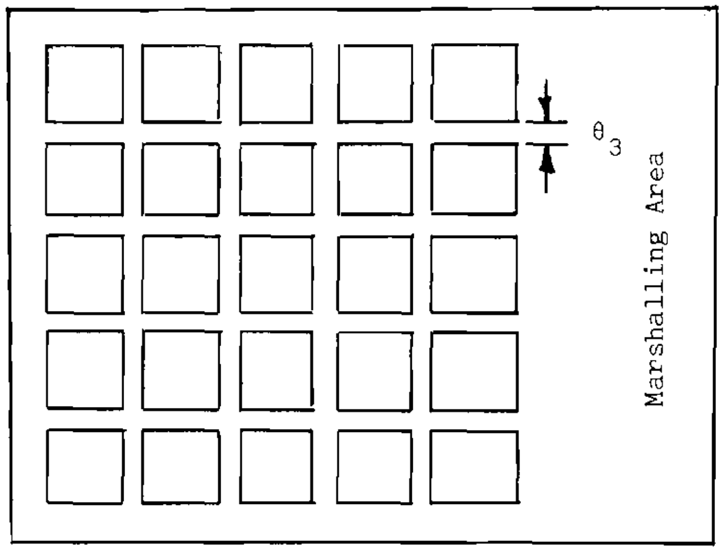

Figure 4. Warehouse Layout, Overhead Crane

Warehouse Layout

In a logically organized warehouse, the marshalling area will be at one or both ends of the storage rows, and cross aisles will only be useful for minimizing time in storage to storage moves (Figures 3 and 4). The other effect of cross aisles is to increase the amount of space required to store the same quantity of goods. Therefore, cross aisles are disallowed, and the area required is:

$$A_1(t) = \sum_i \frac{(l_i+\theta_2)w^*\alpha_i(t)}{e_i} + \frac{(l_i+\theta_2)W_a\alpha_i(t)}{2}$$

or

$$A_1(t) = \sum_i (l_i+\theta_2)\alpha_i(t)\left[\frac{w^*}{e_i} + \frac{W_a}{2}\right] . \qquad (7)$$

Utilized area, A_2, is equal to

$$A_2 = \sum_i a_i \alpha_i(t) \tag{8}$$

Dimensions of the Building

So floor utilization at any time t is equal to A_2/A_1 and is the percent of square footage utilized.

Total cube required, Q_1, is equal to $H^* A_1$. Actual utilized cube, Q_2, is equal to $\sum_i (a_i n_i h_i)$. Cubic utilization is equal to Q_2/Q_1 and represents the percent of cube utilized.

It is advantageous at this point for the practitioner to determine the dimensions of the building. If the number of aisles is equal to j, the width of the building is $(w^* + W_a/2)2j$. The number of rows, 2j, can be varied to give various configurations, as long as 2j is even to avoid half aisles. Support placement, or bay size, can be determined at this time.

The marshalling area should be considered. If it is assumed to have the same width as the storage area, its length can be added to the length of the storage area, and the final building dimensions can be established. Note that the main (cross) aisle is considered as part of the marshalling area (Figure 8).

Variations Over Time

It is entirely possible that the demand for any product i will vary over time. The purpose of a warehouse is to permit economic lot size manufacturing, to smooth production rates, to provide remote storage facilities, or any combination of the foregoing. The amount of product i on hand at any time t is equal to the integral of the warehouse input rate, $\epsilon_i(\tau)$ minus the integral of the demand rate, $\delta_i(\tau)$. The relationship is:

$$n_i(t) = \int_0^t \epsilon_i(\tau)d\tau - \int_0^t \delta_i(\tau)d\tau \tag{9}$$

If demand for a product is subject to seasonal fluctuations and input is steady, then the number of units of product i will be at a maximum when the increasing demand rate equals the input rate (Figure 5) at T.

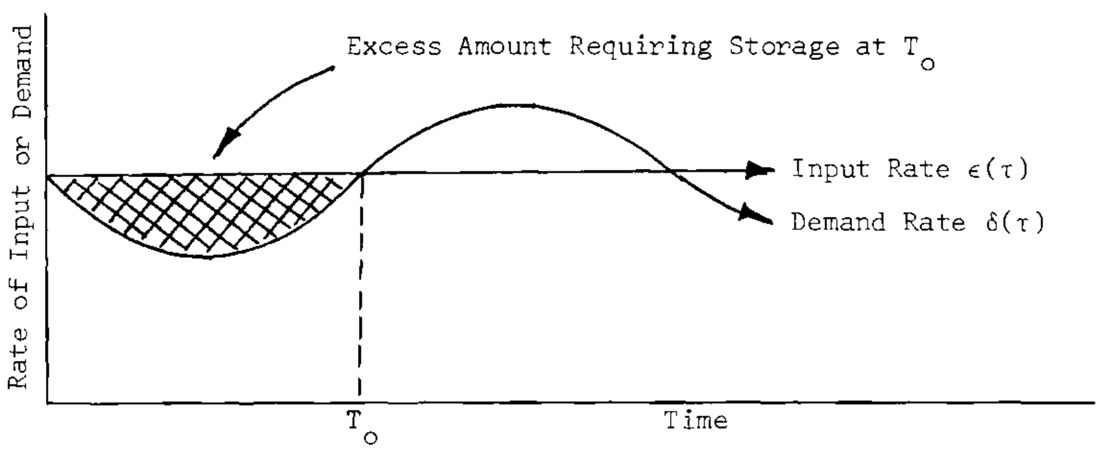

Figure 5. Effect of Seasonal Demand Variations

The area required in the warehouse at any time is dependent on the $\sum n_i(t)$. Thus, if the amount of one product to be stored at any time is increasing while another is decreasing at the same time, the total area required might be approximately the same (Figure 6).

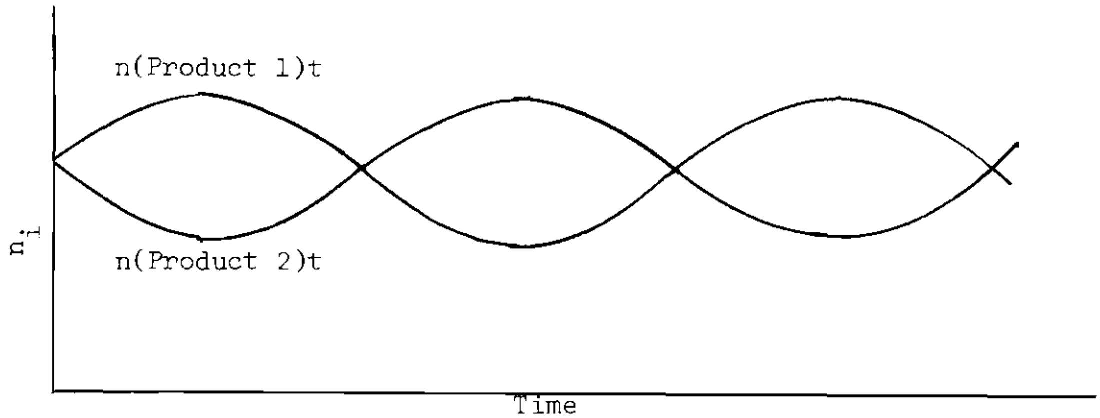

Figure 6. Contraseasonal Products on Hand

If new products are anticipated or more of the same products might be stored in the warehouse at some time in the future, a decision must be made as to how much additional space will be required. If the amount of different product types, n_i, can be considered as a set of independent random variables having mean $E[n_i(t)]$ and variance $V[n_i(t)]$, then the expected value of $A_1(t)$ is

$$\sum_i \frac{(l_i+\theta_2)}{k_i}\left[\frac{w^*}{e_i}+\frac{W_a}{2}\right]E[n_i(t)] \tag{10}$$

and the variance is

$$\sum_i \left[\frac{(l_i+\theta_2)}{k_i}\left[\frac{w^*}{e_i}+\frac{W_a}{2}\right]\right]^2 V[n_i(t)] \tag{11}$$

With this information a band of confidence can be established (Figure 7) for projected storage requirements.

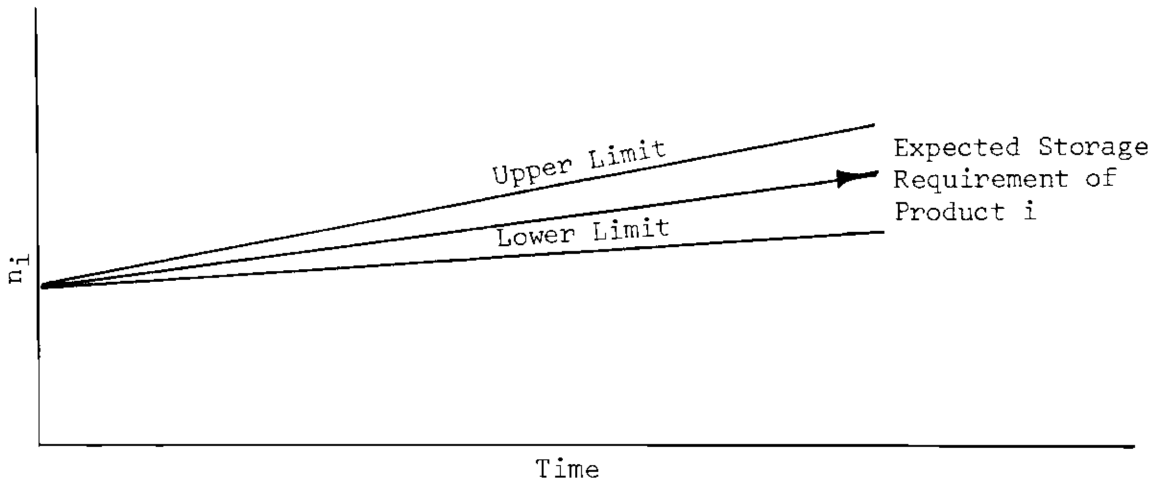

Figure 7. Confidence Limits on n_i

CHAPTER IV

TIME REQUIREMENTS

The next concern is the development of a model to delineate the time required for particular types of handling equipment to complete a cycle of operation. In this formulation it is assumed that either a storage or retrieval will be performed on a single cycle, not both. A cycle of operation consists of selection or deposition of the unit load by the handling equipment, travel to a particular storage location, deposition or selection, and return to a marshalling location. It is assumed that there are as many marshalling locations as there are storage rows, 2j, and that these locations are each the same fixed distance, D_m, from the egress of the storage aisles and opposite each particular row, as shown in Figure 8.

The total time, T_c, to perform a cycle is equal to the sum of the elemental times in performing a selection, a deposition, a travel from and a travel to. Thus, the average cycle time, $\overline{T_c}$, is equal to the sum of the elemental time averages. Therefore, each of the elemental times shall be analyzed, and their averages established. They may be added to give the cycle average.

Simultaneous Operation

Due to differences in operating characteristics of equipment, it will be necessary to make further assumptions. Since an automatically controlled device is going to a programmable or predetermined

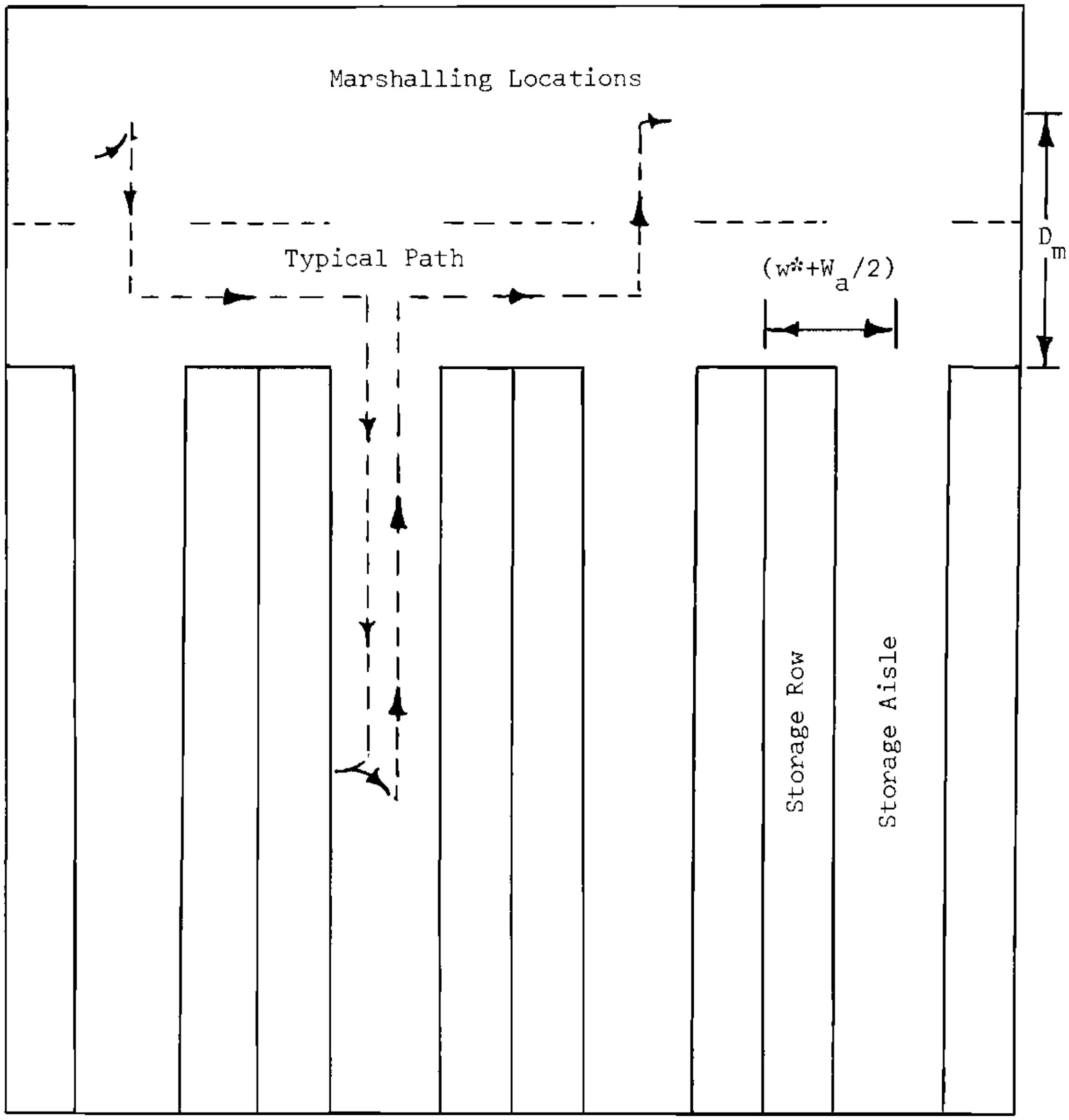

Figure 8. Fixed Path Warehouse Layout and Typical Path

location, it can easily perform simultaneous operations. The operator of a manually controlled device must visually search for the storage location and thus requires a high level of skill to perform simultaneous maneuvers. To illustrate, an ordinary fork truck does not usually position its forks until it has pivoted to pick up or deposit a load. On the other hand, an automatic stacker crane positions its forks as it travels down the aisle. Although a fork truck can perform simultaneous operations, Eaton who has made comprehensive studies of fork truck operations, feel that performing operations separately is more representative of average operator capability.

In analyzing manual material handling operations, it will be assumed that the elements of the cycle of operation are being performed in a non-simultaneous step-by-step sequence. The stacker crane and fork truck will be assumed to use a step-by-step sequence, and the overhead crane will be using simultaneous operations in moving to and from storage locations.

Empirically-Determined Operation Times

All travel time is essentially a function of the distance involved and the rate at which equipment travels. Other factors, such as turns negotiated, stopping and starting, and inclines are elements whose times must be calculated by observation. But since these maneuvers are relatively independent of the particular warehouse, general times may be derived. The aisle will be assumed to be wide enough to permit normal and constant speeds.

It is obvious that it is rather difficult to observe elements in a proposed warehouse. However, the elements are largely independent of the layout and can be obtained from similar operations in existing warehouses.

Average Time for Selection and Deposition

Selection and deposition times are primarily a function of equipment. For instance, most fork trucks must turn perpendicular to the storage aisle in order to lift the unit load. Stacker cranes pivot on their trolley to position their forks, and overhead cranes generally grasp their load from above and lift it above other stacks to transport it. Each of the different types of equipment requires separate evaluation in selection and deposition. Since the time of these phases is highly dependent on the skill of the operator, empirical data will be needed.

Height of Life

One exception in selection and deposition time which may be evaluated on a basis of distance involved is the height of the lift to be made. The overhead crane's sequence of operation is reversed from that of the fork truck and stacker crane in that it lowers, selects, and lifts while the others lift, select, and lower. This makes no difference in the storage area if the overhead crane can pick the unit from the top.

But in the marshalling area where it is assumed that unit loads are only one high, it is necessary for the overhead crane to lower its load from a height which cleared all the storage stacks to the floor, while other equipment types simply deposit theirs.

If it is assumed that rates of lifting and lowering with a load or without are the same and are constant, the time to raise and lower, T_1, is equal to twice the height of the lift, D_1, times the rate of lifting and lowering, G_1. That is,

$$T_1 = 2(D_1)(G_1). \tag{12}$$

The average time is

$$\bar{T}_1 = 2(\bar{D}_1)(G_1), \tag{13}$$

with

$$\bar{D}_1 = \frac{\sum_i (h_i+p)(k_i-1)\alpha_i(t)}{2\sum_i \alpha_i(t)} \tag{14}$$

If h_i is the same for all unit loads, it follows that k_i will be the same for all unit loads and thus,

$$\bar{D}_1 = \frac{(h+p)(k-1)}{2} \tag{15}$$

In the marshalling area it is assumed that the units load will be placed at a height corresponding to the carrying height, which is effectively zero. However, the overhead crane will have to lower its load to ground level. In an actual case it might be advantageous to have the marshalling area somewhat elevated to minimize this aspect of crane handling time and to permit use of gravity feeds. The model for the overhead crane, however, will be structured to use ground level. Therefore, the time involved, T_2, is equal to:

$$T_2 = 2(H^* - \theta_1)G_1 \quad \text{(for overhead cranes only)} \tag{16}$$

Maneuvering and Selection

It is assumed that maneuvering and selection take the same time regardless of location. The time to maneuver equipment into position and back out, T_3, and the time to pick or grasp the load, or deposit same, T_4, are functions of equipment and possibly operator skill, and must be determined empirically.

Therefore, total selection and deposition time, T_5, for stacker cranes and fork trucks on one cycle is equal to:

$$T_5 = 2(T_3+T_4) + 2G_1D_1 \quad \text{(stacker cranes and fork trucks only)} \qquad (17)$$

For overhead cranes, T_5 is equal to:

$$T_5 = 2T_4 + 2G_1D_1 + 2(H^*-\theta_1)G_1 \quad \text{(overhead cranes only)} \qquad (18)$$

or,

$$T_5 = 2[T_4 + G_1(D_1+H^*-\theta_1)] \quad \text{(overhead cranes only).} \qquad (19)$$

Average Travel Time for Fixed Path Equipment

Since the rate of travel for any piece of equipment is given, the travel time can be defined by defining the distances traveled. Determination of the average distance, broken down into elemental averages, will determine the average time. The total distance to be traveled on any one cycle is equal to the distance from a particular marshalling location to a particular storage location and back to a possibly different marshalling location. This may be described in more detail by considering the layout used (Figure 8). For fixed path equipment, the total distance in a cycle is equal to the sum of the distance from the marshalling area to the main aisle, D_m, the distance along the main aisle, D_a, the distance down a storage aisle, D_s, and these distances back to a possibly different marshalling location. D_m is defined as constant due to the characteristics of the marshalling area, so it will be the same going and coming. Obviously, D_s will also be the same going and coming on any particular cycle. D_{ai} is the outgoing distance along the main aisle. D_{a2} is defined as incoming. See Figure 9 for illustration of a typical path. The cycle distance, D_o, is equal to:

$$D_o = 2D_m + 2D_s + D_{a1} + D_{a2} \quad \text{(fixed path equipment only)} \qquad (20)$$

<u>Average Storage Aisle Distance Traveled</u>

The average distance down the storage aisle, $\bar{D}_s$, is the length of the aisle divided by two:

$$\bar{D}_s = \sum_i \frac{(l_i+\theta_2)\alpha_i(t)}{4e_i j} \quad \text{(fixed path equipment only)} \qquad (21)$$

<u>Average Main Aisle Distance Traveled</u>

An expression for the average value of D_a requires a more complex development. An informal procedure will be used. There are j aisles and marshalling locations and they are $(2w^*+W_a)$ apart (Figure 8).

Let b equal $(2w^*+W_a)$. The distance from aisle 1, the outside right aisle, to the other aisles is b, 2b, 3b, ..., (j-1)b, respectively. Since trips down any aisle are assumed equally likely, over a large number of cycles it would be expected that the same number of trips would be made down each aisle and an average value can be developed by considering all the possible combinations, adding the distances of these combinations together, and dividing by the total number of combinations. Starting from aisle 1, the total distance would be:

$$o + b + 2b + 3b \ldots + (j-1)b = (1/2)j(j-1)b. \tag{22}$$

Similarly, from aisle 2, the total distance would be:

$$b + 0 + b + 2b \ldots + (j-2)b = b + (1/2)(j-1)(j-2)b, \tag{23}$$

and from aisle 3:

$$2b + b + 0 + b + 2b \ldots + (j-3)b = 3b + (1/2)(j-2)(j-3)b, \tag{24}$$

and from aisle y, with y = 1,2,3,...,j:

$$(y-1)b \ldots + b + 0 + b + \ldots + (j-y)b = \tag{25}$$

$$\tfrac{1}{2}\{y(y-1) + (j-y)(j-y+1)\}b.$$

Summing this over all aisles, the expression is:

$$\sum_{y=1}^{j} \frac{b}{2} [y(y-1)+(j-y)(j-y+1)],$$

and this expression reduces to:

$$\frac{b}{2} \sum_{y}^{j} (y^2-y+j^2-yj+j-yj+y^2-y) = \frac{b}{2} \sum_{y}^{j} (2y^2-2y-2yj+j^2+j) \quad (26)$$

$$= \frac{b}{2} [2\sum_{y}^{j} y^2-2\sum_{y}^{j} j-2\sum_{y}^{j} yj+\sum_{y}^{j} j^2+\sum_{y}^{j} j]$$

$$= \frac{b}{2} \left[\left[\frac{(j+1)j(2j+1)}{3}\right] -j(j+1)-j^2(j+1)+j^3+j^2\right]$$

$$= \frac{b}{6} (2j^3+j^2+2j^2+j-3j^2-3j)$$

$$= \frac{b}{6} (2j^3-2j)$$

$$= \frac{j(j^2-1)b}{3}$$

Since there were j^2 combinations, the average for D_{a1} or D_{a2} is:

$$\bar{D}_{a1} = \bar{D}_{a2} = [(j^2-1)b/3j] = (j^2-1)(2w^*+W_a)/3j. \quad \text{(fixed path equipment only)} \quad (27)$$

Time for Turning, Stopping and Starting

The time for a turn, T_6, and stopping and starting time, T_7, are functions of the equipment and operator and must be determined empirically. The equipment's speed along the main aisle or laterally is G_a, the speed down the storage aisle is G_s. It is assumed that the direction of a turn makes no significant difference in time.

The number of turns is equal to C. The expected number of turns on a cycle is equal to the number of turns on all possible trips times the expectation that that number of turns will be made. If half the cycle is considered, either incoming or outgoing, there are two possibilities. There will be no turns if the marshalling location is on the same aisle and there will be two turns if the marshalling location is on a different aisle. The probability that the two are the same is 1/j and the probability that they are different is (j-1)j. The expected number of turns is therefore equal to:

$$0(1/j)+2(j-1)/j=2(j-1)/j \qquad (28)$$

Since this describes half the cycle and the two parts of the cycle are independent, the expected number of turns for the total cycle, $\bar{C}$, is:

$$\bar{C} = 4\,\frac{(j-1)}{j} \qquad (29)$$

Summary for Fixed Path Equipment

Total travel time, T_8, is equal to:

$$T_8 = (2D_m+2D_s+D_{a1}+D_{a2})G_a + CT_6 + 2T_7 \text{ (for fixed path equipment only)} \tag{30}$$

For fixed path equipment, the total cycle time, T_c, is equal to:

$$T_c = 2(T_3+T_4) + 2G_1D_1 + (2D_m+2D_s+D_{a1}+D_{a2})G_a + CT_6 + 2T_7 \tag{31}$$

For fixed path equipment, the average cycle time, $\bar{T}_c$, is equal to:

$$\bar{T}_c = 2(\bar{T}_3+\bar{T}_4) + 2G_1\bar{D}_1 + 2(\bar{D}_m+\bar{D}_s+\bar{D}_{a1})G_a + 4(j-1/j)T_6 + 2T_7 \tag{32}$$

where $\bar{D}_1$ is given by Equation 14, $\bar{D}_s$ is given by Equation 21, and, $\bar{D}_{a1} = \bar{D}_{a2}$ is given by Equation 27.

Average Time for Automatic Overhead Crane

Determination of average times for the automatic overhead crane is similar to the fixed path analysis, except for the consideration of simultaneous operation. Only the areas which differ will be analyzed. The crane can move in two directions simultaneously (see Figure 9). It can move up or down the bay at rate G_s while moving across the bay at rate G_a. The travel time up or down the bay, T_s, is equal to the distance involved, D_s, divided by the rate G_s. Similarly, the travel

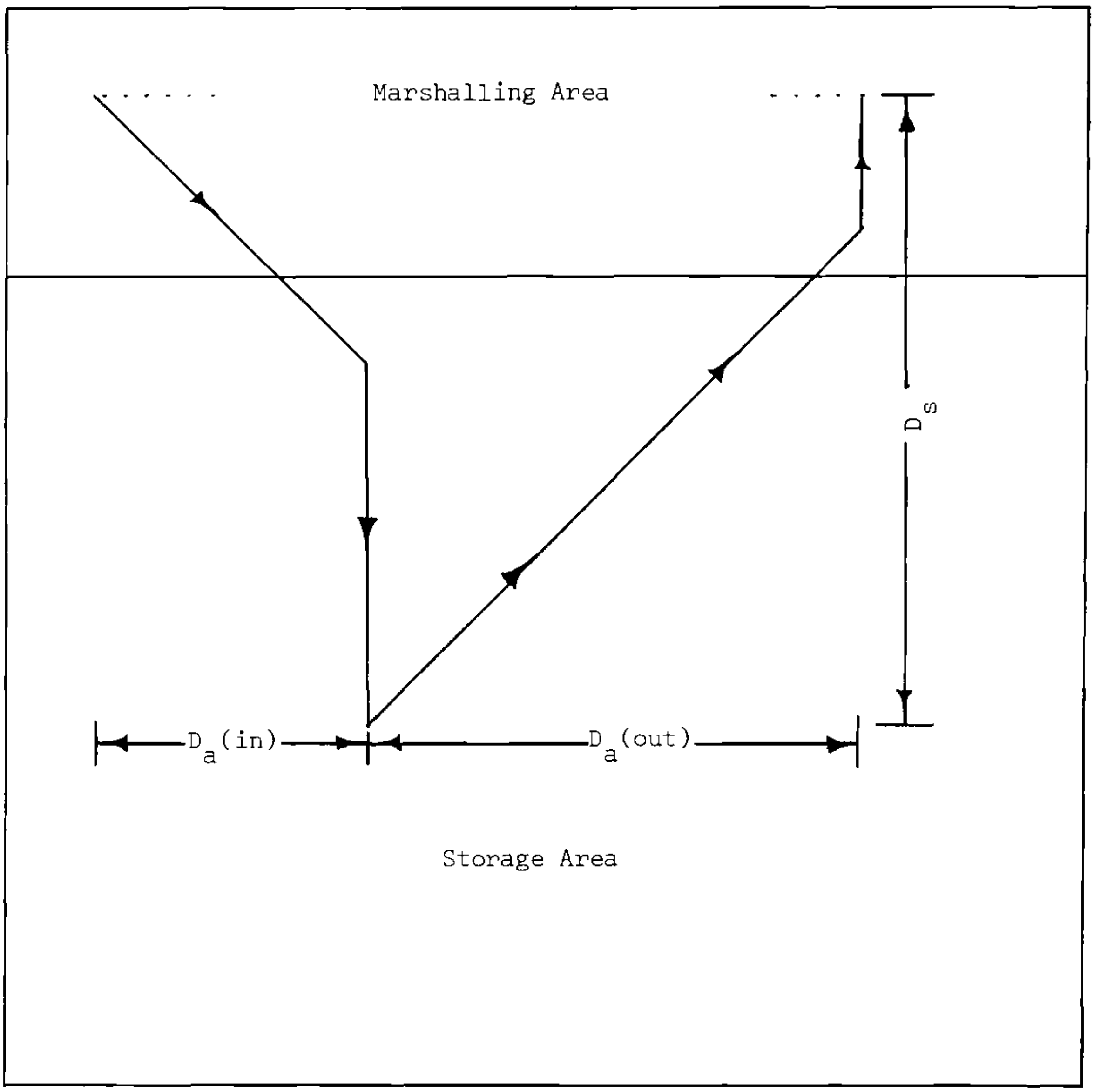

Figure 9. Illustration of Path of Overhead Crane

time across, T_a, is equal to the distance across D_a, divided by G_a. The total time T_9, to travel to or from any marshalling location y, to a storage location is equal to the maximum of the two:

$$T_9 = \max(T_a, T_s) \tag{33}$$

It is assumed that $(l_i+\theta_2)$ is the same for each storage location throughout the warehouse, and this distance, x_s, is obtained by calculating the average value of $(l_i+\theta_2)$. Let x_a equal $(w^*+W_a/2)$, and the warehouse contains a number of locations of length x_s and width x_a.

Let x_{yz} identify a particular storage location in the warehouse with $y = 1,2,3,\ldots,2j$, and $z = 1,2,3,\ldots,\frac{\alpha_i(t)}{2j}$.

The term y also designates the marshalling location. Let y_b and y_f designate the starting and finishing rows, on a trip to or from a storage location.

Then the time to travel to or from a storage location is:

$$T_9 = \max\left[\frac{(y_b-y_f)x_a}{G_a} ; \quad \frac{zx_s + D_m}{G_s}\right] \tag{34}$$

In order to determine the expected time for T_9, $\bar{T}_9$, it is necessary to compute T_a and T_s for all possibilities, select the greater in each instance, sum the maxima together, and divide by the number of possibilities, which is

$$4j^2 \frac{(\alpha_i(t))}{2j}, \quad \text{or} \quad 2j\alpha_i(t).$$

Summary for the Overhead Crane Cycle

For the overhead crane total cycle time T_c is equal to:

$$T_c = T_5 + 2T_9, \tag{35}$$

or

$$T_c = 2T_4 + (D_1+H-\theta)G_1 + 2T_9 + 2T_7 \tag{36}$$

The average total cycle time, $\bar{T}_c$, is equal to:

$$\bar{T}_c = 2\bar{T}_4 + (\bar{D}_1+H-\theta_1)G_1 + 2\bar{T}_9 + 2\bar{T}_7, \tag{37}$$

where $\bar{D}_1$ is given by Equation 14 and $\bar{T}_9$ would be established as described.

Multiple Equipment in Warehouse

All discussion thus far has ignored the possibility of more than one piece of machinery in operation simultaneously in the warehouse. The models developed are valid as long as each piece operates in areas restricted to other pieces. In a practical sense, this would be the only way to operate a stacker crane system--conceivable using one crane per aisle in a high volume operation. Any overhead crane would be severely restricted in operating with another crane in the same bay unless each operated in one half of the bay. The fork truck, least sophisticated and therefore most flexible, suffers from no such problems. If the aisles are of sufficient width, they could pass at will. (It might be possible to operate multiple trucks in a warehouse with only a slight increase in operating time.) If aisles do not permit passing a dispatching system must be developed to prevent trucks from going down the same aisle.

Since differences in operating time averages due to interference are directly dependent on operating methods, such as dispatching, this effect must be evaluated empirically.

Estimations of Variance of Cycle Time

The variance of a set of numbers is equal to the sum of the squares of the differences of the numbers from their mean divided by the size of the set, if each number is equally likely to occur. Therefore, it is possible to give expressions for variances of all time elements for which expected times were computed. Since it is assumed that the time elements are independent, the different elements are additive.

Fixed Path Equipment Variance Estimation

Assuming that speeds are constant permits relating the variances of the time elements to the distances. Accordingly, σ_1^2, is the distance variance in lifting and lowering, σ_a^2 is the distance variance in traveling down the main aisle, and σ_s^2 is the distance variance in traveling down the storage aisle. There is no variance in picking or depositing a load in the marshalling area or in traveling from the main aisle to the marshalling area. Maneuvering the equipment into position and backing it out again has a variance of σ_3^2. Time for picking or depositing the load has a variance of σ_4^2. The variance for turning time is σ_6^2 and for stopping and starting is σ_7^2.

The total variance, σ_8^2, for a complete cycle for fixed path equipment is equal to:

$$\sigma_8^2 = 4G_1^2\sigma_1^2 + 2G_a^2(\sigma_a^2+\sigma_s^2) + 2(\sigma_3^2+\sigma_4^2+\sigma_7^2) + 4(1-1/j)\sigma_6^2 \qquad (38)$$

Overhead Crane Variance Estimation

For overhead cranes, lifting and lowering distance variance is the same as fixed path equipment. The maneuvering time variance is zero. Picking or depositing variance is σ_4^2. Travel time variance is σ_y^2. Stopping and starting time variance is σ_7^2.

The total variance for overhead crane travel time, σ_8^2, is equal to:

$$\sigma_8^2 = 4G_1^2\sigma_1^2 + 2(\sigma_4^2+\sigma_y^2+\sigma_1^2) \qquad (39)$$

Uses of Estimates of Mean and Variance in Cycle Time

If the probability distribution of cycle time is known, then queueing theory or simulation method may be used to compute certain methods of warehouse system effectiveness. For example, it may seem reasonable to assume service (cycle) times are M-Erlang. Then the following can be developed: knowing the mean and variance of the cycle time, the parameter M (for an M-Erlang distribution) can be determined as follows:

$$M = \frac{[\bar{T}_c]^2}{\sigma_8^2} \tag{40}$$

But, in itself, the distribution of cycle times is inadequate, for no account has been taken of demand. It is demand, the rate of arrivals into the system, together with cycle time, that determines waiting time. The sum of waiting time and cycle time establishes the time in the system.

Assuming Poisson arrivals of units with aggregate mean λ and an Erlang distribution of service times, the queue properties can be specified. For a single servicing station:

The average queue time = (Time spent waiting for service)

$$\left(\frac{M+1}{2M}\right) \frac{\lambda \bar{T}_c}{\left[\left(\frac{1}{\bar{T}_c}\right) - \lambda\right]} \tag{41}$$

The average time in the system = (Time in the queue plus time being serviced)

$$\left(\frac{M+1}{2M}\right) \frac{\lambda \bar{T}_c}{\left(\frac{1}{\bar{T}_c}\right) - \lambda} + \bar{T}_c \tag{42}$$

Average queue length =

$$\left(\frac{M+1}{2M}\right) \frac{\lambda^2 \bar{T}_c}{\left(\frac{1}{\bar{T}_c}\right) - \lambda} \tag{43}$$

$$\text{Average number of units in the system} = \frac{M+1}{2M} \; \frac{\lambda^2 \bar{T}_c}{\left(\frac{1}{\bar{T}_c}\right) - \lambda} + \lambda \bar{T}_c \tag{44}$$

CHAPTER V

ILLUSTRATIVE EXAMPLE

This example is structured to indicate a path which a practitioner could use to compare different types of material handling systems in a warehouse. It assumes a certain amount of expertise on the part of the practitioner and the availability of needed factors.

A manufacturer makes four different items, which are packed into boxes and assembled on three different size pallets; 40 x 40, 48 x 40 and 48 x 48 (dimensions in inches unless noted).

The equations can be used for many different types of pallets although in an actual situation items would probably be assembled on standard size pallets with height the only variable. Even this would not be variable if racks were used. If racks are not used, there must be at least two stacks for product in order to permit cycling merchandise.

There are three different heights of the unit loads, (h_i+p): 48, 64, and 72. At peak inventory the unit distribution has been established to be:

40 x 40 x 48 - 2000 units (n_1)

48 x 40 x 64 - 900 units (n_2)

48 x 40 x 72 - 1000 units (n_3)

48 x 48 x 72 - 2500 units (n_4)

Clear Height Determination

In order to determine clear height in the building, the following relationships must be considered:

$$H_1 = (h_1+p)k_1 \text{ with } (h_1+p) = 4'$$

$$H_2 = (h_2+p)k_2 \text{ with } (h_2+p) = 5\ 1/3'$$

$$H_3 = H_4 = (h_3+p)k_3 \text{ with } (h_3+p) = 6'$$

The lowest common multiples for H_1 and H_3 are 24' and 48'. This makes $k_1 = 6$ or 12 and $k_3 = k_4 = 4$ or 8 and k_2 must be 4 or 9.

The following possibilities are feasible for H^*: $H^* = 24$, $24 + \theta$, 48 or $48 + \theta$. Since the stacker crane under consideration is Type 2, and heights over 40' are impractical for this type of overhead crane system, $(48+\theta)$ is eliminated. Judgement determines that the overhead crane system (0) will use $H^* = 24 + \theta$; the stacker crane system(s) will use $H^* = 48$; and the fork truck system (F) will use $H^* = 24$. No other possibilities will be considered.

No product design changes are anticipated in the future which will effect the dimensionality of the units.

Floor Area Determination

The number of stacks required at time t is as follows (assuming no first in-first out requirement)

$$\alpha_1(t) = 334 \text{ or } 167$$

$$\alpha_2(t) = 225 \text{ or } 100$$

$$\alpha_3(t) = 250 \text{ or } 125$$

$$\alpha_4(t) = 625 \text{ or } 312$$

Since the unit loads are rectangular s_i and e_i are equal to 1. $w^* = 4'$. Three-inch clearance is necessary for fork trucks and stacker cranes; 12 inches for overhead crane. The fork truck under consideration requires 8-foot aisles, the stacker crane required 6-foot aisles, and the overhead crane requires 10 feet of clearance in the overhead.

The area required for the fork truck system (F) with a 24-foot clear height is:

$$A_1(t) = \sum_i (l_i + .25)\alpha_i(t)(4 + 8/2)$$

$$A_1(t) = [(3.33+.25)334 + (225+250+625)(4.00+.25)]8$$

$$A_1(t) = 46{,}968 \text{ square feet (F)}$$

The area required for the stacker crane with 48 feet of clear height is:

$$A_1(t) = \sum_i (l_i + .25)\alpha_i(t)(4 + 6/2)$$

$$A_1(t) = [(3.33+.25)167 + (100+125+312)(4.00+.25)]7$$

$$A_1(t) = 20{,}090 \text{ square feet (S)}$$

The area required for the overhead crane system with a clear height of 34 feet is:

$$A_1(t) = \sum_i (l_i+1)\alpha_i(t)(4+1)$$

$$A_1(t) = [(3.33+1)334 + (225+250+625)(4+1)]5$$

$$A_1(t) = 33{,}225 \text{ square feet (O)}$$

The cubic footage required is 1,127,232 for the fork truck system, 964,320 for the stacker crane system, and 1,129,650 for the overhead crane system.

Based on the expertise of the practitioner, it is decided to add a 20 per cent allowance for future expansion, so square footage is 56,000 for the fork truck, 24,000 for the stacker crane, and 40,000 for the overhead crane.

Establishing Dimensions

The length and width of the building can be determined at this point. Width of the building is equal to $(2W^*+W_a)j$, so

$$(2W^{*}+W_a)j(\text{length}) = \text{square footage.}$$

16j (length) = 56,000 (F), so j can be 14, and the length is 250 ft., the width is 224 ft.

14j (length) = 24,000 (S), so j can be 10 and the length is 171 ft., the width is 140 ft.

10j (length) = 40,000 (0), so j can be 10 and the length is 400 ft., the width is 100 ft.

An additional 20 ft. will be added to the length of each building for the marshalling area.

Time Estimations

The following empirical times and rates have been obtained for the fork truck:

$$G_1 = .030 \text{ min./ft.}$$

$$T_3 + T_4 = .300 \text{ min.}$$

$$T_6 = .055 \text{ min.}$$

$$T_7 = .020 \text{ min.}$$

$$G_a = .0024 \text{ min./ft.}$$

So, for fork trucks:

$$\bar{T}_c = .640 + .06\bar{D}_1 + .0024(40+2\bar{D}_s+2\bar{D}_a) + .055\bar{C}$$

The following items have been calculated:

$$\bar{D}_1 = 9.07 \text{ ft.}$$

$$\bar{D}_s = 125 \text{ ft.}$$

$$\bar{D}_a = 74.3 \text{ ft.}$$

So, the expected cycle time for the fork truck system is:

$$\bar{T}_c = .640 + .06(9.07) + .0024(40+250+149) + (.055)4(1 - 1/14)$$

$$\bar{T}_c = 2.44 \text{ min./cycle (F)}$$

The following empirical times and rates have been obtained for the stacker crane:

$$G_1 = .015 \text{ min./ft.}$$

$$T_3 + T_4 = .150 \text{ min.}$$

$$T_6 = .060 \text{ min.}$$

$$T_7 = .020 \text{ min.}$$

$$G_a = .0020 \text{ min./ft.}$$

$$D_m = 20 \text{ ft.}$$

So, $\bar{T}_c$ for stacker cranes is:

$$\bar{T}_c = .290 + .03\overline{D_1} + .002(2D_m+2\overline{D_s}+2\overline{D_a}) + .10\bar{C}$$

The following items are calculated:

$$\overline{D_1} = 21.4 \text{ ft.}$$

$$\overline{D_s} = 86 \text{ ft.}$$

$$\overline{D_a} = 46.2 \text{ ft.}$$

The expected cycle time for the stacker crane system is:

$$\overline{T_c} = .290 + .03(21.4) + (40+172+92).002 + .40(1 - 1/10)$$

$$\overline{T_c} = 1.89 \text{ min/cycle (S)}$$

The following empirical times and rates have been obtained for the overhead crane:

$$G_a = .0025 \text{ min/ft.}$$

$$G_s = .0020 \text{ min/ft.}$$

$$G_1 = .025 \text{ min/ft.}$$

$$T_4 = .300 \text{ min.}$$

$$T_7 = .020 \text{ min.}$$

So $\overline{T_6}$ for overhead cranes is:

$$\overline{T_c} = .16 + (\overline{D_1}+24).050 + 2Exp(T_y)$$

The following data can be calculated:

$$\overline{D_1} = 9.07 \text{ ft.}$$

$$\overline{T_y} = \max(T_a,T_s) = \max(.0025D_a,.0020(D_s+D_m))$$

If the operation was sequential, $\overline{D_a}$ = 33 ft. and $(\overline{D_s}+D_m)$ = 220 ft. would establish $\overline{T_y} = \overline{T_a} + \overline{T_s}$ = (.082+.44) = .52 min. Since it is not sequential, the expected time for T_y must be between .44 and .52. This means that an approximate time can be obtained by averaging these two extremes:

$$\overline{T_c} = .44 + 1.65 + (.44 \text{ to } .52) + .04,$$

so

$$\overline{T_c} \doteq 2.77 \text{ min. } (0)$$

It is interesting to note that the overhead crane spends most of its time in lifting and lowering. If it was possible to place an elevated platform in the marshalling area so the crane need only deposit the load, $\overline{T_c}$ would be reduced to 1.57 (0).

This highlights the advantage of sequential operation. So reconsidering the stacker crane and considering it capable of lifting and lowering while traveling,

$$\overline{T_c} = .700 + \max(G_1, \overline{D_1}, 2(D_m + D_a + D_s)G_a) \text{ (S)}$$

$$\overline{T_c} = 1.29 \text{ to } 1.89 \doteq 1.59 \text{ (S)}$$

Now three possible alternatives have been evaluated. Others can be dealt with just as simply. From this example, it can be seen that the stacker crane possesses a significant advantage in its ability to stack to much greater heights than the fork truck or the overhead crane. The former is limited by the visual position of the operator and the need for counterbalancing. Counterbalancing also limits the potential benefit in building fork trucks which can lift higher. This requires a larger counterweight and therefore would need wider aisles to maneuver adequately. The wider aisles offset the benefit of the higher lift.

The overhead crane's limitation in stacking height is directly dependent on the strength and weight of the products it would be handling. Its major benefit, traveling from point to point without the use of aisles, seems more amenable to a manufacturing rather than a storage environment.

In a warehouse of any size, span width, which is equal to the width of the building $(2_w + W_a)j$, becomes a limiting factor. An overhead crane warehouse necessarily becomes more tunnel like as the storage requirement grows.

Serious cost tradeoffs would probably be made between the stacker crane and the fork truck. The stacker crane costs more than a fork truck and it requires more expensive support to recognize its potential. Racks are almost mandatory, and overhead support certainly is. These costs would probably be more than compensated for by reduced square footage requirements.

In this example, the stacker crane is a winner. It can operate faster with less total investment. In reality, this situation could easily be altered, not only by special circumstances, but by using more than one fork truck to compare against the stacker crane. If interference among trucks is minimal, such that the expected cycle time for fork trucks does not become significantly higher, then it becomes necessary to consider demand and evaluate the queues for both systems in order to establish the expected time for each.

CHAPTER VI

CONCLUSIONS

It is felt that a properly done feasibility study for proposed material handling systems use these models or simplifications of them, since they represent the only formal structuring this problem has received. The study could be for a complete new warehouse, a new handling system, an expansion of existing facilities, or an analysis of an existing system.

These equations provide means for a systematic approach to analyze a material handling system once inventories have been defined. They give the practitioner tools needed to depict alternatives and evaluate effects.

Since the models were developed on a theoretical basis, there is an obvious need for validation before the results can be used with confidence. Despite this weakness, the models are still useful in comparative studies; and it is in evaluating possible configurations that they are most valuable.

Substantial elements of the times are dependent upon empirical data. The only empirical data readily available is the 1954 fork truck study by Yale & Towne (now Eaton, Yale & Towne), excepting manufacturers' specifications in speeds and load capacity. The equations do not provide all the answers: the practitioner must have a certain amount of expertise to obtain plausible results.

The time models are flexible since the elemental averages are separable. Alternate configurations in layout and equipment can be simply resolved. For instance, if a belt onveyor was added to a fork truck system with the belt running the length of the main aisle and into the marshalling area, its effect would be to shorten the effective cycle by the time it would take the fork truck to run into and out of the marshalling area ($2D_mG_a$) and the time used in turning up and down the main aisle (CT_6). The elemental averages are extremely useful in analyzing which part of the operation is taking the most significant portion of time.

Complex configurations can be analyzed by evaluating segments of the warehouse as complete studies. The segments can be combined by multiplying them by their percentage of activity and adding. If storage is completely random, the percentage of activity is solely dependent on the ratio of units in each segment. If not, it also depends on the relative popularity of each.

Each system and subsystem must be evaluated separately and completely to develop comparisons. There is no preliminary point at which the models point out inferior systems. It is, of course, possible for external factors to reduce the number of choices.

Some of the equations developed are extremely cumbersome and were ignored in the example. Although it is feasible to program these models for a computer, it is not felt to be practicable at this time, since there are many areas where an unprogrammable decision must be made.

The equations do not get beyond the marshalling area. Although this was deliberate to avoid decisions concerning delivery systems which should be more specifically tailored to the type of operation, these factors must be considered in designing the total system.

No variations or effects are considered significant from weight or dimensionality.

Multiple handling devices are not considered, since interactive effects are difficult to evaluate. If interaction can be eliminated by scheduling or zoning, the system can be evaluated as before.

Recommendations

There are several areas related to this study which would expand the usefulness of the tools it provides. Foremost is the need for empirical data in maneuvering operations with stacker and overhead cranes. Gathering these data would also permit validation of the models.

The effects of interaction of equipment should be studied so multiple handling systems can be evaluated. It seems more logical to compare two or three fork trucks against one stacker crane.

Decision rules should be developed for judgemental decisions, such as how high to stack, so complete computer programs can be written.

Cost comparisons of the systems should be developed. Although there is little relationship between cost of building and cubic footage or usable height, typical building costs for each system could be established. The cost of power, maintenance and labor must be considered.

Marshalling areas should be studied and models developed in order to obtain the complete model for a warehouse.

GLOSSARY OF TERMS

A line over a term represents the average of that term.

$A_1(t)$	Total area required
A_2	Utilized area
a_i	Cross sectional area of a unit of product i
b	$2w^* + W_a$
c	Number of turns
D_a	Distance along main aisle
D_{a1}	D_a outgoing
D_{a2}	D_a incoming
D_m	Distance from main aisle to marshalling area
D_o	Cycle distance
D_s	Distance from main aisle to storage location along storage aisle
D_l	Height of lift
e_i	Nesting factor of product i
$E[n_i(t)]$	Expected value of n_i in t
(F)	A designator for fork truck systems
G_a	Reciprocal of speed along main aisle
G_s	Reciprocal of speed down storage aisle
G_l	Reciprocal of speed in lifting and lowering
h_i	Height of a unit of product i
H_i	$(h_i+p)k_i + \theta$

H^*	$\max(H_i)$
i	Designator for type of product
j	Number of aisles
l_i	The effective length of a unit of product i
k_i	Number of units in a stack of product i
M	Number designating the member of the Erlang family
$n_i(t)$	Number of units of the i types of product on hand at time t
(O)	Designator for overhead crane system
p	Pallet or rack allowance
Q_1	Total cube required
Q_2	Utilized cube
r	Number of rows
(S)	Designator for stacker crane system
S_i	Shape factor
t	Time
T_a	Travel time across bay (O)
T_c	Total cycle time
T_s	Time up or down bay (O)
T_y	$\max(T_a, T_s)$
T_o	A point in time at which increasing demand rate equals input
T_1	Time to raise and lower
T_2	$2(H^*-\theta_1)G_1$ (O)
T_3	Time to maneuver into position and back out
T_4	Time to pick or grasp
T_5	Total selection and deposition time

T_6	Time for a turn
T_7	Time to stop and start
T_8	Total travel time
$V(n_i(t))$	Variance of n_i over t
W_a	Width of aisles (storage and main)
W_i	Width of unit of product i
W^*	$\max_i (W_i)$
W^*_r	$\max_i (W_i \text{ in row } r)$
x	Designation of starting row
y	Designation of finishing row
$\alpha_i(t)$	Smallest integer larger than $n_i(t)/k_i$
λ	Mean of poisson arrival rate
$\delta_i(\tau)$	Warehouse demand rate
$\epsilon_i(\tau)$	Warehouse input rate
θ_2	Clearance factor in overhead
θ_3	$W_a/2$-Aisle clearance for overhead cranes
σ_9^2	Time variance in traveling main aisle
σ_s^2	Time variance in traveling storage aisle
σ_y^2	Time variance in traveling in bay (0)
σ_1^2	Time variance in lifting and lowering
σ_3^2	Time variance in maneuvering into position
σ_4^2	Time variance in picking or depositing
σ_6^2	Time variance in turning

σ_6^2 Time variance in turning

σ_7^2 Time variance in stopping and starting

σ_3^2 Time variance in total cycle

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