A MULTIVARIABLE SCREENING PROCEDURE ADAPTABLE TO ELECTRONIC COMPUTERS FOR THE EMPIRICAL EXPLORATION OF RESPONSE SURFACES

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PREFACE

The subjective nature of the initial synthesis phase in industrial experimentation has drawn the attention of many writers. In particular the indeterminable most often mentioned in connection with this speculative stage of experimentation is the possible omission of an important variable.

A method of analysis applicable to observational data, not suitable for rigorous statistical analysis, is developed utilizing a Burroughs 220 electronic computer. The method used is essentially the classification of data points by setting class limits on each observed variable and thereby creating levels of a factor. These factors are treated in pairs as in factorial design and the error sum of squares is compared for each pair. The relative magnitudes of the error sum of squares for each pair provide indications of the relative goodness of fit for each pair and thereby assist the investigator in a preliminary screening of factors with which he need not be concerned.

Through this after-the-fact stratification of observational data and through the use of an electronic computer to perform the myriad of calculations, the candidate variables are ranked according to the variation in the response which is removed when the effects of each factor are removed.

The ability to consider up to thirty candidate factors reduces the risk of overlooking an important variable. Hence the latter stages of the experiment are less susceptible to the invitiating omission of an important variable.

In addition, the organization and display in tabular form of the estimates of the mean and variance for each factor-level combination of those factor combinations having a relatively small error sum of squares, provide the experimenter with an estimate of the general contour of the response surface over the observed range of paired factors.

As a result, the experimenter obtains an appreciation for the nature of the response surface. The risk of failing to use appropriate transformations of the candidate variables in subsequent experiments is reduced.

Consideration of each possible three-factor classification of the data by this method is adjudged to be practical only when the number of factors is small, say $n < 10$, or otherwise only if a means is provided for eliminating certain of the less interesting factors prior to performing the calculations associated with all possible three-factor combinations.

The method developed permits the experimenter to lay the data open so as to be able, as Tukey expressed the need, "...to see what they look like inside, even though they do not give definite significance levels."

It is concluded that for a given commitment of resources to an experimental program, the utilization, under the conditions for which designed, of the procedure herein developed will minimize the risk of failure of the experiment as a whole.

It is emphasized that this method is designed as a complement to, rather than a substitute for, existing methods of analysis.

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CHAPTER I

INTRODUCTION

The objective of this work is to permit, under certain conditions to be defined, a reduction in the subjective nature of the initial synthesis phase in industrial experimentation. The hypothesis associated with this work is that an Industrial Engineer, through use of the procedure developed, will be able to design subsequent experiments that are less susceptible to the indeterminables which are associated with this phase of experimentation. These indeterminables are elaborated upon in Chapter II.

In essence, this hypothesis states that for a given commitment of resources to an experimental program, the utilization of the procedure herein developed will minimize the risk of failure of the "experiment as a whole," as defined by Box (1, p. 27).*

The method proposed is essentially to classify data points by setting class limits on each observed variable and thereby create levels of a factor, then to treat these factors in pairs as in factorial design and to compare the error sum of squares for each pair. The relative magnitudes of the error sum of squares for each pair provide indications of the relative goodness of fit for each pair and thereby assist the investigator in a preliminary screening of factors with which he need not be concerned.

*References in parentheses are to items in the Bibliography.

CHAPTER II

THE QUALITATIVE NATURE OF THE INITIAL PHASE OF INDUSTRIAL EXPERIMENTATION

The nature of scientific investigation consists of two essential processes:

> (a) the devising of experiments suggested by the investigator's appreciation of the situation to date and designed to elucidate it further;
> (b) the examination of results of experiments performed to date in the light of all knowledge available, with the object of postulating theories susceptible of test in future experimentation. (2, p. 318)

The qualitative nature of the initial synthesis phase in problem solving has attracted the attention of Box and his co-workers.

> Most investigations first pass through a "speculative" stage. Here statistical methods can rarely be of help but it is nevertheless vital that this early work should be done fully and with imagination, otherwise later efforts may be wasted in detailed investigation of the wrong basic system. (2, p. 319)

Box and Hunter are more specific.

> It has been remarked that the only time an experiment can be properly designed is after it has been completed. The more one considers this paradoxical statement, the more one realizes that it is true. It is not uncommon to find after a set of experiments have been made ...
> (i) one or more important variables have probably been overlooked;
> (ii) more could have been learned if the factors could have been varied over different ranges;
> (iii) some transformation of the variables would have been more appropriate; and/or
> (iv) some more elaborate pattern of experiments is needed to elucidate the situation. (3, p. 139)

Their next statement is or primary significance to this study. "Since the outcome of a group of experiments depends on all of the items mentioned above, and since no two experimenters studying the same problem are likely to have the same opinion about any one of them, it is quite clear that the type of experimentation we are discussing contains many 'indeterminacies.'"

Box continues, "The ultimate success of an <u>experiment as a whole</u> (in contrast to the statistical exercise) must necessarily depend on the skill of the experimenter." (1, p. 27)

The attention of other writers (4, p. 506; 5, p. 5) has also been drawn to the problems associated with, and to the importance of, this "speculative" or qualitative stage of experimentation. The aspect most often mentioned is the possible omission of an important variable.

Budne notes the general failure to consider the risk of omitting one or more of the correct variables.

> ... risks have been associated with the failure to recognize an effect which exists or with the identification of an effect which does not exist - <u>among the variables studied</u>. The question as to whether the correct variables are being studied in a fact-finding situation raises the risk of <u>real success or failure</u> to large and unmeasurable magnitudes. However, this has generally not been considered as a "statistical risk." (6, p. 19)

Brownlee notes a reservoir of pertinent data, often available during the initial phase in industrial experimentation yet infrequently tapped. "In many production processes records are kept of conditions but often little use is made of these records; they are looked at cursorily and then put in files to gather dust till eventually they are thrown out." (7, p. 2068)

In view of the problem outlined above, the objective of this work, as stated in Chapter I, is to permit a reduction in the qualitative nature of the initial synthesis phase in industrial experimentation.

CHAPTER III

SURVEY OF CURRENT LITERATURE

To achieve a reduction in the qualitative nature of the "speculative" stage of industrial experimentation, a review of the problems associated therewith, as outlined in the current literature, is necessary.

The class of problems here considered, and common to all industrial concerns, is the elucidation of functional relationships connecting a response Y — such as yield, profit, or a measure of product quality — with the levels $x_1, x_2, \ldots, x_k$ of a group of k variables or factors such as temperature, sales volume, or raw material. The relationship may be written

$$Y = \varphi(x_1, x_2, \ldots, x_k).$$

As stated by Plackett and Burman, "A problem which often occurs in ... industrial research is that of determining ... or ascertaining the effect of quantitative or qualitative alterations in the various components upon some measured characteristic of the complete assembly." (8, p. 305)

To understand clearly certain of the difficulties inherent in industrial research on problems of this type, G. E. P. Box suggests a careful delineation between "... the problems in experimentation which are statistical and those which are essentially nonstatistical . . . ." (1, p. 26)

As suggested by Dr. Box, the experimenter (by which is meant the biologist, chemist, or engineer who is conducting the experiments) and the statistician will be spoken of as two individuals. If the experimenter is also the statistician in a particular investigation, the terms will differentiate between the work requiring statistical skill and the work requiring the application of other knowledge.

This study is limited to the situations wherein the experimenter has at his disposal pertinent observational data, as defined by Bryant to be "... the class of data represented by observations on a population or a segment thereof, where there has been no attempt to modify or 'control' any of the possible influencing factors." (9, p. 136) Examples in the industrial community are plant logs, quality test results, data processing records, and cost records which provide a series of determinations on a response variable and similar information concerning previous quantitative or qualitative alterations in one or more possible influencing factors.

Unfortunately, these data are not orthogonal, as defined by Chew (10, p. 16) and therefore there is no assurance of independence of the contrasts when the experimenter tries to disentangle the effects of the variables one from another.

Box reports discouragingly upon the analysis of plant records using multiple regression techniques in an attempt to determine the "effects" of the variables.

> In my experience the results of such investigations are nearly always disappointing. The reasons are not far to seek:
> 1. Many of the factors that may vitally affect the efficiency of the process are not in the normal course of events altered at all.
> 2. Those factors which vary naturally do so, not over the ranges we should like, but over ranges dictated by the degree of control which happens to exist .... .
> 3. The fluctuations that occur naturally in the variables are often heavily correlated .... .
> 4. Accidental modifications often tend to happen in phases and so become spuriously correlated with causal unrelated time trends in response .... . (11, p. 98)

The inapplicability of existing methods of analysis in the preliminary stages of experimentation has drawn the attention of various writers. Box and Youle elaborate as follows on the statement, quoted in Chapter II, to the effect that statistical methods can rarely be of help during the speculative stage of an investigation.

> Statistical methods provide efficient tools for investigating a system whose general nature has been broadly decided. They provide no substitute for basic scientific thinking about what the system to be investigated should be. (2, p. 319)
>
> The statistician's function is to advise the experimenter on the best positioning of experimental points in a space which the experimenter <u>must of necessity construct for him</u> and construct purely on the basis of the experimenter's expert background knowledge of the subject in which he is experimenting. (1, p. 26)

The experimenter must decide during this phase:

1. which factors should be varied,
2. in what way the factors should be varied,
3. by how much the factors should be varied, and
4. the probable nature of the response surface.

The aspect of the experimenter's decision, at this stage, most often mentioned in the literature is the possible omission of an important variable. Because the amount of effort which can be exerted on any given problem is in practice limited, the experimenter must often select a few factors which he believes will be important out of a large number which might be important. "In some investigations, particularly in preliminary work, the number of factors of potential importance may be much larger than the number than can be dealt with." (12, p. 134)

Youden places considerable importance upon this problem and recognizes its nonstatistical nature. "The discriminatory powers of trained investigators to dichotomize factors into those worth investigating and those of distinctly secondary interest constitutes our strongest weapon of research." (13, p. 158)

Satterthwaite emphasizes the lack of quantitative guidance in this decision period.

> ... there are often compelling engineering reasons to include a large number of independent variables (i.e., 10 to 100) in a single experimental program with many of these variables at five to ten levels. Historical statistical principles give almost no guidance for such experimental programs. (14, p. 55)

The basic unifying concept of the experimental designs developed by Dr. Box and his co-workers is that of research as an iterative process. (15, p. 63)

> During a complete investigation these processes of synthesis and analysis used in alternation will normally be employed many times and, by what we may call "experimental iteration," the investigator should be led closer and closer to the truth. (2, p. 319)

Davies uses the term "sequential approach" to describe "The idea of using information from the early parts of a series of observations to design the later work ... ." (4, p. 5)

Davies and Hay point out that the circumstances surrounding industrial experimentation lend themselves to the sequential approach in problem solving, more so than the circumstances encountered in agricultural experimentation.

> Once a field experiment in agriculture has been started it is not usually possible to change or modify the design but in most industrial work a high degree of flexibility exists because the situation may be reviewed after every observation or set of observations come to hand. It is not necessary to adhere strictly to the design drawn up at the outset of an experiment but the design may be modified as the result of information gained from the earlier observations. (16, p. 245)

Typical of the interest during the past decade in the sequential nature of experimentation is the observation of Read, "The key to the whole problem ... [estimation of optimum conditions] ... lies in making full use of the sequential nature of the test procedure, by carrying out experiments in a sequence of small groups ... ." (5, p. 5)

Davies accords a permanent role in overall experimental strategy to the sequential approach.

> In addition to its use in sequential experiments for simple comparative trials, the sequential approach can also be employed in a less formal way in the general strategy of experimental design. An investigation may proceed as a series of small experiments instead of as a single comprehensive experiment so that the information obtained in the earlier experiments may be used in the later ones. Industrial research offers a particularly favourable field for the application of methods of this sort. (4, p. 10)

Interest in the sequential approach to experimentation, as far as this study is concerned, arises because the state of knowledge concerning a response variable under investigation is likely to change during the course of the investigation. In different stages of the experimental iteration, the experimenter's knowledge concerning the response is at different levels. Hence a single method of analysis is not necessarily the most applicable in all stages of an investigation.

For example, the method of steepest ascent recommended by Box and Wilson (17, p. 18) for exploring a response surface consists first of performing a pattern of experiments designed to detect, in the initial region explored, any general sloping tendency of the surface. If such a tendency is found, further experiments are performed in the indicated direction of increasing response. After several cycles of this search enable the experimenter to attain a region in which no sloping tendency can be detected, the region so attained is examined by performing a more elaborate pattern of experiments which permits the curvature in the surface and the dependence between variables to be taken into account.

Brooks (18, p. 454) suggests a further sequentialization due to the fact that the method of steepest ascent can find only local maxima. He suggests that the procedure be augmented with a preliminary exploration in experimental regions suspected of having more than one maximum.

From the realization that a single method of analysis is not necessarily the most applicable throughout all stages of an investigation, it follows that a method of analysis appropriate for the requirements of the preliminary stage of experimentation, need not necessarily be applicable in the latter stages wherein the requirements are changed.

Tukey points out the requirements for data analysis during the preliminary stage of investigation and places emphasis upon insight rather than proofs. He includes as a part of "... the current revolutions in statistical thinking ... ,"

> ... a return to an interest in the wider aspects of the data, growth of interest in procedures that are incisive, that lay the data open so that we can see what they look like inside, even though they do not give definite significance of confidence levels. This means emphasis on insight and understanding rather than "proven" knowledge. (19, p. 172)

Box concurs.

> The situation ... [screening a large number of candidate factors] ... is frequently such that groups of experiments <u>should</u> be performed in sequence and the data <u>ought to be</u> viewed from a number of different aspects and points of view ... . There is still a great deal of room for research on <u>how</u> screening experiments ought to be analyzed and standard models are not necessarily appropriate. (20, p. 174)

Thus the conclusions drawn from this survey of current literature are:

1. For an understanding of the difficulties in industrial experimentation, it is necessary to delineate between the problems which are statistical and those which are essentially nonstatistical.

2. The lack of orthogonality of observational data renders it not amenable to the usual statistical methods of analysis.

3. Most industrial research is iterative in nature and employs the process of synthesis and analysis in alternation.

4. A method of analysis appropriate for the requirements of the preliminary stage of experimentation need not necessarily be applicable in the latter stages.

5. The primary requirement for a method of analysis applicable in the preliminary stage of experimentation is that it provide insight and understanding rather than proven knowledge.

CHAPTER IV

A METHOD FOR THE DISPLAY AND ANALYSIS OF OBSERVATIONAL DATA

The methodology relating to the elucidation of the features of the relationship between a response and independent variables is called by Muller (21, p. 11) "response surface methodology." A response surface is a graphical representation of a relationship between a response and a number of factors or variables. Box uses the term "candidate" factors in referring to independent variables whose relationships to a response are being explored.

Brooks (18, p. 454), Box (1, p. 58), and others have noted that the results of a complete factorial experiment provide a desirable, systematic, overall picture of the response surface.

Brownlee (22, p. 17) notes that interactions between factors "can only be detected by one form or another of a factorial experiment." The factorial type method may be thought of as the conduction of trials at the points of a grid in the factor space. For each factor, several levels are selected; and for each combination of these factor-levels, the response is determined from a trial.

The factorial design increases the number of necessary observations rapidly with the number of dimensions or independent factors. Plackett and Burman demonstrate this difficulty:

> ... to carry out a complete factorial experiment (i.e., to make up assemblies of all possible combinations of the n components) would require L^n assemblies where L is the number of values at which each component can appear. For L equal to 2 this number is large for moderate n and quite impractical for n greater than, say, 10. For larger L the situation is even worse. (8, p. 305)

Box and Hunter concur as to the general impracticability of the complete factorial experiment in situations encountered in industry. The carrying out of "a close grid of experiments sufficiently widespread to cover the whole region of possible operation conditions would usually be too prodigal a policy to contemplate." (3, p. 141)

However, in the event of availability of observational data, such as plant logs, the use of an after-the-fact factorial-type display is here considered. In this case simultaneous observations, say 50 or more, are often available on a relatively large number of candidate factors, say six or more.

By arbitrarily segmenting the observed range of each continuous variable into discrete levels, each observation may be classified as a particular factor-level combination and represented by an $n + 1$ dimensional vector where n is the number of candidate factors and the $(n + 1)^{st}$ dimension is the response. Each observation may be considered as falling within a particular cell formed by the intersection of parallel planes drawn through the class limits, or boundaries of each level, and perpendicular to each axis of the n-dimensional factor space, where each of the n axes represents one candidate variable.

Thus each observation provides an indication of the response at a particular point in an n-dimensional space. The mean and variance of each cell provide an indication of the magnitude and variability of the response at a particular point on a n-dimensional grid formed by the intersection of parallel lines drawn through the midpoint of each interval and drawn perpendicular to each axis. The mean value of the response at a point within the grid - that is, at the midpoint of a particular cell - is an estimate of the height of the response surface above that cell. Each cell includes an infinite number of points of which only a few are represented among the observations. In some cases there may be no observations falling into a particular factor level combination.

To illustrate, a hypothetical situation involving only two candidate factors is given. Suppose a plant log contained the following 29 observations on two candidate factors suspected of influencing the yield of a chemical reaction. The observations are recorded as in Table 1.

By constructing an arbitrary 3 x 3 grid - that is, with each candidate factor segmented into three discrete levels - with equal class intervals for each factor, the experimenter may distribute the observations to the appropriate cell, as for example has been done in Table 2. A discussion of the determination of class intervals will follow in the latter portion of Chapter V.

The sample means and variances for the observed yields within each cell are calculated and displayed in similar 3 x 3 grids, Tables 3 and 4. Consideration of a method for determining the relative goodness of fit for and meaningfulness of various two factor surfaces will be discussed in Chapter V.

Table 1. Illustration of Plant Log Data for Use in After-the-Fact Factorial Grid

Yield	Temperature °F	Concentration	Yield	Temperature °F	Concentration
8	174	14%	7	172	20%
13	199	10	4	174	28
10	189	11	12	191	24
12	192	14	17	203	22
19	182	18	11	179	10
5	167	19	5	193	28
10	171	13	16	183	21
10	174	10	11	186	8
15	185	21	17	200	6
19	210	18	6	170	22
23	201	26	10	166	9
12	197	12	3	188	35
13	194	30	14	190	19
14	185	20	11	170	14
17	198	33			

Table 2. Three x Three Grid Containing Observations on Yield

	Level	Class Interval	Observations		
Concentration %	3	26-35	4	13, 5, 3	23,17
	2	16-25	5, 7, 6	19,15,14 12,16,14	19,17
	1	6 -15	8,10,10 11,10,11	10,12,11	13,12,17
	Class Interval		166-180	181-195	196-210
	Level		1	2	3

Temperature °F

Table 3. Three x Three Grid, Cell Means

	Level	Class Interval	Cell Means (Yield)		
Concentration %	3	26-35	4	7	20
	2	16-25	6	15	18
	1	6-15	10	11	14
	Class Interval		166-180	181-195	196-210
	Level		1	2	3

Temperature °F

Table 4. Three x Three Grid, Cell Variances (Yield)

	Level	Class Interval	Cell Variances (Yield)		
Concentration %	3	26-35	-	28	18
	2	16-25	1	5.6	2
	1	6-15	1.2	1	7
	Class Interval		166-180	181-195	196-210
	Level		1	2	3

Temperature °F

By considering the grids such as in Tables 3 and 4 as surfaces viewed from above and by considering the values within the grid as representing the estimated heights of the surface above each grid point or cell mean, a mental image or picture of the estimates of the two surfaces emerges:

1. the response surface (Table 3);

2. the surface representing the variance of the observed responses within each cell. (Table 4) This is not to be confused with the variance of the estimate of the cell means which is a function of the number of observations that happened to be available for each cell as well as the cell response variance.

The geometrical interpretation of three-factor grids is more complicated due to the additional dimension involved. It is difficult to "picture" mentally a fourth dimension.

For example, a three-factor combination involving factors which each have four levels may be mentally pictured as a 4 x 4 x 4 cube, each cell of which contains a number representing the mean or variance of that cell. The magnitude of the cell mean may be considered an estimate of the height of the response surface "above" that point of the grid, as measured in a fourth dimension. An easier interpretation is to consider a three-factor combination as simply a separate two-factor combination for each level of the third factor. A separate grid may be presented for each of the third factor levels. Geometrical interpretations of higher factor combinations are of little assistance to the experimenter.

In the industrial situation considered in this study, there are more than two candidate factors to be examined. If two-factor grids - or tables - and possibly three-factor combination grids are used to aid an experimenter in appreciating the salient features of the response surface, he faces the awesome task of considering each possible two-factor grid and surface. When the number of candidate factors is moderately large, say 15, the total number of two-factor combinations is

$$C_2^{15} = \frac{15!}{2!\ 13!} = 105 .$$

Assuming for the moment that it is possible to obtain the grids for each possible two-factor combination, the experimenter needs a method by which to eliminate the majority of these grids and surfaces from consideration. It is necessary to develop a means for determining which of the many possible surfaces, so displayed, most nearly describes the real or true response surface.

For this purpose, attention is focused upon the variance-surface grid mentioned above. Until one or more of the factor-combination-surface displays are selected for further consideration, the plots of the cell means - the points of the grid - are of little interest.

For a given set of data, the smaller the within cell variances for a particular factor combination, the larger the proportion of the overall variance in the response which may be attributed to that factor combination or to some factor combination correlated thereto.

The similarity between this concept and that of using randomized blocks in experimental design is noteworthy. The total variation among the observed yields is decomposed into one assignable cause and one unassignable - blocks and error. The more the blocks are made to differ from one another in terms of the response, the bigger will be the sum of squares for blocks, and the smaller will be the error sum of squares and within cell variances due to the removal of the block effect from the error sum of squares. Thus the more successful the blocking - that is, the more variation in the response explained by the effects over which blocked - the less the remaining unexplained variance.

Ordinarily, blocking is used in self-defense due to lack of knowledge concerning variability between blocks and is an effort to remove the unknown and unpredictable sources of variation by elimination of effects (other than treatment effects) which only dilute the strength of the statistical conclusions. In this case however, the situation is somewhat reversed. The unexplained or residual variation is that which remains after blocking out the effects present in the particular factor combination under consideration or by blocking out effects correlated with the factor combination over which blocked.

The smaller the remaining unexplained variance after blocking, the larger the variance which may be attributed to or explained by those effects over which blocked. Since the data are not orthogonal, the presence of a correlation between the factors over which blocked and another factor having a real effect may produce a spurious reduction in the unexplained within cell variance.

If the reductions in the unexplained variance which occur when blocking over each two-factor combination are compared, the largest reduction would be expected when blocking over the real effect, as opposed to merely a correlated effect.

Because the blocking method herein used is a type of two-factor factorial, variation in the response caused by a two-factor interaction, as well as the main effects, is blocked out. Thus the variation removed by the two-factor blocking is the sum total of that caused by the main effects of the two factors and the two-factor interaction. In the case of three-factor combination blocking, the variation removed is made up of that caused by the three main effects, all two-factor interactions, and the three-factor interaction.

It is necessary to determine a method for measuring the overall unexplained variation remaining after blocking. This measure is the index herein used as an indication of the most important factor combination. In essence the problem is one of weighting the individual within cell variances to determine the overall unexplained variance. The problem is discussed in the following chapter.

CHAPTER V

AFTER-THE-FACT STRATIFICATION OF OBSERVATIONAL DATA

The analysis underlying factorial design assumes replications of the entire design and thereby the same number of observations in each cell. Observational data, however, are not orthogonal. The number of observations falling into the various cells constructed as outlined in Chapter IV will almost certainly be different. Thus the problem arises as to how to weight the data in each cell in order to arrive at an estimate of the overall surface variance for any given factor combination. This chapter is concerned with that problem.

In this study it is assumed for the calculation of expected values, that sampling is from a finite population of elements even though the size of the population may be large enough to permit the use of limiting distributions. The logic of Madow and Madow, as expressed below, provides the basis for this assumption.

> The same results would be obtained by assuming a correctly defined multivariate normal distribution and using the notions of conditional probability. From a physical point of view, however, there are several factors that lead to the use of the finite population. We are most frequently sampling an existing population whose laws of transformation are either unknown or not mathematically expressed. Consequently, the notion of a normal or other specified distribution from which we sample and use conditional probability is not part of our thinking concerning the physical problem. On the other hand, if we consider the population to be a finite population, and use a table of random numbers to draw our sample from the finite population, we are using only mathematics implicit in our physical problem.

> Furthermore, we do obtain a repeatable experiment; that of selecting a random number, that we know is in a state of statistical control. (23, p. 2)

Using, in general, the symbol structure of Cochran (24, p. 65), consider a heterogeneous population of N units divisible into L subpopulations which are internally homogeneous relative to the entire population. Let the L subpopulations be nonoverlapping strata of size $N_1, N_2, \ldots, N_L$ units and:

y_{hi} = the i^{th} unit in the h^{th} stratum;

N_h = number of items in the h^{th} stratum;

n_h = number of observations in the h^{th} stratum;

$$N = \sum_{h=1}^{L} N_h = \text{total number of units.}$$

Henceforth $\sum$ will be taken to mean $\sum_{h=1}^{L}$.

Also let:

$$n = \sum n_h = \text{sample size;}$$

$$\bar{Y}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} y_{hi} = \text{population stratum mean;}$$

$$\bar{Y} = \frac{1}{N} \sum \sum_{i=1}^{N_h} y_{hi} = \text{population grand mean.}$$

Assuming the finite population correction to be negligible,

$$\sigma_h^2 = \sum_{i=1}^{N_h} \frac{(y_{hi} - \bar{Y}_h)^2}{N_h} = \text{variance within the } h^{th} \text{ cell;}$$

$$\sigma^2 = \frac{1}{N} \sum \sum_{i=1}^{N_h} (y_{hi} - \bar{Y})^2 = \text{population variance.}$$

For a method of weighting the data in each cell, it might be argued that since the experimenter has more confidence in estimates based on larger sample sizes, each cell estimate should be weighted according to the number of observations. The assumption of subpopulation parameter equality, however, and the use of its associated method for weighting stratum estimates according to the number of observations or degrees of freedom may not always lead to tenable estimates for population parameters. Consider, for example, the hypothetical task of estimating the average fractional number of automobiles per person in the entire world. Suppose that stratification according to nationality is deemed advantageous. Suppose also that the availability of recent census data for the United States permits the calculation of what is, for practical purposes, the exact or true fractional number of automobiles per person in the United States.

Unfortunately for the purpose of this hypothetical study, data relating to population and automobile registration in various other nations, for example the Soviet Union, may be scarce, or unreliable, or both scarce and unreliable.

From an extension of the method of weighting subpopulation estimates according to the confidence in the estimate, as for example under a null hypothesis of parameter equality, it follows that complete knowledge regarding the subpopulation parameter from any stratum is equivalent to complete knowledge of the population parameter and the stratum parameters from all strata.

However, in the hypothetical case under consideration, the fact that in a particular stratum, namely that made up of the people living in the United States, the fact that the deviation of the estimate of the subpopulation mean from the true subpopulation mean is negligible does not justify the conclusion that the number of automobiles per person in the world is equal to that in the United States. However, if stratum estimates are weighted in a fashion inversely proportional to the estimated variance of the estimate--that is, weighted in relation to confidence in the estimates--the pooled estimate of the fractional number of automobiles per person would be equal to the estimate of the parameter for the United States stratum, since this estimate was considered to be without error. This result, of course, is illogical and incorrect.

Thus, where the assumption of subpopulation parameter equality is untenable, the weighting of estimates of stratum parameters according to confidence in the within stratum estimates will provide a poor estimate of the population parameter. In a situation where a functional relationship between subpopulation means cannot be stated, such as is the case during the initial stages of industrial experimentation, knowledge of data concerning one or more of the stratum means offers no assistance in the estimation of the remainder of the subpopulation means. No matter how well one mean is known, the other stratum estimates do not benefit.

An alternative estimate of the population mean is

$$\bar{y}^* = \frac{1}{N} \sum N_h \, \bar{y}_h,$$

which is appropriate for estimates based on stratified sampling. In this case the individual stratum estimates are weighted according to the true proportion, N_h / N, of the total population units included in the individual stratum.

It will be helpful at this point to compare in more quantitative detail these two alternative methods for the estimation of population parameters and specifically the population mean.

The more frequently encountered estimate, $\bar{y}$, is merely the sample mean, which is a weighting of stratum means in proportion to the number of observations which happen, through the process of random sampling, to fall within each stratum. Thus,

$$\begin{aligned} \bar{y} &= \frac{1}{n} \sum_{i=1}^{n} y_i \\ &= \frac{1}{n} \sum \sum_{i=1}^{n_h} y_{hi} \\ &= \frac{1}{n} \sum n_h \, \bar{y}_h \, . \end{aligned}$$

Duncan shows (25, p. 824) that for a random sample from the population of all possible responses, the expected value of the estimate is

$$E(\bar{y}) = E(\sum \frac{n_h}{n} \bar{y}_h)$$

$$= \sum E(\frac{n_h}{n}) E(\bar{y}_h)$$

$$= \sum \frac{N_h}{N} \bar{Y}_h$$

$$= \bar{Y} .$$

The assumption of randomness of the sample insures independence between the random variables n_h / n and $\bar{y}_h$. The applicability of this assumption when considering observational data will be discussed later in this chapter.

For the estimate weighted according to the population proportionality, again assuming a random sample, the expected value of the estimate is

$$E(\bar{y}^*) = E(\sum \frac{N_h}{N} \bar{y}_h)$$

$$= \sum \frac{N_h}{N} \bar{Y}_h$$

$$= \bar{Y} .$$

As might be expected, both estimates are unbiased. However, the two estimates are not the same. In the first case above, the population estimate involves two random variables. The first of these is the sample proportion n_h / n, which does not appear in the second weighting method since the random variable is there replaced by the true stratum proportions N_h / N. The random nature of the sampling process insures that in a long series of such samples, the average of n_h/n will approximate very closely the true ratio N_h/N and that deviations for any given sample are strictly random. That is, by the law of large numbers,

$$\lim P\left\{ \left| \frac{\sum_{j=1}^{k} (n_h)_j}{nk} - \frac{N_h}{N} \right| > \varepsilon \right\} = 0 ,$$

where K = number of trials.

It does not insure that the two will be equal. In fact the random nature of the sampling makes it unlikely that the two proportionalities would be equal.

The difference in the two estimates for a <u>given</u> sample is:

$$\bar{y} - \bar{y}^* = \frac{1}{n}\sum \sum_{i=1}^{n_h} y_{hi} - \frac{1}{N}\sum \sum_{i=1}^{n_h} \frac{N_h}{n_h} y_{hi}$$

$$= \sum \sum_{i=1}^{n_h} \left(\frac{y_{hi}}{n} - \frac{N_h\, y_{hi}}{n_h N} \right)$$

$$= \sum \sum_{i=1}^{n_h} \left(\frac{1}{n} - \frac{N_h}{n_h N} \right) y_{hi}$$

$$= \sum \left(\frac{1}{n} - \frac{N_h}{n_h N} \right) \sum_{i=1}^{n_h} y_{hi}$$

$$= \sum \left(\frac{1}{n} - \frac{N_h}{n_h N} \right) \bar{y}_h\, n_h$$

$$= \sum \left(\frac{n_h}{n} - \frac{N_h}{N} \right) \bar{y}_h .$$

The two estimates are identical under the following two conditions.

1. $\bar{y}_h = \bar{y}$ for all h.

In this case $\bar{y}_h$ is a constant and

$$\sum \left(\frac{n_h}{n} - \frac{N_h}{N} \right) \bar{y}_h$$

becomes

$$\bar{y} \sum \left(\frac{n_h}{n} - \frac{N_h}{N} \right) = 0$$

since $\sum \frac{n_h}{n} = 1$ and $\sum \frac{N_h}{N} = 1$.

2. $\left(\frac{n_h}{n} - \frac{N_h}{N} \right) = 0$ for all h.

In this case the sampling fraction is the same as the population fraction in all strata and the sample is, in effect, a proportionally allocated stratified random sample.

Neither of these conditions is assumed to hold in the proposed analysis of observational data. Thus, the two estimates $\bar{y}$ and $\bar{y}^*$ are not, in general, identical.

The term "proportional stratification" is used to describe a method of sampling in which the observations or sample units are allocated among the strata in proportion to the total number of units in each stratum; that is, $n_h = n\, N_h/N$. The use of the word "random" with this method of sampling will be taken to mean that each unit within a particular stratum is equally likely to be included in the sample from <u>that</u> stratum. The variance of the estimate of the population mean for a stratified sample is shown by Cochran (24, p. 67) to be

$$\sigma_{\bar{y}*}^2 = \sum \frac{N_h^2}{N^2} E(\bar{y}_h - \bar{Y}_h)^2 ,$$

with the restrictions that:

1. $\bar{y}_h$ is an unbiased estimate of $\bar{Y}_h$, and

2. The samples are drawn independently in different strata.

As noted by Cochran, (24, p. 68), "The important point about this result is that the variance of ... [$\bar{y}*$] ... depends only on the variances of the estimates of the individual stratum means $\bar{Y}_h$."

But $E(\bar{y}_h - \bar{Y}_h)^2 = \sigma_h^2/n_h$. Thus, for stratified random sampling,** by substituting in Cochran's equation, the estimate $\bar{y}*$ is

$$\sigma_{\bar{y}*}^2 = \sum \frac{N_h^2}{N^2} \frac{\sigma_h^2}{n_h}$$

where

$$\sigma_h^2 = \sum_{i=1}^{N_h} \frac{(y_{hi} - Y_h)^2}{N_h} ,$$

as defined earlier.

In the case of stratified sampling, deviations of the true stratum means from the grand population mean do not reduce the precision of the estimate $\bar{y}*$.

**With proportional allocation,

$$\frac{n_h}{n} = \frac{N_h}{N} \quad \text{and} \quad n_h = n\frac{N_h}{N} .$$

Substituting for n_h in the above equation for $\sigma_{\bar{y}*}^2$, the variance reduces to

$$\sigma_{\bar{y}*}^2 = \sum \frac{N_h}{N} \frac{\sigma_h^2}{n} .$$

Note in contrast the development below of the variance of the estimate $\bar{y}$ obtained by weighting stratum results in relation to sample proportions.

The estimate $\frac{n_h}{n}\bar{y}_h$ is an indication of the contribution of the h^{th} stratum to the population mean $\bar{Y}$. For large samples the variance of the product of two random variables is (26, p. 513):

$$\sigma^2_{xy} = (\bar{X}\bar{Y})^2 \left(\frac{\sigma^2_x}{\bar{X}^2} + \frac{\sigma^2_y}{\bar{Y}^2} + \frac{2 \text{ cov } xy}{\bar{X}\,\bar{Y}} \right),$$

where x and y are random variables,

$\bar{X}$ and $\bar{Y}$ are the parameters estimated,

σ^2_x and σ^2_y are the respective variances of the random variables

and $\text{cov } xy = \rho_{xy}\,\sigma_x\,\sigma_y$ is the covariance of x and y, and rho is the correlation coefficient.

In the case of a sample which is random with respect to the various strata, the random variables $\frac{n_h}{n}$ and $\bar{y}_h$ are independent. Hence the variance of the product reduces to

$$\sigma^2_{\frac{n_h}{n}\bar{y}_h} = \left(\frac{N_h}{N}\bar{Y}_h \right)^2 \left[\frac{\sigma^2_{\frac{n_h}{n}}}{\left(\frac{N_h}{N}\right)^2} + \frac{\sigma^2_{\bar{y}_h}}{(\bar{Y}_h)^2} \right] + \sigma^2_{\frac{n_h}{n}}\,\sigma^2_{\bar{y}_h}$$

$$= \bar{Y}^2_h\,\sigma^2_{\frac{n_h}{n}} + \left(\frac{N_h}{N}\right)^2 \sigma^2_{\bar{y}_h} + \sigma^2_{\frac{n_h}{n}}\,\sigma^2_{\bar{y}_h}\,.$$

Note that the estimate $\frac{N_h}{N}\,\bar{y}_h$ of the h^{th} stratum's contribution to the grand mean is not a product of two random variables. Hence the variance of that estimate is

$$\sigma^2_{\frac{N_h}{N}\bar{y}_h} = \left(\frac{N_h}{N}\right)^2 \sigma^2_{\bar{y}_h} .$$

By substitution,

$$\sigma^2_{\frac{n_h}{n}\bar{y}_h} = \sigma^2_{\frac{N_h}{N}\bar{y}_h} + \bar{Y}_h^2\,\sigma^2_{\frac{n_h}{n}} + \sigma^2_{\frac{n_h}{n}}\,\sigma^2_{\bar{y}_h} ,$$

and

$$\sigma^2_{\frac{n_h}{n}\bar{y}_h} \geq \sigma^2_{\frac{N_h}{N}\bar{y}_h} .$$

Therefore the estimate utilizing the true stratum proportions in determining the individual stratum contribution to the grand mean has a variance equal to or less than the individual cell estimate using the sample proportions.

The variance of the sum of k random variables, u_h, where $h = 1, 2, \ldots, k$, is (26, p. 513):

$$\sigma^2_{\sum\limits_{h=1}^{k} u_h} = \sigma^2_{u_1} + \sigma^2_{u_2} + \cdots + \sigma^2_{u_k} + 2(\sigma_{u_1u_2} + \sigma_{u_1u_3} + \cdots + \sigma_{u_{k-1}\,u_k}).$$

In our case the estimate $\bar{y}$ can be considered to be the sum of k random variables, each of which is the product of two random variables.

$$u_h = \frac{n_h}{n}\,\overline{y}_h \,.$$

Thus,

$$\sigma^2_{\overline{y}} = \sigma^2_{\sum\limits_{h=1}^{k} \frac{n_h}{n}\overline{y}_h}$$

$$= \sigma^2_{\frac{n_1}{n}\overline{y}_1} + \sigma^2_{\frac{n_2}{n}\overline{y}_2} + \cdots + \sigma^2_{\frac{n_k}{n}\overline{y}_k}$$

$$+ 2\left\{\sigma_{(\frac{n_1}{n}\overline{y}_1)(\frac{n_2}{n}\overline{y}_2)} + \sigma_{(\frac{n_1}{n}\overline{y}_1)(\frac{n_3}{n}\overline{y}_3)} + \cdots + \sigma_{(\frac{n_{k-1}}{n}\overline{y}_{k-1})(\frac{n_k}{n}\overline{y}_k)}\right\}.$$

But

$$\sigma^2_{\frac{n_h}{n}\overline{y}_h} = \overline{Y}_h^2\,\sigma^2_{\frac{n_h}{n}} + \left(\frac{N_h}{N}\right)^2 \sigma^2_{\overline{y}_h} + \sigma^2_{\frac{n_h}{n}}\,\sigma^2_{\overline{y}_h} \,.$$

Substituting:

$$\sigma^2_{\overline{y}} = \left\{\overline{Y}_1^2\,\sigma^2_{\frac{n_1}{n}} + \left(\frac{N_1}{N}\right)^2 \sigma^2_{\overline{y}_1}\right\} + \left\{\overline{Y}_2^2\,\sigma^2_{\frac{n_2}{n}} + \left(\frac{N_2}{N}\right)^2 \sigma^2_{\overline{y}_2}\right\} +$$

$$\cdots + \left\{\overline{Y}_k^2\,\sigma^2_{\frac{n_k}{n}} + \left(\frac{N_k}{N}\right)^2 \sigma^2_{\overline{y}_k}\right\}$$

$$+ 2\left\{\sigma_{(\frac{n_1}{n}\overline{y}_1)(\frac{n_2}{n}\overline{y}_2)} + \sigma_{(\frac{n_1}{n}\overline{y}_1)(\frac{n_3}{n}\overline{y}_3)} + \right.$$

$$\cdots + \sigma_{\left(\frac{n_{k-1}}{n}\,\overline{y}_{k-1}\right)\left(\frac{n_k}{n}\,\overline{y}_k\right)} \Bigg\} + \sum_{h=1}^{k} \sigma^2_{\frac{n_h}{n}} \sigma^2_{\overline{y}_h} \; .$$

Similarly,

$$\sigma^2_{\overline{y}*} = \sigma^2_{\sum\limits_{h=1}^{k} u^*_h} \; ,$$

where

$$u^*_h = \frac{N_h}{N}\,\overline{y}_h \; .$$

$$\sigma^2_{\overline{y}*} = \left(\frac{N_1}{N}\right)^2 \sigma^2_{\overline{y}_1} + \left(\frac{N_2}{N}\right)^2 \sigma^2_{\overline{y}_2} + \cdots + \left(\frac{N_k}{N}\right)^2 \sigma^2_{\overline{y}_k}$$

$$+ 2 \Bigg\{ \sigma_{\left(\frac{N_1}{N}\,\overline{y}_1\right)\left(\frac{N_2}{N}\,\overline{y}_2\right)} + \sigma_{\left(\frac{N_1}{N}\,\overline{y}_1\right)\left(\frac{N_3}{N}\,\overline{y}_3\right)} +$$

$$\cdots + \sigma_{\left(\frac{N_{k-1}}{N}\,\overline{y}_{k-1}\right)\left(\frac{N_k}{N}\,\overline{y}_k\right)} \Bigg\} ,$$

but

$$\sigma_{\left(\frac{N_{h-1}}{N}\,\overline{y}_{h-1}\right)\left(\frac{N_h}{N}\,\overline{y}_h\right)} = E\left(\frac{N_{h-1}}{N}\,\overline{y}_{h-1} - \frac{N_{h-1}}{N}\,\overline{Y}_{h-1}\right)\left(\frac{N_h}{N}\,\overline{y}_h - \frac{N_h}{N}\,\overline{Y}_h\right)$$

$$= E\left\{\left(\frac{N_{h-1}}{N}\right)\left(\overline{y}_{h-1} - \overline{Y}_{h-1}\right)\right\}\left\{\frac{N_h}{N}\left(\overline{y}_h - \overline{Y}_h\right)\right\}$$

$$= \left(\frac{N_{h-1}}{N}\right)\left(\frac{N_h}{N}\right) E\left(\overline{y}_{h-1} - \overline{Y}_{h-1}\right)\left(\overline{y}_h - \overline{Y}_h\right) \; .$$

Since the sampling in any cell h is assumed independent of the sampling in other cells, the deviation of the estimate $\bar{y}_{h-1}$ from the cell mean $\bar{Y}_{h-1}$ is independent of the deviation of the estimate $\bar{y}_h$ from the cell mean $\bar{Y}_h$ for all h, that is $\rho = 0$. Thus the equation for $\sigma^2_{\bar{y}*}$ simplifies to:

$$\sigma^2_{\bar{y}*} = \left(\frac{N_1}{N}\right)^2 \sigma^2_{\bar{y}_1} + \left(\frac{N_2}{N}\right)^2 \sigma^2_{\bar{y}_2} + \cdots + \left(\frac{N_k}{N}\right)^2 \sigma^2_{\bar{y}_k}$$

$$= \sum_{h=1}^{k} \frac{N_h^2}{N^2} \sigma^2_{\bar{y}_h} ,$$

which agrees with Cochran's result shown earlier. But the objective here is to compare $\sigma^2_{\bar{y}}$ with $\sigma^2_{\bar{y}*}$. Therefore, substituting the last result in the equation for $\sigma^2_{\bar{y}}$, we have

$$\sigma^2_{\bar{y}} = \sigma^2_{\bar{y}*} + \bar{Y}_1^2 \sigma^2_{\frac{n_1}{n}} + \bar{Y}_2^2 \sigma^2_{\frac{n_2}{n}} + \cdots + \bar{Y}_k^2 \sigma^2_{\frac{n_k}{n}}$$

$$+ 2\left\{\sigma_{(\frac{n_1}{n}\bar{y}_1)(\frac{n_2}{n}\bar{y}_2)} + \sigma_{(\frac{n_1}{n}\bar{y}_1)(\frac{n_3}{n}\bar{y}_3)} + \right.$$

$$\left. \cdots + \sigma_{(\frac{n_{k-1}}{n}\bar{y}_{k-1})(\frac{n_k}{n}\bar{y}_k)}\right\} + \sum_{h=1}^{k} \sigma^2_{\frac{n_h}{n}} \sigma^2_{\bar{y}_h}$$

$$= \sigma^2_{\bar{y}*} + \sum_{h=1}^{k} \bar{Y}_h^2 \sigma^2_{\frac{n_h}{n}} + 2(\text{covariance terms})$$

$$+ \sum_{h=1}^{k} \sigma^2_{\frac{n_h}{n}} \sigma^2_{\bar{y}_h} .$$

Considering any one of the covariance terms,

$$
\begin{aligned}
\sigma_{(\frac{n_h}{n}\bar{y}_h)(\frac{n_i}{n}\bar{y}_i)} &= E\left(\frac{n_h}{n}\bar{y}_h - \frac{N_h}{N}\bar{Y}_h\right)\left(\frac{n_i}{n}\bar{y}_i - \frac{N_i}{N}\bar{Y}_i\right) \\
&= E\left\{\left(\frac{n_h}{n}\bar{y}_h\right)\left(\frac{n_i}{n}\bar{y}_i\right) - \left(\frac{n_h}{n}\bar{y}_h\right)\left(\frac{N_i}{N}\bar{Y}_i\right)\right. \\
&\qquad \left. - \left(\frac{N_h}{N}\bar{Y}_h\right)\left(\frac{n_i}{n}\bar{y}_i\right) + \left(\frac{N_h}{N}\bar{Y}_h\right)\left(\frac{N_i}{N}\bar{Y}_i\right)\right\} \\
&= E\left\{\left(\frac{n_h}{n}\bar{y}_h\right)\left(\frac{n_i}{n}\bar{y}_i\right)\right\} - \left(\frac{N_i}{N}\bar{Y}_i\right) E\left(\frac{n_h}{n}\bar{y}_h\right) \\
&\qquad - \left(\frac{N_h}{N}\bar{Y}_h\right) E\left(\frac{n_i}{n}\bar{y}_i\right) + \left(\frac{N_h}{N}\bar{Y}_h\right)\left(\frac{N_i}{N}\bar{Y}_i\right) \\
&= E\left\{\left(\frac{n_h}{n}\bar{y}_h\right)\left(\frac{n_i}{n}\bar{y}_i\right)\right\} - \left(\frac{N_h}{N}\bar{Y}_h\right)\left(\frac{N_i}{N}\bar{Y}_i\right) \\
&= E\left\{\left(\frac{n_h}{n}\cdot\frac{n_i}{n}\right)\left(\bar{y}_h\,\bar{y}_i\right)\right\} - \left(\frac{N_h}{N}\cdot\frac{N_i}{N}\right)\left(\bar{Y}_h\,\bar{Y}_i\right) .
\end{aligned}
$$

But the sampling proportion n_h/n is independent of the stratum mean $\bar{y}_h$, since the sample is random with respect to the various strata.

$$
E\left\{\left(\frac{n_h}{n}\cdot\frac{n_i}{n}\left(\bar{y}_h\cdot\bar{y}_i\right)\right)\right\} = E\left(\frac{n_h\,n_i}{n^2}\right) E\left(\bar{y}_h\,\bar{y}_i\right) = \left(\bar{Y}_h\,\bar{Y}_i\right) E\left(\frac{n_h\,n_i}{n^2}\right) .
$$

Thus,

$$
\sigma_{(\frac{n_h}{n}\bar{y}_h)(\frac{n_i}{n}\bar{y}_i)} = \left\{E\left(\frac{n_h\,n_i}{n^2}\right) - \left(\frac{N_h\,N_i}{N^2}\right)\right\}\bar{Y}_h\,\bar{Y}_i .
$$

Assuming that the sum of the covariance terms discussed above is nonnegative or at least that the absolute value is small relative to

$$\sum \bar{Y}_h^2 \sigma^2_{\frac{n_h}{n}} + \sum_{n=1}^{k} \sigma^2_{\frac{n_h}{n}} \sigma^2_{\bar{y}_h}, \qquad \sigma^2_{\bar{y}} \geq \sigma^2_{\bar{y}*} .$$

The results for the estimate $\bar{y}^*$ are equally applicable to the case where the data are subjected to after-the-fact stratification.

In the case of observational data, it is noted that although each factor level combination may be equally possible, caution must be exercised in treating observational data due to the fact that management decision, operator practice, or indifference may result in a condition such that each factor-level combination is not equally likely in the data. The process may tend to operate at certain factor-level combinations more often than at others. Some factor-level combinations may not be represented in the data at all.

The estimate $\bar{y}*$ does not require the assumption of independence between n_h/n and $\bar{y}_h$. That is, the validity of the use of after-the-fact stratification and the estimate $\bar{y}*$ are not endangered by the lack of randomness, with respect to factor-level combinations, of observational data. As indicated by the presence of the covariance term $\rho_{xy}\ \sigma_x\ \sigma_y$ in the equation for σ^2_{xy}, the variance of the product of two random variables, the estimate $\bar{y}$ is not free of the assumption of independence between the sample proportion n_h/n and the cell sample mean $\bar{y}_h$. A less restrictive assumption is sufficient when using the estimate $\bar{y}^*$. That is, the observations need only be random with respect to the particular cell or factor-level combination within which they fall.

In summary, the above discussion of a method for weighting the cell variances in order to arrive at an unbiased estimate of the overall surface variance suggests the use of N_h/N.

In order to arrive at this weighting factor, each value between the largest and smallest observed value for each candidate variable is assumed to be equally possible. Thus, if a factor is segmented into four levels of equal intervals, each level is considered equally possible. For two-factor combinations, each cell's weighting factor is determined by the ratio of its area (N_h) to the area of all cells (N). That is, each cell is weighted according to the proportion of the total surface area, measured in a horizontal plane, which is included within the class limits of that cell. Further discussion and specific examples are given in Chapter VII.

A discussion of the physical means used to massage the observational data follows in Chapter VI.

CHAPTER VI

THE COMPUTER, SYMBOLIC PROGRAMMING AID, AND TEST DATA USED

The arithmetic required in calculating means and variances for each possible two-way factor-level combination during the screening stage of industrial experimentation is best accomplished by electronic digital computers.

A Burroughs 220 Datatron system was made available for this study by the Rich Electronic Computer Center, a division of the Engineering Experiment Station, Georgia Institute of Technology. The Burroughs 220 is a general-purpose, stored program, sequentially controlled, decimal computer system.

This system has a magnetic-core internal storage of 5000 44-bit words which use the 8, 4, 2, 1 binary code for each ten-decimal-digit-plus-sign-digit-position word. In addition to various input, output, and auxiliary components, this system includes six magnetic tape units which read and write at the rate of 25,000 characters per second.

One of the most impressive characteristics of digital computers is their operating speed. The Burroughs 220 requires approximately 200 millionths of a second to add two numbers. Thus, analyses requiring multitudinous computations which would preclude consideration when utilizing manual resources may become practical through the use of electronic computers.

The task of translating the desired computations into a language which the computer can understand is called coding or programming. A program is a list of instructions which provide an orderly explanation to the computer of each individual operation it is to perform. The instructions comprising a computer vocabulary usually include the basic arithmetic operations of addition and subtraction, through which multiplication and division are available; operations permitting the transfer of data between designated locations in the computer; and operations controlling input and output equipment. Various other more complex operations which are used frequently may be included in the computer vocabulary.

Difficulty in communicating with the computer was greatly reduced through the use of the Burroughs Algebraic Compiler, generally referred to as Algol. This, in essence, is a set of instructions available for use on the 220 which permit the machine to accept symbolic programs, written in almost plain language, and convert these into machine language programs. A description of the evolution and status of Algol is available elsewhere. (28 ; 29)

Prior to translation into an Algol program, the problem was reduced to a graphical diagram, often referred to as a flow chart, of the general sequence of operations necessary to arrive at the desired output. The flow chart is included as Appendix A.

Through the use of an Algol reference manual (30), the problem was then translated into the symbolic Algol program shown in Appendix B.

The data used to test the program were obtained from quality control logs of four textile plants. The response variable of interest was warp contraction, expressed as a percentage. In the process of weaving cloth, lengthwise and widthwise yarns are interlaced. The lengthwise yarns, individually known as ends, are called the warp. The widthwise yarns are called the filling and individually referred to as picks.

When the yarns interlace to form cloth, they bend around one another. This bending is known as crimp and tends to shorten the horizontal span of a given length of yarn. The amount of this crimp is often referred to as the per cent contraction. This is simply the horizontal length of yarn after interlacing divided by the length of the same yarn prior to the interlacing, all multiplied by one hundred.

This dependent variable was chosen because of the high order interactions of the various variables which were thought to affect the warp contraction. The diameters of both the lengthwise and widthwise yarns were thought to influence the contraction. The number of ends per unit of length widthwise and the number of picks per unit of cloth length were also suspected as affecting warp contraction.

The nature of the relationships between the factors was unknown. However, a positive correlation between the diameters of the warp ends and filling picks and also between the number of ends and picks per unit of perpendicular length was suspected. A negative correlation between the number of ends and the diameter of these yarns was anitcipated. A similar correlation was expected for the filling yarns. In addition, the frequency of interlacings was thought to affect the dependent variable.

Data regarding other candidate variables - some discrete, such as the mill at which the data originated, and some continuous, such as the width of the cloth - were also available from the plant logs and were considered as "candidate" factors.

Each continuous factor was segmented into four equal intervals, each of which was assigned a discrete level number. Each observation was then classified according to the appropriate level of each continuous and discrete variable.

In two cases the number of observations falling into the highest numbered level was small relative to that level's proportionate share (25 per cent) of the total of 170 observations. In view of the desirability of obtaining two-way and possibly three-way factor-level combinations, reasonably even distribution of observations among the levels was felt necessary. Consequently, in these two cases the intervals were redetermined so as to render the interval in the region of few observations larger than the other intervals by a factor of two or three.

For example, the range of the continuous variable picks per inch was 26 to 131. In setting up equal intervals Table 5 was obtained.

Since the eight and four observations in levels three and four respectively provide only limited possibilities for further meaningful

Table 5. Original Intervals for the Factor Picks/Inch

	Interval	Number of Observations
Level 1	26-52	90
Level 2	53-78	68
Level 3	79-104	8
Level 4	105-131	4

subdivision into four two-factor-level combinations, the intervals were recalculated as shown in Table 6.

Table 6. Adjusted Intervals for the Factor Picks/Inch

	Interval	Relative Interval Size	Number of Observations
Level 1	26 - 43	1	59
Level 2	44 - 61	1	83
Level 3	62 - 79	1	16
Level 4	80-131	3	12

This is tantamount to dividing the continuous variable into six intervals and pooling the upper levels for analysis purposes.

Table 7 shows the description of class intervals for each of the 15 "candidate" factors considered in the testing of the computer program.

It will be noted that factors three and five are the same as factors four and six respectively, differing only in the class intervals

Table 7. Factor Level Identification

Factor Number	Description	Identification Level 1	Level 2	Level 3	Level 4
1	Fabric (Name of Cloth)	Broad-cloth and Poplin	Print Cloth and Sheeting	Soft Filled Sheeting, Osnaburg, and Misc. Plain Weave	Nonplain Weave
2	Weave	Plain and Semi-plain	Drill	Twill and Semi-regular Drill	Sateen
3	Ends/Inch (Special)	32-45	46-90	91-113	114-127
4	Ends/Inch	32-55	56-79	80-103	104-127
5	Picks/Inch (Special)	26-43	44-51	52-103	104-131
6	Picks/Inch	26-43	44-61	62-79	80-131
7	Let-Off Motion	Roper	Bartlett	Hunt	-
8	Plant	Mill A	Mill B	Mill C	Mill D
9	Loom Type	40" D	50" X-2 60" X-2 Misc.	46" X-2	-
10	Square Root of Warp Yarn Nbr. (Highly Correlated with Inverse of Yarn Diameter)	3.78 - 4.29	4.30 - 4.81	4.82 - 5.33	5.34 - 5.85
11	Square Root of Filling Yarn Nbr.	2.77 - 3.79	3.80 - 4.82	4.83 - 5.85	5.86 - 6.88
12	Loom Speed	152-164	165-176	177-188	189-200
13	Filling Twist Multiple	3.13 - 3.49	3.50 - 3.86	3.87 - 4.23	4.24 - 4.60
14	Cloth Width	32.0 - 40.2	40.3 48.5	48.6 - 56.7	56.8 - 65.0
15	Yards of Cloth per Pound	0.92 - 1.88	1.89 - 2.85	2.86 - 3.82	3.83 - 5.77

shown in Table 7. These factors will be discussed in the next chapter.

Each of the 170 observations was coded for the applicable level of each factor. The end result was the representation of each observation as a 16-dimensional vector, where the first through the fifteenth dimensions represent the level of the 15 "candidate" factors, each having three or four levels. The sixteenth dimension is the continuous response variable. As an illustration, a coded observation is shown in Table 8.

Table 8. Example of Sixteen-Dimensional Coded Observational Vector

	Candidate Factor Number															Response
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
Level	4	2	3	4	3	4	2	4	3	4	4	4	3	3	2	3.90

Thus, for the observation shown in Table 8 (observation number 35 in the data), a response of 3.90 was observed when factor one was at level four (not a plain weave fabric) and factor two was at level two (a drill weave) and factor three at level three (ends per inch between 91 and 113), and so forth.

A listing of all 170 coded observation vectors is included as Appendix C.

CHAPTER VII

RESULTS

The program developed in this study, in general terms, permits the Burroughs 220 Datatron System to accept data cards, one for each observation on the response variable with each card coded for the appropriate level of each candidate variable, and to calculate the number of observations, mean, sum of squares, and variance in each level of each factor. These initial computations provide the desired information for one-way factor-level classification.

The data concerning observations falling into each level are actually sorted into separate portions of the computer's internal storage, one factor at a time. As the initial one-way calculations are performed, the data associated with the first level of factor one are transferred to magnetic tape, followed in sequence by the observations coded with levels two, three, and four respectively. The computer then considers each of the remaining factors in turn. Thus, as the single factor-level means and variances are calculated, the complete file of data is written on magnetic tape as many times as there are factors being considered. Each time the data are written on tape, they are, in effect, grouped according to the levels of the factor being considered.

To obtain the means and variances for the two-way combinations, the groups of observations, each representing all the observations classified with a particular level of one factor, are brought back into internal storage one at a time. In this phase, the data from a group are treated in a manner similar to the original file of data. That is, the data in a group are sorted according to the levels of other factors, one at a time. In order to avoid duplication of calculations, the data for a level of a particular factor, say arbitrarily numbered w, are sorted for only those factors designated with a number larger than w. For example, the data for the levels of factor two need not be sorted according to the levels of factor one since, in previous manipulations of the data, groups for the levels of factor one were subdivided according to the levels of factor two.

The second series of sortings provides the opportunity for calculating the number of observations, mean, sum of squares, and variance of the dependent variable for each two-way factor-level combination.

At the option of the user, the data may again be placed on magnetic tape in groups which represent the observations falling into cells formed by each two-way combination of factor levels. As before, these may be passed through the arithmetic unit of the computer in order to obtain, through the same sorting action, the means and variances for each possible three-way combination.

The results of these data manipulations and calculations may be printed out or punched into card form at the option of the user. The form of these results is as shown in Appendix D.

The two-factor combination results become more meaningful to the experimenter when presented in the form of a b x c table, where b and c are the number of levels into which the candidate factors are subdivided. As discussed in Chapter IV, by considering the table as a surface viewed from above, and by considering the values within the table as representing the estimated heights of the surface above each cell of the table, a mental image of the estimates of the surface emerges.

The variance surfaces for each two-factor combination were considered first. A 4 x 4 or, where applicable, a 4 x 3 table for each two-factor combination was filled by using the appropriate cell variances which had been calculated and printed out as discussed earlier.

The estimate of overall surface variance was determined by weighting each within cell variance, as outlined in Chapter V, by N_h / N which was determined by the ratio of the area of the h^{th} cell relative to the total area of all cells included in the table and for which at least two observations were available. In so doing, it was assumed that each value of a candidate variable between the highest and the lowest observed values was equally possible. For discrete variables, each observed level was assumed equally possible.

For example, the class intervals for factor ten were equal and the class intervals for factor twelve were approximately equal, as shown in Table 7. The development of cell weighting factors for this factor combination is shown in Table 9.

Since each cell in this factor combination is approximately equally possible, the cell weighting factors are approximately equal.

Table 9. Cell Weighting Factors (N_h), Combination of Factors Ten and Twelve

Factor 12 (Loom Speed)	Class Interval	Class Width	Class Weight	Factor 10 (Square Root of Warp Yarn Number) Level 1	Level 2	Level 3	Level 4
	Class Interval			3.78 - 4.29	4.30 - 4.81	4.82 - 5.33	5.34 - 5.85
		Class Width		.52	.52	.52	.52
			Class Weight	.25	.25	.25	.25
Level 1	152 - 164	13	.265	.066	.066	.066	.066
Level 2	165 - 176	12	.245	.061	.061	.061	.061
Level 3	177 - 188	12	.245	.061	.061	.061	.061
Level 4	189 - 200	12	.245	.061	.061	.061	.061

Since all factor-level combinations are not necessarily represented by two or more observations, relative weighting factors were used. In the case shown in Table 9 where each cell was approximately equally possible, a relative weighting factor of unity was used for each cell.

As an example of the case where relative cell weights other than unity were necessary, Tables 10 and 11 show the development of cell weighting factors for the combination of factors six and ten.

A summary of the overall surface variance for each factor combination is shown in Table 12. Recall that the smaller the remaining within-cell variance, the greater the variation which may be attributed

Table 10. Cell Weighting Factors (N_h), Combination of Factors Six and Ten

Factor 6 (Picks/Inch)	Class Interval	Class Width	Class Weight	Factor 10 (Square Root of Warp Yarn Number) Level 1	Level 2	Level 3	Level 4
	Class Interval			3.78 - 4.29	4.30 - 4.81	4.82 - 5.33	5.34 - 5.85
		Class Width		.52	.52	.52	.52
			Class Weight	.25	.25	.25	.25
Level 1	26-43	18	.1698	.042	.042	.042	.042
Level 2	44-61	18	.1698	.042	.042	.042	.042
Level 3	62-79	18	.1698	.042	.042	.042	.042
Level 4	80-131	52	.4906	.123	.123	.123	.123

Table 11. Relative Cell Weighting Factors (N_h), Combination of Factors Six and Ten

Factor 6 (Picks/Inch)	Factor 10 (Square Root of Warp Yarn Number) Level 1	Level 2	Level 3	Level 4
Level 1	1	1	1	1
Level 2	1	1	1	1
Level 3	1	1	1	1
Level 4	3	3	3	3

Table 12. Summary of Estimates of Overall Response Surface Variance for Each Factor Combination

Factor	Estimate of Response Surface Variance Factor						
	1	2	3	4	5	6	7
1							
2	9.95						
3	13.09	8.31					
4	10.63	7.35	10.86				
5	13.65	9.31	10.84	8.83			
6	5.02	7.55	12.73	9.35	11.33		
7	12.05	10.79	11.83	6.57	9.33	11.70	
8	6.88	7.65	5.75	6.61	12.48	7.26	11.83
9	7.73	9.55	7.30	7.31	8.65	9.96	12.13
10	5.70	8.03	5.83	6.58	9.22	4.93	11.12
11	7.28	7.42	6.69	7.32	9.13	10.53	10.30
12	6.80	8.92	13.85	8.81	12.82	6.97	11.78
13	9.56	10.66	10.19	7.04	11.34	7.33	12.57
14	11.14	8.76	9.73	9.77	8.59	14.81	8.42
15	8.85	6.23	6.28	5.39	5.47	4.57	6.32

Table 12 (Continued)

Factor	Estimate of Response Surface Variance Factor						
	8	9	10	11	12	13	14
9	12.40						
10	11.81	8.10					
11	12.19	8.92	11.45				
12	10.60	11.25	9.36	8.09			
13	12.02	11.59	9.98	16.57	11.18		
14	6.76	7.51	7.14	7.81	10.08	13.62	
15	9.77	4.57	7.94	6.20	7.03	6.47	4.59

to the independent variables forming the combination and to their interaction or to other independent variables correlated with the factors forming the combination. The mean value of the surface variances for all two-factor combinations of which each factor is a part is shown in Table 13. Factor fifteen is associated with the surfaces having the lowest residual variance and is thus suspect as having the largest main effect. This had not been anticipated. Factors twelve through fifteen had been included merely because the data had been available. Factors four, two, and six exhibited the next smallest mean residual variances.

On the other hand, factors seven, twelve, and thirteen were associated with the surfaces having the largest residual variances. Since the removal of the sum of squares due to the main effects of these variables reduced the overall response surface variance by a small amount relative to the reduction effected by removing the sum of squares of other candidate factors, these factors would be the first to be dropped from consideration by the experimenter, provided he agrees to adopt the conservative criterion of minimizing the maximum possible risk of an incorrect decision. (31, p. 471, 481)

Tables 14, 15, and 16 present the estimates of the three response surfaces having the smallest overall within-cell variances among the 105 such two-factor variance surfaces investigated.

Each of these surfaces having a relatively small residual variance has factor fifteen as one component of the two-factor combination.

In picturing these tables as surfaces viewed from above, as discussed in Chapter IV, a decrease in the response is noted in each surface as the level of factor fifteen increases.

Particular attention is called to the surface represented in Table 16. In addition to a pronounced negative slope as the level of factor fifteen increases, there is also an apparent decrease in the response with an increase in the factor level for factor fourteen in levels one and two of factor fifteen.

Table 13. Mean of Estimates of Response Surface Variances for All Two-Factor Combinations of Which Each Factor Is a Part

Factor Number	Description	Mean Surface Variance	Rank
1	Fabric	9.16	7
2	Weave	8.60	4
3	Ends/Inch (Special)	9.52	10
4	Ends/Inch	8.03	2
5	Picks/Inch (Special)	9.95	13
6	Picks/Inch	8.87	5
7	Let-Off Motion	10.48	14
8	Mill	9.57	11
9	Loom Type	9.07	6
10	Square Root of Warp Count	8.37	3
11	Square Root of Filling Count	9.28	9
12	Loom Speed	9.83	12
13	Filling Twist Mult.	10.72	15
14	Cloth Width	9.19	8
15	Cloth Yards /Pound	6.40	1

Table 14. Estimate of the Mean of the Response Within Each Factor-Level Combination for Factors Six and Fifteen

		Factor 15 Level 1	Level 2	Level 3	Level 4
Factor 6	Level 1	6.84	5.82	4.94	2.73
	Level 2	10.29	8.03	6.03	2.48
	Level 3	9.09	8.12	4.24	-
	Level 4	-	8.01	5.84	-

Estimated overall surface variance = 4.57 .

Table 15. Estimate of the Mean of the Response Within Each Factor-Level Combination for Factors Nine and Fifteen

		Factor 15 Level 1	Level 2	Level 3	Level 4
Factor 9	Level 1	10.58	7.72	6.03	3.72
	Level 2	-	5.96	6.07	2.88
	Level 3	8.01	7.28	4.75	2.11

Estimate of overall surface variance = 4.57.

Table 16. Estimate of the Mean of the Response Within Each Factor-Level Combination for Factors Fourteen and Fifteen

		Factor 15 Level 1	Level 2	Level 3	Level 4
Factor 14	Level 1	13.83	8.34	5.64	3.83
	Level 2	10.69	8.26	5.88	2.00
	Level 3	7.75	7.10	5.35	2.29
	Level 4	6.77	5.69	3.73	2.74

Estimate of overall surface variance = 4.59.

To the writer, this surface was the most interesting of all those estimated due to its implications concerning the nature of the response. The higher levels of factor fifteen indicate lighter weight cloth. A decrease in warp contraction with decreasing cloth weight per yard is indicated by Tables 14, 15, and 16. Factor fourteen is cloth width and the higher levels indicate wider cloth. Table 16 suggests that warp contraction tends to decrease as the cloth width increases within a given cloth weight classification. These two indications suggest the hypothesis that warp contraction increases with cloth weight per square unit of length.

The variance surface for the combination of factors fourteen and fifteen is shown in Table 17 and the number of observations falling into each cell is shown in Table 18.

Table 17. Estimate of the Variance Surface for the Combination of Factors Fourteen and Fifteen

		Factor 15 Level 1	Level 2	Level 3	Level 4
Factor 14	Level 1	3.28	8.04	4.13	2.99
	Level 2	25.66	13.39	6.94	0.82
	Level 3	0.28	8.26	2.90	0.41
	Level 4	2.63	4.35	3.48	0.02

Estimated overall surface variance = 4.59.

Table 18. Number of Observations Within Each Factor-Level Combination for Factors Fourteen and Fifteen

		Factor 15			
		Level 1	Level 2	Level 3	Level 4
Factor 14	Level 1	3	12	12	3
	Level 2	9	34	19	6
	Level 3	3	7	5	4
	Level 4	16	30	4	3

The generation of means and variances during the computer run makes available to the experimenter a wealth of information useful during the preliminary stages of experimentation. For example, assuming that the experimenter is interested in pursuing a particular hypothesis developed either prior to or during the analysis of observational data, a substantial amount of relatively quantitative information is available. In the example concerning warp contraction which was used to test the computer program, the experimenter would be particularly interested in factor fifteen after a cursory review of the results discussed above. Table 19 demonstrates the additional information which is readily available concerning this factor or any other factor considered in the analysis. From Table 19, the experimenter may plot the response against the midpoint of each interval and thereby obtain an indication of the nature and slope of the response associated with various levels of the independent factor when considered alone.

The estimate of the mean value of the response applicable to any level of any factor may be plotted against the midpoint of that level

Table 19. Example of Summary Data for Candidate Factor Fifteen

Factor 15 Cloth Yards per Pound					
	Level				Overall Surface Variance
ONE-WAY CLASSIFICATION	1	2	3	4	
Unbiased Estimate of the Variance	14.30	10.10	5.33	1.23	6.44
Mean	8.68	7.24	5.52	2.56	
Number of Observations	31	83	40	16	
Relative Weighting Factor	1	1	1	2	
TWO-WAY CLASSIFICATION					
Factor					
1	38.39	7.53	4.09	0.33	8.85
2	14.80	5.41	3.23	0.75	6.23
3	14.04	8.05	3.48	0.48	6.28
4	7.81	8.81	4.55	0.39	5.39
5	13.23	7.36	3.83	1.53	5.47
6	8.10	14.54	3.30	0.96	4.57
7	13.86	10.60	5.43	0.84	6.32
8	27.71	12.43	11.36	0.92	9.77
9	11.92	7.29	4.89	1.04	4.57
10	14.92	10.75	6.61	0.91	7.94
11	13.75	10.19	4.15	2.39	6.20
12	19.17	12.28	5.08	0.58	7.03
13	7.53	12.45	6.48	1.97	6.47
14	7.96	8.51	4.36	1.06	4.59
Mean Two-Way Variance	15.23	9.73	5.06	1.01	6.40

as the result of the data available from the one-way classification. However, the possibility of correlations among the factors cannot be ignored in viewing these data.

Table 19 also provides information concerning the overall variance within each level of factor fifteen when combined with each other factor. Level one of factor fifteen has a larger variance than the other levels. Level four shows a small variance, regardless of the factor with which combined.

The three-way factor combination proved to be impractical for the large number of factors and observations herein used to test the program. The one- and two-way factor combinations, which involved in this test case the calculation of 1740 means and variances (one for each of the four levels of 15 factors in the one-way classification and one for each of the 16 possible factor-level combinations in each of the 105 two-way combinations), required a total of one hour and fifty minutes of computer time.

However, the number of combinations of fifteen factors taken three at a time is 4.33 times greater than when taken two at a time.

$$C_3^{15} = \frac{15!}{3!\ 12!} = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1} = 4.33\ C_2^{15}$$

In addition the number of possible factor-level combinations for each three-factor combination increases from 16 to 64. Consequently, in the writer's opinion, the time required for a three-factor combination of the test data being used was prohibitive. A one-, two-, and three-way combination of only five factors was run in less than forty minutes during the testing of the program.

To demonstrate a method by which an experimenter may utilize more than four levels for one or more of the candidate factors, the class intervals of factors four and six were arbitrarily modified to create factors three and five respectively. By so doing, the combination of factors three and four, for example, is a further modification of the same factor - ends per inch - which is segmented into seven levels instead of four. A three-factor combination including factors three and four would, in effect, be a two-factor combination involving seven levels of the factor ends per inch as though represented by a single factor. Table 20 illustrates the modified intervals created by the combination of factors three and four. Table 21 gives the means

Table 20. Creation of a Seven-Interval Factor Through Combination of Two, Four-Level Factors Representing the Same Candidate Variable

			Factor 4 Ends/Inch			
		Interval	Level 1	Level 2	Level 3	Level 4
			32-55	56-79	80-103	104-127
Factor 3 Ends/Inch (Special)	Level 1	32-45	32-45			
	Level 2	46-90	46-55	56-79	80-90	
	Level 3	91-113			91-103	104-113
	Level 4	114-127				114-127

and variances for each of the seven levels of the modified factor. By using this approach, the number of levels into which a factor may be segmented for the purpose of this analysis is seven rather than four. In order to take advantage of this method, the number of factors considered must be small enough to permit practical utilization of the three-way combination.

Table 21. Estimates of Means and Variances for the Seven Levels of the Candidate Factor Ends/Inch Created by Combination of Factors Three and Four

Interval	Mean	Variance	Combination Factor 3 Level	Factor 4 Level
32-45	4.51	3.81	1	1
46-55	5.04	10.62	2	1
56-79	5.80	7.19	2	2
80-90	7.62	20.55	2	3
91-103	7.45	6.69	3	3
104-113	10.85	9.10	3	4
114-127	7.33	9.04	4	4

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

In summary, through after-the-fact stratification of observational data, through the treatment of candidate variables in pairs as in factorial design, and through utilization of an electronic computer to perform the myriad of calculations, the candidate variables are ranked according to the variation in the response which is removed when the effects of each factor are removed.

The ability to consider up to thirty candidate factors reduces the risk of overlooking an important variable. Hence the latter stages of the experiment are less susceptible to the invitiating omission of an important variable.

It is concluded that for a given commitment of resources to an experimental program, the utilization of the procedure herein developed will minimize the risk of failure of the experiment as a whole.

In addition, the organization and display in tabular form of the estimates of the mean and variance for each factor-level combination of those factor combinations having a relatively small error sum of squares, provide the experimenter with an estimate of the general contour of the response surface over the observed range of the paired factors.

As a result, the experimenter obtains an appreciation for the nature of the response surface. The risk of failing to vary factors over sufficient ranges and the risk of failing to use appropriate transformations of the candidate variables in subsequent experiments is reduced.

The method herein developed permits the user to lay the data open so as to be able, as Tukey expressed the need, "to see what they look like inside, even though they do not give definite significance levels." (19, p. 172)

The writer's recommendations for further study and investigation may be classified as theoretical and mechanical. Among the former is the determination of optimum class intervals to be used in classifying the observational data so as to obtain the greatest reduction in the residual error. After obtaining an indication of the relative importance of the candidate factors, a more detailed study using various class intervals for the several variables selected by the experimenter might prove worthwhile.

Secondly, the ever present hazard of correlations between factors hopelessly entangling the real effects gives rise to a need for a method of ranking the candidate factors after excluding the reduction in residual error which is attributable to correlation with a higher ranking factor.

Thirdly, the covariance terms in the final equation for $\sigma^2_{\overline{y}}$ might well be the subject of further investigation so as to permit a more precise statement regarding the relative magnitudes of $\sigma^2_{\overline{y}}$ and $\sigma^{2*}_{\overline{Y}}$.

From the standpoint of the mechanics of this method of analysis, an automatic means for the elimination of certain factors, based on the results of the two-way classification would reduce the amount of computer time required for performing the calculations associated with all possible three-factor combinations. For example, if an experimenter begins the analysis using X candidate variables, it is conceivable that the program might be so written as to exclude <u>automatically</u> Y of the variables prior to entering the three-factor phase. This exclusion would be based upon the relative ranking of the candidate factors.

An automatic means for displaying weighted cell variances and cell means in addition to the listing of means and variances would also be helpful. In particular, a display of automatically weighted variances in a fashion similar to that of Tables 12, 13, and 19 would be desirable.

A P P E N D I C E S

APPENDIX A

INPUT-OUTPUT DECLARATIONS

(These declarations associate with identifiers an ordered set of numbers which are read into, or out of, the computer as units.)

Input Data Set Label	Identifiers	Description
PARAM (Parameters)	M	The number of factor combinations desired. For example, if all two-way factor-level combinations are desired, M = 2.
	NBR	The number of observations available.
	FMAX	The number of factors which are to be considered. For example, if data are available on 15 "candidate" variables and it is desired to include all of these in the analysis, FMAX = 15.
	R	The number (10^v) by which the X response variable data should be divided to properly position the decimal.
	S	Same as R except for Y response variable.
	U1	The first tape unit used. Note that after the one-way factor-level classification of the data, U1 is altered to indicate the second tape unit. After the two-way factor-level classification, U1 again refers to the first unit.
	U2	The second tape unit used. After the one-way factor-level classification of the data, U2 is altered to indicate the first tape unit. After the two-way factor-level classification, U2 again refers to the second unit.
	MINM	The minimum number of observations within any cell for which the user desires to obtain a variance estimate. For example, if MINM = 5, no variances would be calculated for cells (factor-level combinations) having less than 5 observations. MINM must be ≥ 2.
	ALLEN	The number of factor-level combinations desired before stopping the program. For example, if the user planned to perform all one-way and

Input Data Set Label	Identifiers	Description
		two-way factor-level combinations, halt the program, and later continue with the three factor classification, ALLEN = 2.
OBSIN (Observation)	TOTAL	A separate number associated with each data unit. This is the number referred to as "count" in output data sets.
	DATA(1)	A 10 digit word, each digit referring to the levels of candidate factors one through ten. For example, a "3" in the first digit and a "1" in the second digit indicate level 3 of factor one and level 1 of factor two.
	DATA(2)	Same as DATA(1) except the digits refer to the levels of candidate factors eleven through twenty inclusive.
	DATA(3)	Same as DATA(1) except the digits refer to the levels of candidate factors twenty-one through thirty inclusive.
	DATA(4)	The observed value for the first response variable (X).
	DATA(5)	The observed value for the second response variable (Y).
LBLIN (Label In)	TOTAL	Same as for OBSIN
	IDIN	The identification for the factor-level combination of the data group being considered. For example, "204" identifies the observations associated with level 2 and factor four. For a two-way classification, "204413" identifies the observations associaated with level 2 of factor four and simultaneously level 4 of factor thirteen.
	NBR	Same as for PARAM.
	MORE	Not used in this program.
LBLOT (Label Out)	COUNT	Equivalent of TOTAL at the time being written on tape.
	TID	Equivalent of IDIN at the time being written on tape.
	TN	Equivalent of NBR at the time being written on tape.
	TMORE	Equivalent of MORE at the time being written on tape.

Input Data Set Label	Identifiers	Description
OBSOT (Observations Out)	COUNT	Output equivalent of TOTAL.
	TDATA (1)	Output equivalent of DATA (1).
	TDATA (2)	Output equivalent of DATA (2).
	TDATA (3)	Output equivalent of DATA (3).
	TDATA (4)	Output equivalent of DATA (4).
	TDATA (5)	Output equivalent of DATA (5).
BUMPR (Bumper)	-	A number (9999999999) used to indicate the end of the last group of data on tape.
RESLT (Result)	RID	Equivalent of IDIN at the time printed.
	RNTOT	Equivalent of NBR at the time printed.
	RMNX	Mean value of the X response.
	RSSX	Sum of squares of the X response.
	RVARX	Variance of the X response.
	RMNY	Mean value of the Y response.
	RSSY	Sum of squares of the Y response.
	RVARY	Variance of the Y response.
LONG (Used upon Detection of a Particular Error)	SK	
	M	
	RMNX	
	RSSX	As defined elsewhere.
	RVARX	
	RMNY	
	RSSY	
	RVARY	

TAPE OPERATIONS

Identifier	Description
REWND (Rewind)	Label for the external machine language statement for re-winding a specified tape unit.
MOW	Label for the external machine language statement for transferring data from internal storage to a specified tape unit using the specified output declaration.
MRD	Label for the external machine language statement for transferring data from a specified tape unit to internal storage using the specified input declaration.

Identifier	Description
SERCH	Label for the external statement for a search of a specified tape for the first word of a ten word block (TOTAL) equal to a specified value (SK).

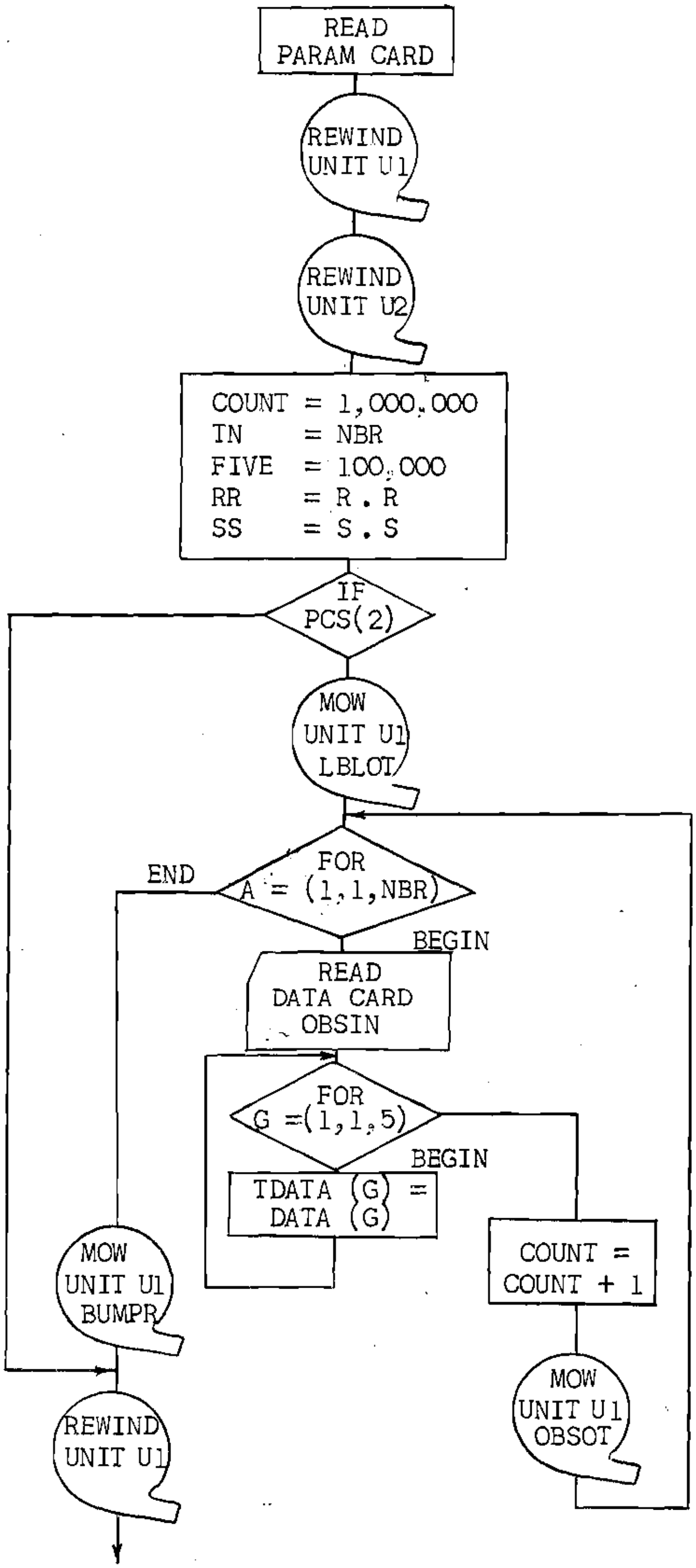
ARRAYS

(These declarations specify the structure of a collection of numbers and enable the user to refer to them with a single identifier.)

Array Identifiers	Description
W(10)	A ten dimensional vector containing specified constants used in the program.
N(4)	A four dimensional vector referring to the number of observations within each pocket or factor-level matrix in internal storage.
ID(4)	A four dimensional vector referring to the factor-level identification within each pocket or factor-level matrix in internal storage.
PKT1(110,5)	A 110 x 5 matrix reserved in internal storage for storing the observations falling into level one of the factor being considered. This matrix is referred to by the writer elsewhere as a pocket.
PKT2(110,5)	Same as PKT1 (110,5) except used for level 2.
PKT3(110,5)	Same as PKT1 (110,5) except used for level 3.
PKT4(110,5)	Same as PKT1(110,5) except used for level 4.
SUMX(4)	A four dimensional vector referring to the sum of the X response within each pocket.
SUMY(4)	Same as SUMX(4) except for Y response.
SSX(4)	A four dimensional vector referring to the sum of squares of the X response within each pocket.
SSY(4)	Same as SSX(4) except for Y response.
MNX(4)	A four dimensional vector referring to the mean of the X response within each pocket.
MNY(4)	Same as MNX(4) except for Y response.
VARX(4)	A four dimensional vector referring to the variance of the X response within each pocket.
VARY(4)	Same as VARX(4) except for Y response.

Array Identifiers	Description
DATA (5)	A five dimensional vector referring to the 5 word data unit DATA (1), DATA (2), DATA (3), DATA (4), and DATA (5).
TDATA (5)	A five dimensional vector referring to the 5 word data unit equivalent to DATA (5). TDATA (5) is used to distinguish output declarations whereas DATA (5) is used in input declarations.

Label	FLOW CHART	Explanation

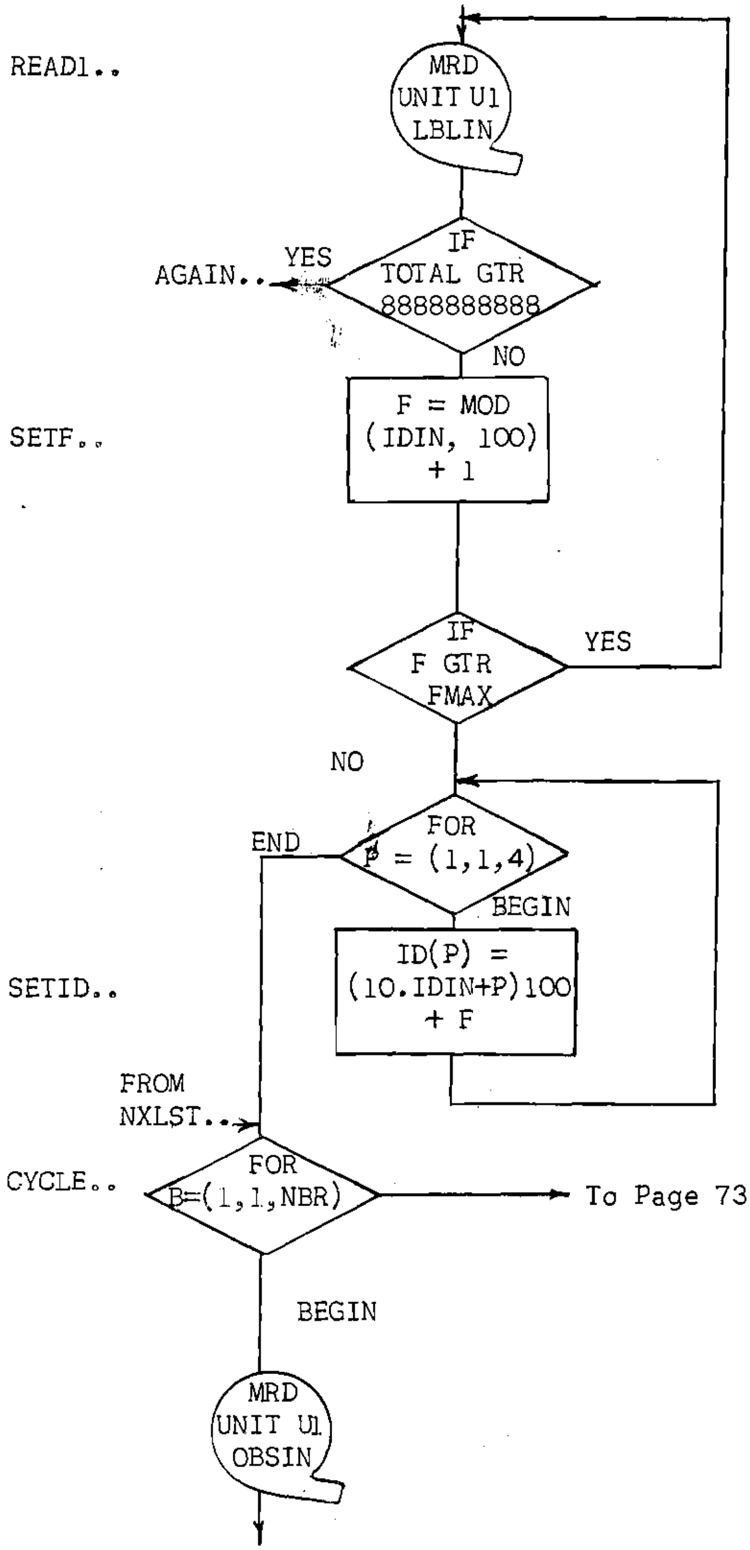


RWND..

Read from the PARAM card: M, NBR, FMAX, R, S, U1, U2, MINM, ALLEN.

Write on tape in the format specified by LBLOT (COUNT, TID,TN, TMORE).

Transfer data from cards to tape, preceded by a label and followed by "BUMPR" = 9999999999. A separate number, "COUNT," is attached to each 5 word data unit.



Read "TOTAL, IDIN, NBR, MORE."

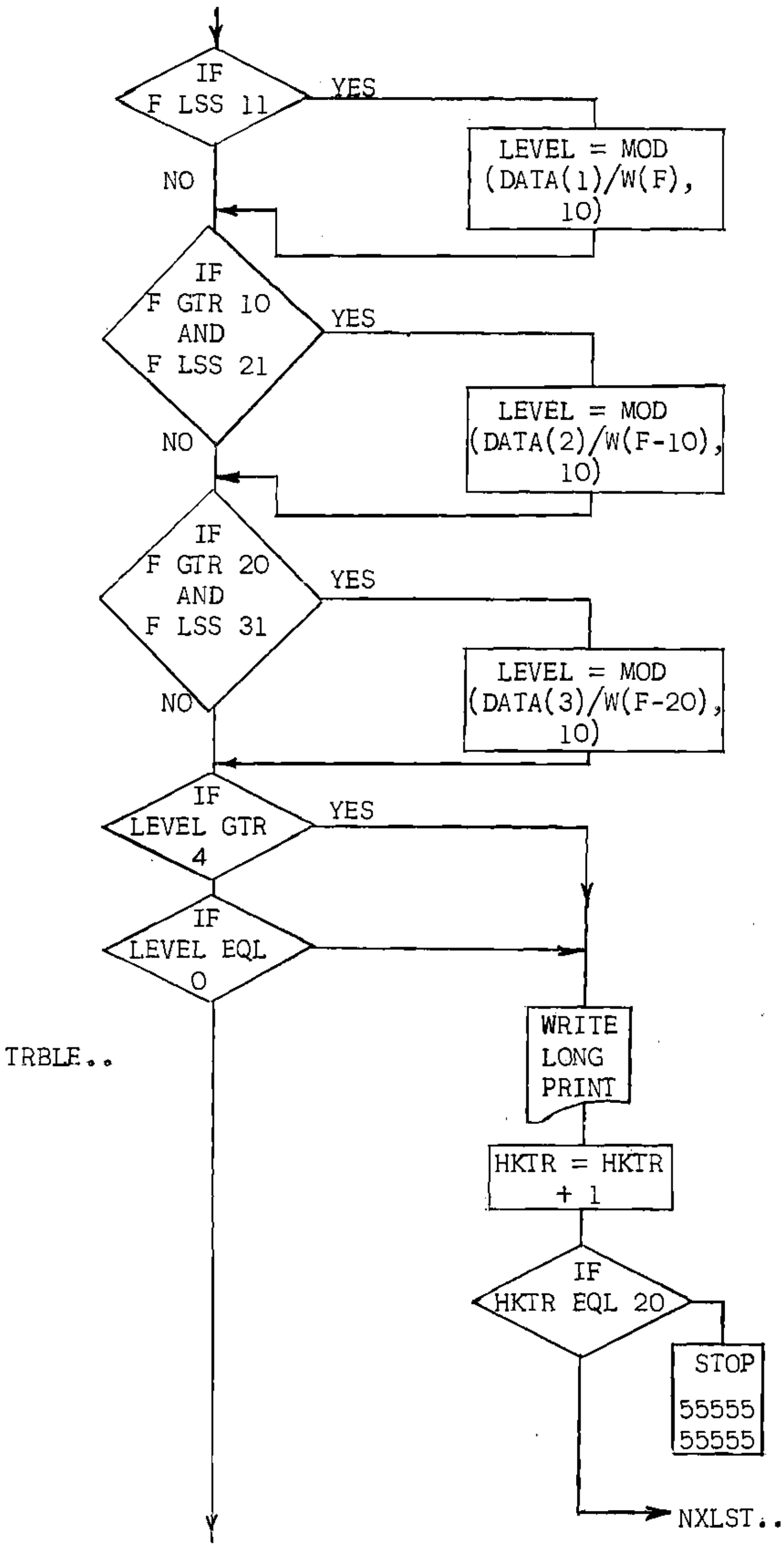
If the bumper (9999999999) is read, the analysis for each group of data on tape is complete.

Take the remainder obtained dividing IDIN by 100, consider this value as an integer; add 1. The net effect is to increase F by 1.

Proceed to the next group of data upon completion of analyses on FMAX factors.

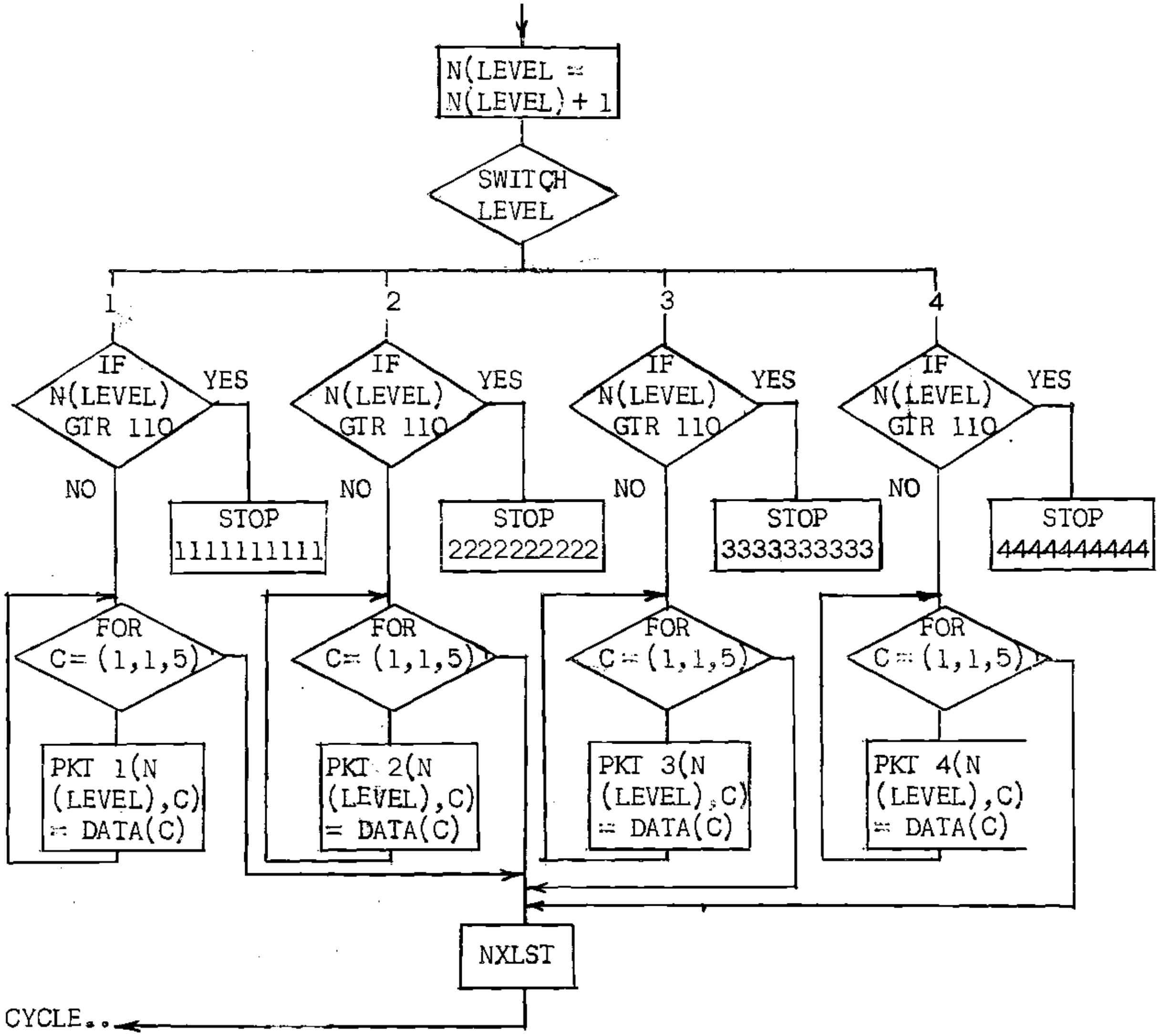
Establishes the current identification for the data which which will be put into each pocket.

Commence reading each of the NBR 5 word data units from tape into internal storage.



These steps detect the level for each observation of the factor currently being analyzed.

These steps detect errors in the data or in transfer of the data.

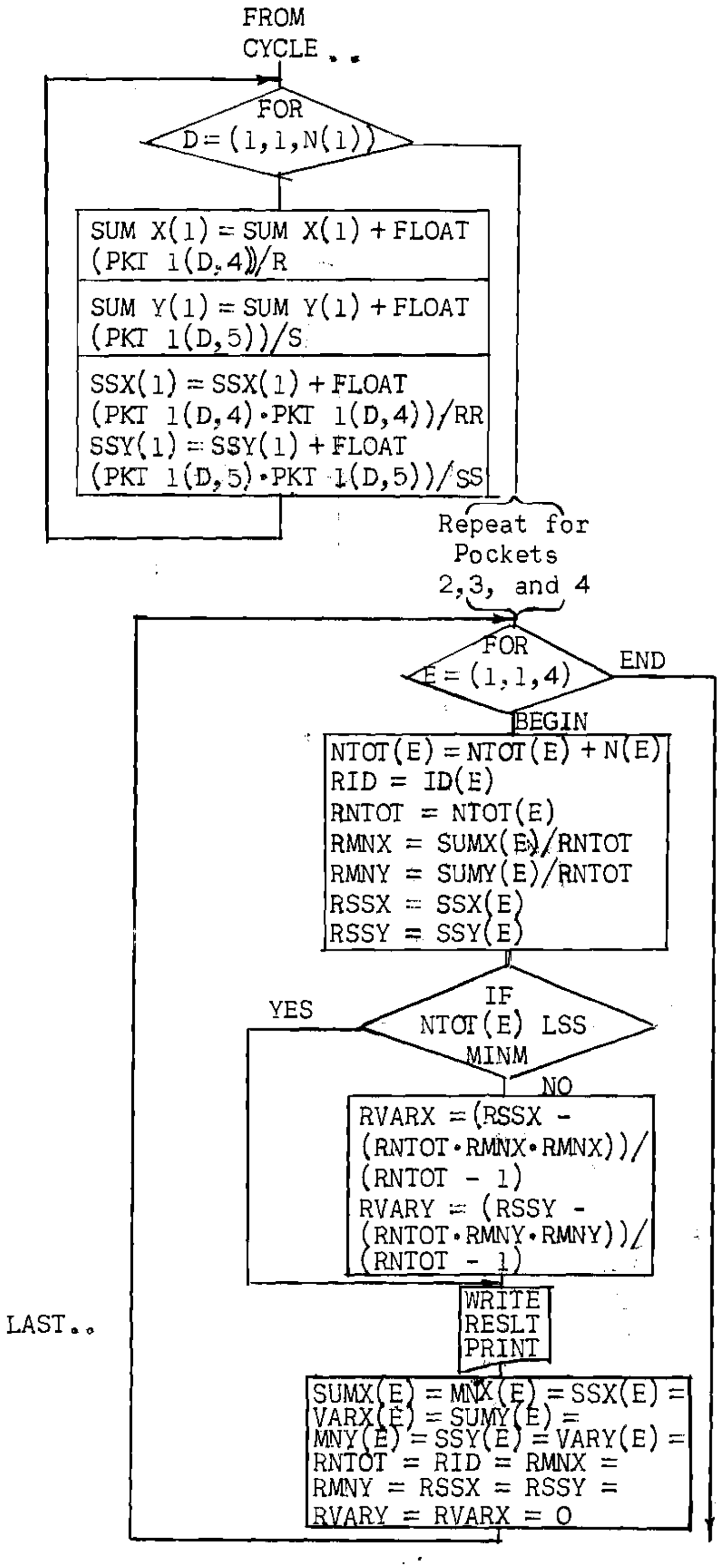


Add 1 to the number of observations which have been detected as having this level of the factor being considered.

Transfer control to one of four statements depending on the level detected.

If the number of observations in one level exceeds the program capacity, detect the difficulty.

Store the 5 word data unit in the appropriate matrix or "pkt."



Calculate, for pocket 1, the sum of observations on the X variable and also for the Y variable as well as the sum of squares for each.

For each pocket.

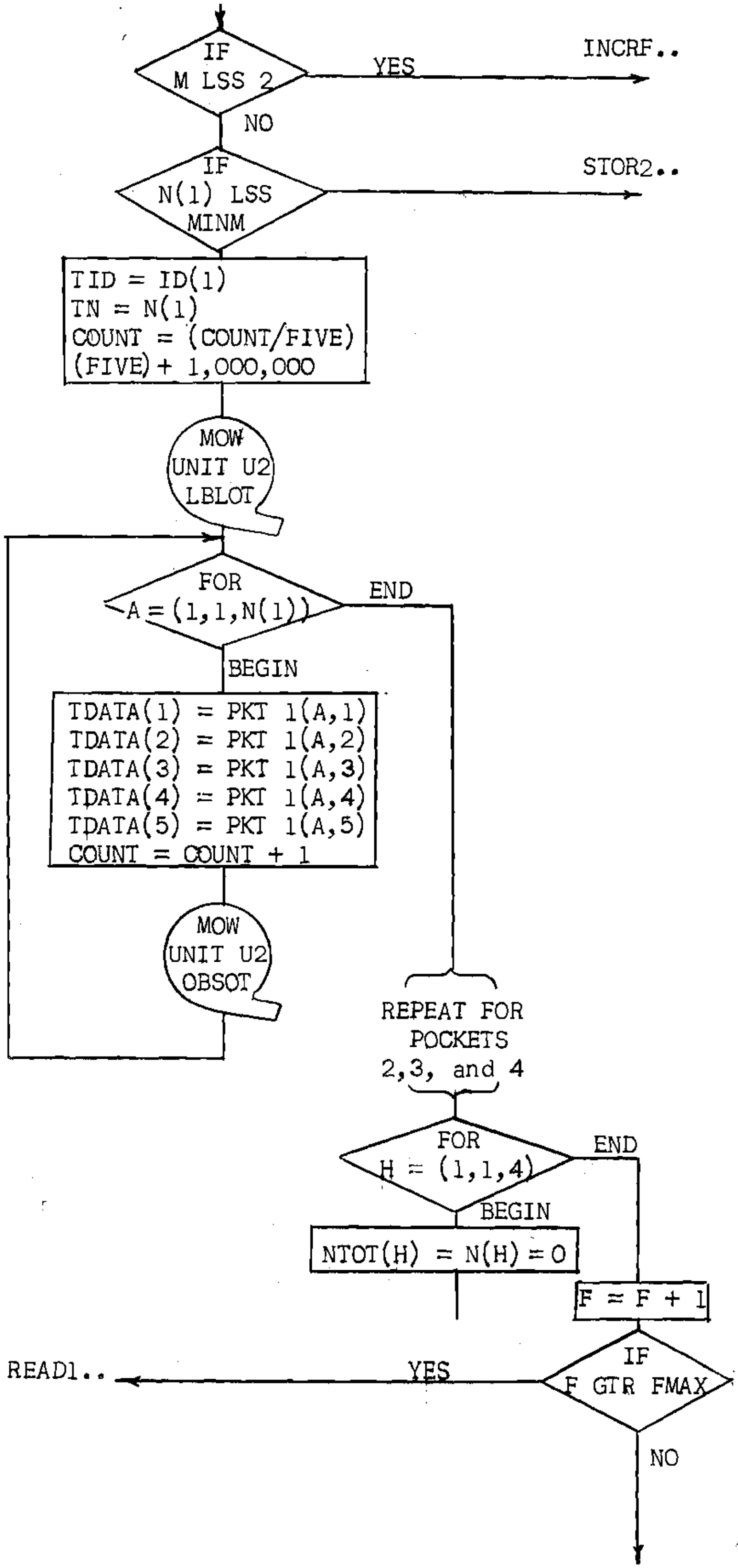
Calculate the mean and sum of squares for both response variables.

If the number of observations is less than a predetermined number, bypass the variance calculation.

Calculate the variance for both response variables.

Print the results as specified by the output declaration "RESLT" and in the format specified by "PRINT."

Clears various identifiers to O.



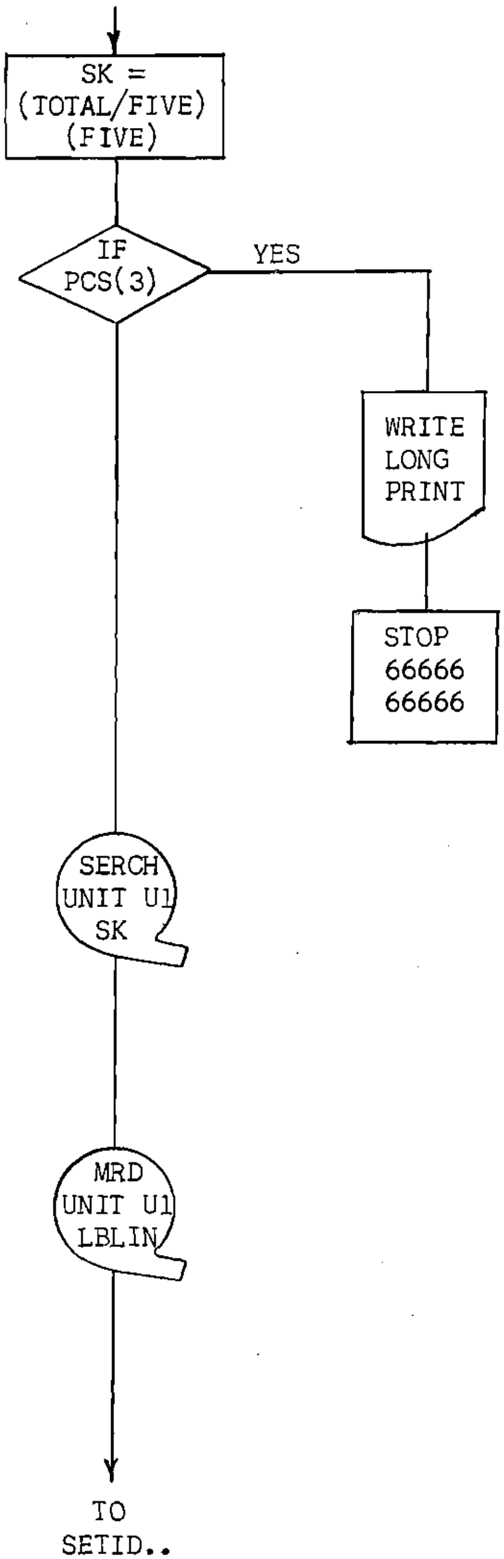
If no further subdivision of the data is desired, bypass the transfer of data to tape.

If the number of observations is less than a predetermined number, bypass the transfer of data to tape.

Transfer all 5 word data units to tape unit U2 with a label showing identification, the number of 5 word data units, and assign each a separate count.

Set the number of observations in each pocket to 0.
Increase F, the factor whose levels are being "sorted" or analyzed, by 1.
If analyses on these data are complete, proceed to start anew on the next group of data.

WHOA..

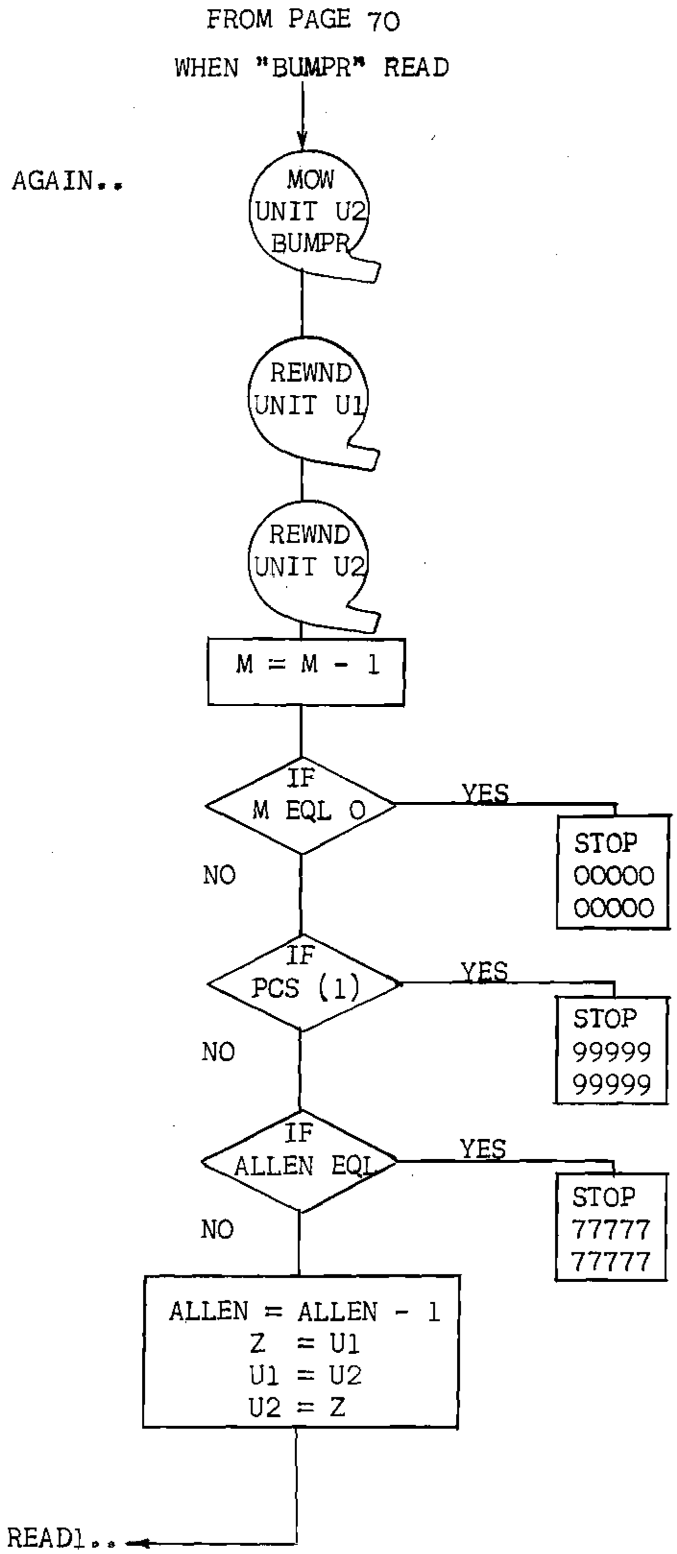


Set "SK" equal to "TOTAL" with the last five digits equal to zero.

If the machine operator has set the program control switch number 3 to "ON," print in accordance with the "LONG" declaration and format "PRINT." STOP with 6's showing in the A register.

Tape search for the data group on tape having "TOTAL" equal to "SK." This is the data group currently being analyzed. In effect this positions the tape back to the label of the current data group.

Read in accordance with declaration LBLIN (Label in).



Write "BUMPR" on tape unit 2.

Reduce by one the number of subdivisions yet to perform.

If all subdivisions are complete, stop with zero's in the A register.

If program control switch 1 is on, stop with 9's in the A register.

If "ALLEN" = 1, stop with 7's in the A register.

Reduce "ALLEN" by one. Reverse tape unit designations. The unit which was formerly U1 is now U2 and vice versa. Start over again.

APPENDIX B

SYMBOLIC ALGOL PROGRAM

```
2                        COMMENT
2        THIS PROGRAM CALCULATES THE MEAN, UNBIASED ESTIMATE OF
2        THE VARIANCE, AND THE NUMBER OF OBSERVATIONS FALLING
2        WITHIN THE CELLS FORMED BY EACH POSSIBLE COMBINATION
2        OF THE VARIOUS LEVELS OF EACH INDEPENDENT VARIABLE.
2        THE PROGRAM PROVIDES FOR AS MANY AS 30 INDEPENDENT VARIABLES
2        AND TWO DEPENDENT VARIABLES.  IT ALSO ALLOWS FOR
2        ONE, TWO, OR THREE WAY CLASSIFICATIONS AT THE OPTION OF THE
2        USER.  THE TOTAL NUMBER OF OBSERVATIONS WITHIN ANY
2        CLASSIFICATION MUST NOT EXCEED 110.                  $COMMENT
2        FLOATING MNX( ),SUMX( ),SSX( ),VARX( ),MNY( ),SUMY( ),
2           SSY( ),VARY( ),RMNX,RMNY,RVARX,RVARY,RSSX,RSSY     $  200
2        INTEGER OTHERWISE                                     $  201
2        ARRAY W(10) = (1000000000,100000000,10000000,1000000,
2           100000,10000,1000,100,10,1),N(4),
2        NTOT(4),ID(4),PKT1(110,5),PKT2(110,5),PKT3(110,5),
2           PKT4(110,5),SUMX(4),SUMY(4),SSX(4),SSY(4),MNX(4),
2           MNY(4),VARX(4),VARY(4),DATA(5),TDATA(5)            $  206
2EXTERNAL PROCEDURE MOW(U$$L1)                                 $
2EXTERNAL PROCEDURE MRD(U$$L1)                                 $
2EXTERNAL PROCEDURE SERCH(U,L,SK)                              $
2EXTERNAL PROCEDURE REWND(U,L)                                 $
2        INPUT PARAM (M,NBR,FMAX, R, S, U1, U2, MINM,ALLEN)    $  207
2        INPUT OBSIN (TOTAL, FOR C = (1,1,5) $ DATA(C))        $  208
2        INPUT LBLIN (TOTAL, IDIN, NBR, MORE)                  $  209
2        OUTPUT LBLOT (COUNT, TID, TN, TMORE)                  $  210
2        OUTPUT OBSOT (COUNT, FOR C = (1,1,5) $ TDATA(C))      $  211
2        OUTPUT BUMPR (9999999999)                             $  212
2        OUTPUT RESLT(RID,RNTOT,RMNX,RSSX,RVARX,RMNY,RSSY,RVARY) $  213
2 OUTPUT LONG (SK,M,RMNX,RSSX,RVARX,RMNY,RSSY,RVARY)           $  213 1
2        FORMAT PRINT (I9, I4, B1, 6F11.5, WO)                 $  214
2        READ ( $$ PARAM)                                      $    1
2        REWND (U1,1)                                          $    2
2        REWND (U2,1)                                          $    3
2        COUNT = 1000000                                       $    4
2        TN = NBR                                              $    4 1
2        FIVE = 100000                                         $    4 2
2        RR = R . R                                            $    4 3
2        SS = S . S                                            $    4 4
2        IF PCS(2)                                             $    4 5
2        GO TO RWND                                            $    4 6
2        MOW (U1 $$ LBLOT)                                     $    5
2        FOR A = (1,1,NBR)                                     $    6
2        BEGIN                                                 $    7
2        READ ( $$ OBSIN)                                      $    8
2        FOR G = (1,1,5)                                       $    9
2        TDATA(G) = DATA(G)                                    $   10
```

```
2          COUNT = COUNT + 1                                    $    11
2          MOW (U1 $$ OBSOT)                                    $    12
2          END                                                  $    13
2          MOW (U1 $$ BUMPR)                                    $    14
2 RWND.. REWND (U1,1)                                           $    15
2 READ1..MRD (U1 $$ LBLIN)                                      $    16
2          IF (TOTAL GTR 8888888888)                            $    17
2          GO TO AGAIN                                          $    18
2 SETF.. F = MOD (IDIN, 100) + 1                                $    19
2          IF (F GTR FMAX)                                      $    20
2          GO TO READ1                                          $    21
2 SETID..FOR P = (1,1,4)                                        $    22
2          ID(P) = (10.IDIN + P) 100 + F                        $    23
2 CYCLE..FOR B = (1,1,NBR)                                      $    24
2          BEGIN                                                $    25
2          MRD (U1 $$ OBSIN)                                    $    26
2          IF (F LSS 11)                                        $    27
2          LEVEL = MOD (DATA(1)/W(F),10)                        $    28
2          IF (F GTR 10) AND (F LSS 21)                         $    29
2          LEVEL = MOD (DATA(2)/W(F - 10), 10)                  $    30
2          IF (F GTR 20) AND (F LSS 31)                         $    31
2          LEVEL = MOD (DATA(3)/W(F - 20), 10)                  $    32
2          IF (LEVEL GTR 4)                                     $    32 1
2          GO TO TRBLE                                          $    32 2
2          IF (LEVEL EQL 0)                                     $    32 3
2          GO TO TRBLE                                          $    32 4
2          N(LEVEL) = N(LEVEL) + 1                              $    33
2          SWITCH LEVEL, (PA, PB, PC, PD)                       $    34
2 PA..     IF (N(LEVEL) GTR 120)                                $    35
2          STOP  1111111111                                     $    36
2          FOR C = (1,1,5)                                      $    37
2          PKT1 (N(LEVEL), C) = DATA(C)                         $    38
2          GO TO NXLST                                          $    39
2 PB..     IF (N(LEVEL) GTR 120)                                $    40
2          STOP  2222222222                                     $    41
2          FOR C = (1,1,5)                                      $    42
2          PKT2 (N(LEVEL), C) = DATA(C)                         $    43
2          GO TO NXLST                                          $    44
2 PC..     IF (N(LEVEL) GTR 120)                                $    45
2          STOP  33333333                                       $    46
2          FOR C = (1,1,5)                                      $    47
2          PKT3 (N(LEVEL),C) = DATA(C)                          $    49
2          GO TO NXLST                                          $    51
2 PD..     IF (N(LEVEL) GTR 120)                                $    52
2          STOP  44444444                                       $    53
2          FOR C = (1,1,5)                                      $    54
2          PKT4 (N(LEVEL), C) = DATA(C)                         $    56
2 NXLST..                                                       $    58
2          END                                                  $    59
```

```
2          IF (MORE EQL 1)
2          GO TO MO
2          FOR D = (1,1,N(1))
2          BEGIN
2          SUMX(1) = SUMX(1) + FLOAT (PKT1(D,4))/R
2          SUMY(1) = SUMY(1) + FLOAT (PKT1(D,5))/S
2          SSX(1) = SSX(1) + FLOAT (PKT1(D,4).PKT1(D,4))/RR
2          SSY(1) = SSY(1) + FLOAT (PKT1(D,5).PKT1(D,5))/SS
2          END
2          FOR D = (1,1,N(2))
2          BEGIN
2          SUMX(2) = SUMX(2) + FLOAT (PKT2(D,4))/R
2          SUMY(2) = SUMY(2) + FLOAT (PKT2(D,5))/S
2          SSX(2) = SSX(2) + FLOAT (PKT2(D,4).PKT2(D,4))/RR
2          SSY(2) = SSY(2) + FLOAT (PKT2(D,5).PKT2(D,5))/SS
2          END
2          FOR D = (1,1,N(3))
2          BEGIN
2          SUMX(3) = SUMX(3) + FLOAT (PKT3(D,4))/R
2          SUMY(3) = SUMY(3) + FLOAT (PKT3(D,5))/S
2          SSX(3) = SSX(3) + FLOAT (PKT3(D,4).PKT3(D,4))/RR
2          SSY(3) = SSY(3) + FLOAT (PKT3(D,5).PKT3(D,5))/SS
2          END
2          FOR D = (1,1,N(4))
2          BEGIN
2          SUMX(4) = SUMX(4) + FLOAT (PKT4(D,4))/R
2          SUMY(4) = SUMY(4) + FLOAT (PKT4(D,5))/S
2          SSX(4) = SSX(4) + FLOAT (PKT4(D,4).PKT4(D,4))/RR
2          SSY(4) = SSY(4) + FLOAT (PKT4(D,5).PKT4(D,5))/SS
2          END
2          FOR E = (1,1,4)
2          BEGIN
2          NTOT(E) = NTOT(E) + N(E)
2          RID = ID(E)
2          RNTOT = NTOT(E)
2          RMNX = SUMX(E)/RNTOT
2          RMNY = SUMY(E)/RNTOT
2          RSSX = SSX(E)
2          RSSY = SSY(E)
2          IF (NTOT(E) LSS MINM)
2          GO TO LAST
2          RVARX = (RSSX - (RNTOT.RMNX.RMNX))/(RNTOT - 1)
2          RVARY = (RSSY - (RNTOT.RMNY.RMNY))/(RNTOT - 1)
2 LAST..   WRITE ( $$ RESLT, PRINT)
2          SUMX(E) = MNX(E) = SSX(E) = VARX(E) = 0
2          SUMY(E) = MNY(E) = SSY(E) = VARY(E) = 0
2          RNTOT = RID = RMNX = RMNY = 0
2          RSSX = RSSY = RVARY = RVARX = 0
2          END
```

```
2         IF (M LSS 2)                                    $
2         GO TO INCRF                                     $
2         IF (N(1) LSS MINM)                              $
2         GO TO STOR2                                     $
2         TID = ID(1)                                     $
2         TN = N(1)                                       $
2         COUNT = (COUNT/FIVE) (FIVE) + 1000000           $
2         MOW (U2 $$ LBLOT)                               $
2         FOR A = (1,1,N(1))                              $
2         BEGIN                                           $
2         TDATA(1) = PKT1(A,1)                            $
2         TDATA(2) = PKT1(A,2)                            $
2         TDATA(3) = PKT1(A,3)                            $
2         TDATA(4) = PKT1(A,4)                            $
2         TDATA(5) = PKT1(A,5)                            $
2         COUNT = COUNT + 1                               $
2         MOW (U2 $$ OBSOT)                               $
2         END                                             $
2 STOR2..IF (N(2) LSS MINM)                               $
2         GO TO STOR3                                     $
2         TID = ID(2)                                     $
2         TN = N(2)                                       $
2         COUNT = (COUNT/FIVE) (FIVE) + 1000000           $
2         MOW (U2 $$ LBLOT)                               $
2         FOR A = (1,1,N(2))                              $
2         BEGIN                                           $
2         TDATA(1) = PKT2(A,1)                            $
2         TDATA(2) = PKT 2(A,2)                           $
2         TDATA(3) = PKT2(A,3)                            $
2         TDATA(4) = PKT2(A,4)                            $
2         TDATA(5) = PKT2(A,5)                            $
2         COUNT = COUNT + 1                               $
2         MOW (U2 $$ OBSOT)                               $
2         END                                             $
2 STOR3..IF (N(3) LSS MINM)                               $
2         GO TO STOR4                                     $
2         TID = ID(3)                                     $
2         TN = N(3)                                       $
2         COUNT = (COUNT/FIVE) (FIVE) + 1000000           $
2         MOW (U2 $$ LBLOT)                               $
2         FOR A = (1,1,N(3))                              $
2         BEGIN                                           $
2         TDATA(1) = PKT3(A,1)                            $
2         TDATA(2) = PKT3(A,2)                            $
2         TDATA(3) = PKT3(A,3)                            $
2         TDATA(4) = PKT3(A,4)                            $
2         TDATA(5) = PKT3(A,5)                            $
2         COUNT = COUNT + 1                               $
```

```
2         MOW (U2 $$ OBSOT)                               $  157
2         END                                             $  158
2 STOR4..IF (N(4) LSS MINM)                               $  159
2         GO TO INCRF                                     $  160
2         TID = ID(4)                                     $  161
2         TN = N(4)                                       $  162
2         COUNT = (COUNT/FIVE) (FIVE) + 1000000           $  163
2         MOW (U2 $$ LBLOT)                               $  164
2         FOR A = (1,1,N(4))                              $  165
2         BEGIN                                           $  166
2         TDATA(1) = PKT4(A,1)                            $  167
2         TDATA(2) = PKT4(A,2)                            $  168
2         TDATA(3) = PKT4(A,3)                            $  169
2         TDATA(4) = PKT4(A,4)                            $  170
2         TDATA(5) = PKT4(A,5)                            $  171
2         COUNT = COUNT + 1                               $  172
2         MOW (U2 $$ OBSOT)                               $  173
2         END                                             $  174
2 INCRF..FOR H = (1,1,4)                                  $  175
2         NTOT(H) = N(H) = 0                              $  176
2         F = F + 1                                       $  177
2         IF (F GTR FMAX)                                 $  178
2         GO TO READ1                                     $  179
2         SK = (TOTAL/FIVE) (FIVE)                        $  179 1
2         IF PCS(3)                                       $  179 2
2         GO TO WHOA                                      $  179 3
2         SERCH (U1,1,SK)                                 $  180
2         MRD (U1 $$ LBLIN)                               $  181
2         GO TO SETID                                     $  182
2 AGAIN..MOW (U2 $$ BUMPR)                                $  183
2         REWND (U1,1)                                    $  184
2         REWND (U2,1)                                    $  185
2         M = M - 1                                       $  186
2         IF (M EQL 0)                                    $  187
2         STOP  0000000000                                $  188
2         IF PCS(1)                                       $  189
2         STOP  9999999999                                $  190
2         IF ALLEN EQL 1                                  $  190 1
2         STOP  7777777777                                $  190 2
2         ALLEN = ALLEN - 1                               $  190 3
2         Z = U1                                          $  191
2         U1 = U2                                         $  192
2         U2 = Z                                          $  193
2         GO TO READ1                                     $  194
2 MO..    MRD (U1 $$ LBLOT)                               $  196
2         GO TO CYCLE                                     $  197
2 WHOA..  WRITE ( $$ LONG, PRINT)                         $  197 1
2         STOP  6666666666                                $  197 2
2 TRBLE..WRITE ( $$ LONG, PRINT)                          $  197 3
```

```
2         HKTR = HKTR + 1                                                $
2         IF (HKTR EQL 20)                                               $
2         STOP   5555555555                                              $  197 33
2         GO TO NXLST                                                    $  197  4
2         FINISH                                                         $  198
2 MRD
6060000001000070028010000800003100028041040000980000100028000014800018111140008
6060000002000680000420009899992100088010052003000000440000100003000008000933002З
6060000003001280000100030805012600128020126002080000310017800003000108000010000 8
6060000004001880000400020804104000120000000000008000046002080000300010800001000 08
6060000005002480410400012800004600208000042000010000300000400000001024000099000 0
2         SERCH
606000000100007001301000080000400011800001000130000148000181111400008800001000 12
606000000200060000148000383211400008800005000118000042000010000300000400000000 03
60100000030012400009900000000*
2         REWND
606000000100007000901000000001480003832114000068000010000900001480001811114000 06
60500000020006000085000008000042000010000300000400000000014000099000000000000 *
2         MOW
606  MOW C1   70042010000804104000128000010004200001480001811114000238111140002 9
606  MOW C2   81111400039811114000408000044004180000300039800004200128999921001 2
606  MOW C3   00000449999100003000008000933002980401260016800004000428220126002 6
606  MOW C4   80000310019800004100268221118004280000350023800003000138010057004 2
606  MOW C5   80000440041800003000398000044003880000300034800003000138010057004 2
606  MOW C6   80000440038800003000348000042000010000300000800004100398041140001 6
606  MOW C7   80000410027822114000260000030999980100530042001015800000000030999 9
606  MOW C8   000000000004000000010040000990000000000
2         FINISH                                                         $
```

APPENDIX C

CODED OBSERVATIONAL TEST DATA

	Coded Levels of Candidate Variables			Response		Obs. No.
5 0	1134321414	32323	000	0860	*	1
5 0	1133321414	32123	000	0830	*	2
5 0	1134321414	32323	000	0910	*	3
5 0	1133321414	32313	000	0690	*	4
5 0	1133321414	42313	000	0620	*	5
5 0	1133321414	42323	000	0690	*	6
5 0	1133321414	42323	000	0540	*	7
5 0	1133322434	34323	000	0560	*	8
5 0	1134322434	34322	000	0770	*	9
5 0	1134322434	33323	000	0840	*	10
5 0	1133322434	34323	000	0820	*	11
5 0	1133322434	44323	000	0560	*	12
5 0	1133322434	34333	000	0610	*	13
5 0	3123221414	32123	000	0190	*	14
5 0	1133221414	32114	000	0580	*	15
5 0	1133222434	34123	000	0370	*	16
5 0	1144322434	24122	000	1260	*	17
5 0	1144222434	23122	000	0570	*	18
5 0	1144222434	23122	000	0670	*	19
5 0	1133222434	34133	000	0550	*	20
5 0	1133112434	24132	000	0680	*	21
5 0	3121111414	42413	000	0120	*	22
5 0	3122322434	44422	000	0260	*	23
5 0	3121222434	44423	000	0140	*	24
5 0	3122222434	44423	000	0150	*	25
5 0	3121222434	44422	000	0160	*	26
5 0	2123341414	42323	000	0510	*	27
5 0	2122322434	43324	000	0150	*	28
5 0	2122322434	43424	000	0110	*	29
5 0	2122222434	43424	000	0110	*	30
5 0	2122322434	44334	000	0160	*	31
5 0	2123342434	44333	000	0510	*	32
5 0	2122332434	44333	000	0270	*	33
5 0	2122322434	44334	000	0210	*	34
5 0	4234342434	44332	000	0390	*	35
5 0	2122321324	42424	000	0310	*	36
5 0	2122321334	41444	000	0260	*	37
5 0	1134221323	22322	000	1090	*	38
5 0	4444332333	32422	000	0470	*	39
5 0	1134321324	42423	000	1020	*	40
5 0	3111111334	21143	000	0220	*	41
5 0	1134322334	42422	000	0900	*	42
5 0	2122321334	41444	000	0290	*	43
5 0	4323441323	32322	000	0430	*	44

		Coded Levels of Candidate Variables			Response		Obs. No.
5	0	4323441324	22222	000	0390	*	45
5	0	4444321323	12121	000	0580	*	46
5	0	4323442333	32321	000	0320	*	47
5	0	3144332323	32322	000	0530	*	48
5	0	1134322333	22322	000	1240	*	49
5	0	1134321332	23322	000	1400	*	50
5	0	1134322332	22321	000	1340	*	51
5	0	1133322333	32232	000	1230	*	52
5	0	4323441323	32312	000	0410	*	53
5	0	2123342234	42433	000	0733	*	54
5	0	2122322234	42434	000	0313	*	55
5	0	2123342234	42332	000	0733	*	56
5	0	2122332234	42443	000	0641	*	57
5	0	3123342232	33422	000	1246	*	58
5	0	2122322233	12442	000	0750	*	59
5	0	2122322234	42443	000	0349	*	60
5	0	1134222233	24222	000	1562	*	61
5	0	3111222232	14132	000	0391	*	62
5	0	3111112234	22144	000	0273	*	63
5	0	2122322234	44434	000	0234	*	64
5	0	3133341232	33422	000	1484	*	65
5	0	3133341232	33412	000	1328	*	66
5	0	2121112232	12342	000	0625	*	67
5	0	1134222234	24222	000	1211	*	68
5	0	1134322233	24222	000	1445	*	69
5	0	3123321232	13421	000	2109	*	70
5	0	2121112232	12342	000	0664	*	71
5	0	4222112233	22342	000	0589	*	72
5	0	4222112233	12242	000	0589	*	73
5	0	4222112233	22342	000	0478	*	74
5	0	4233332233	34322	000	0859	*	75
5	0	4233332233	33222	000	0859	*	76
5	0	4334322233	13321	000	1289	*	77
5	0	4334322232	13321	000	1094	*	78
5	0	4423322233	22342	000	0392	*	79
5	0	4423322233	12341	000	0431	*	80
5	0	4423222233	22342	000	0469	*	81
5	0	4323111111	11312	000	0615	*	82
5	0	4323111111	11212	000	0735	*	83
5	0	4323321111	11211	000	1215	*	84
5	0	4323321111	11211	000	1360	*	85
5	0	4323321111	11211	000	1575	*	86
5	0	4323221111	21322	000	0610	*	87
5	0	4323111112	11321	000	0950	*	88

	Coded Levels of Candidate Variables			Response		Obs. No.
5 0	4323111112	11222	000	0725	*	89
5 0	4323221112	11222	000	0740	*	90
5 0	4423321111	11221	000	0885	*	91
5 0	4433321111	11241	000	1050	*	92
5 0	4444331112	21321	000	1050	*	93
5 0	4433321112	11212	000	0575	*	94
5 0	4433321112	11222	000	0520	*	95
5 0	4433321112	21322	000	0510	*	96
5 0	4433321132	11241	000	0685	*	97
5 0	4433321132	11241	000	0560	*	98
5 0	4433321132	21341	000	0545	*	99
5 0	4433321112	11231	000	0720	*	100
5 0	4433331132	11231	000	0825	*	101
5 0	4433331132	11241	000	0840	*	102
5 0	4433331132	11241	000	0920	*	103
5 0	4322221121	21322	000	0505	*	104
5 0	4322111122	11213	000	0455	*	105
5 0	4322111122	11213	000	0490	*	106
5 0	4322111122	11213	000	0600	*	107
5 0	4322111122	11213	000	0515	*	108
5 0	4322111122	11213	000	0520	*	109
5 0	4322111122	21314	000	0255	*	110
5 0	4322321121	21322	000	0575	*	111
5 0	4322321121	21322	000	0640	*	112
5 0	4221111122	21324	000	0300	*	113
5 0	4322111122	11222	000	0795	*	114
5 0	3122111122	11213	000	0650	*	115
5 0	3111113131	11141	000	0660	*	116
5 0	2111113131	11231	000	0780	*	117
5 0	3111113111	11122	000	0965	*	118
5 0	2121113111	21332	000	0850	*	119
5 0	3111113132	11142	000	0660	*	120
5 0	2111113112	31324	000	0220	*	121
5 0	3111223132	11142	000	0510	*	122
5 0	3111223132	11142	000	0535	*	123
5 0	3111223132	11142	000	0556	*	124
5 0	2111223112	11232	000	0695	*	125
5 0	2122323112	31323	000	0620	*	126
5 0	2122323112	31323	000	0735	*	127
5 0	2122333132	31342	000	0875	*	128
5 0	2122333132	31342	000	0820	*	129
5 0	2122333132	31342	000	0960	*	130
5 0	2122333132	31342	000	0940	*	131
5 0	2122333132	31342	000	0998	*	132

	Coded Levels of Candidate Variables			Response		Obs. No.
5 0	3111113131	11242	000	0290	*	133
5 0	3111113131	11242	000	0280	*	134
5 0	3111113131	11242	000	0335	*	135
5 0	3111113131	11242	000	0370	*	136
5 0	3111113131	11242	000	0365	*	137
5 0	3111113131	11242	000	0340	*	138
5 0	3111113131	11243	000	0280	*	139
5 0	3111113131	11242	000	0430	*	140
5 0	3111113131	11242	000	0340	*	141
5 0	3111113131	11242	000	0335	*	142
5 0	3111113131	11242	000	0525	*	143
5 0	3121113111	11213	000	0955	*	144
5 0	3121113111	11241	000	0720	*	145
5 0	4222333112	31323	000	0360	*	146
5 0	4222113132	11241	000	0695	*	147
5 0	4222113112	21314	000	0315	*	148
5 0	4222113132	11242	000	0615	*	149
5 0	4222113132	11242	000	0695	*	150
5 0	4222113132	11242	000	0730	*	151
5 0	4222113132	11241	000	0550	*	152
5 0	4222113112	11223	000	0465	*	153
5 0	4222113132	11241	000	0615	*	154
5 0	4222113132	11241	000	0695	*	155
5 0	4222113132	11241	000	0740	*	156
5 0	4222113132	11241	000	0450	*	157
5 0	4222113112	21313	000	0410	*	158
5 0	4222223111	11222	000	1055	*	159
5 0	4222323131	13312	000	1115	*	160
5 0	4222323111	11212	000	1265	*	161
5 0	4222113111	11212	000	0640	*	162
5 0	4222113111	11212	000	0800	*	163
5 0	4222113131	21341	000	0670	*	164
5 0	4222113111	11212	000	0695	*	165
5 0	4222113111	11322	000	0710	*	166
5 0	4222113111	11213	000	0740	*	167
5 0	4222223111	11212	000	0895	*	168
5 0	4222223111	11222	000	0915	*	169
5 0	4222223111	11212	000	0930	*	170

APPENDIX D

EXAMPLES OF PRINTED RESULTS

EXAMPLE OF PRINTED RESULTS FOR ONE-WAY

FACTOR-LEVEL CLASSIFICATION

Level and Factor	Obs. No.	Mean	Sum of Squares	Variance
101	30	.88060, 01	.26293, 04	.10447, 02
201	31	.52983, 01	.11137, 04	.81162, 01
301	34	.55005, 01	.16646, 04	.19271, 02
401	75	.68885, 01	.40907, 04	.71876, 01
102	95	.64784, 01	.54077, 04	.15113, 02
202	32	.68215, 01	.16537, 04	.53110, 01
302	25	.71232, 01	.15573, 04	.12035, 02
402	18	.66816, 01	.87969, 03	.44757, 01
103	23	.45021, 01	.55007, 03	.38123, 01
203	95	.61845, 01	.47004, 04	.11349, 02
303	45	.86602, 01	.38178, 04	.10065, 02
403	7	.73285, 01	.43021, 03	.90423, 01
104	32	.46528, 01	.86347, 03	.55070, 01
204	62	.57961, 01	.25215, 04	.71916, 01
304	53	.75269, 01	.36630, 04	.12697, 02
404	23	.97786, 01	.24504, 04	.11416, 02
105	59	.55547, 01	.20599, 04	.41293, 01
205	26	.62150, 01	.13262, 04	.12877, 02
305	81	.77440, 01	.60516, 04	.14925, 02
405	4	.38750, 01	.60750, 02	.22916, 00
106	59	.55547, 01	.20599, 04	.41293, 01
206	83	.71969, 01	.56347, 04	.16288, 02
306	16	.76356, 01	.10165, 04	.55790, 01
406	12	.70700, 01	.78727, 03	.17041, 02
107	59	.71815, 01	.38572, 04	.14040, 02
207	56	.62730, 01	.29974, 04	.14432, 02
307	55	.64925, 01	.26438, 04	.60268, 01
407	0	.00000, 00	.00000, 00	.00000, 00
108	89	.67661, 01	.46739, 04	.68113, 01
208	28	.82642, 01	.25157, 04	.22349, 02
308	18	.69055, 01	.11586, 04	.17664, 02
408	35	.49771, 01	.11502, 04	.83288, 01
109	49	.74142, 01	.30926, 04	.88130, 01
209	20	.55300, 01	.69945, 03	.46230, 01
309	01	.65167, 01	.57064, 04	.14172, 02
409	0	.00000, 00	.00000, 00	.00000, 00
110	40	.72537, 01	.25026, 04	.10203, 02

(Continued next page)

Level and Factor	Obs. No.	Mean	Sum of Squares	Variance
210	59	.72686, 01	.37616, 04	.11111, 02
310	21	.76247, 01	.15241, 04	.15163, 02
410	50	.50594, 01	.17101, 04	.87815, 01
111	77	.71400, 01	.46498, 04	.95323, 01
211	31	.72625, 01	.21144, 04	.15977, 02
311	33	.73557, 01	.21007, 04	.98497, 01
411	29	.39458, 01	.63351, 03	.64997, 01
112	91	.65795, 01	.45696, 04	.70018, 01
212	40	.62072, 01	.18613, 04	.82095, 01
312	15	.95826, 01	.18291, 04	.32269, 02
412	24	.58883, 01	.12384, 04	.17663, 02
113	19	.58157, 01	.75946, 03	.64902, 01
213	64	.72603, 01	.39834, 04	.96811, 01
313	64	.65946, 01	.33311, 04	.86952, 01
413	23	.58639, 01	.14244, 04	.28798, 02
114	30	.73560, 01	.19797, 04	.12291, 02
214	68	.73614, 01	.48021, 04	.16673, 02
314	19	.57284, 01	.75978, 03	.75723, 01
414	53	.56979, 01	.19568, 04	.45416, 01
115	31	.86832, 01	.27662, 04	.14297, 02
215	83	.72413, 01	.51804, 04	.10100, 02
315	40	.55245, 01	.14287, 04	.53327, 01

EXAMPLE OF PRINTED RESULTS FOR TWO-WAY

FACTOR-LEVEL CLASSIFICATION

Level and Factor	Obs. No.	Mean	Sum of Squares	Variance
101102	30	.88060, 01	.26293, 04	.10447, 02
101202	0	.00000, 00	.00000, 00	.00000, 00
101302	0	.00000, 00	.00000, 00	.00000, 00
101402	0	.00000, 00	.00000, 00	.00000, 00
101103	0	.00000, 00	.00000, 00	.00000, 00
101203	0	.00000, 00	.00000, 00	.00000, 00
101303	27	.88585, 01	.23932, 04	.10555, 02
101403	3	.83333, 01	.23614, 03	.13903, 02
101104	0	.00000, 00	.00000, 00	.00000, 00
101204	0	.00000, 00	.00000, 00	.00000, 00
101304	14	.66642, 01	.67399, 03	.40163, 01
101404	16	.10680, 02	.19553, 04	.86913, 01
101105	1	.68000, 01	.46240, 02	.00000, 00
101205	8	.82537, 01	.66440, 03	.17058, 02
101305	21	.91119, 01	.19187, 04	.87574, 01
101405	0	.00000, 00	.00000, 00	.00000, 00
101106	1	.68000, 01	.46240, 02	.00000, 00
101206	29	.88751, 01	.25831, 04	.10672, 02
101306	0	.00000, 00	.00000, 00	.00000, 00
101406	0	.00000, 00	.00000, 00	.00000, 00
101107	11	.83909, 01	.84097, 03	.66489, 01
101207	19	.90463, 01	.17883, 04	.12972, 02
101307	0	.00000, 00	.00000, 00	.00000, 00
101407	0	.00000, 00	.00000, 00	.00000, 00
101108	0	.00000, 00	.00000, 00	.00000, 00
101208	3	.14060, 02	.59943, 03	.31941, 01
101308	7	.11742, 02	.98446, 03	.31995, 01
101408	20	.69900, 01	.10454, 04	.35924, 01
101109	8	.71500, 01	.42212, 03	.18771, 01
101209	2	.10550, 02	.22285, 03	.24500, 00
101309	20	.92940, 01	.19843, 04	.13516, 02
101409	0	.00000, 00	.00000, 00	.00000, 00
101110	0	.00000, 00	.00000, 00	.00000, 00
101210	2	.13700, 02	.37556, 03	.18000, 00
101310	5	.13134, 02	.87664, 03	.35342, 01
101410	23	.74395, 01	.13771, 04	.47349, 01
101111	0	.00000, 00	.00000, 00	.00000, 00

(Continued next page)

Level and Factor	Obs. No.	Mean	Sum of Squares	Variance
101211	11	.11334, 02	.15299, 04	.11675, 02
101311	13	.74000, 01	.76780, 03	.46600, 01
101411	6	.72166, 01	.33161, 03	.38256, 01
101112	0	.00000, 00	.00000, 00	.00000, 00
101212	14	.89571, 01	.12105, 04	.67195, 01
101312	4	.87000, 01	.34394, 03	.13726, 02
101412	12	.86650, 01	.10748, 04	.15804, 02
101113	8	.68875, 01	.42885, 03	.70498, 01
101213	4	.31620, 02	.75072, 03	.29038, 01
101313	16	.84625, 01	.12647, 04	.79278, 01
101413	2	.96000, 01	.18504, 03	.72000, 00
101114	3	.63000, 01	.11969, 03	.31000, 00
101214	23	.93295, 01	.22446, 04	.11033, 02
101314	4	.76750, 01	.26499, 03	.97891, 01
101414	0	.00000, 00	.00000, 00	.00000, 00
101115	1	.13400, 02	.17956, 03	.00000, 00
101215	13	.10790, 02	.16419, 04	.10686, 02
101315	15	.69800, 01	.77419, 03	.30988, 01
101415	1	.58000, 01	.33640, 02	.00000, 00
201102	31	.52983, 01	.11137, 04	.81162, 01
301202	0	.00000, 00	.00000, 00	.00000, 00
201302	0	.00000, 00	.00000, 00	.00000, 00
201402	0	.00000, 00	.00000, 00	.00000, 00

EXAMPLE OF PRINTED RESULTS FOR THREE-WAY

FACTOR-LEVEL CLASSIFICATION

Level and Factor	Obs. No.	Mean	Sum of Squares	Variance
101102103	0	.00000, 00	.00000, 00	.00000, 00
101102203	0	.00000, 00	.00000, 00	.00000, 00
101102303	27	.88585, 01	.23932, 04	.10555, 02
101102403	3	.83333, 01	.23614, 03	.13903, 02
101102104	0	.00000, 00	.00000, 00	.00000, 00
101102204	0	.00000, 00	.00000, 00	.00000, 00
101102304	14	.66642, 01	.67399, 03	.40163, 01
101102404	16	.10680, 02	.19553, 04	.86913, 01
101102105	1	.68000, 01	.46240, 02	.00000, 00
101102205	8	.82537, 01	.66440, 03	.17058, 02
101102305	21	.91119, 01	.19187, 04	.87574, 01
101102405	0	.00000, 00	.00000, 00	.00000, 00
101102106	1	.68000, 01	.46240, 02	.00000, 00
101102206	29	.88751, 01	.25831, 04	.10672, 02
101102306	0	.00000, 00	.00000, 00	.00000, 00
101102406	0	.00000, 00	.00000, 00	.00000, 00
101102107	11	.83909, 01	.84097, 03	.66489, 01
101102207	19	.90463, 01	.17883, 04	.12972, 02
101102307	0	.00000, 00	.00000, 00	.00000, 00
101102407	0	.00000, 00	.00000, 00	.00000, 00
101102108	0	.00000, 00	.00000, 00	.00000, 00
101102208	3	.14069, 02	.59943, 03	.31941, 01
101102308	7	.11742, 02	.98446, 03	.31995, 01
101102408	20	.69900, 01	.10454, 04	.35925, 01
101102109	8	.71500, 01	.42212, 03	.18771, 01
101102209	2	.10550, 02	.22285, 03	.24500, 00
101102309	20	.92940, 01	.19843, 04	.13516, 02
101102409	0	.00000, 00	.00000, 00	.00000, 00
101102110	0	.00000, 00	.00000, 00	.00000, 00
101102210	2	.13700, 02	.37556, 03	.18000, 00
101102310	5	.13134, 02	.87664, 03	.35342, 01
101102410	23	.74395, 01	.13771, 04	.47349, 01
101102111	0	.00000, 00	.00000, 00	.00000, 00
101102211	11	.11334, 02	.15299, 04	.11675, 02
101102311	13	.74000, 01	.76780, 03	.46600, 01
101102411	6	.72166, 01	.33161, 03	.38256, 01
101102112	0	.00000, 00	.00000, 00	.00000, 00

Continued next page)

Level and Factor	Obs. No.	Mean	Sum of Squares	Variance
101102212	14	.89571, 01	.12105, 04	.67195, 01
101102312	4	.87000, 01	.34394, 03	.13726, 02
101102412	12	.86650, 01	.10748, 04	.15804, 02
101102113	8	.68875, 01	.42885, 03	.70498, 01
101102213	4	.13620, 02	.75072, 03	.29038, 01
101102313	16	.84625, 01	.12647, 04	.79278, 01
101102413	2	.96000, 01	.18504, 03	.72000, 00
101102114	3	.63000, 01	.11969, 03	.31000, 00
101102214	23	.93295, 01	.22446, 04	.11033, 02
101102314	4	.76750, 01	.26499, 03	.97891, 01
101102414	0	.00000, 00	.00000, 00	.00000, 00
101102115	1	.13400, 02	.17956, 03	.00000, 00
101102215	13	.10790, 02	.16419, 04	.10686, 02
101102315	15	.69800, 01	.77419, 03	.30988, 01
101102415	1	.58000, 01	.33640, 02	.00000, 00
101303104	0	.00000, 00	.00000, 00	.00000, 00
101303204	0	.00000, 00	.00000, 00	.00000, 00
101303304	14	.66642, 01	.67399, 03	.40163, 01
101303404	13	.11221, 02	.17192, 04	.68526, 01
101303105	1	.68000, 01	.46240, 02	.00000, 00
101303205	6	.89383, 01	.58702, 03	.21532, 02
101303305	20	.89375, 01	.17599, 04	.85460, 01
101303405	0	.00000, 00	.00000, 00	.00000, 00
101303106	1	.68000, 01	.46240, 02	.00000, 00

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