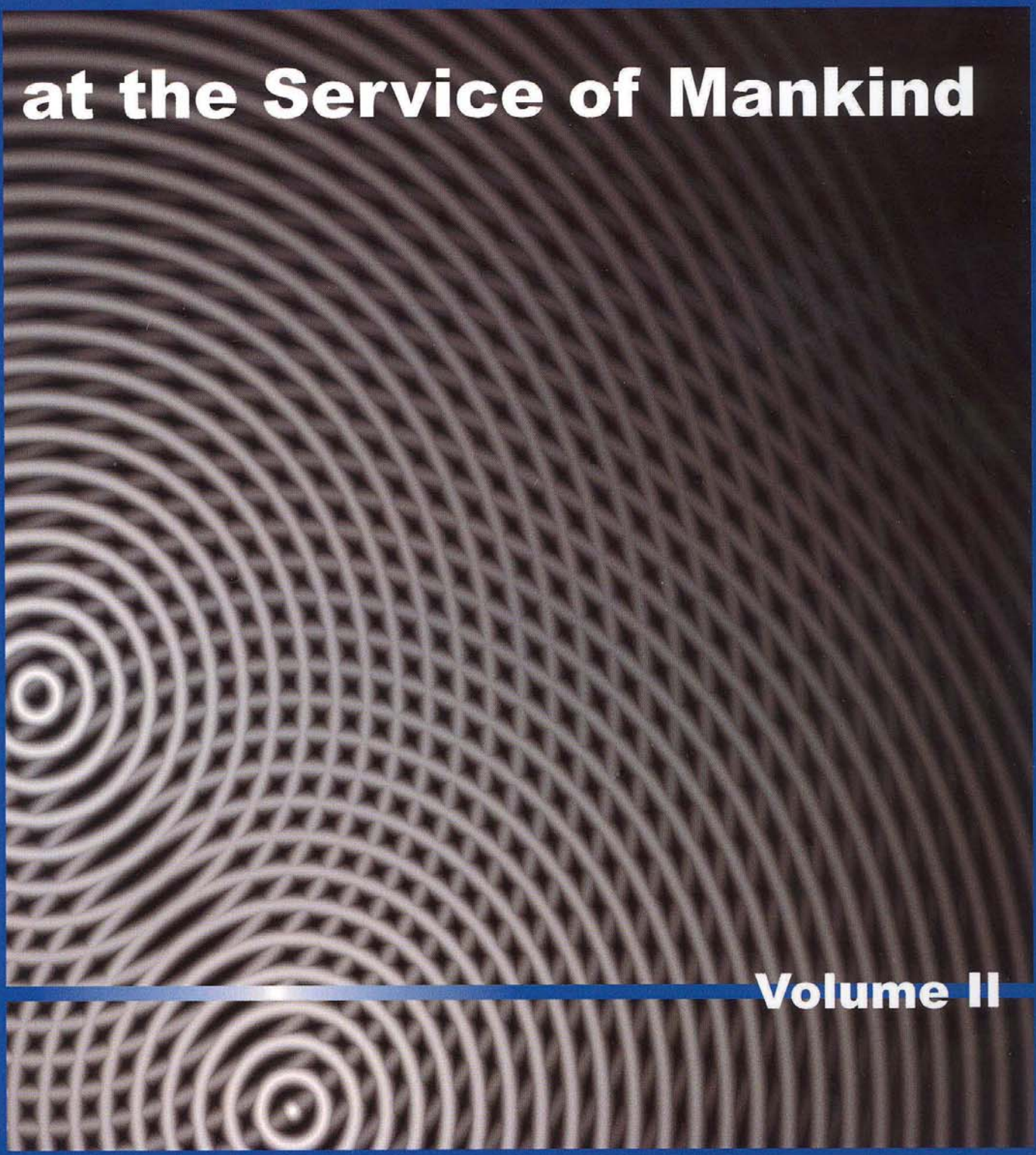
Wilfried Elmenreich, J. Tenreiro Machado
and Imre J. Rudas (Eds)

Intelligent Systems at the Service of Mankind



Volume II

New Concepts Towards the Synthesis of Digital Circuits Through Genetic Algorithms

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Abstract — *This paper analyses the performance of a Genetic Algorithm using two new concepts, namely a static fitness function including a discontinuity measure and a fractional-order dynamic fitness function, for the synthesis of combinational logic circuits. In both cases, experiments reveal superior results in terms of speed and convergence to achieve a solution.*

1 Introduction

In the last decade genetic algorithms (GAs) have been applied in the design of electronic circuits, leading to a novel area of research called Evolutionary Electronics (EE) or Evolvable Hardware (EH) [1]. EE considers the concept for automatic design of electronic systems. Instead of using human conceived models, abstractions and techniques, EE employs search algorithms to develop good designs [2].

One decade ago Sushil and Rawlins [3] applied GAs to the combinational circuit design problem. They combined knowledge-based systems with the GA and defined a genetic operator called masked crossover. This scheme leads to other kinds of offspring that can not be achieved by classical crossover operators.

John Koza [4] adopted genetic programming to design combinational circuits. His goal was the design of functional circuits through AND, OR and NOT logic gates.

In the sequence of this work, Coello, Christiansen and Aguirre [5] presented a computer program that automatically generates high-quality circuit designs. They use five possible types of gates (AND, NOT, OR, XOR and WIRE) with the objective of finding a functional design that minimizes the use of gates other than WIRE.

Miller, Thompson and Fogarty [6] applied evolutionary algorithms for the design of arithmetic circuits. The technique was based on evolving the functionality and connectivity of a rectangular array of logic cells, with a model of the resources available on the Xilinx 6216 FPGA device.

Kalganova, Miller and Lipnitskaya [7] proposed a new technique for designing multiple-valued circuits. The EH is easily adapted to the distinct types of multiple-valued gates, associated with operations corresponding to different types of algebra, and can include other logical expressions. This approach is an extension of EH method for binary logic circuits proposed in [6].

In order to solve complex systems, Torresen [8] proposed the method of increased complexity evolution. The idea is to evolve a system gradually as a kind of divide-and-conquer method. Evolution is first undertaken individually on a large number of simple cells. The evolved functions are the basic blocks adopted in further evolution or assembly of a larger and more complex system.

More recently Hollingworth, Smith and Tyrrell [9] describe the first attempts to evolve circuits using the Virtex family of devices. They implemented a simple 2-bit adder, where the inputs to the circuit are the two 2-bit numbers and the expected output is the sum of the two input values.

A major bottleneck in the evolutionary design of electronic circuits is the problem of scale. This refers to the very fast growth of the number of gates, used in the target circuit, as the number of inputs of the evolved logic function increases. This results in a huge search space that is difficult to explore even with evolutionary techniques. Another related obstacle is the time required to calculate the fitness value of a circuit [10]. A possible method to solve this problem is to use building blocks either than simple gates. Nevertheless, this technique leads to another difficulty, which is how to define building blocks that are suitable for evolution.

Timothy Gordon [11] suggests an approach that allows evolution to search for good inductive bases for solving large-scale complex problems. This scheme generates, inherently, modular and iterative structures, that exist in many real-world circuit designs but, at the same time, allows evolution to search innovative areas of space.

The idea of using memory to achieve better fitness function performances was first introduced by Sano and Kita [12, 13]. Their goal was the optimization of systems with randomly fluctuating fitness function and they developed a Genetic Algorithm with Memory-based Fitness Evaluation (MFEGA). The key ideas of the MFEGA are based on storing the sampled fitness values into memory as a search history, introducing a simple stochastic model of fitness values to be able to estimate fitness values of points of interest using the history for selection operation of the GA.

Bearing these ideas in mind, and looking for better performance GAs, this paper proposes a GA for the design of combinational logic circuits using fractional-order dynamic fitness functions.

The area of Fractional Calculus (FC) deals with the operators of integration and differentiation to an arbitrary (including noninteger) order and is as old as the theory of classical differential calculus [14, 15]. The theory of FC is a well-adapted tool to the modelling of many physical phenomena, allowing the description to take into account some peculiarities that classical integer-order models simply neglect. Nevertheless, the application of FC has been scarce until recently, but the advances on the theory of chaos motivated a renewed interest in this field. In the last two decades we can mention research on viscoelasticity/damping, chaos/fractals, biology, signal processing, system identification, diffusion and wave propagation, electromagnetism and automatic control

[16, 17, 18, 19, 20, 21, 22].

The article is organized as follows. Section 2 describes the adopted GA as well as the fractional-order dynamic fitness functions. Section 3 presents the simulation results and finally, section 4 outlines the main conclusions and addresses perspectives towards future developments.

2 The Adopted Genetic Algorithm

In this section we present the GA developed in the study, in terms of the circuit encoding as a chromosome, the genetic operators and the static and dynamic fitness functions.

2.1 Problem Definition

A GA strategy is adopted to design combinational logic circuits. The circuits are specified by a truth table and the goal is to implement a functional circuit with the least possible complexity. Two sets of logic gates have been defined, as shown in Table 1, being Gset a the simplest one ($i.e.$, a RISC-like set) and Gset b a more complex gate set ($i.e.$, a CISC-like set).

Gate Set	**Logic gates**
Gset a	{AND,XOR,WIRE}
Gset b	{AND,OR,XOR,NOT,WIRE}

Table 1: Gate sets

For each gate set the GA searches the solution space, based on a simulated evolution aiming the survival of the fittest strategy. In general, the best individuals of any population tend to reproduce and survive, thus improving successive generations. However, inferior individuals can, by chance, survive and also reproduce [2]. In our case, the individuals are digital circuits, which can evolve until the solution is reached (in terms of functionality and complexity).

2.2 Circuit encoding

In the GA scheme the each circuit is encoded as a rectangular matrix A (row $\times$ column = $r \times c$) of logic cells as represented in figure 1.

Each cell is represented by three genes: $<$*input1*$><$*input2*$><$*gate type*$>$, where *input1* and *input2* are one of the circuit inputs, if the cell is in the first column of the matrix, or, one of the outputs of a previous cell, if the cell is not in the first column of the matrix. The *gate type* is one of the elements adopted in the gate set. The chromosome is formed by as many triplets of this kind as the matrix size demands. For example, the chromosome that represents a 3×3 matrix is depicted in figure 2.

2.3 The genetic operators

The initial population of circuits (strings) is generated at random. The search is then carried out among this population. The three different operators used are reproduction, crossover and mutation, as described in the sequel.

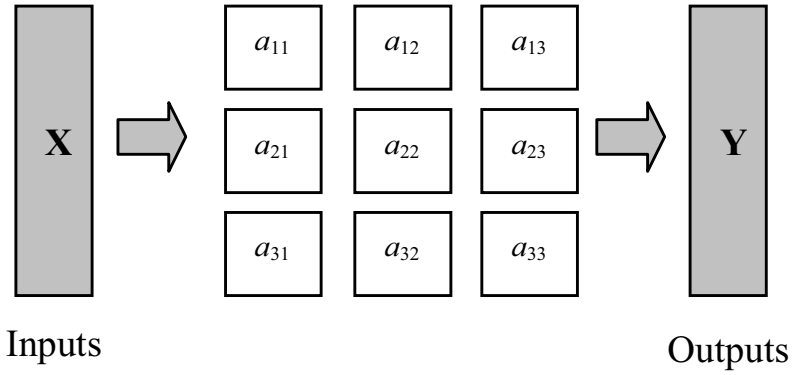

Figure 1: A 3 $\times$ 3 matrix A representing a circuit with input X and output Y

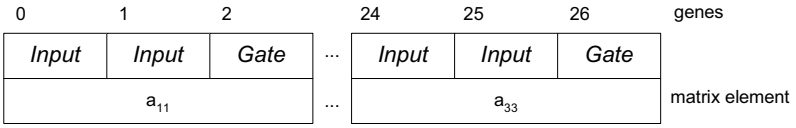

Figure 2: Chromosome for the 3 $\times$ 3 matrix of figure 1

Concerning the reproduction operator, the successive generations of new strings are reproduced on the basis of their fitness function. In this case, it is used a tournament selection to select the strings from the old population, up to the new population.

For the crossover operator, the strings in the new population are grouped together into pairs at random. Single point crossover is then performed among pairs. The crossover point is only allowed between cells to maintain the chromosome integrity.

The mutation operator changes the characteristics of a given cell in the matrix. Therefore, it modifies the gate type and the two inputs, meaning that a completely new cell can appear in the chromosome. Moreover, it is applied an elitist algorithm and, consequently, the best solutions are always kept for the next generation.

To run the GA we have to define the number of individuals to create the initial population P. This population is always the same size across the generations, until the solution is reached.

The crossover rate *CR* represents the percentage of the population P that reproduces in each generation. Likewise, the mutation rate *MR* is the percentage of the population P that can mutate in each generation.

2.4 The Static and the Dynamic Fitness Functions

The initial population of circuits (strings) is generated at random. The search is then carried out among this population. The three different operators used are reproduction, crossover and mutation, as described in the sequel.

The goal of this study is to find new ways of evaluating the individuals of the population in order to achieve better performance GAs.

In this paper we propose two concepts for the fitness functions, namely the static fitness function F_s and the dynamic fitness functionF_d.

The calculation of F_s in (1) is divided in two parts, f_1 and f_2, where f_1 measures the

functionality and the error discontinuity and $f_2$measures the simplicity. In a first phase, we compare the output **Y** produced by the GA-generated circuit with the required values $\mathbf{Y}_R$, according to the truth table, on a bit-per-bit basis. By other words, f_{11} is incremented by *one* for each correct bit of the output until f_{11} reaches the maximum value f_{10}, that occurs, when we have a functional circuit. After this, $f_{11}$is decremented by δ for each $\mathbf{Y}_R - \mathbf{Y}$ error discontinuity, where discontinuity means passing from $\mathbf{Y}_R - \mathbf{Y} = 0$ to $\mathbf{Y}_R - \mathbf{Y} = 1$ or vice-versa when comparing two consecutive levels of the truth table. Once the circuit is functional, in a second phase, the GA tries to generate circuits with the least number of gates. This means that the resulting circuit must have as much genes <*gate type*> $\equiv$ <*wire*> as possible. Therefore, the index f_2, that measures the simplicity (the number of null operations), is increased by *one* (*zero*) for each *wire* (*gate*) of the generated circuit, yielding:

$$f_{10} = 2^{ni} \times no \tag{1a}$$

$$f_{11} = f_{11} + 1 \; if \; \{bit \; i \; of \; \mathbf{Y}\} = \{bit \; i \; of \; \mathbf{Y_R}\}, i = 1, ..., f_{10} \tag{1b}$$

$$f_1 = f_{11} - \delta \; if \; error_i \neq error_{i-1}, i = 1, ..., f_{10} \tag{1c}$$

$$f_2 = f_2 + 1 \; if \; gate \; type = wire \tag{1d}$$

$$F_s = \begin{cases} f_1, & F_s < f_{10} \\ f_1 + f_2, & F_s \geq f_{10} \end{cases} \tag{1e}$$

where *ni* and *no* represent the number of inputs and outputs of the circuit.

The concept of dynamic fitness function F_d results from an analogy with control systems where we have a variable to be controlled similarly with the GA case where we master the population through the fitness function. The simplest control system is the proportional algorithm; nevertheless, there can be other control algorithms, like the differential and the integral schemes. Therefore, applying the static fitness function corresponds to using a kind of proportional algorithm. If we want to implement a proportional-derivative or a proportional-integrative evolution the fitness function needs a scheme of the type:

$$F_d = F_s + K \; D^{\alpha} \left[Fs \right] \tag{2}$$

where $\mathrm{t}1 \leq \alpha \leq 1$ is the differential (integral) fractional-order for positive (negative) values of α and K is the 'gain' of the dynamical term.

The generalization of the concept of derivative $D^{\alpha}[f(x)]$ to noninteger values of α goes back to the beginning of the theory of differential calculus. In fact, Leibniz, in his correspondence with Bernoulli, L'Hôpital and Wallis, had several notes about its calculation for $\alpha = 1/2$ [14, 15]. Nevertheless the adoption of the *FC* in control algorithms has been recently studied using the frequency and discrete-time domains [16, 17, 18, 19].

The mathematical definition of a derivative of fractional order α has been the subject of several different approaches. For example, (3) and (4), represent the Laplace (for zero

initial conditions) and the Grünwald-Letnikov definitions of the fractional derivative of order α of the signal $x(t)$:

$$D^{\alpha}\left[x\left(t\right)\right] = L^{-1}\left\{s^{\alpha}\ X\ \left(s\right)\right\} \tag{3}$$

$$D^{\alpha}\left[x\left(t\right)\right] = \lim_{h\to 0}\left[\frac{1}{h^{\alpha}}\sum_{k=0}^{\infty}\left(-1\right)^{k}\frac{\Gamma\left(\alpha+1\right)}{\Gamma\left(k+1\right)\Gamma\left(\alpha-k+1\right)}x\left(t-kh\right)\right] \tag{4}$$

where Γ is the gamma function and h is the time increment. This formulation [19] inspired a discrete-time calculation algorithm, based on the approximation of the time increment h through the sampling period T and a r-term truncated series yielding the equation:

$$D^{\alpha}\left[x\left(t\right)\right] \approx \frac{1}{T^{\alpha}}\sum_{k=0}^{r}\frac{\left(-1\right)^{k}\Gamma\left(\alpha+1\right)}{k!\ \ \Gamma\left(\alpha-k+1\right)}x\left(t-kT\right) \tag{5}$$

3 Experiments and Simulation Results

Reliable execution and analysis of a GA usually requires a large number of simulations to provide a reasonable assurance that stochastic effects have been properly considered [23]. Therefore, in this study are developed n = 1000 simulations for each case under analysis.

The experiments consist on running the GA to generate a typical combinational logic circuit, namely a 2-to-1 multiplexer ($M2-1$) and a 4-bit parity checker (*PC4*), using the fitness schemes described previously.

Having a superior GA performance means achieving solutions with a smaller number N of generations and a smaller standard deviation in order to reduce the stochastic nature of the algorithm.

3.1 Using the static fitness function

In this sub-section we analyze the GA improvement when adopting a static fitness function including the discontinuity measure δ error.

Figures 3 and 4 show the average number of generations to achieve the solution *AV*(N) and the corresponding standard deviation *SD*(N) versus the discontinuity factor δ = {0, 0.25, 0.5, 0.75, 1}, using Gset a and Gset b, for the $M2-1$ and the *PC4* circuits, respectively.

The results reveal that, as it was expected from previous studies [24], the RISC-like set Gset a presents better performance than the CISC-like gate set Gset b for all values of δ. On the other hand, analysing the influence of δ we conclude that the GA response is best when δ = 0.5 for the two circuits and for the two gate sets.

3.2 Experiments using dynamic fitness function

In this sub-section we analyze the GA performance when we adopt a dynamic scheme for the fitness function.

The simulations investigate an integral scheme (α = {−1, −0.75, −0.5, −0.25, 0}) and a differential scheme (α = {0, 0.25, 0.5, 0.75, 1}) in F_d for gains $10^{-3} \leq K \leq 10^{2}$.

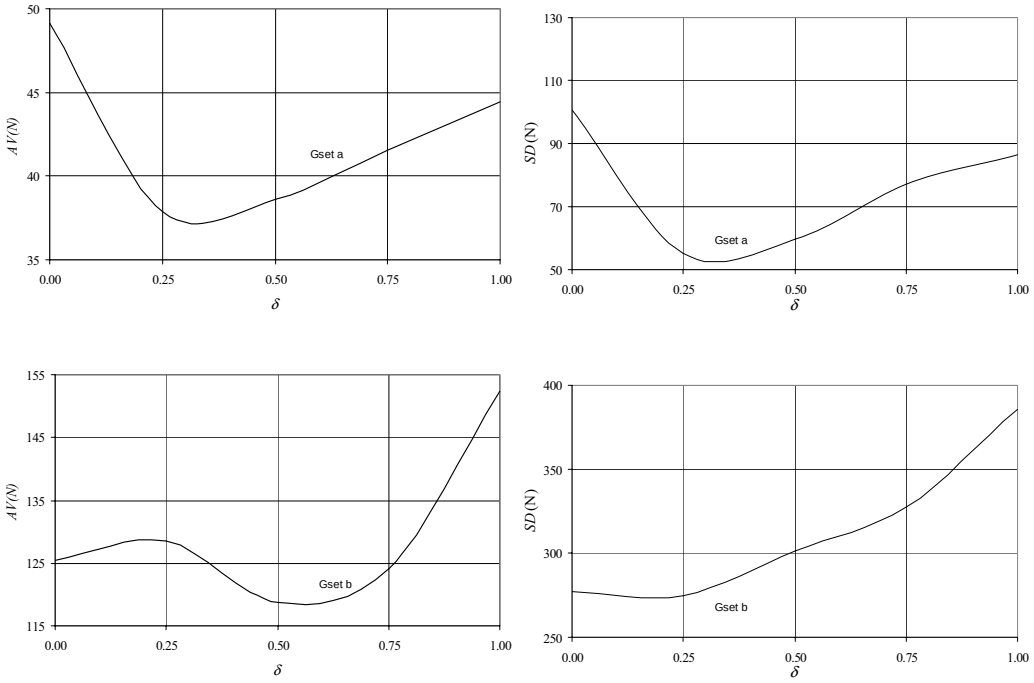

Figure 3: $M2-1$ average number of generations to achieve a solution *AV*(N) and standard deviation *SD*(N) for δ = {0, 0.25, 0.5, 0.75, 1} with Gsets a and b

The implementation of the integral/differential fractional order operator adopts (5) with a series truncation $r = 50$ terms.

Figures 5 to 8 show the average number of generations to achieve a solution *AV*(N) and the standard deviation *SD*(N) for the integral and differential schemes, for the $M2-1$ and *PC4* circuits, using Gset a and Gset b, respectively.

Tables 2 and 3 present the parameters (α, K) pair for each best solution obtained in terms of average number of generations *AV*(N) and in terms of standard deviation *SD*(N), respectively, for the integral and the differential schemes of F_d.

Circuit	**Gset a**	**Gset b**
$M2-1$	(α, K) = (0.5, 0.01)	(α, K) = (1, 0.1)
	(α, K) = ($-$0.5, 0.1)	(α, K) = ($-$1, 0.1)
PC4	(α, K) = (0.5, 0.1)	(α, K) = (0.75, 0.01)
	(α, K) = ($-$0.5, 0.1)	(α, K) = ($-$0.75, 0.01)

Table 2: The (α, K) parameters for each best solution obtained in terms of *AV* (*N*)

In general we conclude that the F_d concept produces better results particularly for the differential scheme. Moreover, once again, the RISC-like gate set is superior to the CISC-like gate set and the best results are for fractional order α.

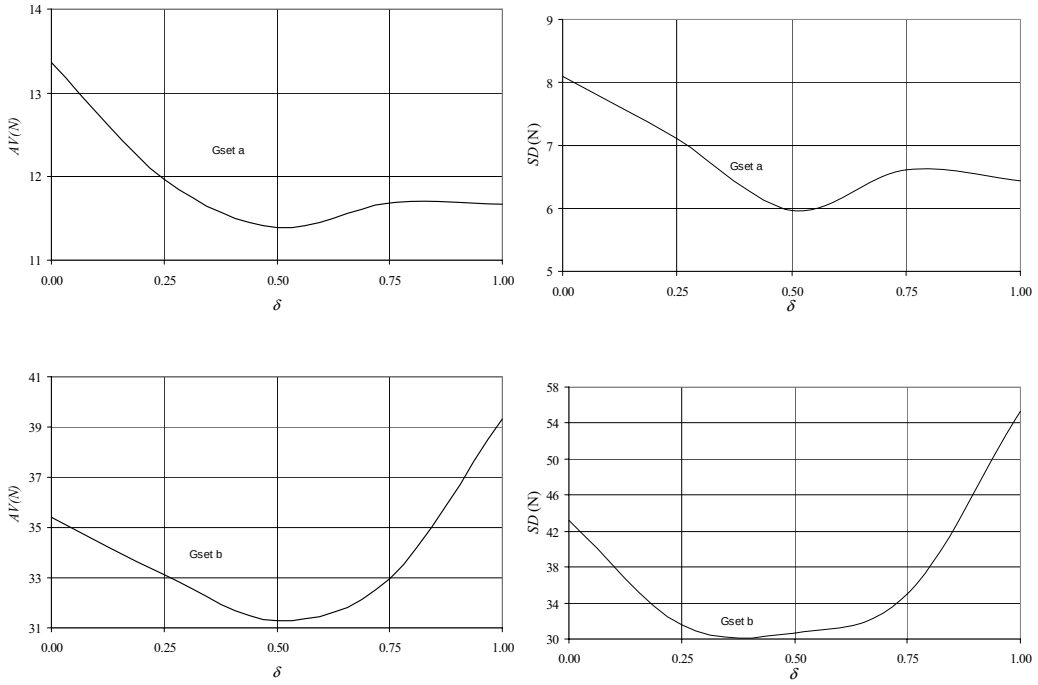

Figure 4: *PC4* average number of generations to achieve a solution *AV*(N) and standard deviation *SD*(N) for $\delta = \{0, 0.25, 0.5, 0.75, 1\}$ with Gsets a and b

Circuit	**Gset a**	**Gset b**
$M2-1$	(α, K) = (1, 10)	(α, K) = (0, 10)
	(α, K) = (−0.5, 0.1)	(α, K) = (0, 100)
PC4	(α, K) = (0.5, 0.1)	(α, K) = (0.5, 0.1)
	(α, K) = (-0.5, 0.1)	(α, K) = (−0.5, 0.1)

Table 3: The (α, K) parameters for each best solution obtained in terms of *SD* (*N*)

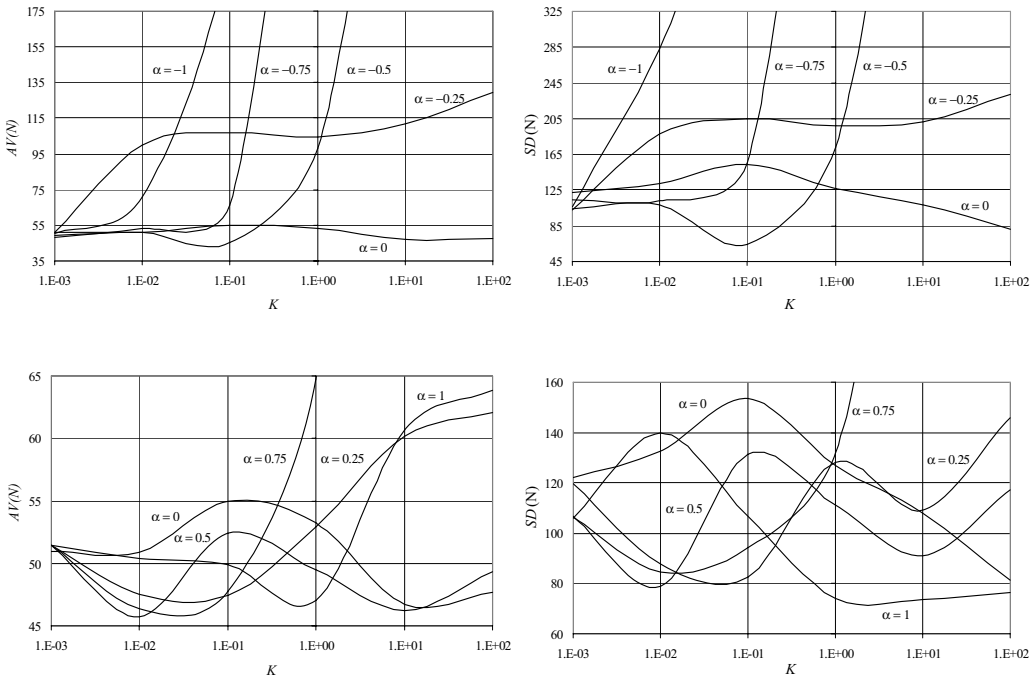

Figure 5: $M2-1$ average number of generations to achieve a solution *AV*(N) and standard deviation *SD*(N) for integral and differential control schemes with Gset a

4 Conclusions

This paper presented two techniques for improving the GA performance. In what concerns to the classical static fitness function we conclude that it is possible to get superior results by measuring the error discontinuity. On the other hand, the new concept of fractional-order dynamic fitness function of the GA, demonstrates to be an important method that outperforms the traditional static fitness function approach. In both cases, the tuning of the 'optimal' parameters δ or (α, K) was established by trial and error. Therefore, future research will address the problem of having a more systematic design method.

These conclusions encourage further studies using not only proportional-differential or proportional-integral control schemes but also proportional-integral-derivative control schemes.

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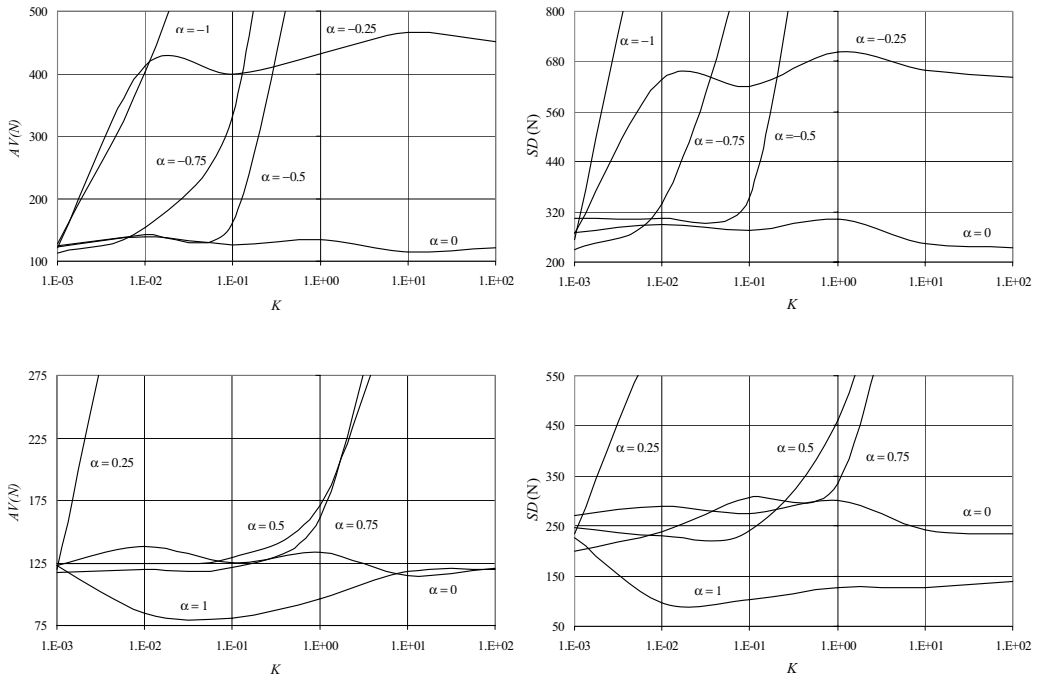

Figure 6: $M2-1$ average number of generations to achieve a solution *AV*(N) and standard deviation *SD*(N) for integral and differential control schemes with Gset b

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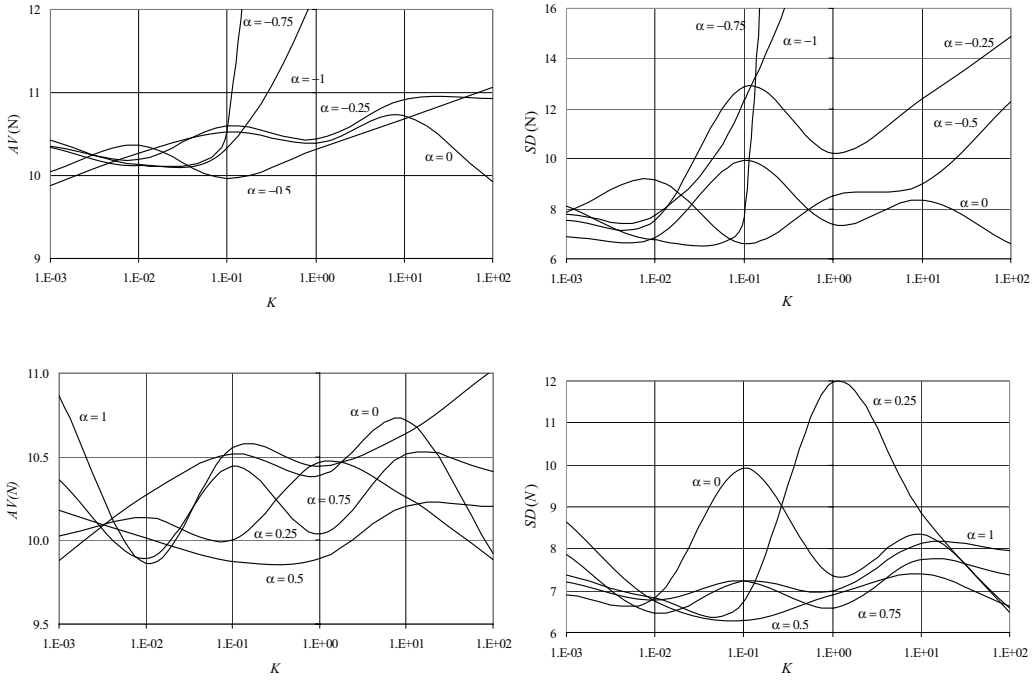

Figure 7: *PC4* average number of generations to achieve a solution $AV(N)$ and standard deviation $SD(N)$ for integral and differential control schemes with Gset a

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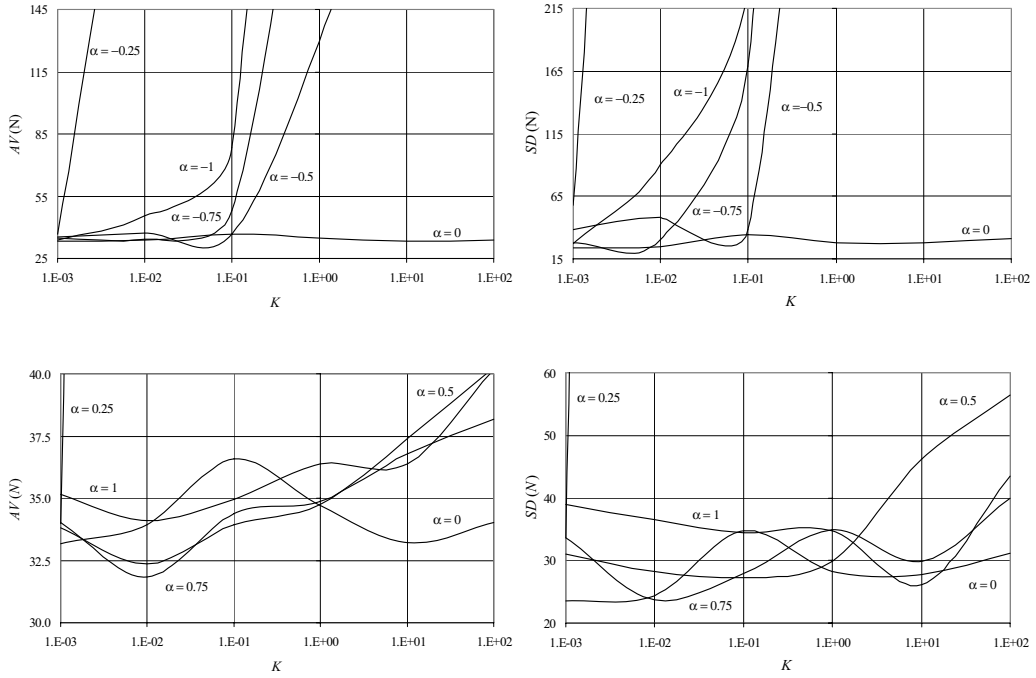

Figure 8: *PC4* average number of generations to achieve a solution $AV(N)$ and standard deviation $SD(N)$ for integral and differential control schemes with Gset b

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Cecília Reis was born in November 24, 1967. She graduated in Electrical Engineering - Industrial Control from Institute of Engineering of Polytechnic Institute of Porto, Portugal, in 1992 and received the Master's degree in Electrical and Computer Engineering from the Faculty of Engineering of the University of Porto, Portugal, in 1995. Presently she teaches at the Institute of Engineering of the Polytechnic Institute of Porto, Department of Electrical Engineering. Her research interests include digital systems, evolvable hardware, genetic algorithms and fractional-order systems.

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