



CISTER

Research Center in
Real-Time & Embedded
Computing Systems

Conference Paper

Performance Model for MRC Receivers with Adaptive Modulation and Coding in Rayleigh Fading Correlated Channels with Imperfect CSIT

Ramiro Robles*

Egons Lavendis

*CISTER Research Center

CISTER-TR-151008

2015/10/05

Performance Model for MRC Receivers with Adaptive Modulation and Coding in Rayleigh Fading Correlated Channels with Imperfect CSIT

Ramiro Robles*, Egons Lavendis

*CISTER Research Center

Polytechnic Institute of Porto (ISEP-IPP)

Rua Dr. António Bernardino de Almeida, 431

4200-072 Porto

Portugal

Tel.: +351.22.8340509, Fax: +351.22.8321159

E-mail: rasro@isep.ipp.pt

<http://www.cister.isep.ipp.pt>

Abstract

This paper presents a performance model of the packet reception process in a wireless link with one antenna transmitter and a multiple-antenna maximum-ratio combining (MRC) receiver. The objective is to address the performance evaluation of multiple antenna systems enabled with adaptive modulation and coding (AMC). Two main assumptions are used: 1) Rayleigh fading correlated channels, and 2) imperfect (outdated) channel state information at the transmitter side (CSIT). The results presented here suggest that spatial correlation not always affects the performance of the MRC receiver: at low signal-to-noise ratio (SNR), correlation can improve performance rather than degrading it. By contrast, at high SNR, correlation is found to always degrade performance. At high SNR, correlation tends to worsen the degrading effects of imperfect CSIT, particularly when the number of antennas increases. Imperfect CSIT causes errors in the assignment of MCSs, thus reducing throughput performance. These errors become more evident at high SNR, particularly when the values of branch correlation and the number of antennas increase.

Performance Model for MRC Receivers with Adaptive Modulation and Coding in Rayleigh Fading Correlated Channels with Imperfect CSIT

Ramiro Sámano-Robles¹ and Egons Lavendelis²

¹Research Centre in Real-time and Embedded Computing Systems, Porto, Portugal

²Department of System Theory and Design, Riga Technical University, Latvia.

emails:¹rasro@isep.ipp.pt ;²egons.lavendelis@rtu.lv

Abstract—This paper presents a performance model of the reception process in a wireless link with one antenna transmitter and a multiple-antenna maximum-ratio combining (MRC) receiver. The objective is to address the performance evaluation of multiple antenna systems enabled with adaptive modulation and coding (AMC). Two main assumptions are used: 1) Rayleigh fading correlated channels, and 2) imperfect (outdated) channel state information at the transmitter side (CSIT). The results presented here suggest that spatial correlation not always affects the performance of the MRC receiver: at low signal-to-noise ratio (SNR), correlation can improve performance rather than degrading it. By contrast, at high SNR, correlation is found to always degrade performance. At high SNR, correlation tends to worsen the degrading effects of imperfect CSIT, particularly when the number of antennas increases. Imperfect CSIT causes errors in the assignment of modulation and coding schemes (MCSs), thus reducing throughput performance. These errors become more evident at high SNR, particularly when the values of branch correlation and the number of antennas increase.

Index Terms—Maximum-ratio combining, cross-layer design, packet reception, compression modelling, imperfect CSIT.

I. INTRODUCTION

Signal processing tools are constantly being developed to cope more efficiently with the interference and fading of wireless links. All the performance details of lower layers cannot be practically used in upper-layer optimization due to complexity constraints. *Compression models* are thus required that provide a fair, but flexible (low-complex) representation of lower layers. This paper deals with the modelling of a wireless link using at the receiver end a multiple antenna system based on maximum-ratio combining (MRC). The objective is to facilitate the design of upper layers of multiple antenna systems enabled with adaptive modulation and coding (AMC).

The simplest multiple antenna system is the MRC receiver, which provides a relatively flexible framework for statistical analysis and compression interface modelling. The literature of MRC receivers has focused on the derivation of outage and bit error probability distributions (see [1]-[9]). The effects of imperfect channel knowledge at the receiving end on the performance of MRC in Rayleigh fading correlated channels

The research from DEWI project (www.dewi-project.eu) leading to these results has received funding from the ARTEMIS Joint Undertaking under grant agreement No. 621353.

can be found in [1]. An extension to Rice fading channels was addressed in [2] following the lines of the analysis with perfect estimation presented in [3]. A series expansion of the statistics of MRC receivers with correlated Rice channels is given in [4]. A unified approach for analysis of two-stage MRC receivers with hybrid selection in generalized Rice correlated channels was proposed in [5]. Extensions under the effects of co-channel interference are given in [6]-[9].

The present work considers the extension of outage probability analysis to the study of AMC-MRC systems in Rayleigh fading correlated channels with imperfect/outdated CSIT. To the best of our knowledge, this is the first attempt in the literature that addresses these issues *simultaneously*. Imperfect CSIT has been addressed in [10] for distributed systems and in [11] for energy efficient MIMO link adaptation. In comparison with these works, which are focused on numerical evaluation, our work provides an analytic framework for obtaining the statistics of errors in MCS assignment for MRC receivers.

This paper is organized as follows. Section II describes the system model and the assumptions of the paper. Section III presents the performance compression model. Section IV presents analytic results and sketches of the statistics of packet reception. Finally, Section V summarizes the conclusions.

II. SYSTEM MODEL

Consider the system depicted in Fig. 1 with one transmitter and one receiver. The transmitter uses a single antenna while the receiver is enabled with a multiple antenna using maximum-ratio combining (MRC). The channel between the transmitter and the multiple antenna receiver is denoted by $\mathbf{h} = [h_1, h_2 \dots h_N]^T$, where $(\cdot)^T$ is the vector transpose operator and N is the number of antennas. All channels will be modelled as zero-mean complex circular symmetrical Gaussian random variables with variance denoted by γ , i.e. $h_n \sim \mathcal{CN}(0, \gamma)$. The received signals of different antennas experience the same statistical correlation, which means that $E[h_k^* h_n] = \rho\gamma$, where $k \neq n$, $(\cdot)^*$ is the complex conjugate operator, $E[\cdot]$ is the statistical average operator, and ρ is the *spatial correlation coefficient*. Signals will use one of M modulation and coding schemes (MCSs). These MCSs are

arranged in increasing order according to their target signal-to-noise ratio (SNR). The target SNR of the m th MCS will be denoted by β_m . The BLER and spectral efficiency (in bps/Hz) performances (considering operation at the target SNR) of the m th MCS will be denoted, respectively, by θ_m and η_m . It is assumed that the receiver monitors the quality of the channel and reports it back to the transmitter. Based on this collected channel state information (CSI), the transmitter selects the most appropriate MCS. This paper considers *perfect channel estimation at the receiver side* and *imperfect channel state information at the transmitter side* (CSIT). Imperfect CSIT is assumed to be caused by feedback channel affected by delay. All the estimated channel variables available at the transmitter side that will be used for MCS selection will be expressed using the following notation $(\hat{\cdot})$. The estimated channel available at the transmitter will be thus denoted by $\hat{\mathbf{h}} = [\hat{h}_1, \hat{h}_2, \dots, \hat{h}_N]^T$, where $E[\hat{h}_k^* \hat{h}_n] = \rho\gamma$, $k \neq n$.

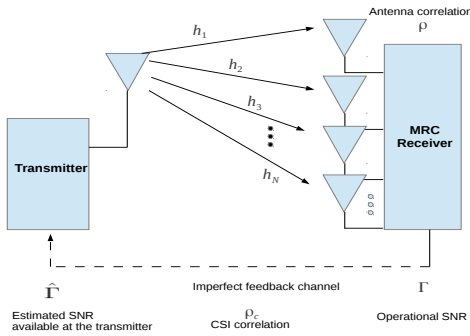


Fig. 1. System model for MRC receiver with imperfect CSIT.

For convenience, all estimated channels will be expressed using a linear correlation model:

$$\hat{h}_n = \sqrt{1 - \rho} Z_n + \sqrt{\rho} G, \quad (1)$$

where the variables Z_n and G are identically and independently distributed (*i.i.d.*) zero-mean complex circular symmetrical Gaussian random variables $\{Z_n, G\} \sim \mathcal{CN}(0, \gamma)$. Note that the model complies with $E[\hat{h}_n^* \hat{h}_n] = \rho$ and $E[\hat{h}_n^* \hat{h}_n] = \gamma$. The relationship between the instantaneous and the estimated CSIT will be also expressed using a linear correlation model that complies with $E[h_n^* \hat{h}_n] = \rho_c \gamma$ and $E[h_n^* h_n] = \gamma$:

$$h_n = \rho_c \hat{h}_n + \sqrt{1 - \rho_c^2} Y_n, \quad (2)$$

where ρ_c is the *temporal correlation coefficient* between the instantaneous (h_n) and the estimated CSI (\hat{h}_n) available at the transmitter. All variables Y_n in (2) are *i.i.d.* zero-mean complex circular symmetrical Gaussian random variables with variance γ , i.e. $Y_n \sim \mathcal{CN}(0, \gamma)$.

III. PERFORMANCE MODEL

The probability that the m th MCS is selected for transmission can be written as $\Pr\{\beta_m \leq \hat{\Gamma} < \beta_{m+1}\}$, where $\hat{\Gamma}$ is the estimated SNR. Link-layer throughput (denoted by T) will be

expressed as a linear contribution of all possible MCSs:

$$T = \sum_{m=1}^M \Pr\{\Gamma \geq \beta_m | \beta_m \leq \hat{\Gamma} < \beta_{m+1}\} \times \Delta_{BW} \eta_m (1 - \theta_m) \Pr\{\beta_m \leq \hat{\Gamma} < \beta_{m+1}\}, \quad (3)$$

where Δ_{BW} is the operational bandwidth in Hz, and $\Pr\{\Gamma \geq \beta_m | \beta_m \leq \hat{\Gamma} < \beta_{m+1}\}$ is the probability of the instantaneous SNR Γ to surpass the threshold β_m provided the estimated SNR $\hat{\Gamma}$ (used for MCS selection) lies in the range $[\beta_m, \beta_{m+1})$.

A. Statistics of $\hat{\Gamma}$

The estimated SNR of the MRC receiver can be written with the help of correlation model described by (1) as follows:

$$\hat{\Gamma} = \sum_{n=1}^N \frac{P|\sqrt{1 - \rho} Z_n + \sqrt{\rho} G|^2}{\sigma_v^2}. \quad (4)$$

The estimated SNR of the MRC receiver $\hat{\Gamma}$ conditional on an instance of random variable G has a non-central chi-square distribution with $2N$ degrees of freedom. The conditional characteristic function (CF) of $\hat{\Gamma}$ can be thus written as [12]:

$$\Psi_{\hat{\Gamma}|x}(i\omega) = (1 - i\omega\tilde{\gamma})^{-N} e^{\frac{i\omega\alpha x}{1 - i\omega\tilde{\gamma}}}. \quad (5)$$

where $\tilde{\gamma} = \frac{P(1-\rho)\gamma}{\sigma_v^2}$, $i = \sqrt{-1}$, ω is the frequency domain variable, $\alpha = \frac{PN\rho}{\sigma_v^2}$, and $x = |G|^2$. The unconditional CF of the estimated SNR can be now obtained by averaging the previous expression over the probability density function (PDF) of $x = |G|^2$, which under the Rayleigh fading assumption is given by $f_x(x) = \frac{1}{\gamma} e^{-\frac{x}{\gamma}}$. This results in the following:

$$\Psi_{\hat{\Gamma}}(i\omega) = (1 - i\omega\check{\gamma})^{-1} (1 - i\omega\tilde{\gamma})^{1-N}, \quad (6)$$

where $\check{\gamma} = \alpha\gamma + \tilde{\gamma}$. The back-transform of yields a complementary cumulative distribution function (CCDF) given by:

$$\bar{F}_{\hat{\Gamma}}(y) = A e^{-\frac{y}{\check{\gamma}}} + e^{-\frac{y}{\tilde{\gamma}}} \sum_{n=1}^{N-1} \sum_{u=0}^{n-1} \frac{B_n y^u}{\tilde{\gamma}^u u!}, \quad (7)$$

where $A = (1 - \tilde{\gamma}/\check{\gamma})^{1-N}$ and $B_n = (-\tilde{\gamma}/\check{\gamma}) (1 - \tilde{\gamma}/\check{\gamma})^{n-N}$. Details of derivation have been omitted due to lack of space. The probability of selecting the m th MCS can be thus written as: $\Pr\{\beta_m \leq \hat{\Gamma} < \beta_{m+1}\} = \bar{F}_{\hat{\Gamma}}(\beta_{m+1}) - \bar{F}_{\hat{\Gamma}}(\beta_m)$

B. Statistics of Γ

The instantaneous SNR at the transmitter side can be written with the help of the correlation model in (2) as follows:

$$\Gamma = \sum_{n=1}^N \frac{P|\rho_c \hat{h}_n + \sqrt{1 - \rho_c^2} Y_n|^2}{\sigma_v^2}. \quad (8)$$

The instantaneous SNR Γ of the MRC receiver conditional on \hat{h}_n has a non-central chi-square distribution with $2N$ degrees of freedom. The conditional CF is thus given by [12]:

$$\Psi_{\Gamma|\hat{\Gamma}}(i\omega) = (1 - i\omega\tilde{\gamma})^{-N} e^{\frac{i\omega\rho_c^2 \hat{\Gamma}}{1 - i\omega\tilde{\gamma}}}. \quad (9)$$

The unconditional CF of the instantaneous SNR can be obtained by averaging the previous expression over the PDF of $\hat{\Gamma}$. The unconditional CF becomes:

$$\Psi_{\Gamma|\beta_m \leq \hat{\Gamma} < \beta_{m+1}}(i\omega) = \frac{A \left(e^{-\frac{\beta_m(1-i\omega\dot{\gamma})}{\dot{\gamma}(1-i\omega\tilde{\gamma})}} - e^{-\frac{\beta_{m+1}(1-i\omega\dot{\gamma})}{\dot{\gamma}(1-i\omega\tilde{\gamma})}} \right)}{(1-i\omega\dot{\gamma})(1-i\omega\tilde{\gamma})^{N-1}} + \sum_{n=1}^{N-1} \sum_{k=0}^{n-1} \frac{B_n \left(\beta_m^k e^{-\frac{\beta_m(1-i\omega\dot{\gamma})}{\dot{\gamma}(1-i\omega\tilde{\gamma})}} - \beta_{m+1}^k e^{-\frac{\beta_{m+1}(1-i\omega\dot{\gamma})}{\dot{\gamma}(1-i\omega\tilde{\gamma})}} \right)}{(n-1)!k!(1-i\omega\dot{\gamma})^{n-k}(1-i\omega\tilde{\gamma})^{N-n+k}}, \quad (10)$$

where $\dot{\gamma} = \rho_c^2 \tilde{\gamma} + \bar{\gamma}$ and $\tilde{\gamma} = \rho_c^2 \tilde{\gamma} + \bar{\gamma}$. The back-transform of this expression can be obtained as an infinite series expansion. Details of this step have been omitted due to lack of space.

IV. RESULTS

A. CDF analysis

The results in Fig. 2 display the cumulative distribution function (CDF) of the instantaneous SNR Γ of the MRC receiver conditional on the estimated SNR $\hat{\Gamma}$ exceeding a hypothetical reception threshold $\beta = 2$ ($\hat{\Gamma} > 2$). The purpose of this setting is to study the statistics of the instantaneous SNR Γ in terms of the imperfection of the estimated SNR for MCS selection. The results have been obtained by using different values of the correlation coefficients: $\rho = 0.2, 0.95$, and $\rho_c = 0.2, 0.95$. A fixed value for average transmit power settings $\frac{P}{\sigma_v^2} = 1$ was used, and also two values for the number of antennas: $N = 2$ and $N = 4$. The first case ($N = 2$, $\rho = 0.2$, $\rho_c = 0.2$) in Fig. 2 also presents the results of simulation work, showing a perfect match with the theoretical model, thus validating the proposed expressions. The results show an interesting behaviour. At low values of the x -axis, the results indicate, as expected, that the CDF for low values of the spatial correlation coefficient has better performance than the CDF for values of ρ closer to one. However, this behaviour changes for larger values of the x -axis, where we can observe that higher values of spatial correlation provide better performance. Larger values of the x -axis correspond to a system where the target SNR of a given MCS is relatively high in comparison with the operational average SNR of the system. This means that in the low SNR regime channel correlation can actually outperform the case of uncorrelated branches. At high SNR values, however, the performance of uncorrelated channels becomes clearly dominant.

The effects of imperfect CSIT on the CDF can be clearly observed as the curves in Fig. 2 spread to the left of the hypothetical SNR decision threshold $\beta = 2$. The larger the spreading of the CDF to the left of this threshold the larger the error due to imperfect CSIT. We recall that the system is supposed to allow MCS transmission when the estimated SNR exceeds the target threshold. Imperfect CSIT causes that the instantaneous SNR is not precisely always above the selection threshold, thus causing the spreading of the CDF as observed in Fig. 2. The value of the CDF at $\hat{\Gamma} = 2$ is thus a measure of the error in the MCS selection process: it

denotes the probability that the instantaneous SNR is below the threshold $\beta = 2$ ($\Gamma < \beta$) when the estimated SNR was actually above this threshold ($\hat{\Gamma} \geq \beta$). We can observe how the CDF curves with higher correlation coefficient ρ_c tend to get closer to the limit of the estimated SNR threshold $\beta = 2$. Note that with perfect CSIT ($\rho_c = 1$), the CDF of the instantaneous SNR only exists for values higher than $\beta = 2$ ($\Gamma > 2$). The figures also show that the higher the number of antennas the effects of imperfect CSIT tend to be reduced for low values of channel correlation. Higher number of antennas means higher diversity gains, which is translated into a higher probability of correct packet reception, even in the presence of an inaccurate CSIT for MCS selection. However, the opposite behaviour is observed for larger values of spatial correlation, which tend to increase the probability of decision error due to imperfect CSIT, particularly when the number of antennas increases. Channel correlation tends to reduce the diversity of the receiver, and therefore it also reduces the probability of the instantaneous SNR to surpass the MCS detection threshold in the presence of imperfect CSIT. In addition, as the number of antennas increases, the lack of diversity due to channel correlation accumulates, leading to higher probabilities of error in the MCS assignment due to imperfect CSIT. As a consequence of this, it can be also observed that the effects of the dominance of correlated channels for low SNR values tends to be reduced with imperfect CSIT, particularly when the number of antennas increases.

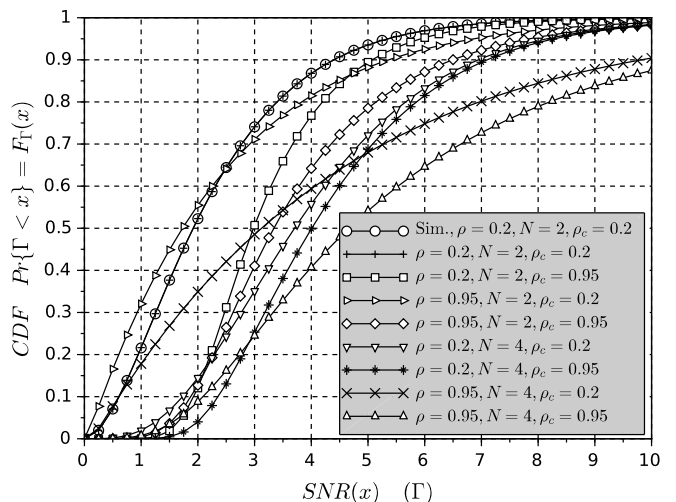


Fig. 2. CDF of the SNR Γ conditional on $\hat{\Gamma} > 2$

B. WiMAX modulation

This subsection employs the analytic formulae derived in previous sections to study a full wireless transmission system (WiMAX) with several MCSs taken from a LUT (look-up-table) that has been previously obtained via PHY-layer simulation. The MCSs used correspond to the WiMAX standard in [13], summarized in Table I, using a block length of 7200

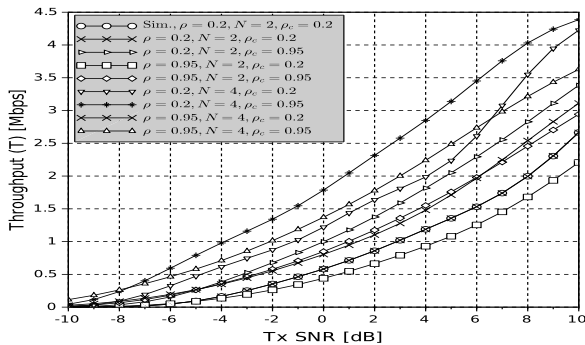


Fig. 3. Throughput vs. transmit SNR ($P\gamma/\sigma_v^2$) in dB

TABLE I
WiMAX MODULATION AND CODING SCHEMES [13].

QPSK 1/3		QPSK 1/2		QPSK 2/3	
SINR	BLER	SINR	BLER	SINR	BLER
-1.14	4.10e-3	1.32	4.13e-3	3.47	6.50e-3
QPSK 3/4		QPSK 4/5		16 QAM 1/3	
SINR	BLER	SINR	BLER	SINR	BLER
4.78	3.30e-3	5.46	4.97e-3	3.66	7.15e-3
16 QAM 1/2		16 QAM 2/3		16 QAM 3/4	
SINR	BLER	SINR	BLER	SINR	BLER
6.52	5.70e-3	9.37	3.80e-3	10.98	1.57e-3

symbols ($Q = 7200$). The spectral efficiency of each MCS is obtained by using the number of bits per constellation times the code rate displayed in Table I and other parameters such as frame length and repetition code rate as presented in the original standard in [13]. Note that the SNR thresholds displayed in Table I are in dBs. Fig. 3 shows the link-layer throughput calculated by using the expression in (3) versus the transmit power to noise ratio settings $\frac{P\gamma}{\sigma_v^2}$ using different values of the correlation coefficients (ρ and ρ_c) and different numbers of antennas of the MRC receiver. The first case ($N = 2, \rho = 0.2, \rho_c = 0.2$) in Fig. 3 also presents the results of simulation work, showing a perfect match with the theoretical model. The results show that at high SNR the total throughput improves, particularly when spatial correlation coefficient is reduced and also by increasing the numbers of the antennas of the MRC receiver. However, performance is generally degraded in case of imperfect CSIT (lower values of ρ_c). It can be observed that at low SNR, the results behave differently: high spatial correlation values lead to higher throughput. This confirms the results presented in the previous subsection. The effects of imperfect CSIT can be observed to be enhanced in the high SNR regime as the values of spatial correlation and the number of antennas increase.

V. CONCLUSIONS

A performance model of the packet reception process of a AMC-MRC receiver with spatially correlated channels and imperfect CSIT was proposed. The reception model is based

on the derivation of analytic expressions of correct packet reception probabilities in Rayleigh fading channels as an extension of outage probability analysis and SNR switching threshold selection. The model is useful for AMC systems, where the correct reception probability of each MCS is weighted by the operational BLER and spectral efficiency considering operation at the SNR threshold of each MCS. The proposed model provides a flexible abstraction/compression of the adaptive PHY layer that can be used in the optimization and design of upper layers. The results point towards an interesting conclusion: spatial channel correlation is not always bad for capacity. In the low SNR regime it can provide higher gains than the case of spatially uncorrelated channel. In the high SNR regime it was confirmed that uncorrelated channels provide more capacity gains than the correlated case. Imperfect CSIT was shown to reduce performance due to decision errors in MCS assignment. The degrading effects of imperfect CSIT were found to be enhanced by channel spatial correlation and by higher numbers of antennas. Correlation was found to reduce diversity gains of the receiver that in turn reduce the probability of correct reception in the presence of imperfect CSIT. This effect was found to be enhanced by increasing the number of antennas as correlation tends to be accumulated.

REFERENCES

- [1] F.A. Dietrich and W. Utschick, "Maximum ratio combining of correlated Rayleigh fading channels with imperfect channel knowledge," *IEEE Communications Letters*, vol. 7, no. 9, pp. 419-421, 2003.
- [2] Y. Ma, R. Schober, and S. Pasupathy, "Effect of channel estimation errors on MRC diversity in Rician fading channels," *IEEE Trans. on Vehicular Technologies*, vol. 54, no. 6, pp. 2137-2142, November 2005.
- [3] Y. Ma, "Impact of correlated diversity branches in Rician fading channels," *IEEE International Conference on Communications (ICC)*, vol. 1, pp. 473-477, 2005.
- [4] H.T. Hui, "The performance of the maximum ratio combining method in correlated Rician-Fading channels for antenna-diversity signal combining," *IEEE Trans. on Ant. and P.*, vol. 53, no. 3, pp. 958-964, 2009.
- [5] P. Loskot and N.C. Beaulieu, "A unified approach to computing error probabilities of diversity combining schemes over correlated fading channels," *IEEE Trans. on Comm.*, vol. 57, no. 7, pp. 2031-2041, 2009.
- [6] N.C. Beaulieu and X. Zhang, "On selecting the number of receiver diversity antennas in Ricean fading cochannel interference," *IEEE Global Telecommunications Conference (Globecom) 2006*, pp. 1-6.
- [7] N.C. Beaulieu and X. Zhang, "On the maximum number of receiver diversity antennas that can be usefully deployed in a cochannel interference dominated environment," *IEEE Transactions on Signal Processing*, vol. 55, no. 7, pp. 3349-3359, July 2007.
- [8] N.C. Beaulieu and X. Zhang, "On the maximum useful number of receiver antennas for MRC diversity in cochannel interference and noise," *IEEE International Conference on Communications (ICC)*, pp. 5103-5108.
- [9] Y. Dong, B. L. Hughes, and G. Lazzi "Performance analysis of maximum ratio combining with imperfect channel estimation in the presence of cochannel interferences," *IEEE Transactions on Wireless Communications*, vol. 8, no. 3, pp. 1080-1085, March 2009.
- [10] R. Samano-Robles and A. Gameiro, "Joint Spectrum Selection and Radio Resource Management for Distributed Antenna Systems with Cognitive Radio and Space Division Multiplexing" *Workshop on Smart Antennas, Stuttgart, Germany*, 2013.
- [11] L.Chen, Y. Yang, X. Chen, and G. Wei, "Energy-Efficient Link Adaptation on Rayleigh Fading Channel for OSTBC MIMO System With Imperfect CSIT" *IEEE Trans. Veh. Tech.*, vol. 62, no. 4, pp. 1577-1585, 2013.
- [12] J. Proakis, *Digital Communications*, McGraw-Hill, 4th edition 2001.
- [13] WiMAX Forum Standard, "WiMAX system level evaluation methodology, V.0.0.1," 2006.