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FRACTIONAL CALCULUS ANALYSIS OF THE COSMIC MICROWAVE BACKGROUND

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Abstract. Cosmic microwave background (CMB) radiation is the imprint from an early stage of the Universe and investigation of its properties is crucial for understanding the fundamental laws governing the structure and evolution of the Universe. Measurements of the CMB anisotropies are decisive to cosmology, since any cosmological model must explain it. The brightness, strongest at the microwave frequencies, is almost uniform in all directions, but tiny variations reveal a spatial pattern of small anisotropies. Active research is being developed seeking better interpretations of the phenomenon. This paper analyses the recent data in the perspective of fractional calculus. By taking advantage of the inherent memory of fractional operators some hidden properties are captured and described.

Key words: Cosmic microwave background, fractional calculus, cosmology, Fourier transform.

1. INTRODUCTION

The cosmic microwave background (CMB) was first detected in 1964 by Arno Penzias and Robert Wilson [1] at the Bell Telephone Laboratories, New Jersey. CMB is a remnant black body thermal radiation of $T_{CMB} = 2.725$ K, left over from the Big Bang, being released approximately 380,000 years after this primordial event and covering today almost uniformly the whole sky. This uniformity of the CMB represents a confirmation of the Big Bang scenario. However, there are tiny deviations of the CMB temperature across the sky, of the order of $0.001\ \%$. These anisotropies are unique relics of the physical conditions in the early Universe, confirming once more the Standard Model of Cosmology, whose inflationary scenario predicts the small fluctuations in the matter-radiation plasma, considered to be the seeds of the structures observed in the Universe today.

The anisotropy of the CMB were for the first time detected and quantified in 1992 by the Cosmic Background Explorer (COBE) mission [2], a National Aeronautics and Space Administration (NASA) satellite which measured large scale CMB temperature fluctuations at a rough angular resolution of about 7 degree. Since then, many efforts (hundreds of ground- and space-based experiments) have been done to measure in detail those properties.

A tremendous improvement in resolution, sensitivity and accuracy of the CMB measurements was attained by the Wilkinson Microwave Anisotropy Probe (WMAP) [3,4], a NASA satellite launched in 2001 for an initial schedule of 27 month of CMB observations, which then has been extended for several years to improve the quality of the data. Until now, there exists four public WMAP data releases, containing CMB measurements over the whole sky for 1, 3, 5 and 7 years, respectively. The measurement error in a given direction of the sky is smaller if the time of observation is longer, being proportional to the inverse of the square root of the number of the observations for a given pixel of sky. The WMAP instrument [5, 6] consists of a set of cooled microwave radiometers which measures the temperature of the CMB sky with an accuracy of μK. In order to facilitate the separation of the contaminant galactic foreground signals from the CMB, the instrument has five frequency bands in the range 22 - 106 GHz. One way to minimize the galactic contamination from the CMB maps measured by WMAP is constructed by a linear combination of maps from different frequencies, weighted such that the galactic signal is minimized. The resulted map is the so called the Internal Linear Combination (ILC) map.

The new information contained in the improved maps of the CMB fluctuations sheds light into several key questions in cosmology. The Standard Model of Cosmology is a theory with a couple of free parameters (to be experimentally determined) responsible for the actual structure and behaviour of the Universe. Their values establish if the Universe will expand forever, or if it will collapse, the percentages for the different forms of matter and energy (barionic matter, dark matter, dark energy) in Universe, the shape of the universe (flat, curved close or curved open), how and when the material structures in Universe were formed and, also, if the expansion of the Universe is accelerating, decelerating or stays at the same rate.

Since the CMB was affected by the physics of matter in conditions set by the cosmological parameters, the data measured with a tremendous accuracy by the WMAP allowed the scientists to determine most of the basic parameters of cosmology. Between the most important outcomes of the WMAP measurements of CMB anisotropies, we can mention here the confirmation of the approximately flat geometry of the Universe, the present accelerated expansion of the Universe, the confirmation of the inflationary scenario with nearly scale invariant primordial adiabatic perturbations and no evidence for primordial gravitational wave background (which would be created by a tensor component of the primordial fluctuations) [7]. Beyond the standard cosmological parameters, the last released WMAP CMB measurements were used, also, to improve the constraints on other parameters [8] like the total mass of neutrinos, $\sum m_v$, the effective number of neutrino species, N_{eff}, the equation of state parameter, w, of the Dark Energy, or on the primordial Helium abundance Y_p.

In this paper is adopted the Fourier analysis supported by the concepts of Fractional Calculus (FC). FC is a generalization of the standard differential calculus that was proposed first by Leibniz [10–14] and presently is being applied successfully to many areas of physics and engineering [15–24]. It is presently recognized that FC dynamics emerges whenever long range dynamical effects occur in the phenomenon under analysis. Given the considerable volume of information and the intricate relationships embedded in the CMB sky maps, it becomes straightforward to expect the association of FC in this dynamical analysis [25–29]).

Bearing these ideas in mind the rest of the paper is organized as follows. Section 2 analyses the microwave sky data covered by satellite measurements in the perspective of fractional calculus. Finally, section 3 outlines the main conclusions.

2. ANALYSIS OF EXPERIMENTAL DATA IN THE PERSPECTIVE OF FRACTIONAL CALCULUS

Several CMB data sets are available for scientific processing at the NASA, Goddard Space Flight Center, website http://lambda.gsfc.nasa.gov. WMAP satellite measured since 2001 the departures of the CMB temperature from the mean value in all directions in the sky, with an angular resolution of 0.23-0.93 degree, in five frequency bands. In the sequel it is considered the internal linear combination (ILC) of all the frequency maps in order to minimize the galactic contamination.

The CMB maps are stored in FITS format (Flexible Image Transport System) binary files. All WMAP maps are pixelized using the HEALPix system. The HEALPix pixelization scheme [30] initially developed by K. Gorski, B. Wandelt, and E. Hivon, is a hierarchical equal area iso-latitude pixelisation of the sphere. The WMAP CMB sky maps contain 12×512^2 equal area pixels, identified by the angular spherical galactic coordinates θ and ϕ (colatitude and longitude, respectively) in radians ($0 < \theta < \pi$ and $0 < \phi < 2\pi$). The FITS files provides the CMB temperature anisotropy for each pixel (the difference between the temperature of the pointing direction and the average temperature (2.725 K)) and, also, the number of observations per pixel, necessary to estimate the measurement error. Are available four data releases denoted $\{1, 2, 3, 4\}$ corresponding to one, three, five and seven years of measurements. For visualization of CMB data, a Mollweide projection [31] is adopted for the representation of the sky sphere in the 2D plane, since such a projection is an area preserving map and the primary scientific interest for CMB anisotropy data is in studying its spatial distribution.

Figure 1 depicts the Mollweide projection of the ILC map of CMB sky data captured by the WMAP satellite during one year of measurements.

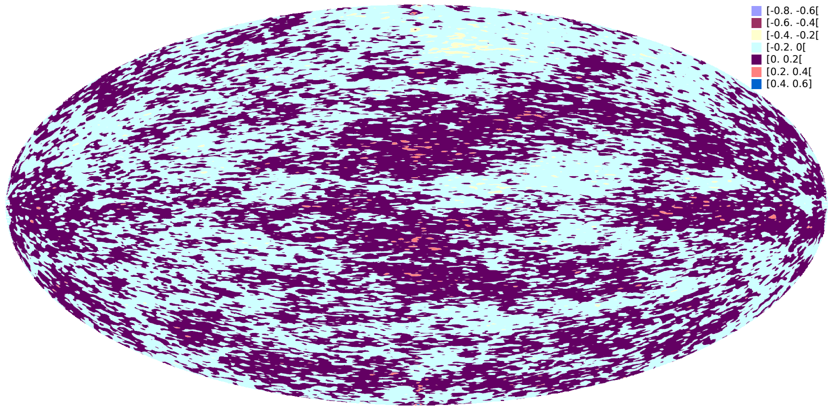

Fig. 1 – Mollweide projection of the ILC map of CMB sky data captured by the WMAP satellite during one year of measurements.

The plots for the other data releases are of the same type. It is clear the fractal nature of the plot and, therefore, it is straightforward to have in mind the estimation of the corresponding fractal dimension f_d. The fractal dimension, f_d, measures how a fractal fills the space when we zoom from large down to smaller measuring scales [32–34]. There are several definitions for fractal dimension that in general do not coincide. The box-counting dimension is adopted frequently because it is easy to obtain by computational means. In a box counting algorithm the number of "boxes" covering the graphical object is a power law function of the "box" size, being f_d the exponent of the power law. For a set S in a n-dimensional space, and for any $\epsilon > 0$, let $N_\epsilon(S)$ be the minimum number of n-dimensional boxes of side-length ϵ, needed to cover S. The box-counting dimension is, therefore, defined as:

$$f_d = -\lim_{\epsilon \to 0} \frac{\ln N_\epsilon(S)}{\ln \epsilon}. \tag{1}$$

Converting the Mollweide projections to black and white figures (colour black/ white for positive/negative values of CMB) and applying the algorithm to the four sets of data we obtain $f_d = \{1.789, 1.801, 1.800, 1.800\}$. As expected we verify that there are a minor variations of f_d and that a few years are not significant when compared with the "age" of the universe. Furthermore, the fractal dimension is a geometrical index not well adapted to capture the dynamical effects. In this line of thought, it was decided to apply the Fourier transform (FT) $F(i\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-i\omega t}\, \mathrm{d}t$, $i = \sqrt{-1}$, where ω represents the angular frequency. In the present case we calculate the FT over the first meridian. Therefore, the units of ω are the inverse of the spatial coordinate.

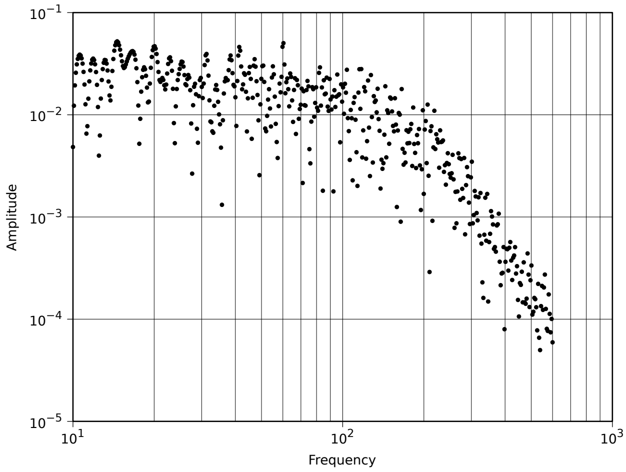

Fig. 2 – Amplitude of the Fourier transform $|F(j\omega)|$ *versus* ω for $\phi = \frac{\pi}{2}$).

Since the resulting plot is depicted in Figure 2 showing a significant level of noise. For reducing this problem is applied a median filter to the amplitude of the FT, and for each set of nine points, both in the horizontal and vertical coordinates, within a window, is replace by its median, resulting a smoother chart $|G(j\omega)|$. After performing several tests was concluded that the following function provided a good fit to the data:

$$|H(i\omega)| = \left| \frac{k}{(i\omega)^{\alpha}\left[1+\left(\frac{i\omega}{p}\right)^{\beta}\right]^{\gamma}} \right|. \tag{2}$$

where $\{k, \alpha, p, \beta, \gamma\}$ are parameters to be estimated.

For adjusting $|H(i\omega)|$ to the numerical data it is adopted a standard genetic algorithm with elitism, crossover within all population and 5% mutation probability. Several experiments demonstrated that the best fitness function J is given by:

$$J = \frac{1}{N}\sum_{k=1}^{N} \frac{|G(i\omega_k)| - |H(i\omega_k)|}{|G(i\omega_k)| + |H(i\omega_k)|}. \tag{3}$$

where N represents the total number of points of the median filtered plot. Furthermore, was adopted a population of 4000 individuals and 4000 iterations of the genetic algorithm.

Figure 3 depicts the amplitude of the median $|G(i\omega)|$ (dots) and approximation

$|H(i\omega)|$ (continuous line) *versus* frequency. The parameters estimated by the genetic algorithm are $\{k, \alpha, p, \beta, \gamma\} = \{0.163, 0.671, 258.855, 1.336, 3.611\}$ revealing clearly a fractional order both at low and high frequencies (α and β, γ, respectively). These results demonstrate that we have memory dynamics of the CMB data both for slow and fast variations in space. While these properties were obtained for the first meridian it remains to be explored the analysis of other coordinates and the case of the 2-dimensional plot itself.

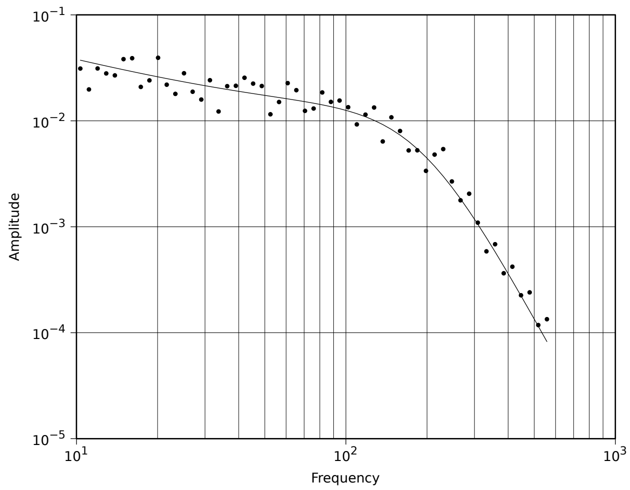

Fig. 3 – Amplitude of the median $|G(i\omega)|$ (dots) and approximation $|H(i\omega)|$ (continuous line) *versus* ω for $\phi = \frac{\pi}{2}$).

3. CONCLUSION

This paper explored the application of FC concepts in the analysis of CMB radiation. This topic is critical to cosmology, and active research is being developed. The observation of the fractal nature of the Mollweide maps triggered the idea of applying Fourier transform to capture the spatial dynamics and the memory properties. The results reveal that the phenomenon is undoubtedly of fractional order, stimulating further study tacking into account all information.

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