

Reply to: Comments on “Particle Swarm Optimization with Fractional-Order Velocity”

J. A. Tenreiro Machado · E. J. Solteiro Pires · Micael S. Couceiro

We agree with Ling-Yun et al. [5] and Zhang and Duan comments [2] about the typing error in equation (9) of the manuscript [8]. The correct formula was initially proposed in [6,7]. The formula adopted in our algorithms discussed in our papers [1,3,4,8] is, in fact, the following:

$$v_{t+1} = \alpha v_t + \frac{1}{2}\alpha(1-\alpha)v_{t-1} + \frac{1}{6}\alpha(1-\alpha)(2-\alpha)v_{t-2} + \frac{1}{24}\alpha(1-\alpha)(2-\alpha)(3-\alpha)v_{t-3} + \phi_1(b-x) + \phi_2(g-x) \quad (1)$$

The results in [5] and [8] are not directly comparable. In [5], the charts are obtained by running a single test; while in [8], the charts are obtained by the median of 201 simulations tests. It should be noted that we

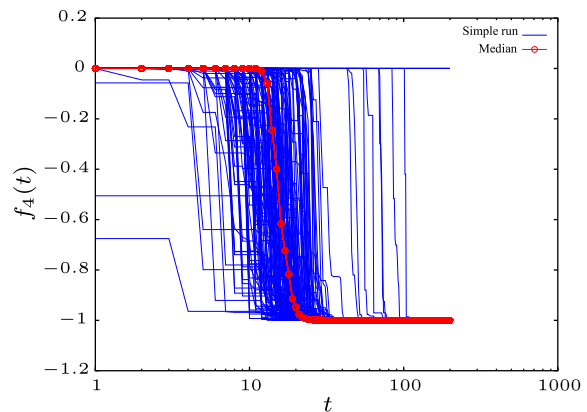


Fig. 1 Evolution of 201 simulations for the Easom function, with $\alpha = 0.5$, and its median

use several simulations, because particle swarm optimization (PSO) is a stochastic algorithm. Therefore, each time it is executed it leads to a distinct trajectory. Due to this fact, a test group of 201 simulation was considered, and the median adopted as the final output. For example, for the Easom function, $f_4(x)$ (2), with $\alpha = 0.5$, the 201 evolution trajectories obtained are depicted in Fig. 1 by blue lines. The corresponding median is depicted by the red line.

In conclusion, with exception of the typing error in the formula the implemented algorithm and the published results are correct.

$$f_4(x) = -\cos(x_1) \cos(x_2) e^{-(x_1-\pi)^2 - (x_2-\pi)^2}$$

$$f^*(x) = -1.0, \quad x_1, x_2 \in [-100, 100] \quad (2)$$

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