Numerical evaluation of three-dimensional scarf repairs in carbon-epoxy structures

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ABSTRACT

The widespread employment of carbon-epoxy laminates in high responsibility and severely loaded applications introduces an issue regarding their handling after damage. Repair of these structures should be evaluated, instead of their disposal, for cost saving and ecological purposes. Under this perspective, the availability of efficient repair methods is essential to restore the strength of the structure. The development and validation of accurate predictive tools for the repairs behaviour are also extremely important, allowing the reduction of costs and time associated to extensive test programmes. Comparing with strap repairs, scarf repairs have the advantages of a higher efficiency and the absence of aerodynamic disturbance. This work reports on a numerical study of the tensile behaviour of three- dimensional scarf repairs in carbon-epoxy structures, using a ductile adhesive (Araldite^S 2015). The finite elements analysis was performed in ABAQUS^S and Cohesive Zone Modelling was used for the simulation of damage onset and growth in the adhesive layer. Trapezoidal cohesive laws in each pure mode were used to account for the ductility of the specific adhesive mentioned. A parametric study was performed on the repair width and scarf angle. The use of over-laminating plies covering the repaired region at the outer or both repair surfaces was also tested as an attempt to increase the repairs efficiency. The obtained results allowed the proposal of design principles for repairing composite structures.

Keywords: Epoxy/epoxides, Composite, Finite element stress analysis, Fracture, Repair

1. Introduction

Carbon-fibre reinforced plastic (CFRP) components are being more and more used in structures demanding a high performance because of their superior characteristics (such as high strength, high stiffness, long fatigue life and low density) [1]. In spite of these advantages, CFRP materials usually show a high sensitivity to temperature, moisture and accidental impacts. Thus, repair strategies should always be included in the quality assurance schedule of these composite structures [2]. Bonding of patches with adhesives, which provides durable and resistant unions [3–8], is being increasingly used in repairs. Typically, the initial strength and stiffness of the damaged components cannot be restored using this technique without a significant weight penalty. Thus, a substantial amount of research has been carried out in the last decades on the development of efficient adhesively bonded repairs and on adhesives technology [9–17]. Several studies are available for the repair of composite panels [10-13], including finite element (FE) works describing predictive techniques for the strength of repaired structures [14-17]. Unlike the single-strap solution, scarf repairs do not cause a substantial bending of the components, which reduces peel stresses [18-22]. Moreover, shear stress distributions along the bond length are more uniform than for single and double-strap repairs, due to the tapering effect at the scarf edges [23]. The outcome of the optimization of stresses is a higher strength for the same bond area than strap repairs [24], which renders scarf repairs more suited to critical applications [25]. The substantial or full strength recovery achieved by this method, provided that the repair is correctly designed, usually makes scarf repairs as permanent [26-29]. Conversely to strap techniques, scarf repairs are also flush with the damaged structure, which is an important advantage since aerodynamic disturbance is prevented and stealth characteristics (if relevant) are not compromised. In specific cases, a flush surface may be imposed to avoid interference between components. The scarf repair technique has become particularly important in the last decades, due to the increasing use of sandwich panels in aircraft structures [28,30].

Despite all of these advantages, scarf repairs are more difficult to execute, which reflects on higher costs. In addition, they require a large repair area, since relatively small angles are necessary to restore the strength of components [27,29,31]. The execution of this repair initiates by machining a tapered cavity to remove the damaged material and to provide the repair geometry. This is usually performed with a hand-held pneumatic router or grinder. Two solutions can be adopted for the patch application: adhesively bonding a conical patch with the complementary shape of the laminate [23,32-34], or filling the scarfed hole with increasing diameter pre-preg plies, followed by simultaneous curing with the adhesive film, in the autoclave or using a vacuumbag [35-38]. The implementation of a pre-cured and machined scarf patch to a composite fuselage skin is described in the work of Bauer and Maier [39]. The high quality repair achieved with this technique met all the structural requirements defined in the repair specifications manual of the aircraft.

In tension, experimental and FE studies show an exponentially increasing strength of scarf assemblies (joints or repairs) with the reduction of the scarf angle (a), due to the corresponding increase of bond area [29,31,40-43]. On the failure modes of these assemblies, it was found that values of a below approximately 21 led to cross-sectional failures of the laminates outside the repaired region, while bigger values typically yielded failures in the adhesive layer [23,33,44]. Charalambides et al. [13] tested experimentally scarf repairs with a¹/₄ 21. Four distinct failures were identified, depending on the temperature and moisture conditions, as well as the type of load (static or dynamic). As a complement, the same authors [14] performed a two-dimensional (2D) FE analysis to simulate the four failure modes of the repairs. The failure loads were estimated by different failure criteria and compared with the experiments, with promising results. Odi and Friend [45] compared the stress distributions between three different FE approaches to simulate stepped and a1/431 CFRP scarf repairs under tension, using equivalent orthotropic elastic properties for the CFRP components. For the scarf repairs, shear stresses in the adhesive were nearly constant, leading to a high efficiency, as the adhesive failed simultaneously at all the bond length. Three-dimensional (3D) ply-level analyses of composite repairs have recently become feasible, as a result of the great evolution in the processing capabilities of common desktop computers [46]. In recent years, good quantitative predictions were achieved on the static strength of adhesively bonded repairs with Cohesive Zone Models (CZMs) coupled with FE simulations [47,48]. The use of this technique, which accounts for the progressive damage evolution, is particular meaningful for scarf repairs due to the difference between damage initiation and failure loads. This is because of singularities at the bond edges of a substantially higher magnitude than the stresses along the adhesive bond [49-52]. The work of Campilho et al. [23] validates with experiments of a developed trapezoidal CZM applied to tensile loaded 2D scarf repairs on CFRP laminates, for values of a between 21 and 451. To account for the experimental fractures, the cohesive failure of the adhesive layer and composite interlaminar and intralaminar (in the transverse and fibre directions) failures were considered at different regions. The corresponding pure modes I and II laws were estimated with an inverse modelling technique. The accurate predictions of the fracture loads and failure mechanisms validated the proposed technique.

This study reports on the tensile behaviour of 3D scarf repairs in CFRP structures, using a ductile adhesive (Araldite^S 2015). Since the proposed numerical methodology was already validated with experiments in previous works, giving accurate predictions [47,48], the present research is restricted to a purely numerical optimization analysis of the repairs that will allow the definition of repair principles for composite structures. Traditionally, the design of scarf repairs is based on 2D models, as a simplification of the 3D geometry [53–56]. The primary motivation for this work stems from the known inconsistencies between the 2D approximations and the 3D repair [17,56-59]. In fact, with the 2D simplification, stresses along the scarf bond are regarded as constant in the width direction of the repairs, thus neglecting the concentrations at the scarf edges near the longitudinal mid-plane of the 3D repair [29,56]. Moreover, the 2D simplified geometry does not capture the typical 3D stress effects of these repairs [60,61], which may result on a premature catastrophic failure at or near the interfaces between the composite and the adhesive layer [51]. The FE analysis was performed in ABAQUS^S and used CZMs for the simulation of damage onset and growth in the adhesive layer. Trapezoidal cohesive laws in each pure mode were used to account for the ductility of the adhesive used. A parametric study was performed on the repair width and scarf angle. The use of over-laminating plies covering the repaired region at the outer or both repair surfaces was also tested as an attempt to increase the repairs efficiency. The results obtained allowed the establishment of design principles for repairing.

2. Numerical analysis

2.1. Cohesive Zone Model

The cohesive fracture of an adhesive layer of Araldite 2015 with thickness ($t_{\rm A}$) of 0.2 mm was simulated with a mixed-mode (I+II+III) CZM. A trapezoidal law between stresses (s) and relative displacements (d_r) between homologous points of the cohesive elements with zero thickness was considered (Fig. 1), to account for the adhesive ductility [23,47,48]. The formulation allows a mixed-mode behaviour, in which damage onset is predicted using a quadratic stress criterion:

$$\begin{pmatrix} \frac{\sigma_{\mathrm{I}}}{\sigma_{\mathrm{u},\mathrm{I}}} \end{pmatrix}^{2} + \left(\frac{\sigma_{\mathrm{II}}}{\sigma_{\mathrm{u},\mathrm{II}}} \right)^{2} + \left(\frac{\sigma_{\mathrm{II}}}{\sigma_{\mathrm{u},\mathrm{II}}} \right)^{2} = 1 \quad \text{if } \sigma_{\mathrm{I}} > 0,$$

$$\left(\frac{\sigma_{\mathrm{II}}}{\sigma_{\mathrm{u},\mathrm{II}}} \right)^{2} + \left(\frac{\sigma_{\mathrm{III}}}{\sigma_{\mathrm{u},\mathrm{III}}} \right)^{2} = 1 \quad \text{if } \sigma_{\mathrm{I}} \le 0,$$

$$(1)$$

where S_i (*i* ¹/₄ I, II, III) represent the stresses in each mode and $S_{u,i}$ (*i* ¹/₄I, II, III) the corresponding local strengths. The first Eq. (1) can be rewritten as a function of the relative displacements:

$$\left(\frac{\delta_{1m,l}}{\delta_{1,l}}\right)^2 + \left(\frac{\delta_{1m,ll}}{\delta_{1,ll}}\right)^2 + \left(\frac{\delta_{1m,ll}}{\delta_{1,ll}}\right)^2 = 1.$$
(2)



Fig. 1. The trapezoidal softening law for pure-mode and mixed-mode.

 $d_{1,i}$ (*i*¹/₄ I, II, III) are the pure mode relative displacements at damage initiation and $d_{1m,i}$ (*i*¹/₄ I, II, III) the corresponding mixed-mode ones. Stress softening onset was predicted using a criterion similar to $\binom{2}{2}$:

$$\left(\frac{\delta_{2m,I}}{\delta_{2,I}}\right)^2 + \left(\frac{\delta_{2m,II}}{\delta_{2,II}}\right)^2 + \left(\frac{\delta_{2m,III}}{\delta_{2,III}}\right)^2 = 1.$$
(3)

stress softening onset and $d_{2m,i}$ (*i*¹/₄ I, II, III) the corresponding mixed-mode ones. Crack growth was simulated by the linear energetic criterion:

$$\frac{J_{\rm I}}{J_{\rm Ic}} + \frac{J_{\rm II}}{J_{\rm IIc}} + \frac{J_{\rm III}}{J_{\rm IIIc}} = 1, \tag{4}$$

being J_{ic} (i¹/4I, II, III) the fracture energy in the respective pure mode. When Eq. (4) is satisfied at a given integration point, damage grows and stresses are released, apart from normal compressive ones. A detailed description of this model can be found in the work of Campilho et al. [62].

2.2. Fracture simulation

In the numerical models, crack propagation with cohesive elements was only considered for the cohesive failure of the adhesive layer. Apart from this possibility, a stress-based criterion was assumed for the tensile failure of the composite parts. For the Ol plies, oriented in the load direction, the experimentally determined strength from unidirectional specimens was considered (1280 7 177 MPa, the average value was used for the prediction of failure). For the 901 plies, i.e., experiencing a matrix failure, typical values from the literature were used (E40 MPa) [23]. In the numerical models, failure was predicted as the load of patch debonding onset [63], guaranteeing that neither the laminate nor the patch attained the mentioned strengths of the two sets of plies. The prospect of interlaminar and intralaminar failures near the scarf was not considered, despite the chance of this occurrence [23], owing to slightly smaller cohesive properties for these propagations compared with the properties of some structural adhesives. Nonetheless, this procedure was adopted due to the modelling difficulties and additional computational time required to simulate these localized fractures, which do not influence by a significant amount the global characteristics of 3D repairs such as the ones proposed in this work. Actually, these are mostly ruled by the laminate properties and instant of patch debonding, rather than by the strength at the repaired region [63]. This is due to the clear difference between the composite tensile properties in the fibres direction and the adhesive and composite (interlaminar and intralaminar) properties. Under this simplification hypothesis, the predictions should be interpreted in relative terms between the different tested solutions, allowing the establishment of design principles for these repairs, instead of being viewed as precise quantitative predictions. The adhesive layer was introduced in the numerical models by the trapezoidal CZM presented previously, with experimentally defined

properties for the specific value of t_A used in the repairs. The adhesive layer elastic stiffness in tension and shear (up to d1,i, Fig. 1) was specified from the experimentally measured values of Young's modulus (E¹/₄1850 MPa) and shear modulus (G¹/₄650 MPa) [64], as detailed in the work of Campilho et al. [62]. The other necessary parameters to define the trapezoidal law are the local strengths $(S_{u,i})$, the second inflexion points $(d_{2,i})$ and the fracture energies (Jic). To account for the known dependency of these quantities with t_A [23], in this work, the cohesive laws of the adhesive layer in pure modes I and II were estimated by Double Cantilever Beam (mode I) and End-Notched Flexure (mode II) tests with the same value of t_A , using inverse modelling [65,66]. The pure mode III cohesive law was equalled to the pure mode II one as an approximation [62]. Table 1 presents the cohesive parameters of the pure modes I and II laws used to simulate an adhesive layer of Araldite^S 2015 with $t_{A^{1/4}}0.2 \text{ mm}$.

2.3. Numerical conditions

Fig. 2(a) shows the scarf repair geometry and characteristic dimensions. The Outer Edge of the Scarf (OES) and Inner Edge of the Scarf (IES) loci are identified in the figure. The characteristic dimensions of the scarf repair are the laminate length (a), width (b) and thickness $(t_{\rm P})$, hole diameter (d), $t_{\rm A}$ and a. The parameter a can be defined as the angle between the scarf and the horizontal plane. The scarf length (L_S) is intrinsically related to the chosen values of a and $t_{\rm P}$. The following dimensions were considered for the numerical analysis: $a^{1}4200 \text{ mm}$, $b^{1}450 \text{ and } 80 \text{ mm}$, $t_{\rm P}$ ¹/₄2.4 mm, d¹/₄10 mm, $t_{\rm A}$ ¹/₄0.2 mm and a¹/₄101, 151, 251 and 451 (for the repairs with b¹/₄50 mm) or a¹/₄51, 101, 151, 251 and 451 (for the repairs with b1/480 mm). The minimum values of a were imposed by the respective values of b. Although small values of a are necessary for the repair to work in the preferential shear plane and to achieve a reasonable bond area, necessary to an efficient repair [31,67], scarf angles up to 451, which may be imposed due to restrictions to the structure dimensions, were also analysed. Sixteen plies and [02,902,02,902]s lay-up laminates and patches were used in this analysis (considering a 0.15 mm ply unit thickness). Fig. 2(b) shows the numerical idealization of the scarf repair tensile test. Only 1/4 of the laminate was modelled, by the use of symmetry conditions at the mid-transversal (A) and midlongitudinal (B) planes. The scarf repairs were simulated in ABAQUS^S with 3D models. The cohesive elements, used to extract stresses along the bond length and to simulate a cohesive failure

Table 1

Cohesive parameters in pure modes I and II to simulate an adhesive layer of Araldite^S 2015 with t_A ^{1/4}0.2 mm.

Cohesive laws	i	J _{ic} (N/mm)	$S_{u,i}$ (MPa)	d _{2,<i>i</i>} (mm)
Adhesive layer	I	0.43	23.0	0.01870
	II	4.70	22.8	0.1710



Fig. 2. Scarf repair geometry (a) and numerical idealization with symmetry conditions (b).



Fig. 3. Detail of the mesh at the repaired region (a) and at the IES (b) for the a¹/₄ 151 repair (b¹/₄ 50 mm).

Table 2 Ply mechanical properties of the CFRP laminates and patches.

E ₁ ¼1.09E+05 MPa	n ₁₂ ¼0.342	G ₁₂ ¼4315 MPa
E ₂ ¼8819 MPa	n ₁₃ ¼0.342	G ₁₃ ¼4315 MPa
E ₃ ¼8819 MPa	n ₂₃ ¼0.380	G ₂₃ ¼3200 MPa

of the adhesive layer, were introduced in the numerical models along the scarf length. Cohesive elements allow the extraction of stresses in the directions tangent and normal to the scarf (coordinate system t-n in Fig. 2(a)). The standard ABAQUS^S solid finite elements, which evaluate stresses at the Gauss points and, additionally, in the coordinate system x-y (Fig. 2(a)), are not suited for these geometries [29,31,68]. A geometrical non-linear analysis was performed, using 8-node hexahedral and 6-node pentahedral solid finite elements available in ABAQUS^S. Fig. 3 shows the mesh at the repaired region (a) and at the IES (b) for the $a^{1/4}$ 151 repair ($b^{1/4}$ 50 mm). The mesh is particularly refined at the scarf region, in the adhesive and in the laminate and patch, with 40 elements along $L_{\rm S}$ to ensure a bigger refinement at the loci of stress concentrations [29,31,69]. Thirty elements were considered for 1/4 of the patch in the radial direction. At the scarf region, each group of two equally oriented and adjacent plies was modelled with five solid elements. Mesh coarsening was applied to reduce the number of elements outside this region. The laminate and patch were modelled as elastic orthotropic, considering the properties of Table 2 [70].

3. Results

3.1. Stress analysis

Normal (s_n) and shear (t_m) stresses in the adhesive layer for scarf assemblies are practically constant for isotropic adherends or unidirectional composites [31,43,45]. This can be justified in light of a smaller load eccentricity than in lap or strap geometries [27,70] and by the adherend tapering effect at the scarfed region [23,27], and helps to the high efficiency of these assemblies relatively to the bond area. For layered composites with differently oriented plies, large variations of stresses develop, due to the stiffness variation between plies, even when the laminates to be joined have identical lay-ups [23,29,68,71,72].

This section presents an elastic analysis of the adhesive layer stresses, for an easier understanding of the repairs fracture behaviour. S_n peel and t_{tn} stresses along the bond line at plane *B* are presented in Figs. 4 and 5, respectively, for the repairs with



Fig. 4. s_n peel stress distributions in the adhesive layer for the different values of a (repairs with b ½ 50 mm).



Fig. 5. t_{tn} shear stress distributions in the adhesive layer for the different values of a (repairs with *b*¹/450 mm).

b ¹/₄ 50 mm and the corresponding values of a. This plane was selected for the analysis since it corresponds to the locus of higher magnitude of stresses [68,73]. Both stresses were normalized by the average shear stress along the scarf length for the respective value of a (t_{avg}). Sn peel stresses are much smaller in magnitude than t_{tn} stresses for the smaller values of a. In fact, under these conditions, the repair is primarily loaded in shear [31,53]. However, Sn stresses gradually increase with a [74]. Inclusively,

for a $a^{1/4}451$ repair, S_n stresses approach t_{avg} in magnitude. S_n stress profiles are wavy, without a direct correspondence with the plies orientation, being tractive along L_S . t_{tn} stress profiles are similar between the different values of a evaluated, despite the small reduction of peak t_{tn} stresses magnitude with the increase of a. t_{tn} stresses clearly reflect the differences in compliance between the plies adjacent to the adhesive [29,68,72]. Actually, they peak near the 0l plies because of their higher stiffness in the loading direction, while the adhesive layer has a constant stiffness [55]. t_{tn} stresses gradients between the regions of 0l and 90l plies gradually diminish with the increase of a. An increasing tendency for S_n and t_{tn} stresses was also found towards the IES and OES, which is consistent with the work of Campilho et al. [31], concerning the tensile behaviour of 2D unidirectional scarf repairs.

3.2. Strength analysis for the standard scarf repairs

Fig. 6 compares the load-displacement (P-d) curves between the different values of a, considering $b \frac{1}{4} 50$ mm. The values of P and d are the direct output of the simulations, i.e., considering half-width and half-length of the repairs. The original curves were shifted by different values ($D^{1}40$, 0.05, 0.10 and 0.15 mm) for an easier visualization. This figure shows an identical stiffness of the repairs and increasing values of P with the decrease of a, whose reasoning was already discussed [29,31,40,43]. The stiffness reduction near the peak load is due to softening of the adhesive layer cohesive elements, in anticipation of patch debonding. Fracture of the repairs was identical for all the studied values of a, with a simultaneous fracture of the adhesive layer at the entire bond near plane B after localized damage at the IES and OES, propagating swiftly in the radial direction of the scarf up to approximately 451 of plane B. An example of this fracture is presented in Fig. 7 for a a¹/₄151 repair (b¹/₄50 mm). These results



Fig. 6. Numerical *P*-d curves for the different values of a (repairs with $b\frac{1}{4}50$ mm).



Fig. 7. Numerical failure for a a¹/₄ 151 repair (b ¹/₄ 50 mm).

can be easily interpreted from the stress analysis presented previously (Figs. 4 and 5), i.e., by the higher magnitude of s_n and t_{tn} stresses near the bond edges. The onset of fracture near plane B and gradual radial propagation is due to peaking of s_n and t_{tn} stresses at plane B, gradually diminishing with the increase of the angle from this plane [68,73]. Fig. 8 plots the efficiency of the repairs (Z) as a function of a for the repairs with $b\frac{1}{4}50$ and 80 mm. The value of Z is defined by the quotient between the patch debonding onset load and the failure load of the undamaged composite. The failure load was determined by experimental tensile tests on three undamaged specimens with b1/415 mm and the same lay-up and thickness of the damaged laminates, giving an average value of failure stress and deviation of 6557134 MPa. For the calculations of Z, the average value was considered in the estimation of the failure load for the b 1/4 50 and 80 mm undamaged laminates, using the respective cross-sectional areas. In all the repairs, including in the study of the following section, it was checked that the failure strength of the 01 and 901 plies was not attained prior to patch debonding onset. The exponential increase of Z with the reduction of a is related to the corresponding increase of the bond area [29,31,40,43]. The values of Z are slightly bigger for the repairs with $b \frac{1}{4} 80 \text{ mm}$, with an increasing difference to the $b \frac{1}{4} 50 \text{ mm}$ repairs as a diminishes. This can be explained by a larger influence of the laminate resistant area at the symmetry plane A on the global characteristics of the repairs for b1/480 mm, since the repair dimensions are similar for a given value of a. The best results are always granted by the smallest value of a, i.e., a1/4101 for the $b^{14}50 \text{ mm}$ repair (Z E 42%) and $a^{14}51$ for the $b^{14}80 \text{ mm}$ repair (Z € 55%).

3.3. Strength analysis for the scarf repairs with over-laminating plies

An alternative to the use of very small values of a, required to fully restore the structure strength, consists on the application of external doublers (or over-laminating plies) adhesively bonded at the repaired region to protect the patch tips and to provide a larger cross-sectional area at the repaired region, thus increasing the strength of the repairs [34]. These plies are generally very thin and designed to follow the parent structure contour as closely as possible [75]. Although the most efficient solution is to bond overlaminating plies on both the laminate faces [13,14,76], a more practical solution consists on their application only on the outer face of the repair (upper surface in Fig. 2(a)) [38,77]. This choice can also be imposed by accessibility difficulties to the inner face of the composite structure, or be rendered unfeasible for sandwich laminates with composite faces [78]. Gunnion and Herszberg [68]



Fig. 8. Z as a function of a for the repairs with $b \frac{1}{50}$ and 80 mm.

investigated the effect of an over-laminate on both faces of scarf joints with 16 and 32 plies laminates applied to the full length and width of the joints. This technique resulted in a significant drop on peak peel and shear stresses within the bond, which otherwise develop near the free edges. Different lav-ups and increasing the number of over-laminating plies from the initial analysis (2 plies) provided no significant differences in reducing peak peel and shear stresses. In the present work, an optimization study was carried out on the influence of using over-laminating plies on the value of Z of the repairs, considering reinforcement only at the outer face of the repair (single reinforcement) and at both faces (double reinforcement). The over-laminate consisted of two plies of circular shape: a 01 ply adjacent to repair surface, covered by a 901 ply. This set-up was selected to account for a typical bi-axial loading, despite not being a symmetric lay-up. Fig. 9 shows the tested geometry, considering double reinforcement. Two values of overlap with the damaged structure at the outer face (e) were tested for the single



Fig. 9. Numerical idealization of the scarf repair with double reinforcement.

reinforcement: 2.5 and 5 mm. For the double reinforcement, only $e \frac{1}{4} 5 \text{ mm}$ was equated, due to evidence of better results. Bigger values were not considered, to guarantee a minimum clearance with the repair edges for the smallest values of a (for each value of b). An example of the mesh refinement for this repair configuration is shown in Fig. 10, illustrating the mentioned limitation. Identical dimensions were considered for the reinforcements on both faces for fabrication simplification purposes and maximum effect of the over-laminates, although the inner face of the repairs may be over-reinforced [76]. Fracture for the different tested solutions depended on the type of reinforcement (single or double) and value of a. For the single reinforcement and bigger values of a, the asymmetry of loading induced by the over-laminating plies led to a slight transverse deflection of the laminate [38] that caused premature crack initiation near plane B at the IES (unreinforced region). This damage then propagated towards the OES and to the overlap region, simultaneously to radial growth of damage in the direction of plane A. Fig. 11(a) shows damage initiation at the IES for a $a_{4}451$ repair with $b_{4}50$ mm and $e_{4}2.5$ mm. For the smaller values of a, fracture was simultaneous over the entire bond and overlap region. Although the transverse deflection of the repairs still subsisted, the bigger taper length in the laminate and patch allowed a slight bending of the scarf tips, enough to avoid a premature fracture at the IES. Fig. 11(b) relates to a a¹/₄10l repair with $b\frac{1}{4}50 \text{ mm}$ and $e\frac{1}{4}2.5 \text{ mm}$. The asymmetry of load induced by the over-laminating plies is prevented using double reinforcement. Thus, for the bigger values of a the entire scarf bond failed simultaneously near plane *B*, whilst the overlapping plies were kept under load (Fig. 12(a)). Increasing further the load causes the detachment of the over-laminate at the outer face, followed by the one at the inner face, both from plane *B* towards



Fig. 10. Detail of the mesh at the repaired region (a) and at the IES (b) for the a¹/₄ 101 repair (b¹/₄ 50 mm) using double reinforcement and e¹/₄ 5 mm.



Fig. 11. Numerical failure initiation for a a^{1/4} 451 repair (a) and failure for a a^{1/4} 101 repair (b) (e^{1/4} 2.5 mm, b^{1/4} 50 mm and reinforcement on the outer face).



Fig. 12. Numerical failure initiation for a a^{1/4} 451 repair (a) and failure for a a^{1/4} 101 repair (b) (e^{1/4} 2.5 mm, b^{1/4} 50 mm and double reinforcement).



Fig. 13. \underline{Z} as a function of a for the repairs with single reinforcement (dimensions in mm).

plane A. For the smaller values of a, the overlap area between the reinforcement plies and the laminate is much larger for the inner plies than for the outer plies, which results on failure along the scarf bond and at the outer over-laminate, whilst the inner one is kept intact (Figs. 12(b)). Figs. 13 and 14 plot the values of Z as a function of a for the repairs with single and double reinforcement, respectively. Fig. 13 globally shows the exponential trends formerly mentioned, with an increasing difference in the value of Z between the $b^{1/4}50$ and 80 mm repairs with the reduction of a, whose cause was already discussed. The value of e showed a large impact on Z, with bigger values being recommended on account of higher repair efficiency. This is due to the larger shear resistant area between the laminate and over-laminating plies [38]. The highest efficiency for the $b\frac{1}{4}50$ mm and 80 mm repairs (single reinforcement) was attained with the a1/4 101 repair $(\mathbb{Z} \in 49\%)$ and the $a^{1/4}51$ repair $(\mathbb{Z} \in 62\%)$, respectively. These results confirm a non-negligible improvement to the standard scarf repair, which will be quantified later on this work. The results of Fig. 14 (double reinforcement) also evidence an increasing difference of Z with the reduction of a between the b1/450 and 80 mm repairs. The improvement of Z was substantial, with the highest values for the $b \frac{1}{4} 50$ and 80 mm repairs being $(\mathbb{Z} \in 72\%)$, respectively. The parameter j is introduced in Fig. 15 as the quotient between the repaired strength with single or double reinforcement and the strength of the corresponding unreinforced repairs (only for e¹/₄5 mm), giving a clear perception of the influence of this technique on the repaired strength. A clear distinction can be made between single and double reinforcements, with the latter ones exceeding more than twice the single reinforcement corresponding values of j. This is



Fig. 14. Z as a function of a for the repairs with double reinforcement (dimensions in mm).



Fig. 15. jas a function of a for the repairs with single and double reinforcement (dimensions in mm).

because of the aforementioned suppression of the premature fracture near the IES and of the load eccentricity using the double reinforcement scheme. As a whole, the strength improvements with this technique are substantial, which makes it an appealing option to increase the efficiency of scarf repairs.

4. Concluding remarks

In this work, a comprehensive numerical analysis was performed on the tensile behaviour of three-dimensional scarf

repairs in carbon-epoxy structures, using a ductile adhesive (Araldite^S 2015). The finite element analysis was performed in ABAQUS^S and used Cohesive Zone Models for the simulation of damage onset and growth in the adhesive layer. Trapezoidal cohesive laws in each pure mode were used to account for the ductility of the adhesive used. Validation of the finite element methodology used was performed in previous works, which assures the legitimacy of the results. A parametric study was performed on the scarf angle, considering two values of width for the laminates to be repaired. The strength improvement increased exponentially with the reduction of the scarf angle, which implies that small angles are always recommended. The use of overlaminating plies at the outer or both of the repair faces was tested as an attempt to increase the repairs efficiency, which for scarf repairs without over-laminate was close to 50% of the undamaged laminates strength, for the smallest scarf angle. Results showed that efficiencies of approximately 70% of the undamaged strength could be attained by the use of over-laminating plies on both the laminate faces, with maximum improvements from the scarf repairs without over-laminate between approximately 30% and 60%, depending on the scarf angle. Reinforcing only at the outer face of the repair, which may be the only feasible option due to accessing or disassembly difficulties, is also recommended, despite a smaller restitution of strength. Efficiencies above 70% could be achieved using smaller scarf angles than the ones tested, which would imply a larger width of the laminates to be repaired. This work allowed the establishment of principles to design scarf repairs. The quantitative predictions presented in this work should be considered valid only for the specific set of conditions selected for the analysis, whilst the generic principles established to increase the efficiency of scarf repairs are always recommended.

Acknowledgements

The authors would like to thank the Portuguese Foundation for Science and Technology for supporting the work here presented, through the individual Grant SFRH/BD/30305/2006 and the research Project PDTC/EME-PME/64839/2006.

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