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Technical Report

Probability Distribution Functions

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Abstract

This technical report describes the PDFs which have been implemented to model the behaviours of certain parameters of the Repeater-Based Hybrid Wired/Wireless PROFIBUS Network Simulator (RHW2PNetSim) and Bridge-Based Hybrid Wired/Wireless PROFIBUS Network Simulator (BHW2PNetSim).

Index

List of Tables

Probability Distribution Functions

1.Introduction

This technical report describes the PDFs which have been implemented to model the behaviours of certain parameters of the Repeater-Based Hybrid Wired/Wireless PROFIBUS Network Simulator (RHW2PNetSim) [1] and Bridge-Based Hybrid Wired/Wireless PROFIBUS Network Simulator (BHW2PNetSim)[2].

The structure of this document is as follows. Section **Error! Reference source not found.** presents the main features of the implemented Probability Distribution Functions and Section 3 describes how to set the parameters of the RHW2PNetSim and the BHW2PNetSim to configure the PDF.

2.Parameterization of Continuous Distributions

For a given family of continuous distributions, e.g., normal or gamma, there are usually several alternative ways to define, or parameterize, the probability density function. However, if the parameters are defined correctly, they can be classified, on the basis of their physical or geometric interpretation, as being on of three basic types: *location*, *scale*, or *shape* parameters[3].

A location parameter γ specifies an abscissa (x axis) location point of a distribution's range of values; usually γ is the midpoint or the lower endpoint of the distribution's range. (in the latter case location parameters are sometimes called shift parameters). As γ changes, the associated distribution merely shifts left or right without otherwise changing. Also, if the distribution of a random variable X has a location parameter of 0, then the distribution of the random variable $Y = X + \gamma$ has location parameter of γ.

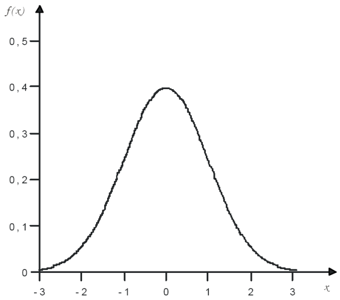
A scale parameter β determines the scale (or unit) of measurement of the values in the ranges of the distribution. (The standard deviation σ is a scale parameter for the normal distribution). A change in β compresses or expands the associated distribution without altering its basic form. Also, if the distribution of a random variable X has a scale parameter of 1, then the distribution of the random variable $Y = \beta X$ has a scale parameter of β.

A shape parameter α determines, distinct from location and scale, the basic form or shape of a distribution within the general family of distributions of interest. A change in α generally alters a distribution's properties (e.g. skewness) more fundamentally than a change in location or scale. Some distributions (e.g., exponential and normal) do not have shape parameter, while others (e,g., beta) may have two.

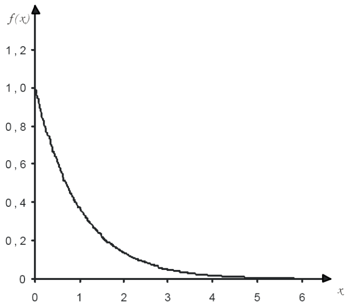
The Table 1 gives the information relevant to the PDFs implemented in both simulators. The range indicates the interval where the associated random variable can take on values. Also listed are the mean (expected value), variance, and mode, i.e., the value at which the density function is maximized.

Table 1 – Probability Distribution Functions features

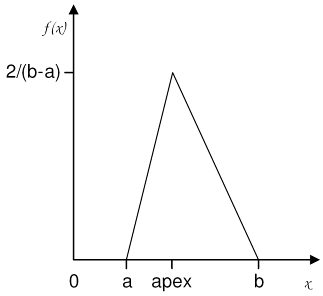
1- Normal	$N(\mu,\sigma^2)$
Possible applications	Errors of various types, e.g., in the impact point of a bomb; quantities that are the sum of a large number of the other quantities (by virtue of central limit theorems)
Density	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ for all real numbers x



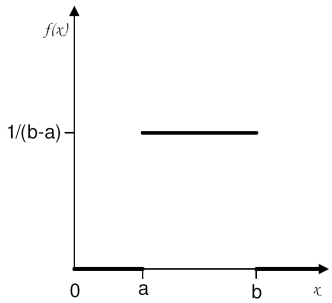
Distribution	No closed form
Parameters	Location parameter $\mu \in (-\infty, \infty)$ Scale parameter $\sigma > 0$
Range	
Mean	$(-\infty, \infty)$
Variance	μ
Mode	σ^2

2- Exponential	$\exp o(\beta)$
Possible Applications	Interarrival times of "customers" to a system that occur at constant rate, time to failure of a piece of equipment
	$f(x) = \begin{cases} \frac{1}{\beta^2} & if \;\; x \geq 0 \\ 0 & otherwise \end{cases}$
Density	
Distribution	$F(x) = \begin{cases} 1 - e^{\frac{-x}{\beta}} & if \;\; x \geq 0 \\ 0 & otherwise \end{cases}$
Parameters	Scale parameter $\beta > 0$
Range	$[0, \infty)$
Mean	β
Variance	β^2
Mode	0

3-Triangular	$triang(a, apex, b)$
Possible applications	Used as rough model in absence of data

Density	$$f(x)=\begin{cases}\dfrac{2(x-a)}{(b-a)(apex-a)} & if\ \ a\le x\le apex\\ \dfrac{2(b-x)}{(b-a)(b-apex)} & if\ \ apex<x\le b\\ 0 & otherwise\end{cases}$$ 
Distribution	$$F(x)=\begin{cases}0 & if\ \ x<a\\ \dfrac{(x-a)^2}{(b-a)(apex-a)} & if\ \ a\le x\le apex\\ 1-\dfrac{(b-x)^2}{(b-a)(b-apex)} & if\ apex<x\le b\\ 1 & if\ \ b<x\end{cases}$$
Parameters	a , b and $apex$ real numbers with $a<apex<b$. a is a location parameter. $b-a$ is a scale parameter. $apex$ is a shape parameter
Range	$[a,b]$
Mean	$\dfrac{a+b+apex}{3}$
Variance	$\dfrac{a^2+b^2+apex^2-ab-aapex-bapex}{18}$
Mode	$apex$

4-Uniform	$uniform(a,b)$
Possible applications	Used as a “first” model for quantity that is felt to be randomly varying between *a* and *b* but about which little else is known.
Density	$$f(x)=\begin{cases}\dfrac{1}{b-a} & if\ \ a\le x\le b\\ 0 & otherwise\end{cases}$$



Distribution	$F(x) = \begin{cases} 0 & if \;\; x < a \\ \frac{x-a}{b-a} & if \;\; a \le x \le b \\ 1 & if \;\; b < x \end{cases}$
Parameters	a and b real numbers with $a < b$. a is a location parameter. $b-a$ is a scale parameter.
Range	$[a, b]$
Mean	$\frac{a+b}{2}$
Variance	$\frac{(b-a)^2}{12}$
Mode	Does not uniquely exists

3.Parameterization of Stochastic Parameters in the Simulators

The RHW2PNetSim and BHW2PNetSim allow setting some parameters using PDFs. The name of all these parameters uses the `_pdf` prefix. For example, the parameters associated to T_{SDR} are the following: `_pdf_ tsdr_type`, `_pdf_tsdr_par1`, `_pdf_tsdr_par2` and `_pdf_tsdr_par3`. Where the `_pdf_tsdr_type` indicates which PDF will be used to generate the value of the T_{SDR} and the other parameters are the arguments of the PDF. Table 2 presents how the simulator parameters must be set according to the PDF.

Table 2 – Probability Distributions Functions simulators parameters

		Probability Distribution Functions			
Parameters	**Constant**	**Normal**	**Exponential**	**Triangular**	**Triangular**
`_pdf_..._type`	0	1	2	3	4
`_pdf_..._par1`	Value	μ	β	a	a
`_pdf_..._par2`		σ^2	–	$apex$	b
`_pdf_..._par3`		–	–	b	–

References

[1] P. Sousa and L. Ferreira, "Repeater-Based Hybrid Wired/Wireless PROFIBUS Network Simulator," Polytechnic Institute of Porto., Porto, Technical-Report Hurray-tr-060402, April 2006.

[2] P. Sousa and L. Ferreira, "Bridge-Based Hybrid Wired/Wireless PROFIBUS Network Simulator," Polytechnic Institute of Porto, Porto, Technical-Report Hurray-tr-050403, April 2005.

[3] A. M. Law and W. D. Kelton, "Simulation Modeling and Analysis", 3rd ed. New York: McGraw-Hill, 2000.