# Optimal virtual cluster-based multiprocessor scheduling

**Arvind Easwaran · Insik Shin · Insup Lee**

**Abstract** Scheduling of constrained deadline sporadic task systems on multiprocessor platforms is an area which has received much attention in the recent past. It is widely believed that finding an optimal scheduler is hard, and therefore most studies have focused on developing algorithms with good processor utilization bounds. These algorithms can be broadly classified into two categories: partitioned scheduling in which tasks are statically assigned to individual processors, and global scheduling in which each task is allowed to execute on any processor in the platform. In this paper we consider a third, more general, approach called cluster-based scheduling. In this approach each task is statically assigned to a processor cluster, tasks in each cluster are globally scheduled among themselves, and clusters in turn are scheduled on the multiprocessor platform. We develop techniques to support such cluster-based scheduling algorithms, and also consider properties that minimize total processor uti-lization of individual clusters. In the last part of this paper, we develop new virtual cluster-based scheduling algorithms. For implicit deadline sporadic task systems, we develop an optimal scheduling algorithm that is neither Pfair nor ERfair. We also show that the processor utilization bound of US-EDF{m/(2m − 1)} can be improved by using virtual clustering. Since neither partitioned nor global strategies dominate over the other, cluster-based scheduling is a natural direction for research towards achieving improved processor utilization bounds.

**Keywords** Multiprocessor scheduling · Virtual processor clustering · Hierarchical scheduling · Compositional schedulability analysis

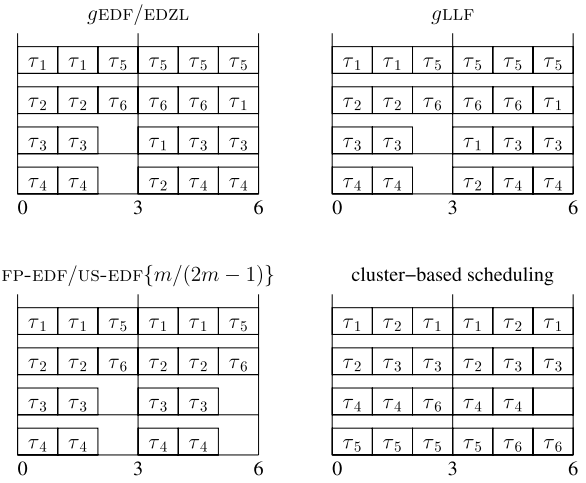
## 1 Introduction

With rapid development in microprocessor technology, multiprocessor and multi-core designs are becoming an attractive solution to fulfill increasing performance demands. In the real-time systems community, there has been a growing interest in multiprocessor scheduling theories. In general, existing approaches over $m$ identical, unit-capacity processors can fall into two categories: *partitioned* and *global* schedul-ing. Under partitioned scheduling each task is statically assigned to a single processor and is allowed to execute on that processor only. Under global scheduling tasks are allowed to dynamically migrate across $m$ processors and execute on any of them.

In this paper we consider another approach using a notion of *processor cluster*. A cluster is a set of $m'$ processors, where $1 \leq m' \leq m$. Under cluster-based schedul-ing, tasks are statically assigned to a cluster and then globally scheduled within the cluster. This scheduling strategy can be viewed as a generalization of partitioned and global scheduling; it is equivalent to partitioned scheduling at one extreme end where we assign tasks to $m$ clusters each of size one, and global scheduling at the other extreme end where we assign tasks to a single cluster of size $m$. Cluster-based scheduling can be further classified into two types: *physical* and *virtual* depending on how a cluster is mapped to processors in the platform. A physical cluster holds a static one-to-one mapping between its $m'$ processors and some $m'$ out of $m$ processors in the platform (Calandrino et al. 2007). A virtual cluster allows a dynamic one-to-many mapping between its $m'$ processors and the $m$ processors in the platform. Scheduling tasks in this virtual cluster can be viewed as scheduling them globally on all the $m$ processors in the platform with amount of concurrency at most $m'$, i.e., at any time instant at most $m'$ of the $m$ processors are used by the cluster. A key difference is that physical clusters share no processors in the platform, while virtual clusters can share some.

*Motivating example* We now illustrate the capabilities of cluster-based scheduling using an example. Consider a sporadic task system comprised of 6 tasks as follows: $\tau_1 = \tau_2 = \tau_3 = \tau_4 = (3, 2, 3)$, $\tau_5 = (6, 4, 6)$ and $\tau_6 = (6, 3, 6)$. The notation followed

**Fig. 1** Motivating example




here is (T, C, D), where T denotes the minimum release separation between successive instances of the task, C denotes the maximum required processor capacity for each instance and D denotes the relative deadline. Let this task set be scheduled on a multiprocessor platform comprised of 4 processors. It is easy to see that this task set is not schedulable under any partitioned scheduling algorithm, because no processor can be allocated more than one task. Figure 1 shows the schedule of this task set under global Earliest Deadline First (gEDF) (Liu 1969), EDZL (Cho et al. 2002), Least Laxity First (gLLF) (Leung 1989), FP-EDF (Baruah 2004) and US-EDF$\{m/(2m-1)\}$ (Srinivasan and Baruah 2002) scheduling algorithms. As shown in the figure, the task set is not schedulable under any of these algorithms. Now consider cluster-based scheduling as follows: tasks $\tau_1$, $\tau_2$ and $\tau_3$ are executed under gLLF on a cluster $\mathcal{C}_1$ comprised of 2 processors, and tasks $\tau_4$, $\tau_5$ and $\tau_6$ are executed under gEDF on another cluster $\mathcal{C}_2$ comprised of 2 processors. The resulting schedule is shown in Fig. 1, and as can be seen all the task deadlines are met.

In addition to being more general than physical clustering, virtual clustering is also less sensitive to task-processor mappings. This can be explained using the same example as above with an additional task $\tau_7 = (6, 1, 6)$. Just for comparison, suppose $\tau_7$ is assigned to the first cluster $\mathcal{C}_1$ along with tasks $\tau_1$, $\tau_2$ and $\tau_3$. Then physical cluster-based scheduling cannot accommodate those two clusters on 4 processors. On the other hand, virtual clustering has a potential to accommodate them on 4 processors by dynamically re-allocating slack from cluster $\mathcal{C}_2$ to cluster $\mathcal{C}_1$ (time interval $(5, 6]$).

Clustering can also be useful as a mechanism to place a restriction on the amount of concurrency. Suppose $m$ tasks can thrash a L2 cache in a multi-core platform, if they run in parallel at the same time. Then one may consider allowing at most $m'$ of these $m$ tasks to run in parallel, in order to prevent them from thrashing the L2 cache. This can be easily done if the $m$ tasks are assigned to a cluster of $m'$ processors. A similar idea was used in (Anderson et al. 2006).

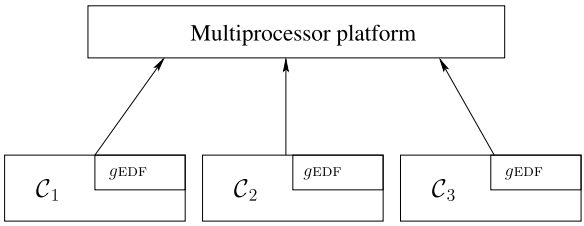



**Fig. 2** Example virtual clustering framework

*Hierarchical scheduling* Physical clustering requires intra-cluster scheduling only. This is because clusters are assigned disjoint physical processors, and hence tasks in different clusters cannot interfere with each others executions. However, the notion of virtual clustering inherently requires a two-level hierarchical scheduling framework; inter- and intra-cluster scheduling. In inter-cluster scheduling physical processors are dynamically assigned to virtual clusters. In intra-cluster scheduling processor allocations given to a virtual cluster are assigned to tasks in that cluster. Consider the example shown in Fig. 2. Let a task set be divided into three clusters $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$, each employing gEDF scheduling strategy. If we use physical clustering, then each cluster can be separately analyzed using existing techniques for gEDF. On the other hand if we use virtual clustering, then in addition to intra-cluster schedulability analysis, there is a need to develop techniques for scheduling the clusters on the multiprocessor platform. Therefore, supporting hierarchical multiprocessor scheduling is cardinal to the successful development of virtual clustering.

There have been considerable studies on hierarchical uniprocessor scheduling. Denoting a collection of tasks and a scheduler as a *component*, these studies employed the notion of a component interface to specify resources required for scheduling the component's tasks (Mok et al. 2001; Shin and Lee 2003; Easwaran et al. 2007). Analogously, we denote a cluster along with the tasks and scheduler assigned to it as a *component* in hierarchical multiprocessor schedulers. To support inter-cluster scheduling, this paper proposes a component interface that specifies resources required by the tasks in the component's cluster. Inter-cluster scheduler can allocate processor supply to the cluster based on its interface. Intra-cluster scheduler can then use this processor supply to schedule the tasks in the cluster. Many new issues arise to adopt the notion of a component interface from uniprocessor to multiprocessor scheduling. One of them is how to enable a component interface to carry information about concurrent execution of tasks in the component. For example, suppose a single task cannot execute in parallel. Then multiple processors cannot be used concurrently to satisfy the execution requirement of this single task. Such an issue needs to be handled for the successful development of component interfaces. In this paper we present one solution to this issue. Our approach is to capture in a component's interface, all the task-level concurrency constraints in that component. The interface demands enough processor supply from inter-cluster scheduler so that the intra-cluster scheduler can handle task-level concurrency constraints. As a result the inter-cluster scheduler does not have to worry about this issue.

*Contributions* The contributions of this paper are five-fold. First, we introduce the notion of general hierarchical multiprocessor schedulers to support virtual cluster-based scheduling. Second, we present an approach to specify the task-level concurrency constraints in a component's interface. In Sect. 2 we introduce a multiprocessor resource model based interface that not only captures the task-level concurrency constraints, but also specifies the total resource requirements of the component. This enables the inter-cluster scheduler to schedule clusters using their interfaces alone. Third, since such interfaces represent *partitioned resource supplies*[1] as opposed to *dedicated resource supplies*,[2] we also extend existing schedulability conditions for gEDF in this direction[3] (see Sect. 4). Such extensions to schedulability conditions are essential for supporting development of component interfaces. Fourth, we consider the optimization problem of minimizing the total resource requirements of the component interface. In Sect. 5, we present an efficient solution to this problem based on the following property of our gEDF schedulability condition: total processor utilization required by a component interface to schedule tasks in the component increases, as number of processors allocated to the component's cluster increases. Thus an optimal solution is obtained when we find the smallest number of processors that guarantee schedulability of the component. Fifth, in Sect. 6 we develop an overhead free inter-cluster scheduling framework based on McNaughton's algorithm (McNaughton 1959). Using this framework we present a new algorithm, called Virtual Clustering-Implicit Deadline Tasks (VC-IDT), for scheduling implicit deadline sporadic task systems on identical, unit-capacity multiprocessor platforms. We show that VC-IDT is an optimal scheduling algorithm, that does not satisfy the property of P-fairness (Baruah et al. 1996) or ER-fairness (Anderson and Srinivasan 2000). The latter feature of our algorithm, as we will see in Sect. 6.2.1, translates into better bounds on the number of preemptions. As an illustration of the capabilities of general task-processor mappings supported by virtual clustering, we also show that the processor utilization bound of US-EDF$\{m/(2m-1)\}$ can be improved by using this framework. In our previous work (Shin et al. 2008) we presented the first four contributions listed above. In this paper we elaborate on (and extend) those contributions, and in the process develop new virtual cluster-based scheduling algorithms (fifth contribution described above).

## 2 Task and resource models

In this section we describe our task model and the multiprocessor platform. We also introduce multiprocessor resource models which we use as component interfaces.

[1] If a processor can be used by a cluster only in some time intervals and not all, then its supply is said to be partitioned.

[2] If a processor can be used by a cluster at all times, then its supply is said to be dedicated.

[3] We have chosen to focus on one scheduling algorithm in this paper. However the issues are the same for other schedulers, and hence techniques developed here are applicable to other schedulers as well.

## 2.1 Task and platform models

*Task model* We assume a constrained deadline sporadic task model (Baruah et al. 1990). In this model a sporadic task is specified as $\tau_i = (T_i, C_i, D_i)$, where $T_i$ is the minimum release separation, $C_i$ is the maximum processor capacity requirement and $D_i$ is the relative deadline. These task parameters satisfy the property $C_i \leq D_i \leq T_i$.[4] Successive instances of $\tau_i$ are released with a minimum separation of $T_i$ time units. We refer to each such instance as a *real-time job*. Each job of $\tau_i$ must receive $C_i$ units of processor capacity within $D_i$ time units from its release. These $C_i$ units must be supplied sequentially to the job. This restriction is useful in modeling many real-world systems, because in general, all portions of a software program cannot be parallelized.

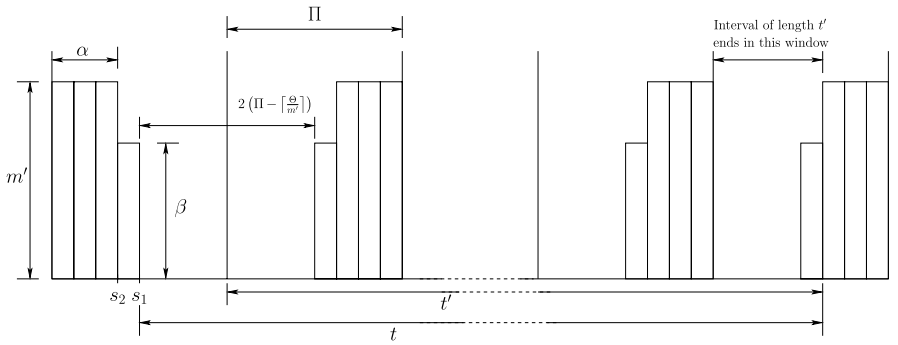
*Multiprocessor platform and scheduling strategy* In this paper we assume an identical, unit-capacity multiprocessor platform having $m$ processors. Each processor in this platform has a resource bandwidth of one, i.e., it can provide $t$ units of processor capacity in every time interval of length $t$. We also assume that a job can be preempted on one processor and may resume execution on another processor with negligible preemption and migration overheads, as in the standard literature of global scheduling (Goossens et al. 2003; Baker 2005a; Bertogna et al. 2005a; Baruah 2007). We assume such a global scheduling strategy within each cluster, and in particular, we assume that the strategy is global EDF (denoted as gEDF). At each time instant, if $m'$ denotes the number of physical processors allocated to the cluster, then gEDF schedules unfinished jobs that have the $m'$ earliest relative deadlines.

## 2.2 Multiprocessor resource model

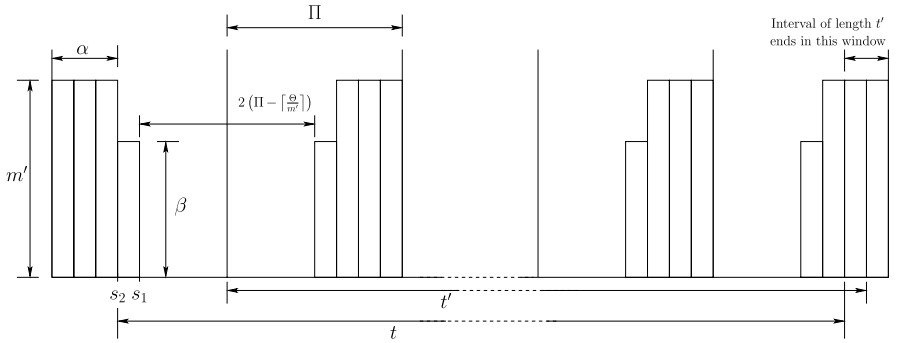
A *resource model* is a model for specifying the characteristics of processor supply. When these models represent component interfaces, they specify total processor requirements of the component. Periodic (Shin and Lee 2003), EDP (Easwaran et al. 2007), bounded-delay (Feng and Mok 2002), etc., are examples of resource models that have been extensively used for analysis of hierarchical uniprocessor schedulers. These resource models can also be used as component interfaces in hierarchical multiprocessor schedulers. One way to achieve this is to consider $m'$ identical resource models as a component interface, where $m'$ is the number of processors allocated to the component's cluster. However, this interface is restrictive because each processor contributes the same amount of resource to the component as any other processor in the cluster. It is desirable to be more flexible in that interfaces should be able to represent the collective processor requirements of clusters, without fixing the contribution of each processor a priori. Apart from increased flexibility, such interfaces can also improve processor utilization in the system.

We now introduce a *multiprocessor resource* model that specifies the characteristics of processor supply provided by an identical, unit-capacity multiprocessor platform. This resource model does not fix the contribution of each processor a priori, and hence is a suitable candidate for cluster interfaces.

[4] If $D_i = T_i$ then the task is called implicit deadline task.




(a) Case 1




(b) Case 2

**Fig. 3** Schedule of $\mu$ w.r.t. $\text{sbf}_\mu(t)$

**Definition 1** (Multiprocessor periodic resource model (MPR)) A *multiprocessor periodic resource* model $\mu = \langle \Pi, \Theta, m' \rangle$ specifies that an identical, unit-capacity multiprocessor platform collectively provides $\Theta$ units of resource in every $\Pi$ time units, where the $\Theta$ units are supplied with concurrency at most $m'$; at any time instant at most $m'$ physical processors are allocated to this resource model. $\frac{\Theta}{\Pi}$ denotes the *resource bandwidth* of model $\mu$.

It is easy to see from the above definition that a *feasible MPR model* must satisfy the condition $\Theta \leq m'\Pi$. The supply bound function of a resource model (sbf) lower bounds the amount of processor supply that the model guarantees in a given time interval. Specifically, $\text{sbf}_R(t)$ is equal to the minimum amount of processor capacity that model $R$ is guaranteed to provide in any time interval of duration $t$. In uniprocessor systems, sbf is used in schedulability conditions to generate resource model based component interfaces. Extending this approach to multiprocessors, in this paper we derive similar schedulability conditions to generate MPR model based component interfaces. Hence we now present the sbf for a MPR model $\mu = \langle \Pi, \Theta, m' \rangle$. Figure 3 shows the schedule for $\mu$ that generates this minimum supply in a time interval of duration $t$, where $\alpha = \lfloor \frac{\Theta}{m'} \rfloor$ and $\beta = \Theta - m'\alpha$. As can be seen, length of the largest

time interval with no supply is equal to $2\Pi - 2\lceil \frac{\Theta}{m'} \rceil$ (shown in the figures). $\text{sbf}_\mu$[5] is given by the following equation.

$$\text{sbf}_\mu(t) = \begin{cases} 0, & t' < 0 \\ \lfloor \frac{t'}{\Pi} \rfloor \Theta + \max\{0, m'x - (m'\Pi - \Theta)\}, & t' \geq 0 \text{ and } x \in [1, y] \\ \lfloor \frac{t'}{\Pi} \rfloor \Theta + \max\{0, m'x - (m'\Pi - \Theta)\} & \\ \quad - (m' - \beta), & t' \geq 0 \text{ and } x \notin [1, y] \end{cases}$$

$$\text{where } t' = t - \left(\Pi - \left\lceil \frac{\Theta}{m'} \right\rceil\right), \qquad x = \left(t' - \Pi \left\lfloor \frac{t'}{\Pi} \right\rfloor\right)$$

$$\text{and} \quad y = \Pi - \left\lfloor \frac{\Theta}{m'} \right\rfloor \tag{1}$$

There are two main cases to consider for $\text{sbf}_\mu$. If $t'$ is as shown in Fig. 3(a), then the interval that generates the minimum supply starts from time instant $s_1$ shown in the same figure. On the other hand, if $t'$ is as shown in Fig. 3(b), then the interval that generates the minimum supply starts from time instant $s_2$ shown in the same figure. In uniprocessor systems although schedulability conditions with sbf have been derived, a linear approximation of sbf is often used to reduce the time-complexity of the interface generation process. Hence, in anticipation, we present the following linear lower bound for $\text{sbf}_\mu$.[6] Functions $\text{sbf}_\mu$ and $\text{lsbf}_\mu$ are plotted in Fig. 4.

$$\text{lsbf}_\mu(t) = \frac{\Theta}{\Pi}\left(t - \left[2\left(\Pi - \frac{\Theta}{m'}\right) + 2\right]\right) \tag{2}$$

The following lemma proves that $\text{lsbf}_\mu$ is indeed a lower bound for $\text{sbf}_\mu$.

**Lemma 1** $\text{lsbf}_\mu(t) \leq \text{sbf}_\mu(t)$ *for all* $t \geq 0$.

*Proof* Consider Fig. 4. Observe that $\text{lsbf}_\mu(t) = 0$ for all $t \leq t_4$. Therefore it is sufficient to show that $\text{lsbf}_\mu(t) \leq \text{sbf}_\mu(t)$ for all $t > t_4$. Suppose $\text{sbf}_\mu(t_4) = 2\beta + \epsilon$ for some $\epsilon \geq 0$.

We now show that $\text{lsbf}_\mu(t) \leq \text{sbf}_\mu(t)$ for all $t$ such that $t_4 < t \leq t_8$, where $t_8 = t_4 + \Pi$. The following statements are true by definition: 1) $\text{sbf}_\mu(t_8) = \Theta + 2\beta + \epsilon$, and 2) $\text{lsbf}_\mu(t_8) = \Theta$. Further, because the slope of $\text{sbf}_\mu$ in the interval $(t_4, t_5]$ is at least as much as the slope of $\text{lsbf}_\mu$ ($\frac{\Theta}{\Pi} \leq m'$), $\text{lsbf}_\mu(t) \leq \text{sbf}_\mu(t)$ for all $t$ such that $t_4 < t \leq t_5$. From the figure, we can see that $\text{sbf}_\mu(t_6) = \Theta = \text{lsbf}_\mu(t_8)$ and $t_6 \leq t_8$. Therefore $\text{lsbf}_\mu(t) \leq \text{sbf}_\mu(t)$ for all $t$ such that $t_6 \leq t \leq t_8$. The last statement follows from the fact that $\text{sbf}_\mu$ is a non-decreasing function. This combined with the facts that $t_6 = t_5 + 1$ and $\text{lsbf}_\mu$ is a linear function, implies $\text{lsbf}_\mu(t) \leq \text{sbf}_\mu(t)$ for all $t$ such that $t_4 < t \leq t_8$.

Observe that in every successive time interval of length $\Pi$ starting from $t_4$, the following holds: 1) both $\text{sbf}_\mu$ and $\text{lsbf}_\mu$ increase by exactly $\Theta$, and 2) they both have

[5] A correction has been made to $\text{sbf}_\mu$ from its original publication in Shin et al. (2008).

[6] $\text{lsbf}_\mu$ has also been modified from its original publication in Shin et al. (2008), in order to be consistent with the new $\text{sbf}_\mu$.

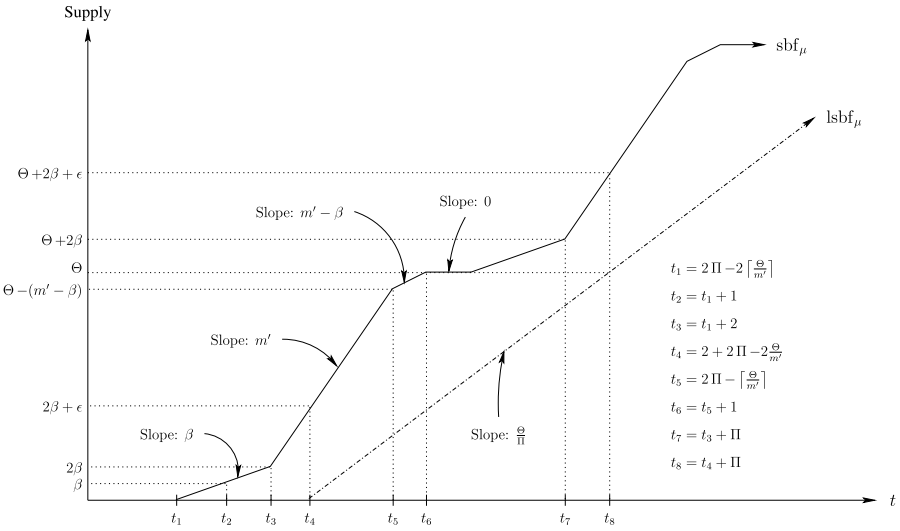



**Fig. 4** $\text{sbf}_\mu$ and $\text{lsbf}_\mu$

slope characteristics identical to those in the interval $(t_4, t_8]$. Therefore the arguments from the previous paragraph hold for each such time interval of length $\Pi$. The result of the lemma then follows. □

Uniprocessor resource models, such as periodic or EDP, allow a view that a component executes over an exclusive share of a physical uniprocessor platform. Extending this notion, MPR models allow a view that a component, and hence the corresponding cluster, executes over an exclusive share of a physical multiprocessor platform. Although this view guarantees a minimum total processor share given by sbf, it does not enforce any distribution of this share over the processors in the platform, apart from the concurrency bound $m'$. In this regard MPR models are general and hence our candidate for component interfaces.

## 3 Related work

*Multiprocessor scheduling* In general, studies on real-time multiprocessor scheduling theory can fall into two categories: *partitioned* and *global* scheduling. Under partitioned scheduling each task is statically assigned to a single processor and uniprocessor scheduling algorithms are used to schedule tasks. Under global scheduling tasks are allowed to migrate across processors and algorithms that simultaneously schedule on all the processors are used. Many partitioning algorithms and their analysis (Oh and Baker 1998; López et al. 2001; Baruah and Fisher 2006; Fisher et al. 2006), and global scheduling algorithms and their analysis (Baruah et al. 1996; Andersson et al. 2001; Cho et al. 2002; Srinivasan and Baruah 2002; Zhu et al. 2003; Goossens et al. 2003; Baker 2003; Baruah 2004; Baker 2005a; Baker 2006; Bertogna et al. 2005a; Cho et al. 2006; Baruah 2007; Cirinei and Baker 2007;

Bertogna and Cirinei 2007; Baruah and Fisher 2007; Baruah and Baker 2008a, 2008b; Funaoka et al. 2008), have been proposed in the past.

For implicit deadline task systems, both Earliest Deadline First (EDF ) (López et al. 2001) and Rate Monotonic (RM) (Oh and Baker 1998) based partitioned scheduling have been proposed along with processor utilization bounds. These studies have since been extended for constrained deadline task systems, and EDF (Baruah and Fisher 2006) and fixed-priority (Fisher et al. 2006) based scheduling have been developed for them. Under global scheduling of implicit deadline task systems, several optimal algorithms such as Pfair (Baruah et al. 1996), BoundaryFair (Zhu et al. 2003), LNREF (Cho et al. 2006), and NVNLF (Funaoka et al. 2008), have been proposed. To reduce the relatively high preemptions in these algorithms and to support constrained deadline task systems, processor utilization bounds and worst-case response time analysis for EDF (Goossens et al. 2003; Baker 2003, 2005a; Bertogna et al. 2005a; Baruah 2007; Bertogna and Cirinei 2007; Baruah and Baker 2008a, 2008b) and Deadline Monotonic (DM) (Baker 2003, 2006; Bertogna and Cirinei 2007; Baruah and Fisher 2007) based global scheduling strategies have been developed. Towards better processor utilization, new global algorithms such as dynamic-priority EDZL (Cho et al. 2002; Cirinei and Baker 2007) and US-EDF$\{m/(2m-1)\}$ (Srinivasan and Baruah 2002), and fixed-priority RM-US$\{m/(3m-2)\}$ (Andersson et al. 2001) and FP-EDF (Baruah 2004), have also been proposed. Partitioned scheduling suffers from an inherent performance limitation in that a task may fail to be assigned to any processor, although the total available processor capacity across the platform is larger than the task's requirements. Global scheduling has been developed to overcome this limitation. However global algorithms are either not known to utilize processors optimally (like in the case of constrained deadline task systems), or if they are known to be optimal, then they have high number of preemptions (like in the case of implicit deadline task systems). Moreover, for constrained deadline tasks, simulations conducted by Baker (2005b) have shown that partitioned scheduling performs much better than global scheduling on an average. These simulations reflect the large pessimism in current schedulability tests for global algorithms. To eliminate the performance limitation of partitioned scheduling and to achieve high processor utilization without incurring high preemption costs, we consider the more general task-processor mappings that virtual cluster-based scheduling proposes.

Algorithms that support slightly more general task-processor mappings than either partitioned or global scheduling have been proposed in the past. Andersson and Tovar (2006), Andersson and Bletsas (2008), Andersson et al. (2008) and Kato and Yamasaki (2007) have developed algorithms that allow a task to be scheduled on at most two processors in the platform. Virtual cluster-based scheduling framework that we propose generalizes all these task-processor mappings and therefore can lead to higher processor utilization. Baruah and Carpenter (2003) introduced an approach that restricts processor migration of jobs, in order to alleviate the performance limitation of partitioned scheduling and the processor migration overheads of global scheduling. It has been shown that the worst-case processor utilization of this approach is no better than partitioned scheduling (roughly 50%). Calandrino et al. (2007) presented a physical clustering framework in which tasks are first assigned to physical processor clusters and then scheduled globally within those clusters. They

experimentally evaluated this framework to show that cache-access related overheads can be reduced in comparison to both partitioned and global scheduling strategies. Virtual clustering is again a generalization of this framework, and moreover, unlike their work, we develop efficient schedulability analysis techniques with a focus on achieving high processor utilization. Recently, virtual clustering has also been considered in the context of tardiness guarantees for soft real-time systems (Leontyev and Anderson 2008).

Moir and Ramamurthy (1999), Holman and Anderson (2001) and Anderson et al. (2006) presented an approach that upper bounds the amount of concurrent execution within a group of tasks. They developed their approach using a two-level Pfair-based scheduling hierarchy. These studies are most related to our work on virtual clustering, but they differ from our technique mainly in the following aspect. We introduce a multiprocessor resource model that makes it possible to clearly separate intra- and inter-cluster scheduling. This allows development of schedulability analysis techniques for virtual clustering that are easily extensible to many different schedulers. However their approaches do not employ such a notion. Therefore their analysis techniques are bound to Pfair scheduling, and do not generalize to other algorithms and task models such as the one considered in this paper. This flexibility provides a powerful tool for the development of various task-processor mappings and intra- and inter-cluster scheduling algorithms.

*Hierarchical scheduling* For uniprocessor platforms there has been a growing attention to hierarchical scheduling frameworks. Since a two-level framework was introduced (Deng and Liu 1997), its schedulability has been analyzed under fixed-priority (Kuo and Li 1999) and EDF-based (Lipari et al. 2000) scheduling. For multi-level frameworks many resource model based component interfaces such as bounded-delay (Mok et al. 2001; Shin and Lee 2004), periodic (Lipari and Bini 2003; Sin and Lee 2003, 2008) and EDP (Easwaran et al. 2007), have been introduced, and schedulability conditions have been derived under fixed-priority and EDF scheduling (Feng and Mok 2002; Lipari and Bini 2003; Shin and Lee 2003; Almeida and Pedreiras 2004; Davis and Burns 2005; Easwaran et al. 2007). As discussed in the introduction, these studies do not provide any technique to capture task-level concurrency constraints in interfaces, and therefore are not well suited for virtual clustering.

## 4 Component schedulability condition

In this section we develop a schedulability condition for components in hierarchical multiprocessor schedulers, such that this condition accommodates the notion of a partitioned resource supply. Specifically, we extend existing gEDF schedulability conditions for dedicated resource, with the supply bound function of a MPR model. Any MPR model that satisfies this condition can be used as an interface for the component.

We consider a component comprising of cluster $\mathcal{C}$ and sporadic tasks $\mathcal{T} = \{\tau_1 = (\mathrm{T}_1, \mathrm{C}_1, \mathrm{D}_1), \ldots, \tau_n = (\mathrm{T}_n, \mathrm{C}_n, \mathrm{D}_n)\}$ scheduled under gEDF. To keep the presentation simple, we use notation $\mathcal{C}$ to refer to the component as well. We now develop a schedulability condition for $\mathcal{C}$ assuming it is scheduled using MPR model

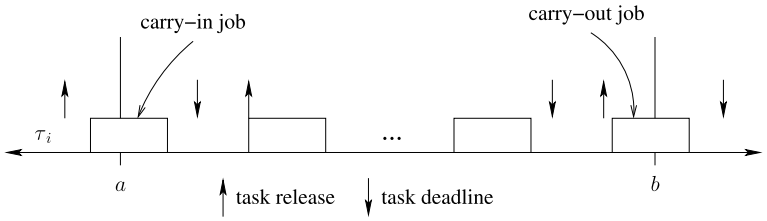



**Fig. 5** Workload of task $\tau_i$ in interval $[a, b]$

$\mu = \langle \Pi, \Theta, m' \rangle$, where $m'$ denotes number of processors in the cluster. This condition uses the total processor demand of task set $\mathcal{T}$ for a given time interval. Existing studies (Bertogna et al. 2005a) have developed an upper bound for this demand which we can use. Only upper bounds are known for this demand, because unlike the synchronous arrival sequence in uniprocessors, no notion of worst-case arrival sequence is known for multiprocessors (Baruah 2007). Hence we first summarize this existing demand upper bound and then present our schedulability condition.

### 4.1 Component demand

*Workload* The workload of a task $\tau_i$ in an interval $[a, b]$ gives the cumulative length of all intervals in which $\tau_i$ is executing, when task set $\mathcal{T}$ is scheduled under $\mathcal{C}$'s scheduler. This workload consists of three parts (illustrated in Fig. 5): (1) the *carry-in* demand generated by a job of $\tau_i$ that is released prior to $a$, but did not finish its execution requirements until $a$, (2) the demand of a set of jobs of $\tau_i$ that are both released and have their deadlines within the interval, and (3) the *carry-out* demand generated by a job of $\tau_i$ that is released in the interval $[a, b)$, but does not finish its execution requirements until $b$.

*Workload upper bound for $\tau_i$ under gEDF* If workload in an interval $[a, b]$ can be efficiently computed for all $a, b \geq 0$ and for all tasks $\tau_i$, then we can obtain the exact demand of task set $\mathcal{T}$ in all intervals. However, since no such efficient computation technique is known (apart from task set simulation), we use an upper bound for this workload obtained by Bertogna et al. (2005a). This bound is obtained under two assumptions: (1) some job of some task $\tau_k$ has a deadline at time instant $b$, and (2) this job of $\tau_k$ misses its deadline. In the schedulability conditions we develop, these assumptions hold for all time instants $b$ that are considered. Hence this is a useful bound and we present it here. Figure 6 illustrates the dispatch pattern corresponding to this bound. A job of task $\tau_i$ has a deadline that coincides with time instant $b$. Jobs of $\tau_i$ that are released prior to time $b$ are assumed to be released as late as possible. Also, the job of $\tau_i$ that is released before $a$ but has a deadline in the interval $[a, b]$, is assumed to execute as late as possible. This imposes maximum possible interference on the job of $\tau_k$ with deadline at $b$. Let $\mathcal{W}_i(t)$ denote this workload bound for $\tau_i$ in a time

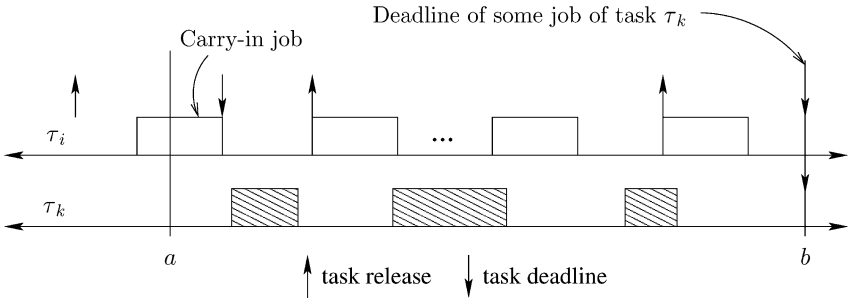



**Fig. 6** Dispatch and execution pattern of task $\tau_i$ for $\mathcal{W}_i(b-a)$

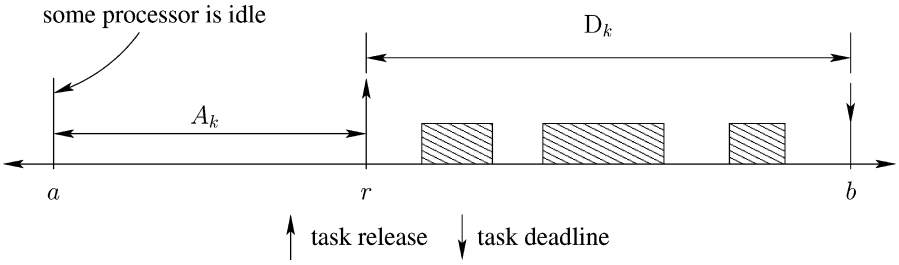



**Fig. 7** Example time instant $a$ under dedicated resource

interval of length $t$ $(= b - a)$. Also let $CI_i(t)$ denote the carry-in demand generated by the execution pattern shown in Fig. 6. Then

$$\mathcal{W}_i(t) = \left\lfloor \frac{t + (\mathrm{T}_i - \mathrm{D}_i)}{\mathrm{T}_i} \right\rfloor C_i + CI_i(t),$$

$$\text{where} \quad CI_i(t) = \min\left\{\mathrm{C}_i, \max\left\{0, t - \left\lfloor \frac{t + (\mathrm{T}_i - \mathrm{D}_i)}{\mathrm{T}_i} \right\rfloor \mathrm{T}_i\right\}\right\} \tag{3}$$

It has been shown that the actual workload of $\tau_i$ can never exceed $\mathcal{W}_i(b-a)$ in the interval $[a, b]$, provided tasks are scheduled under gEDF and a deadline miss occurs for that job of $\tau_k$ whose deadline is at $b$ (Bertogna et al. 2005a). This follows from the observation that no job of $\tau_i$ with deadline greater than $b$ can execute in the interval $[a, b]$. In the following section we develop a schedulability condition for $\mathcal{C}$ using this workload bound.

## 4.2 Schedulability condition

We now present a schedulability condition for component $\mathcal{C}$ when it is scheduled using MPR model $\mu = \langle \Pi, \Theta, m' \rangle$. For this purpose we extend (with the notion of $\mathrm{sbf}_\mu$) an existing condition that checks the schedulability of $\mathcal{C}$ on a dedicated resource comprised of $m'$ unit-capacity processors.

When task $\tau_k$ is scheduled on $m'$ unit-capacity processors under gEDF, existing work identifies different time intervals that must be checked to guarantee schedulability of $\tau_k$ (Baruah 2007). In particular, it assumes $b$ denotes the missed deadline

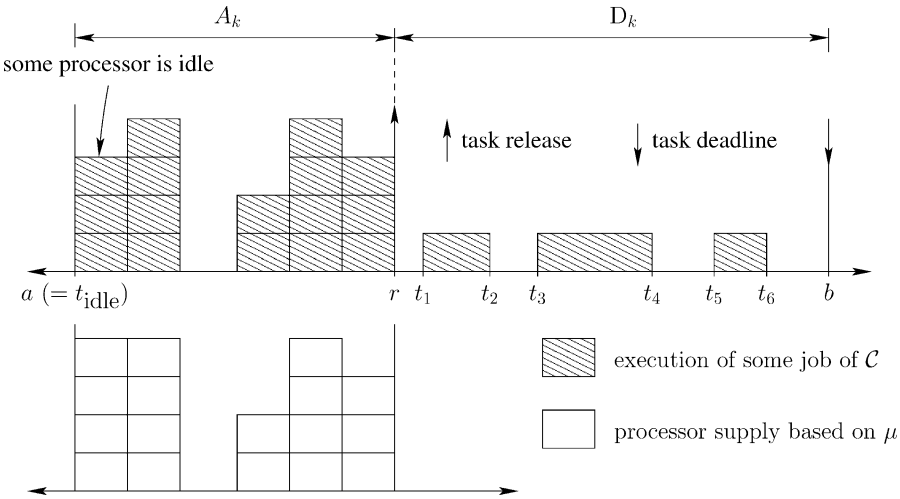



**Fig. 8** Example time instant $t_{\text{idle}}$

of some job of task $\tau_k$ (henceforth denoted as job $\tau_k^b$), and then specifies different values of $a$, corresponding to the interval $[a, b]$, that need to be considered. Figure 7 gives one such time instant $a$. It corresponds to a point in time such that: (1) at least one of the $m'$ processors is idle at that instant, (2) it is prior to the release time of job $\tau_k^b$ ($r$ in the figure), and (3) no processor is idle in the interval $(a, r]$. Observe that at each such time instant $a$ there can be at most $m' - 1$ tasks that contribute towards carry-in demand. This is because at most $m' - 1$ processors are executing jobs at $a$. This observation is used to develop an efficient schedulability condition in the dedicated resource case. Informally, the study derives a condition on the total higher priority workload in the interval $[a, b]$ that guarantees a deadline miss for $\tau_k^b$. In the following discussion we extend this notion of time instant $a$ for the case when $\tau_k$ is scheduled under the partitioned resource supply $\mu$.

When task $\tau_k$ is scheduled using $\mu$, we denote a time instant as $t_{\text{idle}}$ if at least one of the $m'$ processors is idle at that instant, even though it is available for use as per supply $\mu$. Figure 8 illustrates one such time instant, where $r$ denotes the release time of job $\tau_k^b$, $A_k$ denotes the length of the interval $(a, r]$ and $A_k + D_k$ denotes the length of the interval $(a, b]$. To check schedulability of task $\tau_k$ we consider all time instants $a$ such that: (1) $a$ is $t_{\text{idle}}$, (2) $a \leq r$, and (3) no time instant in the interval $(a, r]$ is $t_{\text{idle}}$. The time instant illustrated in Fig. 8 satisfies these properties.

To derive the schedulability condition for component $\mathcal{C}$, we consider all intervals $[a, b]$ as explained above and derive conditions under which a deadline miss occurs for job $\tau_k^b$. If $\tau_k^b$ misses its deadline, then the total workload of jobs having priority at least $\tau_k^b$ must be greater than the total processor supply available to $\mathcal{C}$ in $[a, b]$. Let $I_i$ $(1 \leq i \leq n)$ denote the total workload in interval $[a, b]$ of jobs of $\tau_i$ that have priority at least $\tau_k^b$. Since $\text{sbf}_\mu(b - a)$ denotes a lower bound on the processor supply

available to $\mathcal{C}$ in $[a, b]$, whenever $\tau_k^b$ misses its deadline it must be true that

$$\sum_{i=1}^{n} I_i > \text{sbf}_\mu(b-a) = \text{sbf}_\mu(A_k + \text{D}_k) \tag{4}$$

This inequality can be derived from the following observations: (1) the actual processor supply available to component $\mathcal{C}$ in $[a, b]$ is at least $\text{sbf}_\mu(A_k + \text{D}_k)$ and (2) there are no $t_{\text{idle}}$ time instants in the interval $(a, b]$, i.e., all available processor supply is used by $\mathcal{C}$ to schedule tasks from $\mathcal{T}$. For $\mathcal{C}$ to be schedulable using $\mu$, it then suffices to show that for all tasks $\tau_k$ and for all values of $A_k$ (4) is invalid.

We now derive an upper bound for each workload $I_i$. We separately consider the workload of $\tau_i$ in the following two interval classes: (1) time intervals in $[a, b]$ in which $\tau_k^b$ executes (intervals $[t_1, t_2]$, $[t_3, t_4]$ and $[t_5, t_6]$ in Fig. 8) and (2) the other time intervals in $[a, b]$. Let $I_{i,1}$ denote the workload of $\tau_i$ in intervals of type (1) and $I_{i,2}$ denote the workload of $\tau_i$ in intervals of type (2). We bound $I_i$ using upper bounds for $I_{i,1}$ and $I_{i,2}$. In the dedicated resource case, only intervals of type (2) were considered when deriving the schedulability condition (Baruah 2007). We however consider the contiguous interval $[a, b]$, because sbf of MPR models are only defined over such contiguous time intervals.

Since the cumulative length of intervals of type (1) is at most $\text{C}_k$ and there are at most $m'$ processors on which $\mathcal{C}$ executes, the total workload of all the tasks in intervals of type (1) is clearly upper bounded by $m'\text{C}_k$. Therefore, $\sum_{i=1}^{n} I_{i,1} \leq m'\text{C}_k$. To bound $I_{i,2}$ we use the workload upper bound $\mathcal{W}_i$ presented in Sect. 4.1. Recall that $\mathcal{W}_i(b-a)$ $(= \mathcal{W}_i(A_k + \text{D}_k))$ upper bounds the workload of all jobs of $\tau_i$ that execute in the interval $[a, b]$ and have priority higher than $\tau_k^b$. Therefore $\mathcal{W}_i(A_k + D_k)$ also upper bounds $I_{i,2}$. Further, there is no need for $I_{i,2}$ to be larger than $A_k + \text{D}_k - \text{C}_k$, because we have already considered a total length of $\text{C}_k$ for intervals of type (1). Also this bound can be further tightened for $i = k$, because in $I_{k,2}$ we do not consider the executions of $\tau_k^b$. These executions are already considered for intervals of type (1). Thus we can subtract $\text{C}_k$ from $\mathcal{W}_k(A_k + \text{D}_k)$ and $I_{k,2}$ cannot be greater than $A_k$:

$$I_{i,2} \leq \bar{I}_{i,2} = \min\{\mathcal{W}_i(A_k + \text{D}_k),\, A_k + \text{D}_k - \text{C}_k\} \quad \text{for all } i \neq k$$

$$I_{k,2} \leq \bar{I}_{k,2} = \min\{\mathcal{W}_k(A_k + \text{D}_k) - \text{C}_k,\, A_k\}$$

Now by definition of time instant $a$ at most $m' - 1$ tasks can be active, and hence have carry-in demand, at $a$. This follows from the fact that at least one processor is not being used by $\mathcal{C}$ at $a$ even though that processor is available as per supply $\mu$. Hence we only need to consider $m' - 1$ largest values of $CI_i$ when computing an upper bound for $\sum_{i=1}^{n} I_{i,2}$ using the above equations, where $CI_i$ denotes the carry-in demand in $\mathcal{W}_i$. Let us now define the following two terms:

$$\hat{I}_{i,2} = \min\{\mathcal{W}_i(A_k + \text{D}_k) - CI_i(A_k + \text{D}_k),\, A_k + \text{D}_k - \text{C}_k\} \quad \text{for all } i \neq k$$

$$\hat{I}_{k,2} = \min\{\mathcal{W}_k(A_k + \text{D}_k) - \text{C}_k - CI_k(A_k + \text{D}_k),\, A_k\}$$

Let $L_{(m'-1)}$ denote a set of task indices such that if $i \in L_{(m'-1)}$, then $(\bar{I}_{i,2} - \hat{I}_{i,2})$ is one of the $m' - 1$ largest values among all tasks. Then an upper bound on the worst-case resource demand in the interval $(a, b]$ can be defined as,

$$\mathrm{DEM}(A_k + \mathrm{D}_k, m') = m'\mathrm{C}_k + \sum_{i=1}^{n} \hat{I}_{i,2} + \sum_{i:i \in L_{(m'-1)}} (\bar{I}_{i,2} - \hat{I}_{i,2})$$

The following theorem gives our schedulability condition and its proof follows from the above discussions.

**Theorem 1** *A component comprising of cluster $\mathcal{C}$ with $m'$ processors and sporadic tasks $\mathcal{T} = \{\tau_1 = (\mathrm{T}_1, \mathrm{C}_1, \mathrm{D}_1), \ldots, \tau_n = (\mathrm{T}_n, \mathrm{C}_n, \mathrm{D}_n)\}$ is schedulable under* gEDF *using MPR model $\mu = \langle \Pi, \Theta, m' \rangle$, if for all tasks $\tau_k \in \mathcal{T}$ and all $A_k \geq 0$,*

$$\mathrm{DEM}(A_k + \mathrm{D}_k, m') \leq \mathrm{sbf}_\mu(A_k + \mathrm{D}_k) \tag{5}$$

In Theorem 1 if we set $\Theta = m'\Pi$, then we get the schedulability condition under dedicated resource that was proposed earlier (Baruah 2007). This shows that our condition is no more pessimistic than the one under dedicated resource. Although this theorem gives a schedulability test for component $\mathcal{C}$, it would be highly inefficient if we were required to check for all values of $A_k$. The following theorem shows that this is not the case.

**Theorem 2** *If (5) is violated for some $A_k$, then it must also be violated for a value satisfying the condition*

$$A_k < \frac{\mathrm{C}_\Sigma + m'\mathrm{C}_k - \mathrm{D}_k(\frac{\Theta}{\Pi} - U_\mathcal{T}) + U + B}{\frac{\Theta}{\Pi} - U_\mathcal{T}}$$

*where $\mathrm{C}_\Sigma$ denotes the sum of $m' - 1$ largest $\mathrm{C}_i$'s,*

$$U_\mathcal{T} = \sum_{i=1}^{n} \frac{\mathrm{C}_i}{\mathrm{T}_i}, \qquad U = \sum_{i=1}^{n} (\mathrm{T}_i - \mathrm{D}_i)\frac{\mathrm{C}_i}{\mathrm{T}_i} \quad \textit{and} \quad B = \frac{\Theta}{\Pi}\left[2 + 2\left(\Pi - \frac{\Theta}{m'}\right)\right]$$

*Proof* It is easy to see that $\hat{I}_{i,2} \leq \mathrm{dbf}_{\tau_i}(A_k + \mathrm{D}_k)$ and $\bar{I}_{i,2} \leq \mathrm{dbf}_{\tau_i}(A_k + \mathrm{D}_k) + \mathrm{C}_i$, where $\mathrm{dbf}_{\tau_i}(t) = \lfloor \frac{t + \mathrm{T}_i - D_i}{T_i} \rfloor C_i$. Then the left-hand side of (5) is less than or equal to $\mathrm{C}_\Sigma + m'\mathrm{C}_k + \sum_{i=1}^{n} \mathrm{dbf}_{\tau_i}(A_k + \mathrm{D}_k)$. For this equation to be violated it must be true that

$$\begin{aligned}
&\mathrm{C}_\Sigma + m'\mathrm{C}_k + \sum_{i=1}^{n} \mathrm{dbf}_{\tau_i}(A_k + \mathrm{D}_k) > \mathrm{sbf}_\mu(A_k + \mathrm{D}_k) \\
&\qquad \text{(using } \mathrm{dbf}_{\tau_i} \text{ bound from Baruah et al. 1990)} \\
&\Rightarrow \mathrm{C}_\Sigma + m'\mathrm{C}_k + (A_k + \mathrm{D}_k)U_\mathcal{T} + U > \mathrm{sbf}_\mu(A_k + \mathrm{D}_k) \quad \text{(from (2))} \\
&\Rightarrow \mathrm{C}_\Sigma + m'\mathrm{C}_k + (A_k + \mathrm{D}_k)U_\mathcal{T} + U > \frac{\Theta}{\Pi}(A_k + \mathrm{D}_k) - B \quad \text{(rearranging)} \\
&\Rightarrow A_k < \frac{\mathrm{C}_\Sigma + m\mathrm{C}_k - \mathrm{D}_k(\frac{\Theta}{\Pi} - U_\mathcal{T}) + U + B}{\frac{\Theta}{\Pi} - U_\mathcal{T}}
\end{aligned}$$

□

It can also be observed that (5) only needs to be evaluated at those values of $A_k$ for which at least one of $\hat{I}_{i,2}$, $\bar{I}_{i,2}$ or $\text{sbf}_\mu$ change. Therefore Theorem 1 gives a pseudo-polynomial time schedulability condition whenever utilization $U_\mathcal{T}$ is strictly less than the resource bandwidth $\frac{\Theta}{\Pi}$. In our techniques described later we compute minimum possible $\Theta$ and minimum required concurrency $m'$ for a given value of $\Pi$. Since $\Theta$ appears inside floor and ceiling functions in $\text{sbf}_\mu$, these computations may be intractable. We therefore replace $\text{sbf}_\mu$ in Theorem 1 with $\text{lsbf}_\mu$ from (2) before using it to generate MPR interfaces.

*Discussion* We have only focused on one intra-cluster scheduling algorithm in this paper. However our analysis technique can be easily extended to other intra-cluster scheduling algorithms. Specifically, in the schedulability condition given in (5), $\text{DEM}(A_k + \text{D}_k, m')$ depends on gEDF and $\text{sbf}_\mu(A_k + \text{D}_k)$ depends on MPR model $\mu$. Suppose there exists a function $\text{DEM}_{\text{DM}}(A_k + \text{D}_k, m')$ that can compute the workload upper bound for a task set scheduled under global DM. Then we can plug in $\text{DEM}_{\text{DM}}(A_k + \text{D}_k, m')$ into (5) to derive a schedulability condition for global DM intra-cluster scheduling. In fact, such a $\text{DEM}_{\text{DM}}$ can be obtained by extending current results over dedicated resource (Bertogna et al. 2005b).

Bertogna and Cirinei (2007) have derived an upper bound for the worst-case response time of tasks scheduled under gEDF or global DM. They have also used this bound to improve the carry-in demand $CI_i$ that we use in our schedulability condition. However this improvement to the carry-in demand cannot be applied in our case. Since we schedule tasks using MPR model, any response time computation depends on the processor supply in addition to task demand. Then to use the response time bounds presented in Bertogna and Cirinei (2007), we must extend it with sbf of MPR model. However, since we are computing the MPR model (capacity $\Theta$ and concurrency $m'$), its sbf is unknown and therefore the response time is not computable. One way to resolve this issue is to compute $\Theta$ and $m'$ using binary search. However, since $\Theta$ belongs to the domain of non-negative real numbers, binary search for the minimum $\Theta$ can take a prohibitively long time.

## 5 Component interface generation

In this section we develop a technique to generate interface $\mu = \langle \Pi, \Theta, m' \rangle$ for a cluster $\mathcal{C}$ comprising of sporadic tasks $\mathcal{T} = \{\tau_1 = (\text{T}_1, \text{C}_1, \text{D}_1), \ldots, \tau_n = (\text{T}_n, \text{C}_n, \text{D}_n)\}$ scheduled under gEDF. For this purpose we use the schedulability condition given by Theorem 1. We assume that period $\Pi$ of interface $\mu$ is specified a priori by the system designer. For instance, one can specify this period taking into account preemption overheads in the system. We then compute values for capacity $\Theta$ and number of processors $m'$ so that resource bandwidth of the interface is minimized. Finally, we also develop a technique that transforms MPR interfaces to periodic tasks,[7] in order to schedule clusters on the multiprocessor platform (inter-cluster scheduling).

[7] A periodic task $\tau = (\text{T}, \text{C}, \text{D})$ is a special case of the identically defined sporadic task; T in the periodic case denotes the exact separation between successive job releases instead of minimum separation.

5.1 Minimum bandwidth interface

It is desirable to minimize the resource bandwidth of $\mu$ when generating an interface for $\mathcal{C}$, because $\mathcal{C}$ then consumes the minimum possible processor supply. We now give a lemma which states that the resource bandwidth required to guarantee schedulability of task set $\mathcal{T}$ monotonically increases as number of processors in the cluster increases.

**Lemma 2** *Consider interfaces* $\mu_1 = \langle \Pi_1, \Theta_1, m'_1 \rangle$ *and* $\mu_2 = \langle \Pi_2, \Theta_2, m'_2 \rangle$, *such that* $\Pi_1 = \Pi_2$ *and* $m'_2 = m'_1 + 1$. *Suppose these two interfaces guarantee schedulability of the same component* $\mathcal{C}$ *with their smallest possible resource bandwidth, respectively. Then* $\mu_2$ *has a higher resource bandwidth than* $\mu_1$ *does*, i.e., $\Theta_1 < \Theta_2$.

*Proof* We prove this lemma by contradiction. Consider $\mu'_2 = \langle \Pi_2, \Theta'_2, m'_2 \rangle$ such that $\Theta'_2 \leq \Theta_1$. Suppose $\mu'_2$ guarantees schedulability of component $\mathcal{C}$ as per Theorem 1.

Let $\delta_d$ denote the difference in processor requirements of $\mathcal{C}$ on $m'_1$ and $m'_2$ processors for some interval length $A_k + D_k$, i.e., difference in function DEM used in Theorem 1. Then

$$\begin{aligned}
\delta_d &= \mathrm{DEM}(A_k + \mathrm{D}_k, m'_2) - \mathrm{DEM}(A_k + \mathrm{D}_k, m'_1) \\
&= \sum_{i:i \in L_{(m'_2-1)}} (\bar{I}_{i,2} - \hat{I}_{i,2}) - \sum_{i:i \in L_{(m'_1-1)}} (\bar{I}_{i,2} - \hat{I}_{i,2}) + (m'_2 - m'_1)\mathrm{C}_k \\
&= \sum_{i:i \in L_{(m'_2-1)}} (\bar{I}_{i,2} - \hat{I}_{i,2}) - \sum_{i:i \in L_{(m'_1-1)}} (\bar{I}_{i,2} - \hat{I}_{i,2}) + \mathrm{C}_k > 0. \qquad (6)
\end{aligned}$$

It is indicated by $\delta_d > 0$ that the same component has a greater upper bound on processor demand when it executes on more processors. Now let $\delta_s$ denote the difference in the linear supply bound function between $\mu_1$ and $\mu'_2$ for interval length $A_k + D_k$, i.e.,

$$\begin{aligned}
\delta_s &= \mathrm{lsbf}_{\mu'_2}(A_k + \mathrm{D}_k) - \mathrm{lsbf}_{\mu_1}(A_k + \mathrm{D}_k) \\
&= \frac{\Theta'_2}{\Pi}\left(t - 2\left(\Pi + 1 - \frac{\Theta'_2}{m'_2}\right)\right) - \frac{\Theta_1}{\Pi}\left(t - 2\left(\Pi + 1 - \frac{\Theta_1}{m'_1}\right)\right) \\
&\leq \frac{\Theta_1}{\Pi}\left(t - 2\left(\Pi + 1 - \frac{\Theta_1}{m'_2}\right)\right) - \frac{\Theta_1}{\Pi}\left(t - 2\left(\Pi + 1 - \frac{\Theta_1}{m'_1}\right)\right) \\
&\leq \frac{2(\Theta_1)^2}{\Pi_1}\left(\frac{1}{m'_2} - \frac{1}{m'_1}\right) \\
&= -\frac{2(\Theta_1)^2}{m'_1 m'_2 \Pi_1} < 0. \qquad (7)
\end{aligned}$$

It is indicated by $\delta_s < 0$ that MPR models provide less processor supply with more available processors, when values of period and capacity are fixed. Thus $\delta_d > 0$ and $\delta_s < 0$ for all $A_k + D_k$. Since $\mu_1$ guarantees schedulability of component $\mathcal{C}$ using the

smallest possible resource bandwidth, $\text{DEM}(A_k + D_k, m'_1) = \text{lsbf}_{\mu_1}(A_k + D_k)$ for some $A_k + D_k$. Then $\text{DEM}(A_k + D_k, m'_2) > \text{lsbf}_{\mu'_2}(A_k + D_k)$ for that $A_k + D_k$, and therefore $\mu'_2$ does not guarantee schedulability of $\mathcal{C}$ according to Theorem 1. This contradicts the assumption $\Theta'_2 \leq \Theta_1$. □

Lemma 2 suggests that when we generate interface $\mu$, we should use the smallest number of processors to minimize resource bandwidth of $\mu$. However an arbitrarily small number for $m'$, say $m' = 1$, may result in an *infeasible* $\mu$. Recall that a MPR model $\mu = \langle \Pi, \Theta, m' \rangle$ is defined to be feasible if and only if $\Theta \leq m'\Pi$. Therefore we find a feasible interface $\mu$ for $\mathcal{C}$ that: (1) guarantees schedulability of $\mathcal{C}$ based on Theorem 1 and (2) uses the smallest possible number of processors ($m^*$). We can find such $m^*$ through search. Since bandwidth is monotonic with number of processors, a binary search can be performed to determine $m^*$. For this search to terminate a lower and upper bound on $m^*$ should be known. $\lceil U_{\mathcal{T}} \rceil$ is clearly a lower bound on the number of processors necessary to schedule $\mathcal{C}$ where $U_{\mathcal{T}} = \sum_i \frac{\text{C}_i}{\text{T}_i}$. If the number of processors on the multiprocessor platform is known, then that number can be used as an upper bound for $m^*$. Otherwise, the following lemma gives an upper bound for $m^*$ as a function of task parameters.

**Lemma 3** *If* $m' \geq \frac{\sum_{i=1}^{n} \text{C}_i}{\min_{i=1,\ldots,n}\{\text{D}_i - \text{C}_i\}} + n$, *then feasible MPR model* $\mu = \langle \Pi, m'\Pi, m' \rangle$ *guarantees schedulability of* $\mathcal{C}$ *as per Theorem 1.*

*Proof*

$$
\begin{aligned}
m' &\geq \frac{\sum_{i=1}^{n} \text{C}_i}{\min_{i=1,\ldots,n}\{\text{D}_i - \text{C}_i\}} + n \quad (\text{since } \forall k,\, A_k \geq 0 \text{ in Theorem 1}) \\
&\Rightarrow m' \geq \frac{\sum_{i=1}^{n} \text{C}_i}{A_k + \text{D}_k - \text{C}_k} + n \quad \forall k \text{ and } \forall A_k \\
&\Rightarrow m'(A_k + \text{D}_k - \text{C}_k) \geq \sum_{i=1}^{n} \text{C}_i + n(A_k + \text{D}_k - \text{C}_k) \quad \forall k \text{ and } \forall A_k \qquad (8)
\end{aligned}
$$

Now consider the function $\text{DEM}(A_k + \text{D}_k, m')$ from Theorem 1.

$$
\begin{aligned}
&\text{DEM}(A_k + \text{D}_k, m') = \sum_{i=1}^{n} \hat{I}_{i,2} + \sum_{i : i \in L_{(m'-1)}} (\bar{I}_{i,2} - \hat{I}_{i,2}) + m'\text{C}_k \\
&\left( \text{since each } \hat{I}_{i,2} \leq A_k + \text{D}_k - \text{C}_k \text{ and } \sum_{i : i \in L_{(m'-1)}} (\bar{I}_{i,2} - \hat{I}_{i,2}) \leq \sum_{i=1}^{n} \text{C}_i \right) \\
&\Rightarrow \text{DEM}(A_k + \text{D}_k, m') \leq n(A_k + \text{D}_k - \text{C}_k) + \sum_{i=1}^{n} \text{C}_i + m'\text{C}_k \quad (\text{from (8)}) \\
&\Rightarrow \text{DEM}(A_k + \text{D}_k, m') \leq m'(A_k + \text{D}_k - \text{C}_k) + m'\text{C}_k \\
&\Rightarrow \text{DEM}(A_k + \text{D}_k, m') \leq \text{sbf}_{\mu}(A_k + \text{D}_k)
\end{aligned}
$$

**Table 1** Clusters $\mathcal{C}_1, \mathcal{C}_2$ and $\mathcal{C}_3$

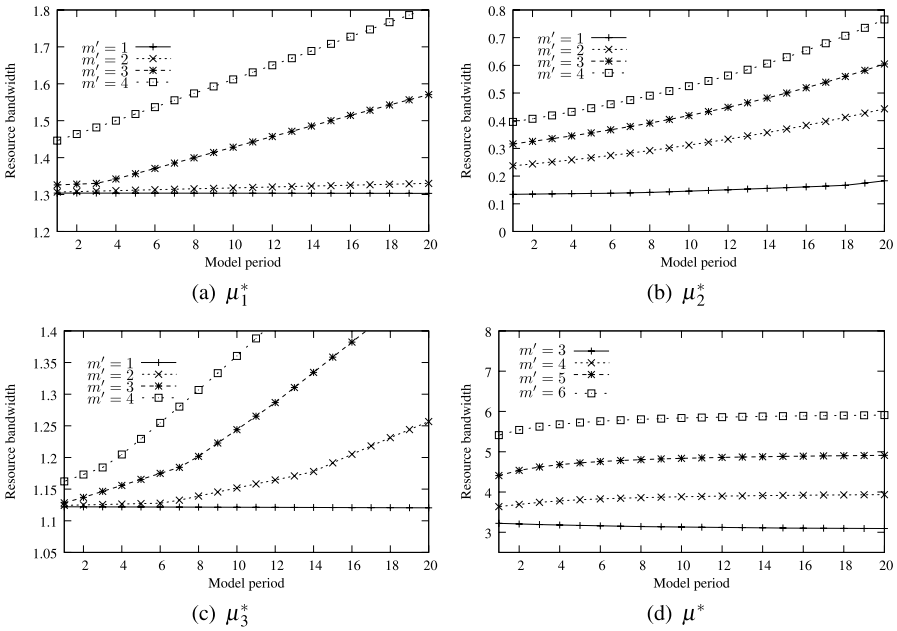
| Cluster | Task set | $\sum_i \frac{C_i}{T_i}$ | $\sum_i \frac{C_i}{D_i}$ |
|---|---|---|---|
| $\mathcal{C}_1$ | {(60, 5, 60), (60, 5, 60), (60, 5, 60), (60, 5, 60), (70, 5, 70), (70, 5, 70), (80, 5, 80), (80, 5, 80), (80, 10, 80), (90, 5, 90), (90, 10, 90), (90, 10, 90), (100, 10, 100), (100, 10, 100), (100, 10, 100)} | 1.304 | 1.304 |
| $\mathcal{C}_2$ | {(60, 5, 60), (100, 5, 100)} | 0.1333 | 0.1333 |
| $\mathcal{C}_3$ | {(45, 2, 40), (45, 2, 45), (45, 3, 40), (45, 3, 45), (50, 5, 45), (50, 5, 50), (50, 5, 50), (50, 5, 50), (70, 5, 60), (70, 5, 60), (70, 5, 65), (70, 5, 65), (70, 5, 65), (70, 5, 65), (70, 5, 70)} | 1.1222 | 1.1930 |

Since this inequality holds for all $k$ and $A_k$, from Theorem 1 we get that $\mu$ is guaranteed to schedule $\mathcal{C}$. □

Since $\mu$ in Lemma 3 is feasible and guarantees schedulability of $\mathcal{C}$, $\frac{\sum_{i=1}^{n} C_i}{\min_{i=1}^{n}\{D_i - C_i\}} + n$ is an upper bound for $m^*$. Thus we generate an interface for $\mathcal{C}$ by doing a binary search for $m^*$ in the range $[\lceil U_{\mathcal{T}} \rceil, \frac{\sum_{i=1}^{n} C_i}{\min_{i=1}^{n}\{D_i - C_i\}} + n]$. For each value of the number of processors $m'$, we compute the smallest value of $\Theta$ that satisfies (5) in Theorem 1, assuming $\text{sbf}_\mu$ is replaced with $\text{lsbf}_\mu$. $\Theta$ $(= \Theta^*)$, corresponding to the smallest value of $m'$ $(= m^*)$ that guarantees schedulability of $\mathcal{C}$ and results in a feasible interface, is then chosen as the capacity of $\mu$. Also $m^*$ is chosen as the number of processors in the cluster, i.e., $\mu = \langle \Pi, \Theta^*, m^* \rangle$.

*Algorithm complexity* To bound $A_k$ as in Theorem 2 we must know the value of $\Theta$. However, since $\Theta$ is being computed, we use its smallest (0) and largest $(m'\Pi)$ possible values to bound $A_k$. For each value of $m' > U_{\mathcal{T}}$, $\Theta$ can then be computed in pseudo-polynomial time using Theorem (1), assuming sbf is replaced with lsbf. This follows from the fact that the denominator in the bound of $A_k$ in Theorem 2 is non-zero. The only problem case is when $m' = \lceil U_{\mathcal{T}} \rceil = U_{\mathcal{T}}$. However in this case, we now show that $\mu = \langle \Pi, m'\Pi, m' \rangle$ can schedule $\mathcal{C}$ if and only if, $m' = 1$ and $D_i \geq T_i$ for each task $\tau_i$ in $\mathcal{T}$. Clearly, if some $D_i < T_i$, then a resource bandwidth of $U_{\mathcal{T}}$ is not sufficient to guarantee schedulability. Now suppose $m' > 1$. Then the left-hand side of (5) is $> \sum_{i=1}^{n} \text{dbf}_{\tau_i}(A_k + D_k) \geq (A_k + D_k)U_{\mathcal{T}}$, because $\hat{I}_{i,2} = \text{dbf}_{\tau_i}(A_k + D_k)$, $\bar{I}_{i,2} \geq \text{dbf}_{\tau_i}(A_k + D_k)$, and $m'C_k > 0$. Hence in this case $m' > U_{\mathcal{T}}$ and this is a contradiction. Therefore computing the interface for $m' = U_{\mathcal{T}}$ can be done in constant time. The number of different values of $m'$ to be considered is polynomial in the input size, because the search interval is bounded by numbers that are polynomial in the input parameters. Therefore the entire interface generation process has pseudo-polynomial complexity.

*Example 1* Consider the example virtual clustering framework shown in Fig. 2. Let clusters $\mathcal{C}_1, \mathcal{C}_2$ and $\mathcal{C}_3$ be assigned tasks as shown in Table 1. Interfaces $\mu_1^*$, $\mu_2^*$ and $\mu_3^*$, for clusters $\mathcal{C}_1, \mathcal{C}_2$ and $\mathcal{C}_3$, are shown in Figs. 9(a), 9(b) and 9(c) respectively. In the figures we have plotted the resource bandwidth of these interfaces for varying periods and $m'$ denotes the number of processors in the cluster.




(a) $\mu_1^*$ (b) $\mu_2^*$ (c) $\mu_3^*$ (d) $\mu^*$

**Fig. 9** MPR model based interfaces

Figures 9(a) and 9(c) show that when $m' = 1$ interfaces $\mu_1^*$ and $\mu_3^*$ are not feasible; their resource bandwidths are greater than 1 for all period values. This shows that clusters $\mathcal{C}_1$ and $\mathcal{C}_3$ are not schedulable on clusters having one processor. This is as expected because the utilization of task sets in these clusters is also greater than one. However when $m' = 2$, $\mu_1^*$ and $\mu_3^*$ are feasible, i.e., their respective resource bandwidths are at most two. Therefore for clusters $\mathcal{C}_1$ and $\mathcal{C}_3$, we choose MPR interfaces $\mu_1^*$ and $\mu_3^*$ with $m' = 2$. Similarly, Fig. 9(b) shows that $\mu_2^*$ is a feasible interface for cluster $\mathcal{C}_2$ when $m' = 1$. These plots also show that resource overheads[8] incurred by our interfaces are small for the non-trivial examples presented here.

## 5.2 Inter-cluster scheduling

As discussed in the introduction, virtual clustering involves two-level scheduling; scheduling of tasks within each cluster (intra-cluster scheduling) and scheduling of clusters on the multiprocessor platform (inter-cluster scheduling). MPR interfaces generated in the previous section capture task-level concurrency constraints within a cluster. Hence inter-cluster scheduling need not worry about these constraints when it schedules cluster interfaces. However there is no known scheduling algorithm for MPR interfaces. Therefore we now develop a technique to transform a MPR model into periodic tasks such that processor requirements of these tasks are at least as much as those of the resource model.

[8] Difference between $\max_k \max_{A_k} \frac{\text{DEM}(A_k+D_k,m')}{A_k+D_k}$ and resource bandwidth of MPR interface.

**Definition 2** Consider a MPR model $\mu = \langle \Pi, \Theta^*, m^* \rangle$ and let $\alpha = \Theta^* - m^* \lfloor \frac{\Theta^*}{m^*} \rfloor$ and $k = \lfloor \alpha \rfloor$. Define the transformation from $\mu$ to a periodic task set $\mathcal{T}_\mu$ as $\mathcal{T}_\mu = \{\tau_1 = (\mathrm{T}_1, \mathrm{C}_1, \mathrm{D}_1), \ldots, \tau_{m^*} = (\mathrm{T}_{m^*}, \mathrm{C}_{m^*}, \mathrm{D}_{m^*})\}$, where

$$\tau_1 = \cdots = \tau_k = \left(\Pi, \left\lfloor \frac{\Theta^*}{m^*} \right\rfloor + 1, \Pi\right)$$

$$\tau_{k+1} = \left(\Pi, \left\lfloor \frac{\Theta^*}{m^*} \right\rfloor + \alpha - k \left\lfloor \frac{\alpha}{k} \right\rfloor, \Pi\right) \quad \text{and}$$

$$\tau_{k+2} = \cdots = \tau_{m^*} = \left(\Pi, \left\lfloor \frac{\Theta^*}{m^*} \right\rfloor, \Pi\right)$$

In this definition it is easy to see that the total processor demand of $\mathcal{T}_\mu$ is $\Theta^*$ in every period $\Pi$. Further, we have assumed that whenever $\Theta^*$ is not an integer, processor supply from $\mu$ fully utilizes one processor before using another. For example, if $\Theta^* = 2.5$ and $m^* = 3$, then $\mu$ will provide two units of resource from two processors and the remaining 0.5 units from the third processor. The following theorem proves correctness of this transformation.

**Theorem 3** *If all the deadlines of task set $\mathcal{T}_\mu$ in Definition 2 are met by some processor supply with concurrency at most $m^*$ at any time instant, then its supply bound function is lower bounded by* $\mathrm{sbf}_\mu$.

*Proof* Since $\mathcal{T}_\mu$ has $m^*$ tasks, it can utilize at most $m^*$ processors at any time instant. Therefore if some processor supply provides more than $m^*$ processors at any time instant, then we can ignore these additional processor allocations. Hence we only need to consider processor supplies with concurrency at most $m^*$.

Total processor demand of all the tasks in $\mathcal{T}_\mu$ is $\Theta^*$ in every period of $\Pi$ time units. Then to meet all the deadlines of task set $\mathcal{T}_\mu$, any processor supply must provide at least $\Theta^*$ processor units in every period of $\Pi$ time units, with amount of concurrency at most $m^*$. But this is exactly the definition of MPR model $\mu = \langle \Pi, \Theta^*, m^* \rangle$. Therefore the supply bound function of this processor supply is lower bounded by $\mathrm{sbf}_\mu$. □

Thus MPR interfaces generated in the previous section can be transformed into periodic tasks using Definition 2. Once such tasks are generated for each virtual cluster, inter-cluster scheduling can be done using existing multiprocessor algorithms like gEDF, Pfair (Baruah et al. 1996), etc.

*Example 2* For MPR interfaces $\mu_1^*$, $\mu_2^*$ and $\mu_3^*$ generated in Example 1, we select periods 6, 8, and 5 respectively, i.e., interfaces $\langle 6, 8.22, 2 \rangle$, $\langle 8, 2.34, 1 \rangle$ and $\langle 5, 5.83, 2 \rangle$. Using Definition 2 we get task sets $\mathcal{T}_{\mu_1^*} = \{(6, 5, 6), (6, 4, 6)\}$, $\mathcal{T}_{\mu_2^*} = \{(8, 3, 8)\}$ and $\mathcal{T}_{\mu_3^*} = \{(5, 3, 5), (5, 3, 5)\}$. Suppose the three clusters $\mathcal{C}_1, \mathcal{C}_2$ and $\mathcal{C}_3$ (i.e., task set $\{\mathcal{T}_{\mu_1^*}, \mathcal{T}_{\mu_2^*}, \mathcal{T}_{\mu_3^*}\}$) are scheduled on a multiprocessor platform using gEDF. Then the resulting MPR interface $\mu^*$ is plotted in Fig. 9(d). As shown in the figure, $\mu^*$ is not feasible for $m' = 3$; its resource bandwidth is greater than 3 for all period values.

However these three clusters are schedulable on a multiprocessor platform having 4 processors (in the figure $\mu^*$ is feasible when $m' = 4$).

The above example clearly illustrates the advantage of virtual clustering over physical clustering. The three components $\mathcal{C}_1, \mathcal{C}_2$ and $\mathcal{C}_3$, would require 5 processors under physical clustering (2 each for $\mathcal{C}_1$ and $\mathcal{C}_3$ and 1 for $\mathcal{C}_2$). On the other hand, a gEDF based virtual clustering technique can schedule these clusters using only 4 processors. Although total utilization of tasks in the three clusters is 2.56, our analysis requires 4 processors to schedule the system. This overhead is as a result of the following factors: (1) gEDF is not an optimal scheduling algorithm on multiprocessor platforms (both for intra- and inter-cluster scheduling), (2) the schedulability conditions we use are only sufficient conditions, and (3) capturing task-level concurrency constraints in a component interface leads to some increase in processor requirements (resource overhead of abstracting a cluster into MPR interface).

## 6 Virtual cluster-based scheduling algorithms

In this section we propose new virtual-cluster based scheduling algorithms for implicit deadline sporadic task systems. Prior to presenting these algorithms, we eliminate resource overheads from the virtual clustering framework proposed in Sect. 5.

### 6.1 Improved virtual-clustering framework

In this section we present an inter-cluster scheduling algorithm that is optimal whenever all the MPR interfaces being scheduled under it have identical periods. We also present another transformation from MPR models to periodic tasks, which along with the optimal inter-cluster scheduler, results in an improved sbf for MPR models. These two together, eliminate the resource overheads described at the end of previous section.

McNaughton (1959) presented an algorithm for scheduling real-time jobs in a given time interval on a multiprocessor platform. This algorithm can be explained as follows: Consider $n$ jobs to be scheduled on $m$ processors in a time interval $(t_1, t_2]$ of length $t$, such that no job is simultaneously scheduled on more than one processor. The job set need not be sorted in any particular order. McNaughton's algorithm schedules the $i$th job on the first non-empty processor, packing jobs from left to right. Suppose the $(i-1)$st job was scheduled on processor $k$ up to time instant $t_3$ $(t_1 \leq t_3 \leq t_2)$. Then up to $t_2 - t_3$ time units of the $i$th job are scheduled on processor $k$ and the remaining time units are scheduled on processor $k+1$ starting from $t_1$. Figure 10 illustrates this schedule for a job set $\{J_1, \ldots, J_5\}$ on 4 processors. Note that if the total resource demand of a job is at most $t_2 - t_1$, then (1) the job is scheduled on at most two processors by McNaughton's algorithm, and (2) the job is never scheduled simultaneously on both the processors. The following theorem establishes conditions under which this algorithm can successfully schedule job sets.

**Theorem 4** (Theorem 3.1 in McNaughton 1959) *Let* $c_1, \ldots, c_n$ *denote the number of processor units of the n jobs that must be scheduled in the interval* $(t_1, t_2]$ *on m*

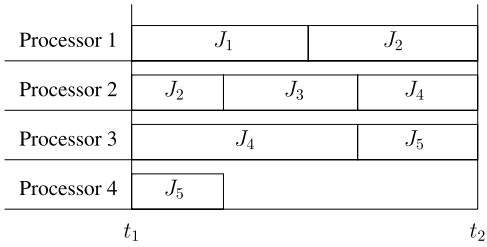



**Fig. 10** Schedule of job set under McNaughton's algorithm in the interval $(t_1, t_2]$

*identical, unit-capacity processors. If* $\sum_{i=1}^{n} \mathrm{c}_i \leq m(t_2 - t_1)$*, then a necessary and sufficient condition to guarantee schedulability of this job set is that for all* $i$, $\mathrm{c}_i \leq t_2 - t_1$.

If $\mathrm{c}_i > t_2 - t_1$ then the $i$th job cannot be scheduled in the interval $(t_1, t_2]$ by any scheduling algorithm, unless the job is simultaneously scheduled on more than one processor. Likewise, if $\sum_{i=1}^{n} \mathrm{c}_i > m(t_2 - t_1)$, then also the job set cannot be scheduled by any scheduling algorithm, because the total processor demand in the interval $(t_1, t_2]$ is greater than the total available processing capacity. Hence Theorem 4 in fact shows that McNaughton's algorithm is optimal for scheduling job sets in a given time interval.

Consider a periodic task set $\mathcal{T} = \{(\mathrm{T}, \mathrm{C}_1, \mathrm{T}), \ldots, (\mathrm{T}, \mathrm{C}_n, \mathrm{T})\}$. Tasks in $\mathcal{T}$ have identical periods and implicit deadline. Suppose we use McNaughton's algorithm in the intervals $(k\mathrm{T}, (k+1)\mathrm{T}]$, $k \in \mathbb{I}$, to schedule jobs of $\mathcal{T}$ on $m$ identical, unit-capacity processors. Then for each interval $(k\mathrm{T}, (k+1)\mathrm{T}]$ (1) all jobs of $\mathcal{T}$ are released at the beginning of the interval $(k\mathrm{T})$ and (2) all jobs of $\mathcal{T}$ have deadline at the end of the interval $((k+1)\mathrm{T})$. Therefore, from Theorem 4, we get that McNaughton's algorithm optimally schedules these jobs in each interval and this leads to the following direct corollary.

**Corollary 1** *Let* $\mathcal{T} = \{\tau_1 = (\mathrm{T}_1, \mathrm{C}_1, \mathrm{D}_1), \ldots, \tau_n = (\mathrm{T}_n, \mathrm{C}_n, \mathrm{D}_n)\}$ *denote a periodic task set to be scheduled on m identical, unit-capacity processors. If* $\mathrm{T}_1 = \cdots = \mathrm{T}_n = \mathrm{D}_1 = \cdots = \mathrm{D}_n \ (= \mathrm{T})$*, then a necessary and sufficient condition for* $\mathcal{T}$ *to be schedulable using McNaughton's algorithm is that* $\sum_{i=1}^{n} \mathrm{C}_i \leq m\mathrm{T}$ *and* $\mathrm{C}_i \leq \mathrm{T}_i$ *for each* $i$.

Consider the virtual clustering framework proposed in Sect. 5. Suppose all the MPR interfaces in this framework have identical periods. Then all the periodic tasks generated using Definition 2 also have identical periods. And from Corollary 1 we get that McNaughton's algorithm is optimal for scheduling these tasks on the physical platform, i.e., the algorithm does not incur any resource overhead for inter-cluster scheduling.

Another source of resource overhead is the abstraction of a cluster into MPR interface and its transformation to a periodic task set. This overhead results from the sub-optimality of sbf of MPR models which can be explained as follows. Consider the two functions, $\mathrm{sbf}_\mu$ and $\mathrm{usbf}_\mu$, shown in Fig. 11. The resource bandwidth used by $\mu$ is equal to the slope of line $\mathrm{usbf}_\mu$ $(\frac{\Theta}{\Pi})$. Suppose $\mu$ is used to abstract the processor demand of cluster $\mathcal{C}$ in Theorem 1. Since $\mathrm{sbf}_\mu$ has a non-zero

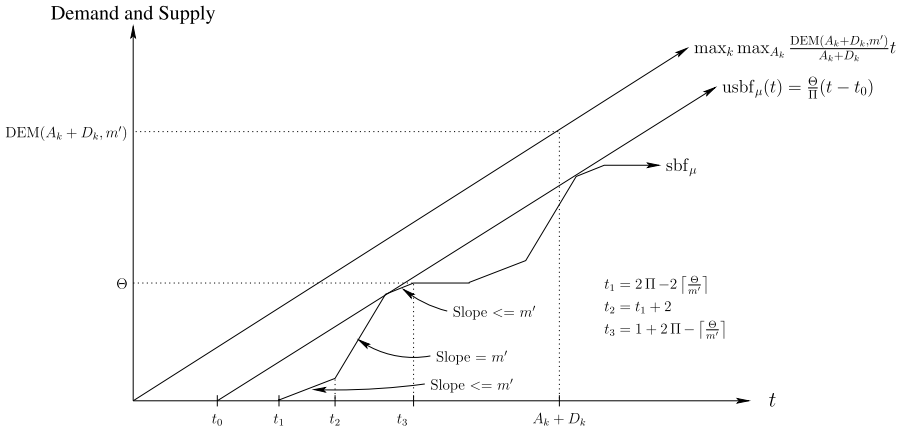



**Fig. 11** Bandwidth of $\mathrm{sbf}_\mu$ and schedulability load of cluster $\mathcal{C}$

x-axis intercept, the bandwidth of $\mu$ is strictly larger than the schedulability load, $\max_k \max_{A_k} \mathrm{DEM}(A_k + \mathrm{D}_k, m')/(A_k + \mathrm{D}_k)$, of cluster $\mathcal{C}$. If not then, as shown in Fig. 11, there exists some $A_k + D_k$ for which Theorem 1 is not satisfied. This explains the resource overhead in the abstraction of clusters to MPR interfaces. Now suppose $\mu$ is transformed into the periodic task set $\mathcal{T}_\mu$ using Definition 2. Then from Theorem 3 we get that the total processor demand of $\mathcal{T}_\mu$ is at least as much as $\mathrm{sbf}_\mu$. However, since $\mathrm{sbf}_\mu$ does not guarantee $\Theta$ resource units in an interval of length $\Pi$ (see Fig. 11), a processor supply with supply bound function exactly $\mathrm{sbf}_\mu$ cannot schedule $\mathcal{T}_\mu$. This explains the resource overhead in the transformation of MPR interfaces to periodic tasks.

To eliminate the aforementioned overheads, we must modify the transformation presented in Definition 2. This is because the schedule of $\mathcal{T}_\mu$ determines the processor supply from the multiprocessor platform to $\mu$, and this in turn determines $\mathrm{sbf}_\mu$. We now present a new transformation from MPR models to periodic tasks as follows.

**Definition 3** Given a MPR model $\mu = \langle \Pi, \Theta^*, m^* \rangle$, we define its transformation to a periodic task set $\mathcal{T}_\mu$ as

$$\mathcal{T}_\mu = \{\tau_1 = (\mathrm{T}_1, \mathrm{C}_1, \mathrm{D}_1), \ldots, \tau_{m^*} = (\mathrm{T}_{m^*}, \mathrm{C}_{m^*}, \mathrm{D}_{m^*})\}, \quad \text{where}$$
$$\tau_1 = \cdots = \tau_{m^*-1} = (\Pi, \Pi, \Pi) \quad \text{and} \quad \tau_{m^*} = (\Pi, \Theta^* - (m^* - 1)\Pi, \Pi)$$

In this definition it is easy to see that the total processor demand of $\mathcal{T}_\mu$ is $\Theta^*$ in every $\Pi$ time units, with concurrency at most $m^*$. Therefore Theorem 3 holds in this case as well, i.e., if all the deadlines of task set $\mathcal{T}_\mu$ are met by some processor supply with concurrency at most $m^*$ at any time instant, then its supply bound function is lower bounded by $\mathrm{sbf}_\mu$.

Now suppose a cluster is abstracted into MPR interface $\mu = \langle \Pi, \Theta, m' \rangle$, which is then transformed into task set $\mathcal{T}_\mu$ using Definition 3. Let $\mathcal{T}_\mu$ be scheduled on the multiprocessor platform using McNaughton's algorithm, along with periodic tasks

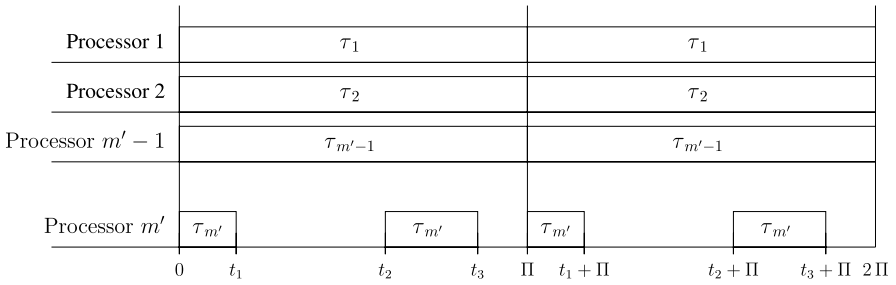



**Fig. 12** McNaughton's schedule of implicit deadline periodic tasks with identical periods

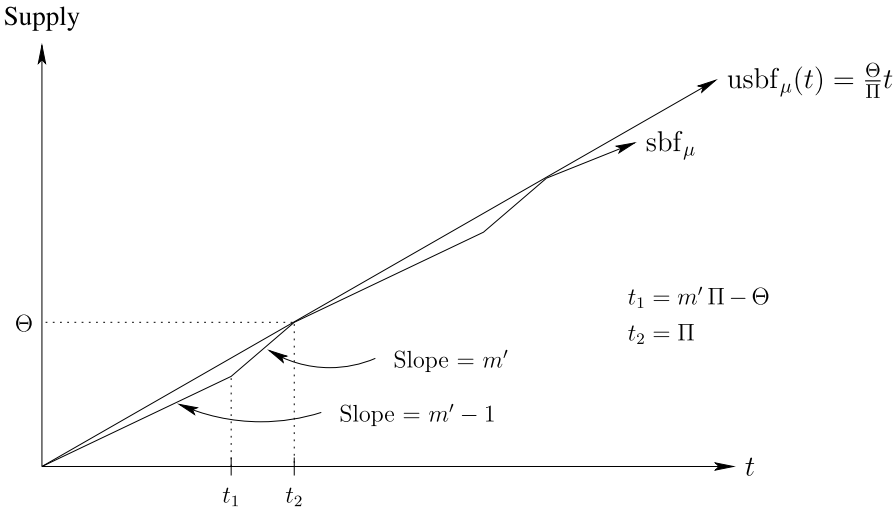



**Fig. 13** Improved $\text{sbf}_\mu$ and its linear upper bound $\text{usbf}_\mu$

that all have period and deadline $\Pi$ (implicit deadline task system with identical periods). Figure 12 illustrates the McNaughton schedule for task set $\mathcal{T}_\mu$. As can be seen in the figure, tasks $\tau_1, \ldots, \tau_{m'-1}$ completely utilize $m' - 1$ processors on the platform. Further, every job of task $\tau_{m'}$ is scheduled in an identical manner within its execution window (intervals $(0, t_1]$ and $(t_2, t_3]$ relative to release time). Since this schedule of $\mathcal{T}_\mu$ is used as the processor supply for the underlying MPR interface, $\mu$ guarantees $\Theta$ processor units in any time interval of length $\Pi$, $2\Theta$ processor units in any time interval of length $2\Pi$, and so on. In other words, the blackout interval of $\text{sbf}_\mu$ (described in Sect. 2.2) reduces to zero. The resulting sbf is plotted in Fig. 13 and it is given by the following equation.

$$\text{sbf}_\mu(t) = \left\lfloor \frac{t}{\Pi} \right\rfloor \Theta + \left( t - \left\lfloor \frac{t}{\Pi} \right\rfloor \Pi \right) m' - \min\left\{ t - \left\lfloor \frac{t}{\Pi} \right\rfloor \Pi, m'\Pi - \Theta \right\} \quad (9)$$

$\text{sbf}_\mu$ guarantees $\Theta$ resource units in any time interval of length $\Pi$. Then a processor supply with supply bound function equal to $\text{sbf}_\mu$ can successfully schedule task

set $\mathcal{T}_\mu$. Thus we have eliminated the resource overhead that was present in the previous transformation given in Definition 2.

Now consider the schedulability condition for cluster $\mathcal{C}$ given by (5) in Theorem 1. This equation needs to be evaluated for all values of $A_k$ up to the bound given in Theorem 2 and for all tasks $\tau_k$ in cluster $\mathcal{C}$. In this equation it is easy to see that $\text{DEM}(A_k + \text{D}_k, m')$ increases by at most $m' - 1$ for every unit increase in $A_k$, as long as $A_k + 1 + \text{D}_k$ does not coincide with the release or deadline of some task in cluster $\mathcal{C}$. In other words, $\text{DEM}(A_k + 1 + \text{D}_k, m') \leq \text{DEM}(A_k + \text{D}_k, m') + m' - 1$, whenever $A_k + 1 + \text{D}_k$ is not equal to $l\text{T}_i$ or $l\text{T}_i + \text{D}_i$ for any $l$ and $i$ (denoted as property *bounded increase*). This is because over such unit increases in $A_k$, $m'\text{C}_k$ and each $\hat{I}_{i,2}$ remain constant and $\sum_{i:i \in L_{(m'-1)}} (\bar{I}_{i,2} - \hat{I}_{i,2})$ increases by at most $m' - 1$. However $\text{sbf}_\mu$ increases by at least $m' - 1$ over each unit time interval (see Fig. 13). Therefore to generate interface $\mu$, it is sufficient to evaluate (5) at only those values of $A_k$ for which $A_k + \text{D}_k$ is equal to $l\text{T}_i$ or $l\text{T}_i + \text{D}_i$ for some $l$ and $i$. Now suppose period $\Pi$ of $\mu$ is equal to the GCD (greatest common divisor) of the periods and deadlines of all the tasks in cluster $\mathcal{C}$. Then all the required evaluations of (5) will occur at time instants $t$ for which $\text{sbf}_\mu(t) = \text{usbf}_\mu(t) = \frac{\Theta}{\Pi} t$ (see Fig. 13). In other words, the right-hand side of (5) can be replaced with $\frac{\Theta}{\Pi} t$. This means that the resource bandwidth of the resulting interface $\mu$ ($\frac{\Theta}{\Pi}$) will be equal to the schedulability load, $\max_k \max_{A_k} \frac{\text{DEM}(A_k + D_k, m')}{A_k + D_k}$, of cluster $\mathcal{C}$. Thus we have eliminated the resource overhead that was previously present in the cluster abstraction process.

We now summarize the contributions of this section. The following theorem, which is a direct consequence of the above discussions, states the fundamental result of this section. This theorem states that our improved virtual-clustering framework does not incur any resource overheads in transforming MPR interfaces to periodic tasks or in scheduling the transformed tasks on the multiprocessor platform.

**Theorem 5** *Consider MPR interfaces* $\mu_1 = \langle \Pi, \Theta_1, m'_1 \rangle, \ldots, \mu_p = \langle \Pi, \Theta_p, m'_p \rangle$. *Suppose they are transformed to periodic tasks using Definition 3. McNaughton's algorithm can successfully schedule the transformed tasks on m identical, unit-capacity processors if and only if,*

$$\sum_{i=1}^{p} \frac{\Theta_i}{\Pi} \leq m$$

Suppose (1) we want to schedule a constrained deadline sporadic task set $\mathcal{T}$ using virtual clusters on $m$ identical, unit-capacity processors, (2) task-cluster mapping is given, and (3) each intra-cluster scheduler is such that the corresponding schedulability condition satisfies *bounded increase* property described above (e.g., gEDF). Let (1) each virtual cluster be abstracted into an MPR interface whose period $\Pi$ is equal to the GCD of the periods and deadlines of all the tasks in $\mathcal{T}$, (2) these interfaces be transformed into periodic tasks using Definition 3, and (3) these periodic tasks be scheduled on the multiprocessor platform using McNaughton's algorithm. Then, in addition to the results stated in Theorem 5, the resource bandwidth of each MPR interface will be equal to the schedulability load of the corresponding cluster.

## 6.2 Virtual clustering of implicit deadline task systems

In this section we propose two virtual cluster-based scheduling algorithms for implicit deadline sporadic task sets. We consider the problem of scheduling an implicit deadline sporadic task set $\mathcal{T} = \{\tau_1 = (\mathrm{T}_1, \mathrm{C}_1, \mathrm{T}_1), \ldots, \tau_n = (\mathrm{T}_n, \mathrm{C}_n, \mathrm{T}_n)\}$ on $m$ identical, unit-capacity processors. We first present a new virtual-clustering technique that is optimal like the well known Pfair algorithm (Baruah et al. 1996), but unlike Pfair, has a non-trivial bound on the number of preemptions. The second technique extends the well known algorithm US-EDF$\{m/(2m-1)\}$ (Srinivasan and Baruah 2002) with virtual clusters. We show that the presently known processor utilization bound of US-EDF$\{m/(2m-1)\}$ can be improved by using virtual clusters.

### *6.2.1 VC-IDT scheduling algorithm*

In VC-IDT (*Virtual Clustering-Implicit Deadline Tasks*) scheduling algorithm we consider a trivial task-processor mapping that assigns each task $\tau_i \in \mathcal{T}$ to its own virtual cluster $\mathcal{C}_i$ having one processor. Since each cluster has only one processor, we assume that each cluster uses EDF for intra-cluster scheduling.[9] Each cluster $\mathcal{C}_i$ is abstracted into a MPR interface $\mu_i = \langle \Pi, \Theta_i, 1 \rangle$, where $\Pi$ is equal to the GCD of $\mathrm{T}_1, \ldots, \mathrm{T}_n$ and $\Theta_i/\Pi = \mathrm{C}_i/\mathrm{T}_i$. Further, each interface $\mu_i$ is transformed into periodic tasks using Definition 3 and the resulting task set is scheduled on the multiprocessor platform using McNaughton's algorithm. The following theorem proves that VC-IDT is an optimal algorithm for scheduling implicit deadline sporadic task systems on identical, unit-capacity multiprocessor platforms.

**Theorem 6** *Consider sporadic tasks $\mathcal{T} = \{\tau_1 = (\mathrm{T}_1, \mathrm{C}_1, \mathrm{T}_1), \ldots, \tau_n = (\mathrm{T}_n, \mathrm{C}_n, \mathrm{T}_n)\}$. A necessary and sufficient condition to guarantee that $\mathcal{T}$ is schedulable on $m$ identical, unit-capacity processors using VC-IDT algorithm is*

$$\sum_{i=1}^{n} \frac{\mathrm{C}_i}{\mathrm{T}_i} \leq m \tag{10}$$

*Proof* In VC-IDT each virtual cluster $\mathcal{C}_i$, comprising of task $\tau_i$, is abstracted to interface $\mu_i = \langle \Pi, \Theta_i, 1 \rangle$, where $\Pi$ is equal to the GCD of $\mathrm{T}_1, \ldots, \mathrm{T}_n$ and $\frac{\Theta_i}{\Pi} = \frac{\mathrm{C}_i}{\mathrm{T}_i}$. The interface set $\mu_1, \ldots, \mu_n$, all having identical periods, are then transformed to periodic tasks using Definition 3 and scheduled on the platform using McNaughton's algorithm. Therefore, from Theorem 5, we get that this interface set is schedulable on the multiprocessor platform if and only if,

$$\sum_{i=1}^{n} \frac{\Theta_i}{\Pi} \leq m \quad \Rightarrow \quad \sum_{i=1}^{n} \frac{\mathrm{C}_i}{\mathrm{T}_i} \leq m$$

[9] Since each cluster also has only one task, any work conserving algorithm can be used for intra-cluster scheduling.

To prove this theorem we then need to show that for each $i$, interface $\mu_i$ can schedule cluster $\mathcal{C}_i$. $\mathcal{C}_i$ comprises of sporadic task $\tau_i$ and uses EDF scheduler. Therefore any processor supply that can guarantee $C_i$ processor units in all time intervals of length $T_i$ can be used to schedule $\tau_i$. But from the sbf of model $\mu_i$ (see (9)), it is easy to see that $\mu_i$ guarantees $C_i$ processor units in any time interval of length $T_i$. This proves the theorem. □

Equation (10) is known to be a necessary and sufficient feasibility condition for scheduling implicit deadline sporadic task systems on $m$ identical, unit-capacity processors (Srinivasan and Anderson 2006). Hence VC-IDT is an optimal scheduling algorithm for this problem domain. The other known optimal schedulers for this problem, to the best of our knowledge, are the $PD^2$ Pfair/ERfair algorithm (Srinivasan and Anderson 2006) and the task-splitting algorithm (Andersson and Bletsas 2008).

$PD^2$ algorithm is known to incur a high number of preemptions in order to guarantee P-fairness/ER-fairness, because fairness is a stricter requirement than deadline satisfaction. It can potentially incur $m$ preemptions in every time unit, which is the maximum possible on this multiprocessor platform. In contrast, the number of preemptions incurred by VC-IDT has a non-trivial upper bound which can be explained as follows. When interfaces $\mu_1, \ldots, \mu_n$ are scheduled using McNaughton's algorithm (after being transformed into periodic tasks), there are at most $m - 1$ of them that use more than one processor. Each such interface $\mu_i$ is preempted once in every $\Pi$ time units and this may result in a preemption in the execution of task $\tau_i$. Each of the other $n - (m - 1)$ tasks may also experience preemption once in every $\Pi$ time units, because the execution requirements of a job of this task cannot be entirely satisfied by a single job of the corresponding interface. The entire sporadic task set will thus incur at most $n$ preemptions in every $\Pi$ time units. Therefore when $\Pi$, the GCD of task periods, is very small VC-IDT does not offer any advantage over $PD^2$ algorithm. This can happen for instance even if two task periods are co-prime (the GCD in this case is one). However, in real-world systems, it has been observed that task periods are typically harmonic to (multiples of) each other. For example, harmonic task periods can be found in avionics real-time applications; see ARINC-653 standards (ARINC specification 2006) and sample avionics workloads in the appendix of this technical report (Easwaran et al. 2009). In this case, the GCD of task periods is equal to the smallest task period (typically a few milliseconds as indicated by the workloads in Easwaran et al. 2009), and then VC-IDT incurs far fewer preemptions than Pfair/ERfair algorithms. It is worth noting that although the BoundaryFair algorithm (Zhu et al. 2003) incurs fewer preemptions than VC-IDT, it is only optimal for scheduling periodic (not sporadic) task systems.

The task splitting algorithm proposed by Andersson and Bletsas (2008) has also been shown to be optimal for implicit deadline sporadic task systems (see Theorem 3 in Andersson and Bletsas 2008). Suppose $jobs(t)$ denotes the maximum number of jobs that will be released by the task system in any time interval of length $t$. Then this algorithm is known to incur at most $\frac{3mt}{\text{GCD}} + 2m + jobs(t)$ number of preemptions, where GCD denotes the greatest common divisor of task periods (derived from Theorems 2 and 3 in Andersson and Bletsas 2008). In contrast, VC-IDT algorithm incurs at most $\frac{nt}{\text{GCD}}$ number of preemptions. Clearly, our algorithm outperforms the

task splitting approach whenever $n < 3m$. When $n > 3m$, either algorithm can incur fewer preemptions depending on the value of GCD and the relation between task periods. The runtime complexity of the dispatcher under task splitting is the same as that of partitioned EDF (roughly logarithmic in the number of tasks for every scheduling decision). In contrast, under VC-IDT, the entire interface schedule based on McNaughton's algorithm can be generated and stored offline for intervals of length GCD. Therefore at runtime the tasks can be scheduled in constant time. This vastly improved runtime complexity at the expense of increased storage requirements is particularly useful in embedded systems, where cheaper ROM and Flash memory is still preferred over the more expensive RAM (for instance, MICAz, the sensor node from crossbow, has 512 k of Flash memory whereas only 4 k of RAM (MICAz 2009)). Finally, a practical limitation of the task splitting approach is that they do not provide any error isolation mechanism, i.e., a task that executes for more than its stated worst-case execution time can cause other tasks in the system to miss deadlines. In contrast, VC-IDT provides automatic error isolation, because a mis-behaving task will never get more processor share than already provided by its MPR interface.

### *6.2.2 Virtual clustering for* US-EDF$\{m/(2m-1)\}$

US-EDF$\{m/(2m-1)\}$, proposed by Srinivasan and Baruah (2002), is a global scheduling algorithm for implicit deadline sporadic task systems. Under this algorithm each task with utilization $(\frac{C}{T})$ greater than $\frac{m}{2m-1}$ is given the highest priority, and the remaining tasks are scheduled based on gEDF. It has been shown that this algorithm has a processor utilization bound of $\frac{m^2}{2m-1}$, i.e., any sporadic task set with total utilization $(\sum_i \frac{C_i}{T_i})$ at most $\frac{m^2}{2m-1}$ can be scheduled by US-EDF$\{m/(2m-1)\}$ on $m$ identical, unit-capacity processors (Srinivasan and Baruah 2002).

Now consider the following virtual cluster-based US-EDF$\{m/(2m-1)\}$ scheduling algorithm. Let each task with utilization greater than $\frac{m}{2m-1}$ be assigned to its own virtual cluster having one processor and using EDF (denoted as *high utilization cluster*), and all the remaining tasks be assigned to a single cluster using gEDF (denoted as *low utilization cluster*). Each cluster is abstracted to a MPR interface such that period $\Pi$ of each interface is equal to the GCD of $T_1, \ldots, T_n$. Each high utilization cluster is abstracted to interface $\langle \Pi, \Theta, 1 \rangle$, where $\frac{\Theta}{\Pi}$ is equal to the utilization of task in the cluster (Theorem 6 proves correctness of this abstraction). The low utilization cluster is abstracted to interface $\mu_{\text{low}} = \langle \Pi, \Theta', m' \rangle$, where $\Theta'$ and $m'$ are generated using techniques in Sects. 5 and 6.1. Finally, these interfaces are transformed to periodic tasks using Definition 3 and the resulting task set is scheduled on the multiprocessor platform using McNaughton's algorithm.

We now derive a utilization bound for the virtual cluster-based US-EDF$\{m/(2m-1)\}$ algorithm described above. Suppose $\alpha$ denotes the total utilization of all the high utilization tasks, i.e., the total resource bandwidth of all the MPR interfaces that represent high utilization clusters is $\alpha$. Since all the interfaces that we generate have identical periods, from Theorem 5 we get that the maximum resource bandwidth available for $\mu_{\text{low}}$ is $m - \alpha$. This means that $\frac{\Theta'}{\Pi} \leq m - \alpha$ and $\alpha \leq m$ are necessary and sufficient conditions to guarantee schedulability of task set $\mathcal{T}$ under virtual cluster-based US-EDF$\{m/(2m-1)\}$.

Suppose $\alpha > m - 1$. Then $m - \alpha < 1$ and $m' \leq 1$. The last inequality can be explained as follows. $m' = \lceil \frac{\Theta'}{\Pi} \rceil$ because $m'$ is the smallest number of processors upon which the low utilization cluster is schedulable. Then $\frac{\Theta'}{\Pi} \leq m - \alpha < 1$ implies $m' \leq 1$. In this case the low utilization cluster is scheduled on a uniprocessor platform and gEDF reduces to EDF, an optimal uniprocessor scheduler with utilization bound $m - \alpha$. Therefore virtual cluster-based US-EDF$\{m/(2m-1)\}$ is optimal whenever $\alpha > m - 1$, i.e., it can successfully schedule task set $\mathcal{T}$ if $\sum_{i=1}^{n} \frac{C_i}{T_i} \leq m$.

Now suppose $\alpha \leq m - 1$. To derive the utilization bound in this case, we use a utilization bound of gEDF that was developed by Goossens et al. (2003). As per this bound $\mu_{\text{low}}$ can support a low utilization cluster whose total task utilization is upper bounded by $(\frac{\Theta'}{\Pi} - (\frac{\Theta'}{\Pi} - 1)U_{\max})$, where $U_{\max}$ is the maximum utilization of any task in the cluster. Therefore, in this case, the utilization bound of virtual cluster-based US-EDF$\{m/(2m-1)\}$ is

$$\alpha + \left(\frac{\Theta'}{\Pi} - \left(\frac{\Theta'}{\Pi} - 1\right)U_{\max}\right) = \alpha + (m - \alpha - (m - \alpha - 1)U_{\max})$$

Since $m - \alpha \geq 1$, the bound in the above equation is minimized when $U_{\max}$ is maximized. Substituting $U_{\max} = \frac{m}{2m-1}$ (largest utilization of any task in the low utilization cluster), we get a utilization bound of

$$\begin{aligned}
\alpha + \left(m - \alpha\left(1 - \frac{m}{2m-1}\right) + \frac{m}{2m-1}\right) &= \frac{\alpha(2m-1) + (m-\alpha)(m-1) + m}{2m-1} \\
&\geq \frac{\alpha(2m-1) + (m-\alpha)(m-1) + m}{2m-1} \\
&= \frac{m^2 + \alpha m}{2m-1}
\end{aligned}$$

Thus the processor utilization bound of virtual cluster-based US-EDF$\{m/(2m-1)\}$ is $\min\{m, \frac{m^2+\alpha m}{2m-1}\}$. It is easy to see that whenever $\alpha > 0$, this bound is greater than the presently known utilization bound of $\frac{m^2}{2m-1}$ for US-EDF$\{m/(2m-1)\}$. This shows that virtual clustering, unlike the earlier US-EDF$\{m/(2m-1)\}$ algorithm, allows one to use the leftover processing capacity from high utilization clusters for scheduling tasks in the low utilization cluster. It also shows that the improvement in utilization bound is achievable even when clusters are scheduled on the platform using non-trivial abstractions such as MPR models. This gain however comes at a cost; since $\Pi$ is equal to the GCD of task periods, the resulting schedule can potentially incur more preemptions when compared to the original algorithm.

## 7 Conclusions

In this paper we have considered the idea of cluster-based scheduling on multiprocessor platforms as an alternative to existing partitioned and global scheduling strategies. Cluster-based scheduling can be viewed as a two-level scheduling strategy. Tasks

in a cluster are globally scheduled within the cluster (intra-cluster scheduling) and clusters are then scheduled on the multiprocessor platform (inter-cluster scheduling). We have further classified clustering into physical (one-to-one) and virtual (many-to-many), depending on the mapping between clusters and processors on the platform. Virtual clustering is more general and less sensitive to task-processor mappings than physical clustering.

Towards supporting virtual cluster-based scheduling, we have developed techniques for hierarchical scheduling in this paper. Resource requirements and concurrency constraints of tasks within each cluster are first abstracted into MPR interfaces. These interfaces are then transformed into periodic tasks which are used for inter-cluster scheduling. We have also developed an efficient technique to minimize processor utilization of individual clusters under gEDF. Finally, we developed a new optimal scheduling algorithm for implicit deadline sporadic task systems, and also illustrated the power of general task-processor mappings by virtualizing US-EDF$\{m/(2m-1)\}$ algorithm.

We only focused on gEDF for intra-cluster and McNaughton's for inter-cluster scheduling. However, our approach of isolating the inter-cluster scheduler from task-level concurrency constraints is general, and can be adopted to other scheduling algorithms as well. Moreover, this generality also means that our technique enables clusters with different intra-cluster schedulers to be scheduled on the same platform. It would be interesting to generalize this framework by including other intra and inter-cluster scheduling algorithms, with an aim to solve some open problems in multiprocessor scheduling.

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