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Research Article

Power Law Analysis of Financial Index Dynamics

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Power law (PL) and fractional calculus are two faces of phenomena with long memory behavior. This paper applies PL description to analyze different periods of the business cycle. With such purpose the evolution of ten important stock market indices (DAX, Dow Jones, NASDAQ, Nikkei, NYSE, S&P500, SSEC, HSI, TWII, and BSE) over time is studied. An evolutionary algorithm is used for the fitting of the PL parameters. It is observed that the PL curve fitting constitutes a good tool for revealing the signal main characteristics leading to the emergence of the global financial dynamic evolution.

1. Introduction

Business cycles are the usual trend behavior found in the economic activity over a significant period of time (i.e., several months or years). Such fluctuations tend to involve shifts between periods of economic growth (expansions) and periods of stagnation or decline (recessions); see Table 1. The complex reasons behind business cycles are studied by macroeconomics and are mainly grounded on the gears of the economic activity (e.g., monetary policy, business sentiment, inflation). In the United States the business cycle is followed by the National Bureau of Economic Research (NBER), a research organization which is dedicated to promote a greater understanding of how the economy works. The NBER's Business Cycle Dating Committee defines the official US business cycle's peaks and troughs. An economy expansion corresponds to a period from a trough to a peak, and a recession corresponds to the period from a peak to a trough. For NBER a recession is defined as "a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real Gross Domestic Product (GDP), real income, employment, industrial production, and

Table 1: Business cycle reference dates, in US, since 1970.

Period	Main characteristics	Main causes
Dec. 1969 to Nov. 1970	(i) Followed one of the longest economic expansion in the US history (Feb. 1961 to Dec. 1969) (ii) Relatively mild recession	(i) Fiscal tightening to close the budget deficits of the Vietnam War (ii) Monetary tightening to face the rising inflation
Nov. 1973 to Mar. 1975	(i) Simultaneous increase of inflation and unemployment (ii) Long and deep recession	(i) 1973 oil crisis (ii) Wage and price control policies implemented to mask inflation pressures and fight unemployment (iii) Abnormal long decline in productivity growth (iv) Emergence of newly industrialized countries
Jan. 1980 to Jul. 1980 and Jul. 1981 to Nov. 1982	(i) Conjunction of two recessions, separated by a very short expansion (w-shaped) (ii) Deepest and longest recession in the postwar period	(i) Contractionary monetary policy to control high inflation
Jul. 1990 to Mar. 1991	(i) Hit much of the world—not particularly deep or long	(i) Fed tightened monetary policy (Feb. 1988 to May 1989) to counter a rising inflation rate (ii) Oil price shock after Iraq invaded Kuwait gave momentum to the starting recession (iii) Serious solvency problems among thrift institutions due to savings and loan crisis (iv) Consumer pessimism
Mar. 2001 to Nov. 2001	(i) Ended the longest period of growth in the American history (ii) Predicted by economists for years (iii) Affected all the developed world	(i) Collapse of the speculative dot-com bubble (ii) Fall in business investments (iii) September 11th attacks
Since Dec. 2007	(i) Worst financial crisis since the Great Depression (1930s) (ii) Unprecedented responses by governments and central banks (fiscal stimulus, monetary policy expansion, and institutional bailouts) (iii) Ongoing (iv) Fears of a new w-shaped recession	(i) Collapse of the housing bubble (ii) Financial crisis (iii) Return to tight monetary policy

wholesale-retail sales” [1–3]. Since 2000 there has been two recessions (Figure 1), which are now briefly described.

Economy recessions are the primary factor that drives fluctuations in the volatility of stock returns. It is not surprising that changes in economic activity have strong consequences on stock markets.

Stock values are based on corporate earnings which are greatly determined by the business cycle. Therefore, the stock market growth and the GDP tend to correlate quite well. However, it is clear that the correlation is not direct because of the following.

(i) Stock markets tend to behave in a magnified way when compared with the GDP fluctuations. When the GDP falls/increases, the stock market falls/increases even more.

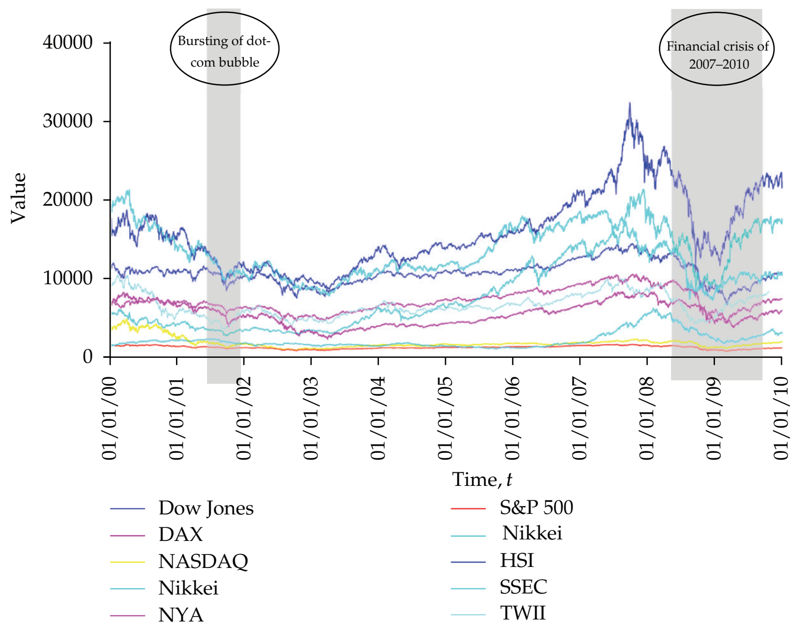


Figure 1: The temporal evolution of the daily closing value for the DAX, DJI, NASDAQ, Nikkei, NYA, BSE, SSEC, TWII, HIS, and S&P 500 indices and the main crisis, from Jan, 2000 to Dec, 2009.

(ii) Stock markets are normally faster to react than the economy and are, therefore, considered by many as a leading indicator of the business cycle. Almost without exception, the stock market turns down prior to recessions and rises before economic recoveries. In fact Siegel [4] shows that out of the 46 recessions from 1802, 42 of them (91.3%) have been preceded (or accompanied) by declines of 8 percent or more in the total stock returns index.

Business cycle forecasting is a popular effort in stock markets not because it is successful, but because the potential gains are so large. In fact, such prevision is a very difficult task and most of the times it is not correct, as illustrated by a famous Samuelson [5] quote "Wall Street indices predicted nine out of the last five recessions!". Therefore, although the stock markets normally identify coming recessions, there is a tendency to be many false alarms. The gains of being able to predict the turning points of the economic cycle are enormous. If an investor could identify the turning points of the economic cycle, he would switch stocks for government bonds before the business downturn begins (stocks fall prior to a recession while treasury bills tend to valorize) and return to stocks when prospects for economic recovery are positive. Nevertheless, if the investor lacks forecasting effort and just follows the established business sentiment about economic activity, he will be buying when prices are high (because everyone is optimistic) and selling when they are low (because everyone is pessimistic) resulting in big losses.

The development of mathematical tools for describing, analyzing and forecasting financial markets has been the subject of considerable research during the last decades, in different perspectives such as in the case of statistics, stochastic systems, signal processing, nonlinear dynamics, and chaos. However, only recently intelligent and evolutionary algorithms were considered for this task, but the results seem promising and motivate

Table 2: The ten stock markets adopted in the study.

k	Stock market index	Abbreviation	Country
1	Deutscher Aktienindex	DAX	Germany
2	Dow Jones Industrial	DJI	USA
3	NASDAQ	NDX	USA
4	New York Stock Exchange	NYA	USA
5	Standard & Poor's	SP500	USA
6	Tokyo Stock Exchange	NIKKEI	Japan
7	Stock market index in Hong Kong	HSI	Hong Kong
8	Bombay Stock Exchange Index	BSE	India
9	Shanghai Stock Exchange	SSEC	China
10	Stock market index in Taipei	TWII	Taiwan

further work [6–8]. Bearing these ideas in mind, the present work establishes a link between classical methods, namely those of system dynamics, and intelligent algorithms, through the development of a adaptive trendline scheme with genetic algorithms and is expected to contribute to the improvement of business cycle forecasting practices by developing an intelligent method of analysis of the trend.

The remainder of this paper is as follow. Section 2 presents the data, namely the financial indices, the fundamental concepts adopted in the study, and the methodology of analysis. Finally, Section 3 draws the main conclusions.

2. Financial Data and Methodology of Analysis

In this section we analyze the stock market indices from January 2000 to December 2009. Our data comprises daily close values of $S = 10$ stock markets to be denoted as $x_k(t),\ 1 \le t \le n$, where t is time, n is the total number of samples, and $k = 1, \ldots, S$. The stock markets in study are listed in Table 2. The data is obtained from the Yahoo Finance website [9] and corresponds to indices in local currencies.

Figure 1 depicts the time evolution, of daily, closing price of the indices versus time with the well-know noisy, and "chaotic-like" characteristics [10].

These signals have a strong variability which makes difficult their direct comparisons in the time domain.

In order to examine the behavior of the signal spectrum, we superimpose a trendline over to the Fourier transform (FT); that is, we approximate the modulus of the FT amplitude through the power law in the frequency domain (ωPL):

$$\begin{gathered}
\mathcal{F}\{x_k(t)\} = \int_{-\infty}^{+\infty} x_k(t) e^{-j\omega t} dt, \\
|\mathcal{F}\{x_k(t)\}| \approx p_k\ \omega^{q_k}, \quad p_k \in \mathbb{R}^+,\ q_k \in \mathbb{R},\ k = 1, \ldots, S,
\end{gathered} \tag{2.1}$$

where $\mathcal{F}$ is the Fourier operator, ω is the frequency, p_k a positive constant that depends on the signal amplitude, and q_k is the trendline slope [11, 12] presented in Table 3. According to the values of q_k, the signals can exhibit an integer or fractional order behavior.

Table 3: Parameter values for the ωPL (left) and average parameter values for tPL, with $\varepsilon = 0.05$ (right) for the ten stock markets adopted in the study.

k	p_k	q_k	a_k_av	b_k_av	c_k_av	d_k_av
1	16388.5	−0.94	1604.7	−0.09	−475.7	364.1
2	59112.7	−0.88	2405.0	−0.54	331.2	4665.2
3	5703.0	−0.97	1546.5	0.02	−1055.9	1334.9
4	39112.2	−0.88	1938.1	0.14	162.3	1815.2
5	5307.9	−0.90	3498.2	0.19	−1516.2	618.9
6	31180.2	−0.97	1816.2	−0.26	−901.8	−1231.6
7	32323.6	−0.98	1744.8	−0.53	−393.8	2426.3
8	19344.8	−0.99	2434.1	−0.76	100.6	1442.8
9	11800.1	−0.91	1529.4	0.21	211.8	1279.6
10	9251.3	−1.01	1562.4	−1.27	780.6	2514.5

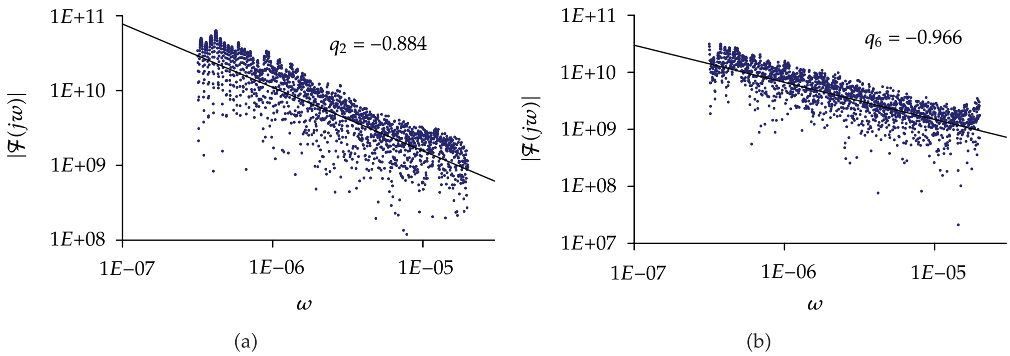


Figure 2: $|FT\{x_k(t)\}|$ and the power trendline $p_k\omega^{q_k}$ for the indices Dow Jones (a) and Nikkei (b).

For example, Figure 2 depicts the amplitude of the FT of the Dow Jones and Nikkei indices and the corresponding ωPL slope values $q_2 = -0.884$, $q_6 = -0.966$, respectively.

We verify that we get a fractional order spectrum in between the white and pink noises, typical in fractional systems, and corresponding to a considerable volatility.

FT is not capable of characterizing signal variations in a limited time window and leads to a portrait of the overall signal characteristics. Therefore, given the signal volatility it is important to develop an intelligent method capable of capturing evolutions and trends in finite width time windows [13, 14].

In order to examine the behavior of the signal, for small time partitions, a power law in the time domain (tPL) trendline is calculated according the following equation:

$$x_k(t) \approx a_k(t - c_k)^{b_k} + d_k, \quad a_k, d_k \in \mathbb{R}^+, \ b_k, c_k \in \mathbb{R}, \qquad (2.2)$$
$$k = 1, \dots, S, \ 1 \le t \le n,$$

where t represents time and a_k, b_k, c_k, d_k are fitting parameters.

It should be noted that, a priori, there is no formal link between expression (2.1) and (2.2), but this study may clarify any dependence in case it exists.

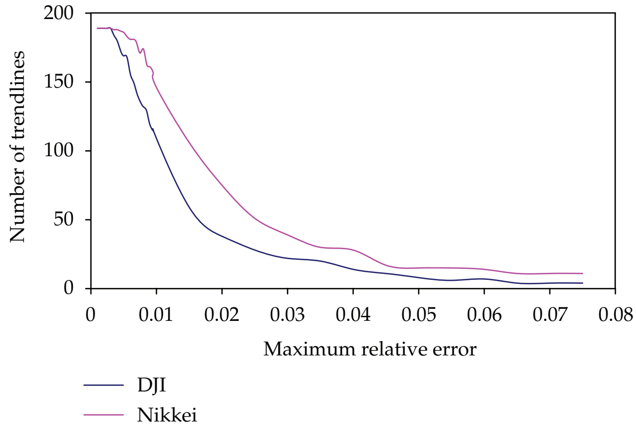


Figure 3: Number of trendlines N_w versus $0.001 \leqslant \varepsilon \leqslant 0.06$ for the Dow Jones and Nikkei indices.

In this approximation the parameter a_k describes the "magnitude," b_k is related with dynamics of evolution, and c_k and d_k coordinate offsets. Therefore, from the point of view of financial dynamics, the parameter b_k is clearly the most relevant one.

We must mention that tPL and fractional dynamics may be manifestations of the same type of phenomena, that is to say, of dynamical systems with long memory. In fact, the power law behavior can emerge even in systems lacking such property [15]. Nevertheless, while the relation between the two faces is not yet clearly understood, the mathematical complexities underlying fractional calculus can be softened with the tPL approximation [16].

Based on a visual analysis of the pattern of the indices chart, we see that we can subdivide each of them into several different partitions. According to the pattern of the indices chart we decided to adopt a variable number of the trendlines N_w according with a maximum relative error ε, defined as

$$\varepsilon_k = \sum_{t=1}^{n} \left| \frac{x_k(t) - \left[a_k(t-c_k)^{b_k} + d_k\right]}{x_k(t)} \right|. \tag{2.3}$$

Figure 3 shows the number of trendlines N_w, for the Dow Jones and Nikkei indices, versus the value of the maximum relative error ε. Obviously, the larger the N_w the smaller the ε.

In our case we consider $0.001 \leqslant \varepsilon \leqslant 0.075$. Having calculated the tPL approximations, for each one of the partitions, we superimpose the corresponding values of the tPL trendline over the original data. For example, Figure 4 depicts the partitions and the trendlines approximation for the Dow Jones and the Nikkei indices.

For the calculation of the parameters $\{a, b, c, d\}$ in (2.2), it is adopted a genetic Algorithm (GA). GAs are a class of computational techniques to find approximate solutions in optimization and search problems [17, 18]. GAs are simulated through a population of candidates of size N_{GA} that evolve computationally towards better solutions. Once the genetic representation and the fitness function are defined, the GA proceeds to initialize a population randomly and then to improve them through the repetitive application of mutation, crossover, and selection operators. During the successive iterations, a part or

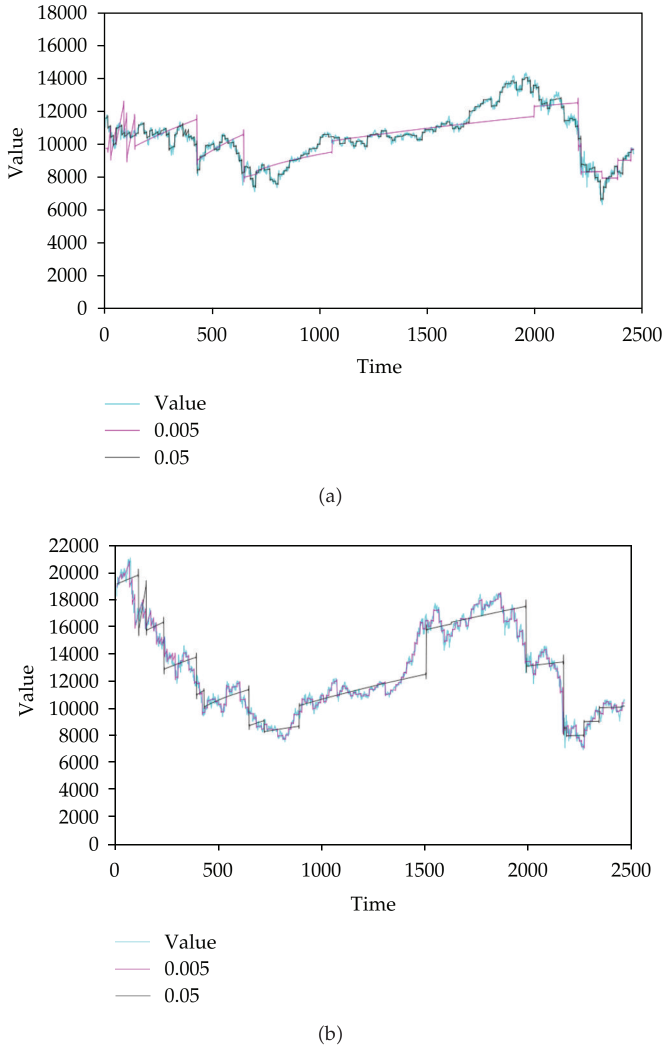
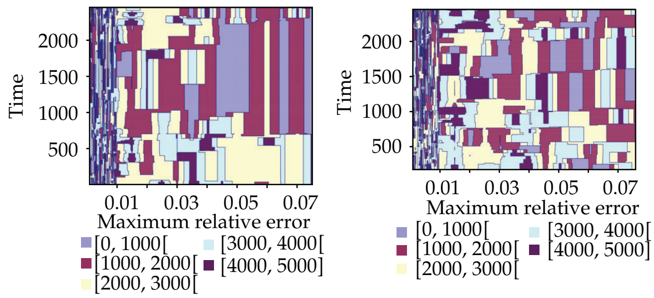


Figure 4: The temporal evolution and *t*PL trendline for the the Dow Jones $x_2(k)$ (a) and Nikkei $x_6(k)$ (b) indices from Jan. 2000 to Dec. 2009 with $\varepsilon = \{0.005, 0.05\}$.

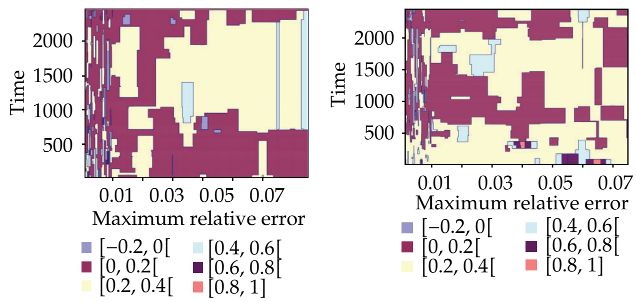
the totality of the population is selected to breed a new generation. Individual solutions are selected through a fitness-based process, where fitter solutions (measured by a fitness function) are usually more likely to be selected. The GA terminates when either the maximum number of generations is produced or a satisfactory fitness level has been reached. In the experiments was considered a GA population of $N_{\mathrm{GA}} = 400$ elements, the crossover of all population elements and the adoption of elitism, a mutation probability of 5%, and an evolution with 60 iterations. This scheme leads to a fast convergence and reduced computational time.

For the purpose of checking the convergence of the GA towards nonoptimal values, several executions of the algorithm were performed, and the results compared.

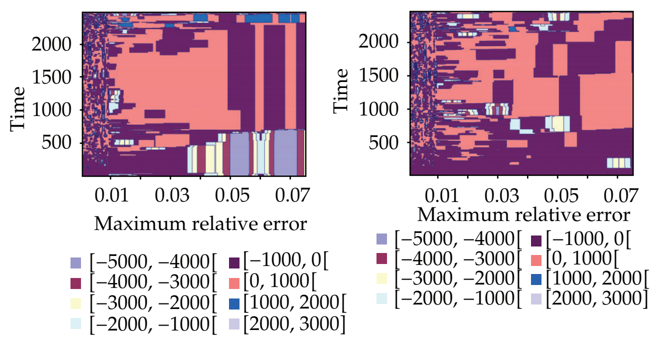
Figure 5 shows the charts of the parameters $\{a, b, c, d\}$ of the Dow Jones and Nikkei indices, for the *t*PL approximation. Again the charts reveal that for small/large values of ε



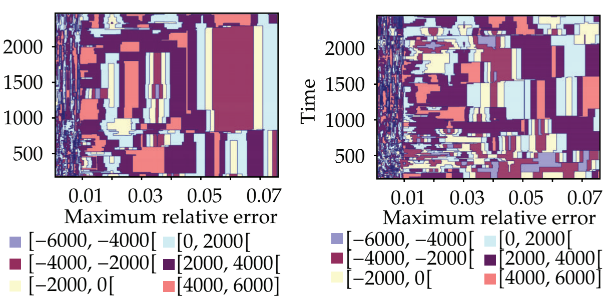
(a)



(b)

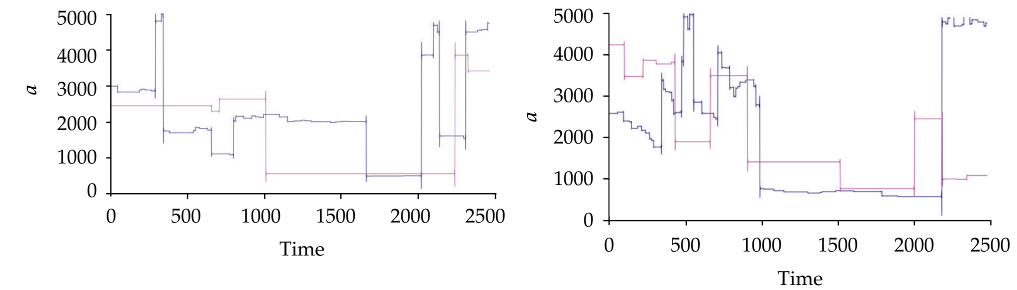


(c)

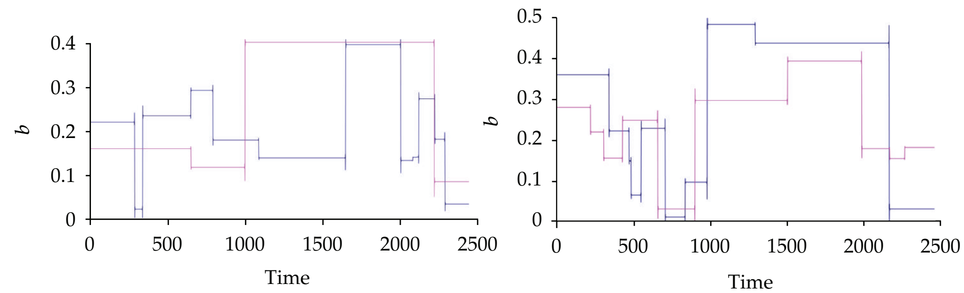


(d)

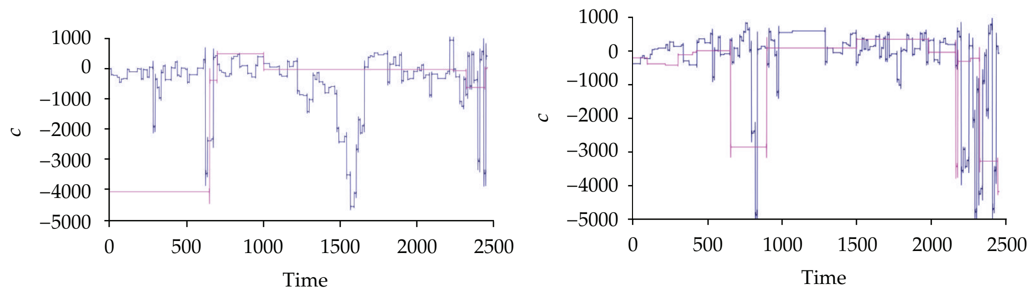
Figure 5: Locus of the parameters $\{a, b, c, d\}$ versus $(\varepsilon,\ t)$, based on (2.2), for the Dow Jones (left) and Nikkei (right) indices, from Jan. 2000 to Dec. 2009.



(a)



(b)



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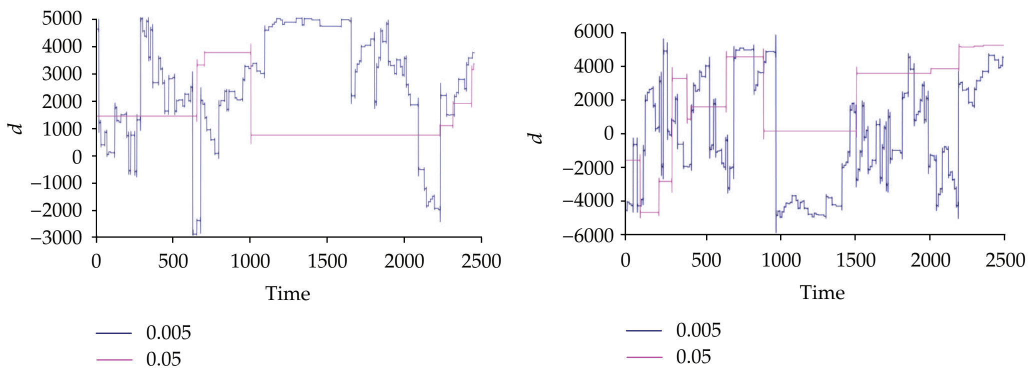
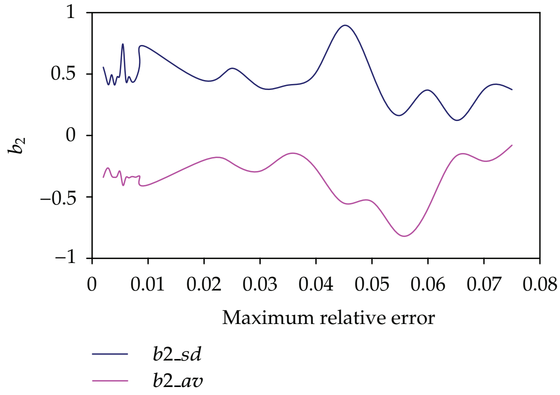
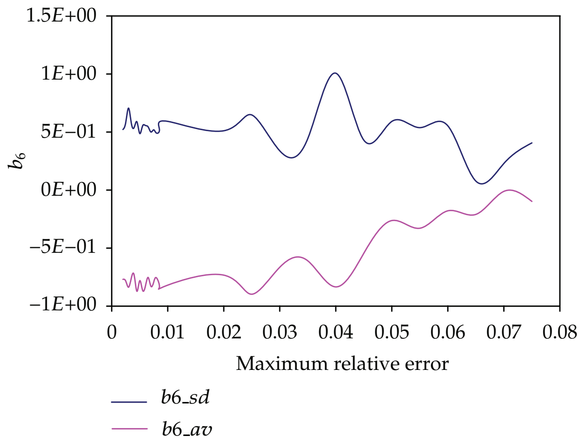


(d)

Figure 6: Values of the parameters $\{a, b, c, d\}$ versus t, for the Dow Jones (left) and Nikkei (right) indices, with $\varepsilon = \{0.005, 0.05\}$ from Jan. 2000 to Dec. 2009.



(a)



(b)

Figure 7: Average and standard deviation value for parameter b versus ε, for the Dow Jones (a) and Nikkei (b) indices.

we get an high/low number of N_w. By other words, the smaller the number of windows N_w the larger the generalization and scope of the conclusions, but the higher the error ε. Furthermore, we observe that the estimation of the model parameters is robust since we have only volatile results for small values of ε.

From the point of view of dynamics, parameter b is the most relevant one. Figure 6 depicts the variation of the tPL parameters $\{a_k, b_k, c_k, d_k\}$, $k = \{2, 6\}$ through time for the Dow Jones, and Nikkei signals, and the cases of $\varepsilon = \{0.005, 0.05\}$. On the other hand, Figure 7 shows the evolution of the average and standard deviation for the parameter b_k over time (b_k_av,b_k_sd), $k = \{2, 6\}$ versus ε for the two indices Dow Jones and Nikkei.

We should note that the power law approximation fits well the time evolution of the stock markets. Since we are doing a splitting of the signal in adequate time windows, an interesting question is if any kind of function could be fitted for any time signal. Several experiments with linear approximations, quadratic polynomials and rational fractions revealed that we could get a good fit with a given approximation for a particular financial index. However, when considering the approximation of all indices, numerical experiments demonstrated problems, both in the resulting plots and the GA convergence.

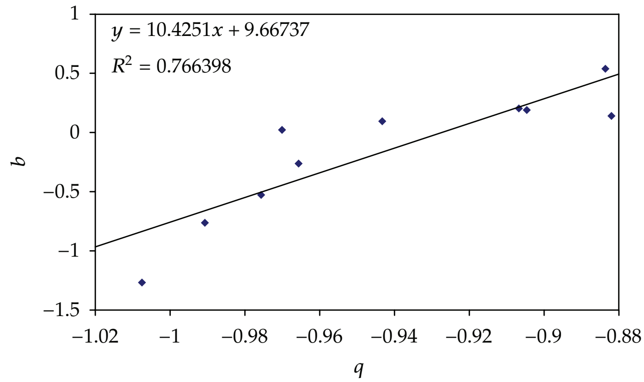


Figure 8: Parameter b values versus parameter q values, for all indices.

Therefore, further research is necessary to explore the type of functions that constitute good approximations, having in mind both the numerical convergence of the fitting procedure and the characteristics of the resulting plots.

Finally, Table 3 shows the ωPL parameters and average parameter's values for tPL, with $\varepsilon = 0.05$ for the ten indices. We verify that the tPL leads to a much more detailed description of the signal, being capable of adapting to its time variability, while capturing its trend within the time window under analysis. In this perspective the tPL establishes a good compromise between time adaptation and trend estimation.

We can note that while formally there is no relationship between the parameters of the ωPL and tPL, there is some degree of correlation as can be seen in Figure 8 that depicts b versus q. In fact, we verify not only that b varies from negative (economic recession) to positive (economic expansion) for $q \approx 0.9$ but also the sensitivity of the time model that "dilutes" the transients into the final result.

3. Conclusions

Economy cycles are the cumulative result of a plethora of different phenomena. Therefore, financial indices reveal a complex behavior, and their dynamical analysis poses problems not usual in other types of systems. In this paper it was studied a PL trendline as a manifestation of the long memory property of systems with fractional dynamics.

For that purpose we developed an intelligent algorithm with a sliding time window having width proportional to a predefined threshold error. Moreover, for the parameter estimation we adopt a genetic algorithm that demonstrates to pose a low computational load while leading to a fast convergence.

The PL trendline proved to constitute a tool capable of retaining the dynamical properties of the economic cycles while providing a global perspective of its evolution.

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